

Latest results of the $K \rightarrow \pi \bar{\nu} \nu$ branching ratio calculations

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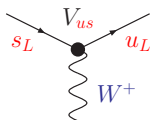
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This talk

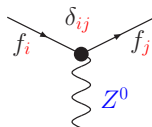
- ▶ Introduction to $K \rightarrow \pi \bar{\nu} \nu$
 - ▶ Status of Perturbative Calculations
 - ▶ Theory Prediction
- ▶ Perturbative Calculations for New Physics
 - ▶ Constrained by perturbative unitarity renormalisability
 - ▶ Define generic Lagrangian
 - ▶ Renormalisation for extra charged vectors
- ▶ Conclusions

Neutral & Charged Current Interactions

Mass \neq flavour eigenstates



SM: Only charged currents
change the flavour ($\propto V_{us}$)

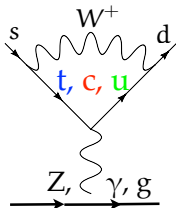


SM: Neutral currents do not
change the flavour ($i=j$) at tree-level

$$V_{\text{CKM}} = \begin{pmatrix} 1 - \frac{\lambda^2}{2} & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - \frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1 \end{pmatrix}$$

- ▶ CKM matrix parametrises CP and flavour violation in the SM
- ▶ Standard Model: Higgs sector is the source of flavour violation

Rare Kaon Decays



Using the GIM mechanism,
we can eliminate either $V_{cs}^* V_{cd}$ or
 $V_{us}^* V_{ud} \rightarrow -V_{cs}^* V_{cd} - V_{ts}^* V_{td}$

$$\begin{aligned} \text{Im} V_{ts}^* V_{td} &= -\text{Im} V_{cs}^* V_{cd} = \mathcal{O}(\lambda^5) & \text{Im} V_{us}^* V_{ud} &= 0 \\ \text{Re} V_{us}^* V_{ud} &= -\text{Re} V_{cs}^* V_{cd} = \mathcal{O}(\lambda^1) & \text{Re} V_{ts}^* V_{td} &= \mathcal{O}(\lambda^5) \end{aligned}$$

Z-Penguin and Boxes (high virtuality):

power expansion in: $A_c - A_u \propto 0 + \mathcal{O}(m_c^2/M_W^2)$

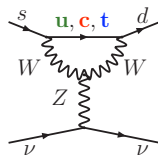
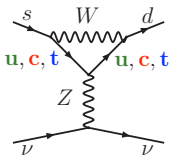
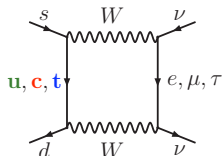
γ/g -Penguin (momentum expansion + e.o.m.):

power expansion in: $A_c - A_u \propto \mathcal{O}(\text{Log}(m_c^2/m_u^2))$

$K \rightarrow \pi \bar{\nu} \nu$ transmitted by Z-Penguin and box:

- ▶ Good theory control & $V_{ts}^* V_{td} \frac{1}{16\pi^2}$ suppression
- ▶ Sensitivity to New Physics

$K \rightarrow \pi \bar{\nu} \nu$ at M_W



$$\chi_i = \frac{m_i^2}{M_W^2}$$

$$\sum_i V_{is}^* V_{id} F(\chi_i) = V_{ts}^* V_{td} (F(\chi_t) - F(\chi_u)) + V_{cs}^* V_{cd} (F(\chi_c) - F(\chi_u))$$

Quadratic GIM:

$$\lambda^5 \frac{m_t^2}{M_W^2}$$

$$\lambda \frac{m_c^2}{M_W^2} \ln \frac{M_W}{m_c}$$

$$\lambda \frac{\Lambda_{\text{QCD}}^2}{M_W^2}$$

Matching (NLO + EW):

$$Q_\nu = (\bar{s}_L \gamma_\mu d_L) (\bar{\nu}_L \gamma^\mu \nu_L)$$

Operator
Mixing (RGE)

ChiPT &
Lattice

- ▶ Below the charm: Only Q_ν , ME from K_{l3}
- ▶ semi-leptonic $(\bar{s} \gamma_\mu u_L) (\bar{\nu} \gamma^\mu \ell_L)$ operator: χ PT gives small contribution (10% of charm contribution)

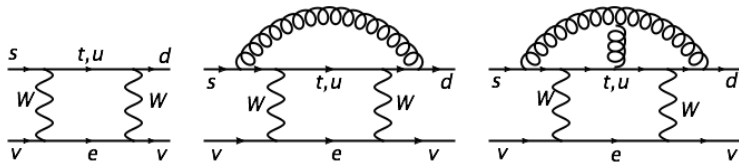
Expressions for $K \rightarrow \pi \bar{\nu} \nu$

$$\mathcal{B}(K^+ \rightarrow \pi^+ \nu \bar{\nu}) = \kappa_+ (1 + \Delta_{\text{EM}}) \cdot \left[\left(\frac{\text{Im} \lambda_t}{\lambda^5} X(x_t) \right)^2 + \left(\frac{\text{Re} \lambda_c}{\lambda} P_c(X) + \frac{\text{Re} \lambda_t}{\lambda^5} X(x_t) \right)^2 \right]$$

$$\mathcal{B}(K_L \rightarrow \pi^0 \nu \bar{\nu}) = \kappa_L \cdot \left(\frac{\text{Im} \lambda_t}{\lambda^5} X(x_t) \right)^2$$

- ▶ $\text{Im} \lambda_t = \eta A^2 \lambda^5$, $\text{Re} \lambda_t = \frac{\lambda^2 - 2}{2} A^2 \lambda^2 (1 - \bar{\rho})$, $\text{Re} \lambda_c = \lambda \frac{\lambda^2 - 2}{2}$
- ▶ $\kappa_+, \kappa_L, \Delta_{\text{EM}}$ strong and em iso-spin breaking [0705.2025]
- ▶ $P_c = P_c^{\text{pert.}} + \delta P_{c,u} = 0.372(15) + 0.04(2) \leftarrow (\text{NNLO} + \text{EW})$ [ph/0603079] [0805.4119] + χ PT & Lattice [ph/0503107] [1806.11520]

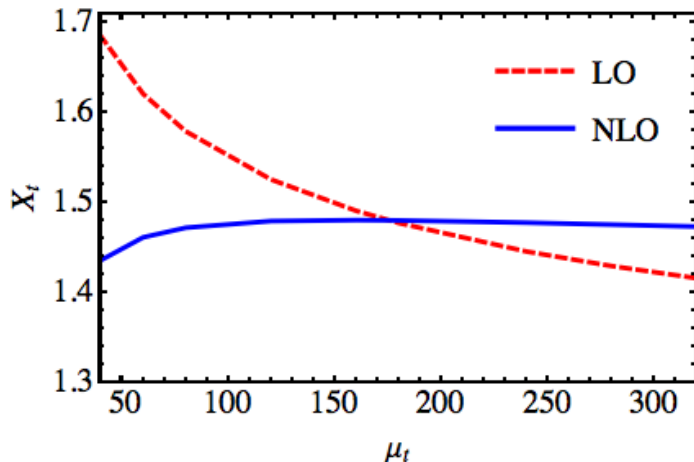
Higher order corrections for X_t



- ▶ $X_t = X_t^{\text{NLO}} + X_t^{\text{EW}} = 1.469(30)$ up to now
- ▶ NLO [Buchalla, Buras; Bobeth, Misiak], NNLO Penguin [Hermann, Misiak, Steinhauser] and EW [Brod, Gorbahn, Stamou]
- ▶ NNLO-Boxes [Cerdeña-Sevilla, Gorbahn, Leck] (related to electron-boxes): Use known master integrals and numerical evaluation
- ▶ Matching result should be independent of μ_t (order by order)

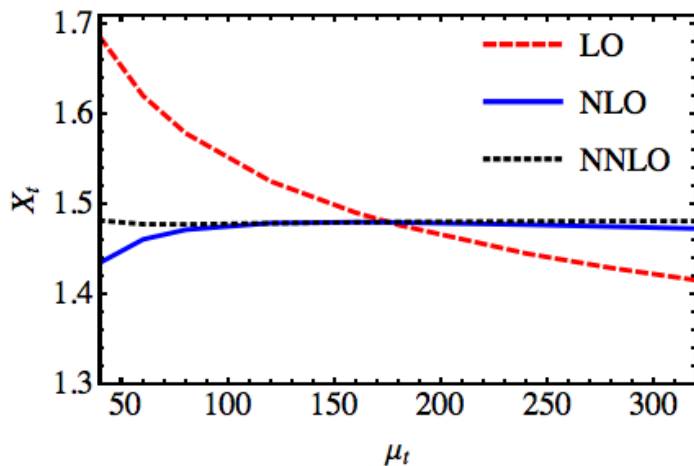
Scale Dependence @ NLO

- ▶ Residual μ_t dependence estimate higher order corrections
- ▶ Potential $\pm 2\%$ NNLO corrections



Possible Scale Dependence @ NNLO

- ▶ NNLO finite result with correct μ_t dependence
- ▶ Numerics not checked: **Toy numerics**
 - ▶ fix $X_t^{\text{NNLO}}(\mu_t = 170 \text{ GeV}) = X_t^{\text{NLO}}(\mu_t = 170 \text{ GeV})$
- ▶ **Absolute size** of NNLO corrections **blinded**



Uncertainty Analysis using UFit values

$\mathcal{B}_+ \cdot 10^{11}$	Central:	8.510	$\mathcal{B}_L \cdot 10^{11}$	Central:	2.858
Error:	-0.543	0.555	Error:	-0.256	0.264
A	-0.34	0.352	A	-0.162	0.17
$\delta P_{c,u}$	-0.246	0.250	η	-0.162	0.167
X_t	-0.236	0.240	X_t	-0.113	0.115
ρ	-0.161	0.162	κ_I	-0.017	0.002
P_c	-0.185	0.187	λ	-0.001	0.00
κ_+	-0.041	0.041			
η	-0.037	0.039			
λ	-0.003	0.003			

- Precise theory prediction, suppression in standard model and current measurement at NA62 → classify new physics contributions

CKM input: $A = 0.826(12)$, $\bar{\rho} = 0.148(13)$, $\bar{\eta} = 0.348(10)$

Descriptions of new physics

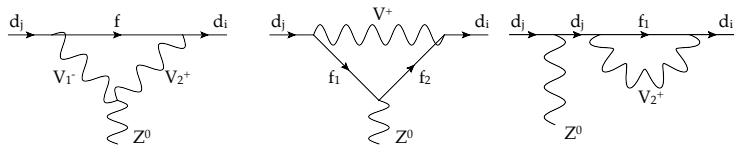
- ▶ Effective theories:
 - ▶ good separation of scales
 - ▶ parameterise heavy new physics (except for huge number of independent operators)
 - ▶ only if momenta are not too large
- ▶ Explicit models:
 - ▶ correlate observables – low energy \leftrightarrow high p_T
 - ▶ falsify validity of effective theory
 - ▶ light weakly coupled new physics
- ▶ Generic models / Simplified models
 - ▶ cover larger set of allowed model space
 - ▶ correlate low energy \leftrightarrow high p_T ?
 - ▶ often neither renormalisable nor unitary

Remainder of this talk

- ▶ Goal: Constrain generic model to achieve
 - ▶ perturbative unitarity
 - ▶ renormalisability
- ▶ For the example of a FCNC Z-Penguin
- ▶ Define generic Lagrangian
- ▶ Renormalisation for extra charged vectors
- ▶ Extensions to arbitrary fermions/scalars/vectors

Toy example: extra vectors

Consider tower of vectors V and $d_j d_i Z$ Green's function:



We only need cubic $\psi - \psi - V$ and $V - V - V$ interactions:

$$\begin{aligned} \mathcal{L}_3^V = & \sum_{f_1 f_2 v_1 L/R} g_{v_1 \bar{f}_1 f_2}^{L/R} V_{v_1, \mu} \bar{\psi}_{f_1} \gamma^\mu P_{L/R} \psi_{f_2} \\ & + \frac{i}{6} \sum_{v_1 v_2 v_3} g_{v_1 v_2 v_3}^{abc} \left(V_{v_1, \mu}^a V_{v_2, \nu}^b \partial^{[\mu} V_{v_3}^{c, \nu]} \right. \\ & \left. + V_{v_3, \mu}^c V_{v_1, \nu}^a \partial^{[\mu} V_{v_2}^{b, \nu]} + V_{v_2, \mu}^b V_{v_3, \nu}^c \partial^{[\mu} V_{v_1}^{a, \nu]} \right). \end{aligned}$$

Couplings in the Standard Model

$$\mathcal{L}_3^V = \sum_{f_1 f_2 v_1 L/R} g_{v_1 \bar{f}_1 f_2}^{L/R} v_{v_1, \mu} \bar{\psi}_{f_1} \gamma^\mu P_{L/R} \psi_{f_2} \\ + \frac{i}{6} \sum_{v_1 v_2 v_3} g_{v_1 v_2 v_3}^{abc} \left(v_{v_1, \mu}^a v_{v_2, \nu}^b \partial^{[\mu} v_{v_3}^{c, \nu]} + \dots \right).$$

In SM we would **need** the following couplings:

- ▶ $g_{W^+ \bar{u}_j d_k}^L = \frac{e}{s_w \sqrt{2}} V_{jk}, \quad y_{G^+ \bar{u}_j d_k}^L = \frac{m_{uj}}{M_W} \frac{e}{s_w \sqrt{2}} V_{jk}$
- ▶ $g_{Z \bar{f}_j f_k}^L = \frac{2e}{s_{2w}} (T_3^f - Q_f s_w^2) \delta_{jk}, \quad g_{Z \bar{f}_j f_k}^R = -\frac{2e}{s_{2w}} Q_f s_w^2 \delta_{jk}$
- ▶ $g_{ZW^+ W^-} = \frac{e}{t_w}, \quad g_{ZW^+ G^-} = -t_w^2 \frac{e}{t_w}, \quad g_{ZG^+ G^-} = \left(1 - \frac{1}{2c_w^2}\right) \frac{e}{t_w}$

Generic Lagrangian

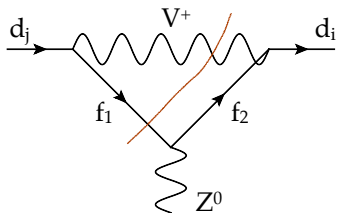
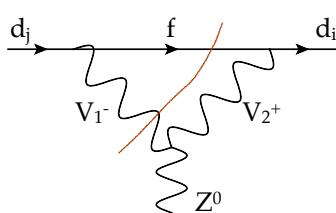
Incorporating Goldstones we arrive at:

$$\begin{aligned}\mathcal{L}_3 = & \sum_{f_1 f_2 v_1 L/R} y_{s_1 \bar{f}_1 f_2}^{L/R} h_{s_1} \bar{\psi}_{f_1} P_{L/R} \psi_{f_2} + \frac{1}{2} \sum_{v_1 v_2 s_1} g_{v_1 v_2 s_1} V_{v_1, \mu} V_{v_2}^{\mu} h_{s_1} \\ & - \frac{i}{2} \sum_{v_1 s_1 s_2} g_{v_1 s_1 s_2} V_{v_1}^{\mu} \left(h_{s_1} \partial_{\mu} h_{s_2} - (\partial_{\mu} h_{s_1}) h_{s_2} \right) + \mathcal{L}_3^V.\end{aligned}$$

- ▶ h extra physical scalars (Goldstones $h \rightarrow \phi$)
- ▶ Add R_{ξ} gauge-fixing
- ▶ Adding $SU(3) \times U(1) \rightarrow$ higher order corrections
- ▶ Using Lagrangian will give divergent results

Finite FCNC Z-Penguin at one-loop?

- ▶ Perturbative Unitary \leftrightarrow massive vectors from SSB
[Llewellyn Smith '73; Cornwall et.al. 73/74]
- ▶ Need correct high-energy behaviour in loops:
 - ▶ Gauge-structure from Slavnov-Taylor (STIs)
 - ▶ Traditionally used in high-energy scattering (“Goldstone-boson Equivalence Theorem”)
 - ▶ UV behaviour controls renormalization properties



Remnants of gauge symmetry

- ▶ Massive vector bosons originate from a spontaneously broken gauge symmetry
- ▶ Fix the gauge for massive vector ($\sigma_{V^\pm} = \pm i$, $\sigma_V = 1$)

$$\mathcal{L}_{\text{fix}} = - \sum_V (2\xi_V)^{-1} F_{\bar{V}} F_V, \quad F_V = \partial_\mu V_V^\mu - \sigma_V \xi_V M_V \phi_V,$$

- ▶ BRST invariant field combination $s(\dots)_{\text{ph}} = 0$
- ▶ STIs from $s\langle T\{\bar{u}_V(\dots)_{\text{ph}}\}\rangle = 0$ at required order:

$$\langle T\left\{k^\mu \underline{V}_V^\mu - i\sigma_{\bar{V}} M_V \underline{\phi}_V\right\}(\dots)_{\text{ph}}\rangle,$$

- ▶ E.g. for $(\dots)_{\text{ph}} = \bar{f}_1 f_2$ we have

$$y_{\phi_1 \bar{f}_1 f_2}^{L/R} = -i\sigma_{v_1} \frac{1}{M_{v_1}} \left(m_{f_1} g_{v_1 \bar{f}_1 f_2}^{L/R} - g_{v_1 \bar{f}_1 f_2}^{R/L} m_{f_2} \right)$$

Elimination of gauge boson couplings

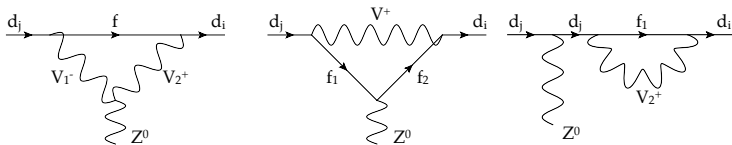
$$\begin{array}{c} \text{f}_2 \longrightarrow \text{f}_1 \\ \text{wavy line } V_{\mu}^+ \text{ with } k^\mu \end{array} + \begin{array}{c} \text{f}_2 \longrightarrow \text{f}_1 \\ \text{dashed line } G^+ \text{ with } M_{V^+} \end{array} = 0$$

► From $(VV)_{\text{ph}}, (Vh)_{\text{ph}}, (hh)_{\text{ph}}$ we obtain 3-point STIs:

$$\begin{aligned}
 g_{v_1 \phi_2 \phi_3} &= \sigma_{v_2} \sigma_{v_3} \frac{M_{v_2}^2 + M_{v_3}^2 - M_{v_1}^2}{2 M_{v_2} M_{v_3}} g_{v_1 v_2 v_3}, & g_{v_1 \phi_2 s_1} &= -i \sigma_{v_2} \frac{1}{2 M_{v_2}} g_{v_1 v_2 s_1}, \\
 g_{v_1 v_2 \phi_3} &= -i \sigma_{v_3} \frac{M_{v_1}^2 - M_{v_2}^2}{M_{v_3}} g_{v_1 v_2 v_3}, & g_{\phi_1 s_1 s_2} &= i \sigma_{v_1} \frac{M_{s_1}^2 - M_{s_2}^2}{M_{v_1}} g_{v_1 s_1 s_2}, \\
 g_{\phi_1 \phi_2 s_1} &= -\sigma_{v_1} \sigma_{v_2} \frac{M_{s_1}^2}{2 M_{v_1} M_{v_2}} g_{v_1 v_2 s_1}, & g_{\phi_1 \phi_2 \phi_3} &= 0.
 \end{aligned}$$

► Allows us to eliminate all Goldstone couplings

Results in terms of physical parameters



$$\sum_{f_1 f_2 V_1} \left[\tilde{k}_{f_1 f_2 V_1}^L \left(\tilde{C}_0(m_{f_1}, m_{f_2}, M_{V_1}) - \frac{1}{2} \right) + k_{f_1 f_2 V_1}^L C_0(m_{f_1}, m_{f_2}, M_{V_1}) + k_{f_1 f_2 V_1}'^L \right] \\ + \sum_{f_1 V_1 V_2} \left[\tilde{k}_{f_1 V_1 V_2}^L \left(\tilde{C}_0(m_{f_1}, M_{V_1}, M_{V_2}) + \frac{1}{2} \right) + k_{f_1 V_1 V_2}^L C_0(m_{f_1}, M_{V_1}, M_{V_2}) + k_{f_1 V_1 V_2}'^L \right]$$

The divergent loop functions \tilde{C}_0 are multiplied with:

$$\tilde{k}_{f_1 f_2 V_1}^L = \left(g_{Z \bar{f}_2 f_1}^L + \frac{m_{f_1} m_{f_2}}{2M_{V_1}^2} g_{Z \bar{f}_2 f_1}^R \right) g_{\bar{V}_1 d_i f_2}^L g_{V_1 \bar{f}_1 d_j}^L, \\ \tilde{k}_{f_1 V_1 V_2}^L = - \left(3 + \frac{m_{f_1}^2 (M_{V_1}^2 + M_{V_2}^2 - M_Z^2)}{4M_{V_1}^2 M_{V_2}^2} \right) g_{Z V_1 \bar{V}_2}^L g_{\bar{V}_1 d_i f_1}^L g_{V_2 \bar{f}_1 d_j}^L \\ - \frac{1}{2} \left(1 + \frac{m_{f_1}^2}{2M_{V_1}^2} \right) (g_{Z \bar{d}_i d_i}^L g_{V_1 \bar{d}_i f_1}^L g_{\bar{V}_1 \bar{f}_1 d_j}^L + g_{V_1 \bar{d}_i f_1}^L g_{\bar{V}_1 \bar{f}_1 d_j}^L g_{Z \bar{d}_j d_j}^L) \delta_{V_1 V_2},$$

Consider SM fermions and extra vectors

Derive STIs for $f - f - V - V$ function:

- Relations between products of trilinear couplings

$$\sum_{v_3} g_{v_3 \bar{f}_1 f_2}^{L/R} g_{v_1 v_2 \bar{v}_3} = \sum_{f_3} (g_{v_1 \bar{f}_1 f_3}^{L/R} g_{v_2 \bar{f}_3 f_2}^{L/R} - g_{v_2 \bar{f}_1 f_3}^{L/R} g_{v_1 \bar{f}_3 f_2}^{L/R})$$

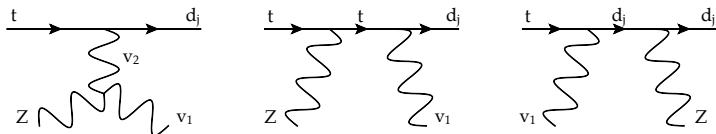
- $v_1 \rightarrow W_1^+, v_2 \rightarrow W_2^-, f_1 \rightarrow d_i, f_2 \rightarrow d_j$ and $g_{Z \bar{d}_i d_j} = 0$:

$$0 = \sum_{f_3} g_{W_2^- \bar{s} f_3}^L g_{W_1^+ \bar{f}_3 d}^L \quad \text{CKM unitarity}$$

- We still obtain a divergence proportional to

$$\sum_{v_1, v_2} \left(\frac{1}{2M_{v_1}^2} (g_{Z \bar{t} t}^R - g_{Z \bar{d} d}^L) \delta_{v_1 v_2} - \frac{(M_{v_1}^2 + M_{v_2}^2 - M_Z^2)}{4M_{v_1}^2 M_{v_2}^2} g_{Z v_1 \bar{v}_2} \right) g_{\bar{v}_1 \bar{d}_i t}^L g_{v_2 \bar{t} d_j}^L$$

Two additional STIs:



Setting $v_3 = Z$, $f_2 = d_j$ there are two additional STIs:

$$g_{Z\bar{t}t}^L g_{v_1^+ \bar{t} d_j}^L = g_{v_1^+ \bar{t} d_j}^L g_{Z\bar{d}_j d_j}^L + \sum_{v_2} g_{Zv_1^+ v_2^-} g_{v_2^+ \bar{t} d_j}^L$$

$$g_{Z\bar{t}t}^R g_{v_1^+ \bar{t} d_j}^L = \frac{1}{2} g_{v_1^+ \bar{t} d_j}^L \left(g_{Z\bar{t}t}^L + g_{Z\bar{d}_j d_j}^L \right) + \sum_{v_2} \frac{M_{v_1}^2 - M_Z^2}{2M_{v_2}^2} g_{Zv_1^+ v_2^-} g_{v_2^+ \bar{t} d_j}^L$$

Which can be used to eliminate $g_{Z\bar{t}t}^{L/R}$ from the expression

Results for extra vectors

The resulting expression comprises less parameters

$$\hat{C}_{d_j d_i Z}^L = \sum_{v_1 v_2} f_V(m_t, M_{v_1}, M_{v_2}) g_{Z v_2^+ v_1^-} g_{v_1^+ \bar{t} d_j}^L g_{v_2^- \bar{d}_i t}^L$$

and a finite loop function

$$\begin{aligned} f_V(m_i, m_j, m_k) = & m_i^2 C_0(m_i, m_k, m_k) - \frac{m_i^2 (m_j^2 + m_k^2 - M_Z^2)}{4m_j^2 m_k^2} \\ & + \frac{m_i^2 (-3m_j^2 + m_k^2 - M_Z^2) + 4m_k^2 (m_j^2 - m_k^2 + M_Z^2)}{4m_j^2 m_k^2} m_i^2 C_0(m_i, m_i, m_k) \\ & + \frac{-M_Z^2 (3m_j^2 + 4m_k^2) - 13m_j^2 m_k^2 + 3m_j^4 + 4m_k^4}{4m_j^2 m_k^2} m_i^2 C_0(m_i, m_j, m_k). \end{aligned}$$

SM couplings: $g_{W^+ \bar{u}_j d_k}^L = \frac{e}{s_w \sqrt{2}} V_{jk}$ and $g_{ZW^+ W^-} = \frac{e}{t_w}$.

Including extra scalars

$$\begin{aligned}
 \hat{C}_{d_j d_i Z}^L = & \sum_{s_1 s_2} f_S(m_t, M_{s_1}, M_{s_2}) y_{s_2^+ \bar{t} d_j}^L \left(\delta_{s_1 s_2} y_{s_2^- \bar{d}_i t}^R \left(g_{Z \bar{d}_j d_j}^L - g_{Z \bar{t} t}^L \right) + g_{Z s_1^+ s_2^-} y_{s_1^- \bar{d}_i t}^R \right) \\
 & + \sum_{v_1 v_2} f_V(m_t, M_{v_1}, M_{v_2}) g_{Z v_2^+ v_1^-} g_{v_1^+ \bar{t} d_j}^L g_{v_2^- \bar{d}_i t}^L \\
 & + \sum_{s_1 v_1} f_{VS}(m_t, M_{s_1}, M_{v_1}) y_{s_1^+ \bar{t} d_j}^L g_{v_1^- \bar{d}_i t}^L g_{Z v_1^+ s_1^-} \\
 & + \sum_{s_1 v_1} f_{VS'}(m_t, M_{s_1}, M_{v_1}) y_{s_1^- \bar{d}_i t}^R g_{v_1^+ \bar{t} d_j}^L g_{Z v_1^- s_1^+} .
 \end{aligned}$$

- ▶ $f_{S,VS,VS'}$ are again functions of C_0 and provide results for LR-Models.
- ▶ E.g. for one charged Higgs we reproduce 2HDM type II results in Literature by specifying:

$$g_{Zh^-h^+} = -e \frac{c_{2W}}{2s_W c_W}, \quad y_{h^+ \bar{t} d_i}^L = \frac{m_t}{t_\beta} \frac{V_{td} e}{\sqrt{2} s_W M_W}.$$

Generic fermions, vectors and scalars

- ▶ Renormalisation procedure works also for the most generic Lagrangian
- ▶ Simplified formulas given for charged particles in [1903.05116]
- ▶ Checked against results in the literature
 - ▶ general MSSM reproduce Literature (but explicitly finite)
 - ▶ Vector-like-quarks reproduce SMEFT logs
- ▶ Method also works for neutral particles

Generic fermions, vectors and scalars

$$\begin{aligned}
 \hat{C}_{d_j d_i Z}^L = & \sum_{f_1 f_2 v_1} g_{Z \bar{f}_2 f_1}^L g_{v_1 \bar{f}_1 d_j}^L g_{\bar{v}_1 \bar{d}_i f_2}^L F_V(m_{f_1}, m_{f_2}, M_{v_1}) \\
 & + \sum_{f_1 f_2 v_1} g_{Z \bar{f}_2 f_1}^R g_{v_1 \bar{f}_1 d_j}^L g_{\bar{v}_1 \bar{d}_i f_2}^L F_{V'}(m_{f_1}, m_{f_2}, M_{v_1}) \\
 & + \sum_{f_1 v_1 v_2} g_{Z v_2 \bar{v}_1} g_{v_1 \bar{f}_1 d_j}^L g_{\bar{v}_2 \bar{d}_i f_1}^L F_{V''}(m_{f_1}, m_{f_1}, M_{v_1}, M_{v_2}) \\
 & + \sum_{f_1 f_2 s_1} g_{Z \bar{f}_2 f_1}^L y_{s_1 \bar{f}_1 d_j}^L y_{\bar{s}_1 \bar{d}_i f_2}^R F_S(m_{f_1}, m_{f_2}, M_{s_1}) \\
 & + \sum_{f_1 s_1 s_2} \left(g_{Z s_2 \bar{s}_1} + \delta_{s_1 s_2} g_{Z \bar{d}_j d_j}^L \right) y_{s_1 \bar{f}_1 d_j}^L y_{\bar{s}_2 \bar{d}_i f_1}^R F_{S'}(m_{f_1}, M_{s_1}, M_{s_2}) \\
 & + \sum_{f_1 f_2 s_1} g_{Z \bar{f}_2 f_1}^R y_{s_1 \bar{f}_1 d_j}^L y_{\bar{s}_1 \bar{d}_i f_2}^R F_{S''}(m_{f_1}, m_{f_2}, M_{s_1}) \\
 & + \sum_{f_1 s_1 v_1} g_{Z v_1 \bar{s}_1} y_{s_1 \bar{f}_1 d_j}^L g_{\bar{v}_1 \bar{d}_i f_1}^L F_{SV}(m_{f_1}, M_{s_1}, M_{v_1}) \\
 & + \sum_{f_1 s_1 v_1} g_{Z \bar{v}_1 s_1} g_{v_1 \bar{f}_1 d_j}^L y_{\bar{s}_1 \bar{d}_i f_1}^R F_{SV'}(m_{f_1}, M_{s_1}, M_{v_1}) ,
 \end{aligned}$$

Penguins come with boxes

- ▶ Results for $\Delta F = 2$ boxes known [Senjanovic et.al.]
- ▶ Both $\Delta F = 2$ and $\Delta F = 1$ boxes given for our most general fermion-scalar-vector interactions [1903.05116]
- ▶ Results are finite by power counting.
- ▶ See also [1904.05890] [Arnan, Crivellin, Fedele, Mescia] for boxes from fermion-scalar interactions.

Conclusions

- ▶ SM Theory prediction under good control
- ▶ Experimental measurements will:
 1. Test the Standard Model
 2. Constrain New Physics
- ▶ Provide general formulas for New Physics Contribution
 - ▶ Extend to different operators and develop code for numerical evaluation [F. Bishara, J. Brod, MG, U. Moldanazarova]