Latest results of the $K \rightarrow \pi\bar{\nu}\nu$ branching ratio calculations

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This talk

- Introduction to $K \rightarrow \pi \bar{\nu} \nu$
  - Status of Perturbative Calculations
  - Theory Prediction
- Perturbative Calculations for New Physics
  - Constrained by perturbative unitarity renormalisibility
  - Define generic Lagrangian
  - Renormalisation for extra charged vectors
- Conclusions
Neutral & Charged Current Interactions

Mass ≠ flavour eigenstates

SM: Only charged currents change the flavour (∝ \( V_{us} \))
SM: Neutral currents do not change the flavour (i=j) at tree-level

\[
V_{\text{CKM}} = \begin{pmatrix}
1 - \frac{\lambda^2}{2} & \lambda & A\lambda^3(\rho - i\eta) \\
-\lambda & 1 - \frac{\lambda^2}{2} & A\lambda^2 \\
A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1
\end{pmatrix}
\]

▶ CKM matrix parametrises CP and flavour violation in the SM
▶ Standard Model: Higgs sector is the source of flavour violation
Rare Kaon Decays

Using the GIM mechanism, we can eliminate either $V_{cs}^* V_{cd}$ or
$V_{us}^* V_{ud} \rightarrow - V_{cs}^* V_{cd} - V_{ts}^* V_{td}$

$\text{Im} V_{ts}^* V_{td} = -\text{Im} V_{cs}^* V_{cd} = \mathcal{O}(\lambda^5)$  $\text{Im} V_{us}^* V_{ud} = 0$
$\text{Re} V_{us}^* V_{ud} = -\text{Re} V_{cs}^* V_{cd} = \mathcal{O}(\lambda^1)$  $\text{Re} V_{ts}^* V_{td} = \mathcal{O}(\lambda^5)$

Z-Penguin and Boxes (high virtuality):
power expansion in: $A_c - A_u \propto 0 + \mathcal{O}(m_c^2/M_W^2)$

$\gamma/g$-Penguin (momentum expansion + e.o.m.):
power expansion in: $A_c - A_u \propto \mathcal{O}(\text{Log}(m_c^2/m_u^2))$

$K \rightarrow \pi \bar{\nu}\nu$ transmitted by Z-Penguin and box:
- Good theory control & $V_{ts}^* V_{td} \frac{1}{16\pi^2}$ suppression
- Sensitivity to New Physics
\( K \rightarrow \pi \bar{\nu} \nu \) at \( M_W \)

\[
\sum_i V_{is}^* V_{id} F(\chi_i) = V_{ts}^* V_{td} (F(x_t) - F(x_u)) + V_{cs}^* V_{cd} (F(x_c) - F(x_u))
\]

**Quadratic GIM:**

\[
\chi_i = \frac{m_i^2}{M_W^2}
\]

**Matching (NLO + EW):**

\[
Q_\nu = (\bar{s}_L \gamma_\mu d_L)(\bar{\nu}_L \gamma^\mu \nu_L)
\]

- Above the charm: \( Q_\nu \), ME from \( K_{l3} \)
- Below the charm: Only \( Q_\nu \), ME from \( K_{l3} \)
- Semi-leptonic \((\bar{s}_\gamma_\mu u_L)(\bar{\nu}_\gamma^\mu \ell_L)\) operator: \( \chi \) PT gives small contribution (10% of charm contribution)
Expressions for $K \rightarrow \pi \bar{\nu} \nu$

\[
\mathcal{B}(K^+ \rightarrow \pi^+ \nu \bar{\nu}) = \kappa_+(1 + \Delta_{EM}) \cdot \left[ \left( \frac{\text{Im}\lambda_t}{\lambda^5} X(x_t) \right)^2 \right.
\]
\[
\left. + \left( \frac{\text{Re}\lambda_c}{\lambda} P_c(X) + \frac{\text{Re}\lambda_t}{\lambda^5} X(x_t) \right)^2 \right]
\]

\[
\mathcal{B}(K_L \rightarrow \pi^0 \nu \bar{\nu}) = \kappa_L \cdot \left( \frac{\text{Im}\lambda_t}{\lambda^5} X(x_t) \right)^2
\]

- $\text{Im}\lambda_t = \eta A^2 \lambda^5$, $\text{Re}\lambda_t = \frac{\lambda^2-2}{2} A^2 \lambda^2 (1 - \bar{\rho})$, $\text{Re}\lambda_c = \lambda \frac{\lambda^2-2}{2}$
- $\kappa_+, \kappa_L, \Delta_{EM}$ strong and em iso-spin breaking
  [0705.2025]
- $P_c = P_c^{\text{pert.}} + \delta P_{c,u} = 0.372(15) + 0.04(2) \leftarrow (\text{NNLO} + \text{EW})$ [ph/0603079] [0805.4119] + $\chi$ PT & Lattice
  [ph/0503107] [1806.11520]
Higher order corrections for $X_t$

$x_t = x_t^{NLO} + x_t^{EW} = 1.469(30)$ up to now

- NLO [Buchalla, Buras; Bobeth, Misiak], NNLO Penguin [Hermann, Misiak, Steinhauser] and EW [Brod, Gorbahn, Stamou]
- NNLO-Boxes [Cerda-Sevilla, Gorbahn, Leak] (related to electron-boxes): Use known master integrals and numerical evaluation
- Matching result should be independent of $\mu_t$ (order by order)
Scale Dependence @ NLO

- Residual $\mu_t$ dependence estimate higher order corrections
- Potential $\pm 2\%$ NNLO corrections

![Graph showing scale dependence at NLO with LO and NLO curves.](image-url)
Possible Scale Dependence @ NNLO

- NNLO finite result with correct $\mu_t$ dependence
- Numerics not checked: Toy numerics
  - fix $X_t^{NNLO}(\mu_t = 170 \text{ GeV}) = X_t^{NLO}(\mu_t = 170 \text{ GeV})$
- Absolute size of NNLO corrections blinded
Uncertainty Analysis using UTfit values

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- Precise theory prediction, suppression in standard model and current measurement at NA62 \( \rightarrow \) classify new physics contributions

CKM input: \( A = 0.826(12), \bar{\rho} = 0.148(13), \bar{\eta} = 0.348(10) \)
Descriptions of new physics

- Effective theories:
  - good separation of scales
  - parameterise heavy new physics
    (except for huge number of independent operators)
  - only if momenta are not too large

- Explicit models:
  - correlate observables – low energy ↔ high $p_T$
  - falsify validity of effective theory
  - light weakly coupled new physics

- Generic models / Simplified models
  - cover larger set of allowed model space
  - correlate low energy ↔ high $p_T$?
  - often neither renormalisable nor unitary
Goal: Constrain generic model to achieve
  - perturbative unitarity
  - renormalisibility

For the example of a FCNC Z-Penguin
Define generic Lagrangian
Renormalisation for extra charged vectors
Extensions to arbitrary fermions/scalars/vectors
Toy example: extra vectors

Consider tower of vectors $V$ and $djdZ$ Green’s function:

![Diagram of toy example](image)

We only need cubic $\psi - \psi - V$ and $V - V - V$ interactions:

$$
\mathcal{L}_3^V = \sum_{f_1 f_2 v_1 L/R} g^{L/R}_{v_1 \bar{f}_1 f_2} V_{v_1,\mu} \bar{\psi}_{f_1} \gamma^\mu P_{L/R} \psi_{f_2} \\
+ \frac{i}{6} \sum_{v_1 v_2 v_3} g^{abc}_{v_1 v_2 v_3} \left( V^a_{v_1,\mu} V^b_{v_2,\nu} \partial^{[\mu} V^c_{v_3,\nu]} + V^c_{v_3,\mu} V^a_{v_1,\nu} \partial^{[\mu} V^b_{v_2,\nu]} + V^b_{v_2,\mu} V^c_{v_3,\nu} \partial^{[\mu} V^a_{v_1,\nu]} \right).
$$
Couplings in the Standard Model

\[ \mathcal{L}_3^V = \sum_{f_1 f_2 v_1 L/R} g^{L/R}_{v_1 \bar{f}_1 f_2} V_{v_1,\mu} \bar{\psi}_{f_1} \gamma^{\mu} P_{L/R} \psi_{f_2} \]

\[ + \frac{i}{6} \sum_{v_1 v_2 v_3} g^{abc}_{v_1 v_2 v_3} \left( V^{a}_{v_1,\mu} V^{b}_{v_2,\nu} \partial[\mu V^{c,\nu}_{v_3}] + \ldots \right). \]

In SM we would need the following couplings:

\[ g^{L\bar{u}_j d_k}_{W+} = \frac{e}{s_w \sqrt{2}} V_{jk}, \quad y^{L\bar{u}_j d_k}_{G+} = \frac{m_{uj}}{M_W} \frac{e}{s_w \sqrt{2}} V_{jk} \]

\[ g^{L\bar{f}_j f_k}_{Z} = \frac{2e}{s_{2w}} \left( T^f_3 - Q_f s_w^2 \right) \delta_{jk}, \quad g^{R\bar{f}_j f_k}_{Z} = -\frac{2e}{s_{2w}} Q_f s_w^2 \delta_{jk} \]

\[ g^{ Zw+ w} = \frac{e}{t_w}, \quad g^{ Zw+ G} = -t_w^2 \frac{e}{t_w}, \quad g^{ Z G+ G} = \left( 1 - \frac{1}{2c_w^2} \right) \frac{e}{t_w} \]
Incorporating Goldstones we arrive at:

\[ \mathcal{L}_3 = \sum_{f_1 f_2 v_1 L/R} y^{L/R}_{s_1 f_1 f_2} h_{s_1} \bar{\psi}_{f_1} P_{L/R} \psi_{f_2} + \frac{1}{2} \sum_{v_1 v_2 s_1} g_{v_1 v_2 s_1} V_{v_1,\mu} V_{v_2}^{\mu} h_{s_1} \]

\[ - \frac{i}{2} \sum_{v_1 s_1 s_2} g_{v_1 s_1 s_2} V_{v_1}^{\mu} \left( h_{s_1} \partial_{\mu} h_{s_2} - \left( \partial_{\mu} h_{s_1} \right) h_{s_2} \right) + \mathcal{L}_3^V . \]

- \textit{h extra physical scalars (Goldstones } h \rightarrow \phi \textit{)}
- \textit{Add } R_\xi \textit{ gauge-fixing}
- \textit{Adding } SU(3) \times U(1) \rightarrow \textit{ higher order corrections}
- \textit{Using Lagrangian will give divergent results}
Finite FCNC Z-Penguin at one-loop?

- Perturbative Unitary ↔ massive vectors from SSB
  [Llewellyn Smith ’73; Cornwall et.al. 73/74]
- Need correct high-energy behaviour in loops:
  - Gauge-structure from Slavnov-Taylor (STIs)
  - Traditionally used in high-energy scattering ("Goldstone-boson Equivalence Theorem")
  - UV behaviour controls renormalization properties
Remnants of gauge symmetry

- Massive vector bosons originate from a spontaneously broken gauge symmetry
- Fix the gauge for massive vector \( (\sigma V^\pm = \pm i, \sigma V = 1) \)

\[
\mathcal{L}_{\text{fix}} = - \sum_v (2\xi_v)^{-1} F_v F_v, \quad F_v = \partial_\mu V^\mu_v - \sigma_v \xi_v M_v \phi_v,
\]

- BRST invariant field combination \( s(\ldots)_{ph} = 0 \)
- STIs from \( s\langle T\{\bar{u}_v(\ldots)_{ph}\}\rangle = 0 \) at required order:

\[
\langle T\left\{ k^\mu V^\mu_v - i\bar{\sigma}_v M_v \phi_v \right\}(\ldots)_{ph} \rangle,
\]

- E.g. for \( (\ldots)_{ph} = \bar{f}_1 f_2 \) we have

\[
y_{\phi_1\bar{f}_1 f_2}^{L/R} = -i\sigma_{v_1} \frac{1}{M_{v_1}} \left( m_{f_1} g_{v_1 \bar{f}_1 f_2}^{L/R} - g_{v_1 \bar{f}_1 f_2}^{R/L} m_{f_2} \right)
\]
Elimination of gauge boson couplings

\[ V!^+ G!^+ k! \cdot M!^+ f_2 f_1 f_2 f_1 + = 0 \]

- From \((VV)_\text{ph}, (Vh)_\text{ph}, (hh)_\text{ph}\) we obtain 3-point STIs:
  \[
  g_{v_1 \phi_2 \phi_3} = \sigma_{v_2} \sigma_{v_3} \frac{M_{v_2}^2 + M_{v_3}^2 - M_{v_1}^2}{2 M_{v_2} M_{v_3}} \ g_{v_1 v_2 v_3}, \quad g_{v_1 \phi_2 s_1} = -i \sigma_{v_2} \frac{1}{2 M_{v_2}} \ g_{v_1 v_2 s_1}, \]
  \[
  g_{v_1 v_2 \phi_3} = -i \sigma_{v_3} \frac{M_{v_1}^2 - M_{v_2}^2}{M_{v_3}} \ g_{v_1 v_2 v_3}, \quad g_{\phi_1 s_1 s_2} = i \sigma_{v_1} \frac{M_{s_1}^2 - M_{s_2}^2}{M_{v_1}} \ g_{v_1 s_1 s_2}, \]
  \[
  g_{\phi_1 \phi_2 s_1} = -\sigma_{v_1} \sigma_{v_2} \frac{M_{s_1}^2}{2 M_{v_1} M_{v_2}} \ g_{v_1 v_2 s_1}, \quad g_{\phi_1 \phi_2 \phi_3} = 0. \]

- Allows us to eliminate all Goldstone couplings
Results in terms of physical parameters

\[
\sum_{f_1 f_2 v_1} \left[ \tilde{k}_{f_1 f_2 v_1}^L \left( \tilde{C}_0(m_{f_1}, m_{f_2}, M_{v_1}) - \frac{1}{2} \right) + k_{f_1 f_2 v_1}^L C_0(m_{f_1}, m_{f_2}, M_{v_1}) + k'_{f_1 f_2 v_1}^L \right] \\
+ \sum_{f_1 v_1 v_2} \left[ \tilde{k}_{f_1 v_1 v_2}^L \left( \tilde{C}_0(m_{f_1}, M_{v_1}, M_{v_2}) + \frac{1}{2} \right) + k_{f_1 v_1 v_2}^L C_0(m_{f_1}, M_{v_1}, M_{v_2}) + k'_{f_1 v_1 v_2}^L \right]
\]

The divergent loop functions \( \tilde{C}_0 \) are multiplied with:

\[
\tilde{k}_{f_1 f_2 v_1}^L = \left( g_{Z f_1 f_2}^L + \frac{m_{f_1} m_{f_2}}{2M_{v_1}^2} g_{Z f_1 f_2}^R \right) g_{v_1 f_2}^L g_{v_1 f_1 d_j}^L,
\]

\[
\tilde{k}_{f_1 v_1 v_2}^L = -\left( 3 + \frac{m_{f_1}^2 (M_{v_1}^2 + M_{v_2}^2 - M_Z^2)}{4M_{v_1}^2 M_{v_2}^2} \right) g_{Z v_1 v_2} g_{v_1 f_2}^L g_{v_1 f_1 d_j}^L g_{v_2 f_2 d_j}^L \delta_{v_1 v_2},
\]

\[
- \frac{1}{2} \left( 1 + \frac{m_{f_1}^2}{2M_{v_1}^2} \right) \left( g_{Z d_j d_f}^L g_{v_1 f_1 d_j}^L + g_{v_1 f_1 d_j}^L g_{Z d_j f}^L \right) \delta_{v_1 v_2},
\]
Consider SM fermions and extra vectors

Derive STIs for $f - f - V - V$ function:

- **Relations between products of trilinear couplings**

  \[
  \sum_{v_3} g_{v_3 f_1 f_2}^{L/R} g_{v_1 v_2 \bar{v}_3} = \sum_{f_3} \left( g_{v_1 \bar{f}_1 f_3}^{L/R} g_{v_2 \bar{f}_3 f_2}^{L/R} - g_{v_2 \bar{f}_1 f_3}^{L/R} g_{v_1 \bar{f}_3 f_2}^{L/R} \right)
  \]

- **$v_1 \rightarrow W_1^+, v_2 \rightarrow W_2^-, f_1 \rightarrow d_i, f_2 \rightarrow d_j$ and $g_{Z\bar{d}_i d_j} = 0$:**

  \[
  0 = \sum_{f_3} g_{W_2^- \bar{s}_f_3}^L g_{W_1^+ \bar{f}_3 d}^L \quad \text{CKM unitarity}
  \]

- **We still obtain a divergence proportional to**

  \[
  \sum_{v_1, v_2} \left( \frac{1}{2M_{v_1}^2} (g_{Z\bar{t}t}^R - g_{Z\bar{d}d}^L) \delta_{v_1 v_2} - \frac{(M_{v_1}^2 + M_{v_2}^2 - M_Z^2)}{4M_{v_1}^2 M_{v_2}^2} g_{Zv_1 \bar{v}_2} \right) g_{\bar{v}_1 \bar{d}_i t}^L g_{v_2 \bar{d}_j}^L
  \]
Two additional STIs:

\[
\begin{align*}
\text{Setting } v_3 &= Z, \quad f_2 = d_j \text{ there are two additional STIs:} \\
\sum_{v_2} g_{Z\bar{t}t} v_1^{-} v_2^{+} d_j^{+} &= \sum_{v_2} g_{Z\bar{t}t} v_1^{-} v_2^{+} d_j^{+} + \sum_{v_2} g_{Z\bar{t}t} v_1^{-} v_2^{+} d_j^{+} \\
\sum_{v_2} g_{Z\bar{t}t} v_1^{-} v_2^{+} d_j^{+} &= \frac{1}{2} \left( g_{Z\bar{t}t} v_1^{-} d_j^{+} + g_{Z\bar{t}t} v_1^{-} d_j^{+} \right) + \sum_{v_2} \frac{M_{v_1}^2 - M_Z^2}{2 M_{v_2}^2} g_{Z\bar{t}t} v_1^{-} v_2^{+} d_j^{+}
\end{align*}
\]

Which can be used to eliminate \( g_{Z\bar{t}t}^{L/R} \) from the expression.
Results for extra vectors

The resulting expression comprises less parameters

\[
\hat{C}_{djdZ}^L = \sum_{v_1v_2} f_V(m_t, M_{v_1}, M_{v_2}) g_{Zv_2^+v_1^-} g_{v_1^+\bar{t}_d}^L g_{v_2^-\bar{d}_t}^L
\]

and a finite loop function

\[
f_V(m_i, m_j, m_k) = m_i^2 C_0(m_i, m_k, m_k) - \frac{m_i^2 \left( m_j^2 + m_k^2 - M_Z^2 \right)}{4m_j^2 m_k^2} m_i^2 C_0(m_i, m_i, m_k) + \frac{m_i^2 \left( -3m_j^2 + m_k^2 - M_Z^2 \right)}{4m_j^2 m_k^2} m_k^2 C_0(m_i, m_i, m_k) + \frac{m_i^2 \left( -3m_j^2 + m_k^2 - M_Z^2 \right)}{4m_j^2 m_k^2} m_k^2 C_0(m_i, m_i, m_k) + \frac{4m_k^2 \left( m_j^2 - m_k^2 + M_Z^2 \right)}{4m_j^2 m_k^2} m_i^2 C_0(m_i, m_i, m_k)
\]

\[
- M_Z^2 \left( 3m_j^2 + 4m_k^2 \right) - 13m_j^2 m_k^2 + 3m_j^4 + 4m_k^4
\]

\[
+ \frac{4m_k^2 \left( 3m_j^2 + 4m_k^2 \right) - 13m_j^2 m_k^2 + 3m_j^4 + 4m_k^4}{4m_j^2 m_k^2} m_i^2 C_0(m_i, m_i, m_k) .
\]

SM couplings: \( g_{W^+\bar{u}_jd_k}^L = \frac{e}{s_w \sqrt{2}} V_{jk} \) and \( g_{ZW^+w^-} = \frac{e}{t_w} \).
Including extra scalars

\[
\hat{C}^L_{d_jd_iZ} = \sum_{s_1s_2} f_S(m_t, M_{s_1}, M_{s_2}) y^L_{s_2^-d_j} \left( \delta_{s_1s_2} y^R_{s_2^-d_i} \left( g^L_{Zd_jd_j} - g^L_{Z\bar{t}t} \right) + g_{Zs_1} y^R_{s_1^-d_i} \right) \\
+ \sum_{v_1v_2} f_V(m_t, M_{v_1}, M_{v_2}) g_{Zv_2^+} g^L_{v_2^-d_j} g^L_{v_1^-d_i} \\
+ \sum_{s_1v_1} f_{VS}(m_t, M_{s_1}, M_{v_1}) y^L_{s_1^-d_j} g^L_{v_1^-d_i} g_{Zv_1^+} \\
+ \sum_{s_1v_1} f_{VS'}(m_t, M_{s_1}, M_{v_1}) y^R_{s_1^-d_i} g^L_{v_1^-d_j} g_{Zv_1^-}.
\]

- \( f_S,VS,VS' \) are again functions of \( C_0 \) and provide results for LR-Models.
- E.g. for one charged Higgs we reproduce 2HDM type II results in Literature by specifying:

\[
g_{Zh^-h^+} = -e \frac{c_{2w}}{2s_w c_w}, \quad y^L_{h^+d_i} = \frac{m_t}{t^\beta} \frac{V_{td} e}{\sqrt{2}s_w M_W}.
\]
Generic fermions, vectors and scalars

- Renormalisation procedure works also for the most generic Lagrangian
- Simplified formulas given for charged particles in [1903.05116]
- Checked against results in the literature
  - general MSSM reproduce Literature (but explicitly finite)
  - Vector-like-quarks reproduce SMEFT logs
- Method also works for neutral particles
Generic fermions, vectors and scalars

\[
\hat{C}_{djdiz}^L = \sum_{f_1 f_2 v_1} g_{Zf_2 f_1}^L g_{v_1 \bar{f}_j}^L g_{\bar{v}_1 d_i f_2}^L \big F_V (m_{f_1}, m_{f_2}, M_{v_1})
+ \sum_{f_1 f_2 v_1} g_{Zf_2 f_1}^R g_{v_1 \bar{f}_j}^L g_{\bar{v}_1 d_i f_2}^L \big F_{V'} (m_{f_1}, m_{f_2}, M_{v_1})
+ \sum_{f_1 v_1 v_2} g_{z v_2 \bar{v}_1}^L g_{v_1 \bar{f}_j}^L g_{\bar{v}_2 d_i f_1}^L \big F_{V''} (m_{f'}, m_{f_1}, M_{v_1}, M_{v_2})
+ \sum_{f_1 f_2 s_1} g_{Zf_2 f_1}^L y_{s_1 \bar{f}_j}^L y_{\bar{s}_1 d_i f_2}^R \big F_S (m_{f_1}, m_{f_2}, M_{s_1})
+ \sum_{f_1 s_1 s_2} \left( g_{Zs_2 \bar{s}_1}^L + \delta_{s_1 s_2} g_{Z \bar{d}_j d_i}^L \right) y_{s_1 \bar{f}_j}^L y_{\bar{s}_2 \bar{d}_i f_1}^R \big F_{S'} (m_{f_1}, M_{s_1}, M_{s_2})
+ \sum_{f_1 f_2 s_1} y_{s_1 \bar{f}_j}^L y_{s_1 \bar{d}_i f_2}^R \big F_{S''} (m_{f_1}, m_{f_2}, M_{s_1})
+ \sum_{f_1 s_1 v_1} g_{z v_1 \bar{s}_1}^L y_{s_1 \bar{f}_j}^L g_{\bar{v}_1 d_i f_1}^L \big F_{SV} (m_{f_1}, M_{s_1}, M_{v_1})
+ \sum_{f_1 s_1 v_1} g_{z v_1 \bar{s}_1}^L y_{s_1 \bar{d}_i f_1}^R \big F_{SV'} (m_{f_1}, M_{s_1}, M_{v_1}),
\]
Penguins come with boxes

- Results for $\Delta F = 2$ boxes known [Senjanovic et.al.]
- Both $\Delta F = 2$ and $\Delta F = 1$ boxes given for our most general fermion-scalar-vector interactions [1903.05116]
- Results are finite by power counting.
- See also [1904.05890] [Arnan, Crivellin, Fedele, Mescia] for boxes from fermion-scalar interactions.
Conclusions

- SM Theory prediction under good control
- Experimental measurements will:
  1. Test the Standard Model
  2. Constrain New Physics
- Provide general formulas for New Physics
- Contribution
  - Extend to different operators and develop code for numerical evaluation [F. Bishara, J. Brod, MG, U. Moldanazarova]