

Extraction of V_{us} from experimental measurements

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Outline :

1. Introduction and Motivation
2. V_{us} from K_{l3} decays
3. V_{us}/V_{ud} from K_{l2}/π_{l2} decays
4. V_{us} and Unitarity of the CKM matrix
5. Conclusion and outlook

1. Introduction and Motivation

1.1 Test of the Standard Model: V_{us} and CKM unitarity

- Extraction of the Cabibbo-Kobayashi-Maskawa matrix element V_{us}
 - Fundamental parameter of the Standard Model

Description of the **weak interactions**:

$$\mathcal{L}_{EW} = \frac{g}{\sqrt{2}} W_\alpha^+ \left(\bar{D}_L V_{CKM} \gamma^\alpha U_L + \bar{e}_L \gamma^\alpha \nu_{e_L} + \bar{\mu}_L \gamma^\alpha \nu_{\mu_L} + \bar{\tau}_L \gamma^\alpha \nu_{\tau_L} \right) + \text{h.c.}$$

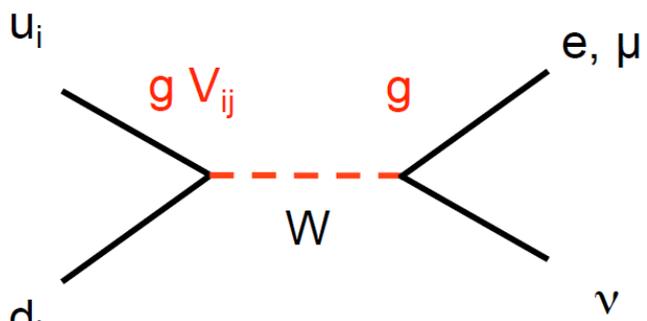
Unitary matrix

	I	II	III		
Quarks	u	c	t	γ	H
	d	s	b	g	
	ν_e	ν_μ	ν_τ	Z	
Leptons	e	μ	τ	W	
	3 generations				Forces

- Check unitarity of the first row of the CKM matrix:

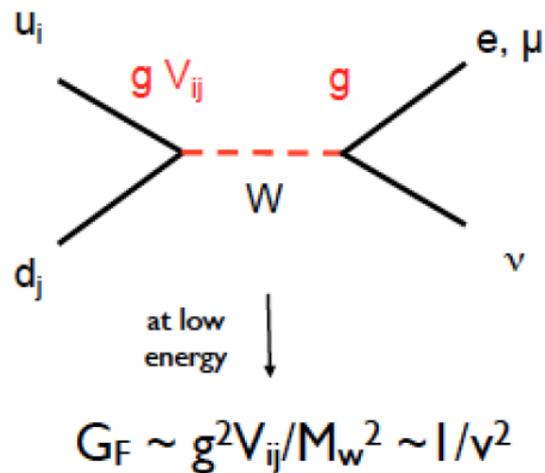
➔ **Cabibbo Universality:** $|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1$

Negligible $\sim 2 \times 10^{-5}$
(B decays)



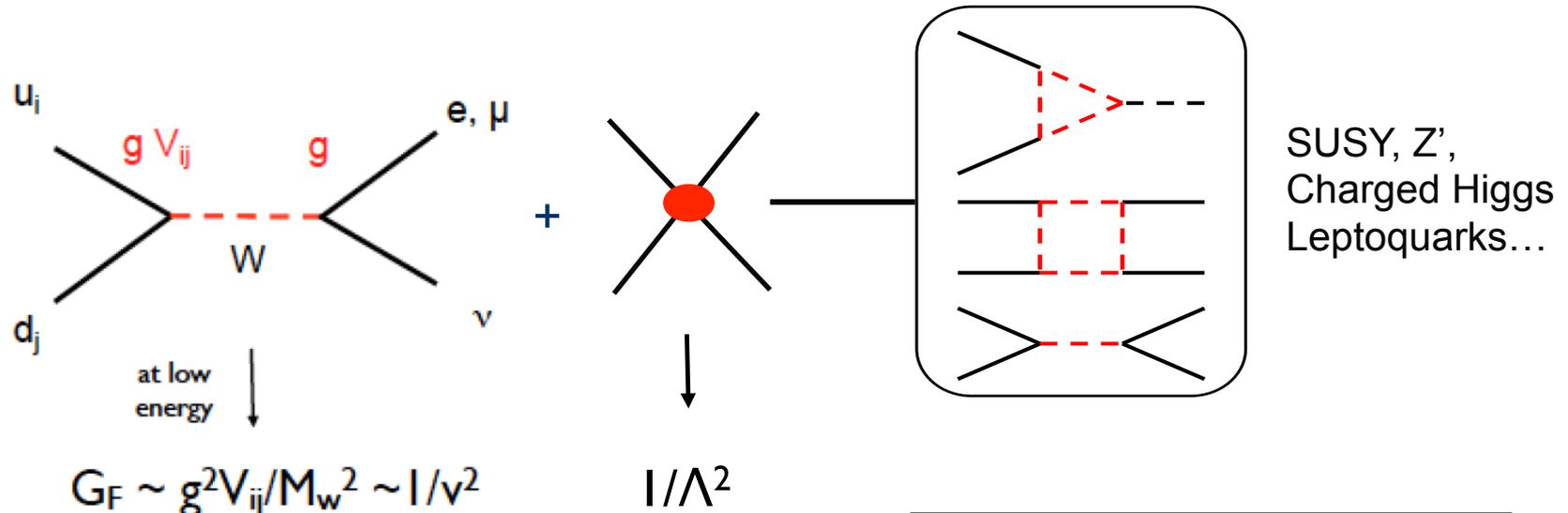
1.2 Constraining New Physics

- In the SM, W exchange \Rightarrow V-A currents, universality



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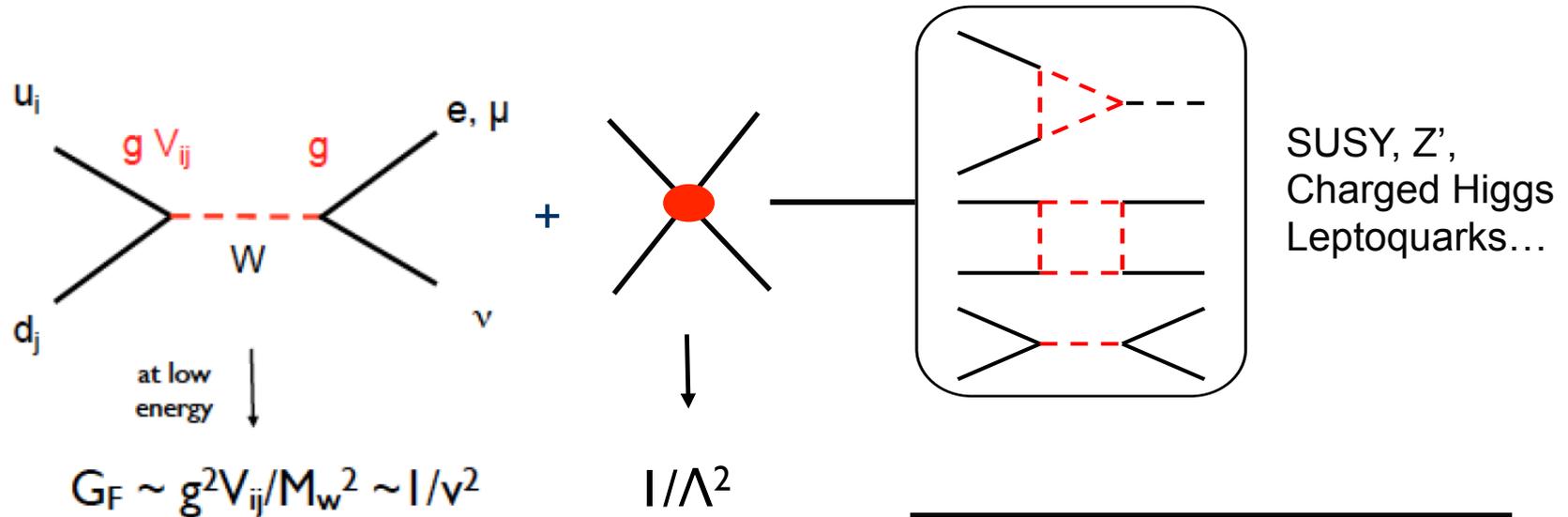
$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1 + \Delta_{CKM}$$

- Broad sensitivity to BSM scenarios :

$$\Delta \sim \frac{c_n}{g^2} \frac{M_W^2}{\Lambda^2} \leq 10^{-2} - 10^{-3} \leftrightarrow \Lambda \sim 1-10 \text{ TeV}$$

1.2 Constraining New Physics

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- Look for new physics by comparing the extraction of V_{us} from different processes: helicity suppressed $K_{\mu 2}$, helicity allowed $K_{l 3}$, hadronic τ decays

1.3 Some history

- 2002 Old K \rightarrow $\pi l \nu_l$ data gave

$$\Delta_{CKM} = 1 - |V_{ud}|^2 - |V_{us}|^2 = 0.0035(15)$$

➡ PDG 2004: a 2.3σ hint of *unitarity violation*?

- 2003: BNL E865 measured: $BR(K^+ \rightarrow \pi^0 e^+ \nu_e) = 5.13(10)\%$

➡ Value of V_{us} consistent with unitarity

- 2004 – present: Many new measurements from **KTeV**, **ISTRA+**, **KLOE**, **NA48**

➤ BRs, lifetimes, form-factors

➤ Much higher statistics than older measurements

➤ Proper account of correlations between measurements

➡ Isospin breaking, radiative corrections start to matter:
computed within ChPT

- 2008 – beyond: Progress in the computation of hadronic elements from lattice QCD as well as isospin breaking and EM effects

1.4 Experiment, Theory & Evaluation

- V_{us} from K_{l3} & K_{l2} :
 - ~100 measurements of ~10 experimental parameters
 - 40+ lattice results for 2 hadronic matrix elements
 - Radiative and SU(2) breaking corrections computed using ChPT

FlaviA
net **Kaon WG**
2006-2010 (EU 6FP)

Experimental averages, fits, etc
Selection of results (experiments, corrections)
Evaluation, discussion and interpretation

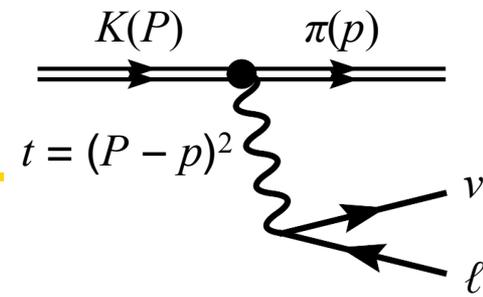
Final report: EPJC 69 (2010) 399

➔ Here update based on
Moulson & Passemar@CKM'2018
Moulson@UMass, Amherst'2019

- Corresponding effort to synthesize results from lattice QCD:
Flavor Lattice Averaging Group (FLAG)
FLAG Review 2019: **arXiv 1902.08191**

2. V_{us} from K_{l3} decays

2.1 V_{us} from K_{l3} : Master Formula



- Master formula for $K \rightarrow \pi l \nu$: $K = \{K^+, K^0\}$, $l = \{e, \mu\}$

$$\Gamma(K \rightarrow \pi l \nu [\gamma]) = Br(K_{l3}) / \tau = C_K^2 \frac{G_F^2 m_K^5}{192 \pi^3} S_{EW}^K |V_{us}|^2 \left| f_+^{K^0 \pi^-}(0) \right|^2 I_{KI} \left(1 + 2\Delta_{EM}^{KI} + 2\Delta_{SU(2)}^{K\pi} \right)$$

Experimental inputs:

$\Gamma(K_{l3})$ Rates with well-determined treatment of radiative decays

- Branching ratios
- Kaon lifetimes

$I_{KI}(\lambda_{KI})$ Integral of form factor over phase space: λ s parametrize evolution in $t=q^2$

Inputs from theory:

S_{EW}^K Universal short distance EW corrections

$f_+^{K^0 \pi^-}(0)$ Hadronic matrix element (form factor) at zero momentum transfer ($t=0$)

Δ_{EM}^{KI} Form-factor correction for long-distance EM effects

$\Delta_{SU(2)}^{K\pi}$ Form-factor correction for SU(2) breaking

2.2 Updates: K^\pm BRs and lifetimes

KLOE-2
PLB 738 (2014)

$$\mathbf{BR}(\pi^+\pi^+\pi^-) = \mathbf{0.05565(31)(25)} \quad (0.7\%)$$

- **No good measurements of $\mathbf{BR}(\pi^+\pi^+\pi^-)$ in 2010 fit**
- Reconstruct 2 tracks in small fiducial volume near interaction region; evaluate missing mass for 3rd track
- Fully inclusive of radiation, but radiative corrections handled differently from other KLOE measurements
- Significant impact on value of $\mathbf{BR}(\mu\nu)$ from fit
Correlation between $\mathbf{BR}(\mu\nu)$, $\mathbf{BR}(\pi^+\pi^+\pi^-) = -0.75$

ISTRA+
PAN 77 (2014)

$$\mathbf{BR}(K^-_{e3}/\pi^-\pi^0) = \mathbf{0.2423(15)(37)} \quad (1.6\%)$$

- Claimed to be fully inclusive for $K_{e3\gamma}$
 - No mention of radiative corrections
 - Many cuts, mainly topological
 - 3 different selections, at least 1 may be largely inclusive
- Included in PDG '15 fit
- **Treated as preliminary here (not in K^\pm BR fit)**

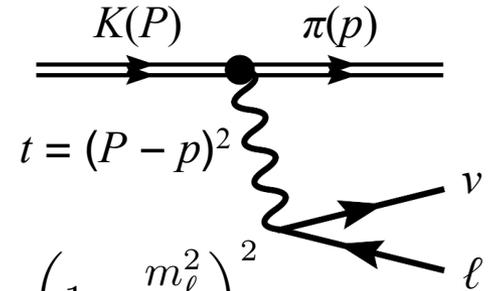
2.3 Updates on Phase Space Integrals

- Master formula for $K \rightarrow \pi l \nu$: $K = \{K^+, K^0\}$, $l = \{e, \mu\}$

$$\Gamma(K \rightarrow \pi l \nu [\gamma]) = \frac{Br(K_{l3})}{\tau} = C_K^2 \frac{G_F^2 m_K^5}{192 \pi^3} S_{EW}^K |V_{us}|^2 |f_+^{K^0 \pi^-}(0)|^2 I_{KI} \left(1 + 2\Delta_{EM}^{KI} + 2\Delta_{SU(2)}^{K\pi}\right)$$

Hadronic matrix element:

$$\langle \pi^-(p) | \bar{s} \gamma_\mu u | K^0(P) \rangle = f_+^{K^0 \pi^-}(0) \left[(P+p)_\mu \bar{f}_+^{K^0 \pi^-}(t) + (P-p)_\mu \bar{f}_-^{K^0 \pi^-}(t) \right]$$



- Phase space integrals:
$$I_{K\ell} = \frac{2}{3} \int_{m_\ell^2}^{t_0} \frac{dt}{M_K^8} \bar{\lambda}^{3/2} \left(1 + \frac{m_\ell^2}{2t}\right) \left(1 - \frac{m_\ell^2}{2t}\right)^2 \times \left(\bar{f}_+^2(t) + \frac{3m_\ell^2 \Delta_{K\pi}^2}{(2t + m_\ell^2) \bar{\lambda}} \bar{f}_0^2(t) \right),$$

- In K_{e3} decays: only vector FF $\bar{f}_+^{K^0 \pi^-}(t)$

- In $K_{\mu 3}$ decays, also need the scalar FF
$$\bar{f}_0(t) = \bar{f}_+(t) + \frac{t}{m_K^2 - m_\pi^2} \bar{f}_-(t)$$

- For V_{us} , need integral over phase space of squared matrix element: Parameterize form factors and fit distributions in t (or related variables)

$K\pi$ form factor parametrizations

- Parametrizations based on Taylor expansion:

$$\bar{f}_{+,0}(t) = 1 + \lambda_{+,0} \left(\frac{t}{m_{\pi^\pm}^2} \right) \quad \text{or} \quad \bar{f}_{+,0}(t) = 1 + \lambda'_{+,0} \left(\frac{t}{m_{\pi^\pm}^2} \right) + \lambda''_{+,0} \left(\frac{t}{m_{\pi^\pm}^2} \right)^2$$

Very simple parametrization but limited in energy range and not physically motivated: many parameters and strong correlations between them

➡ unstable fits

- Physically motivated parametrizations:

- Pole parametrization

$$\bar{f}_{+,0}(t) = \left(\frac{M_{V,S}^2}{M_{V,S}^2 - t} \right)$$

Well motivated for the vector (K^* resonance)
But for the scalar M_S ?

- Dispersive parametrization

Bernard, Oertel, E.P., Stern'06,'09

$$\bar{f}_+(t) = \exp \left[\frac{t}{m_\pi^2} \left(\Lambda_+ - H(t) \right) \right] \quad \text{and} \quad \bar{f}_0(t) = \exp \left[\frac{t}{m_K^2 - m_\pi^2} \left(\ln C - G(t) \right) \right]$$

$K\pi$ scattering phase

$K_{\ell 3}$ form factor data

- Form-factor parameter measurements in FlaviaNet 2010 fit:

K_L : **KTeV**, **KLOE**, **NA48** (K_{e3} only)

K^- : **ISTRA+**

- Even if not in the original publications, all experiments have:
 - Obtained results for Taylor, pole, and dispersive parameterizations
 - Supplied parameter correlation coefficients

New measurements:

NA48/2

JHEP 1810 (2018)

$2.3 \times 10^6 K_{\mu 3}^{\pm}$

$4.4 \times 10^6 K_{e3}^{\pm}$

OKA

JETPL 107 (2018)

$5.25 \times 10^6 K_{e3}^+$

Updating 2012 preliminary

K^+ and K^- simultaneously acquired in dedicated minimum-bias run

Taylor, pole, and dispersive fits with complete investigation of systematics

Extraordinarily high precision claimed, esp. for λ_+' , λ_+''

Rudimentary discussion of systematics

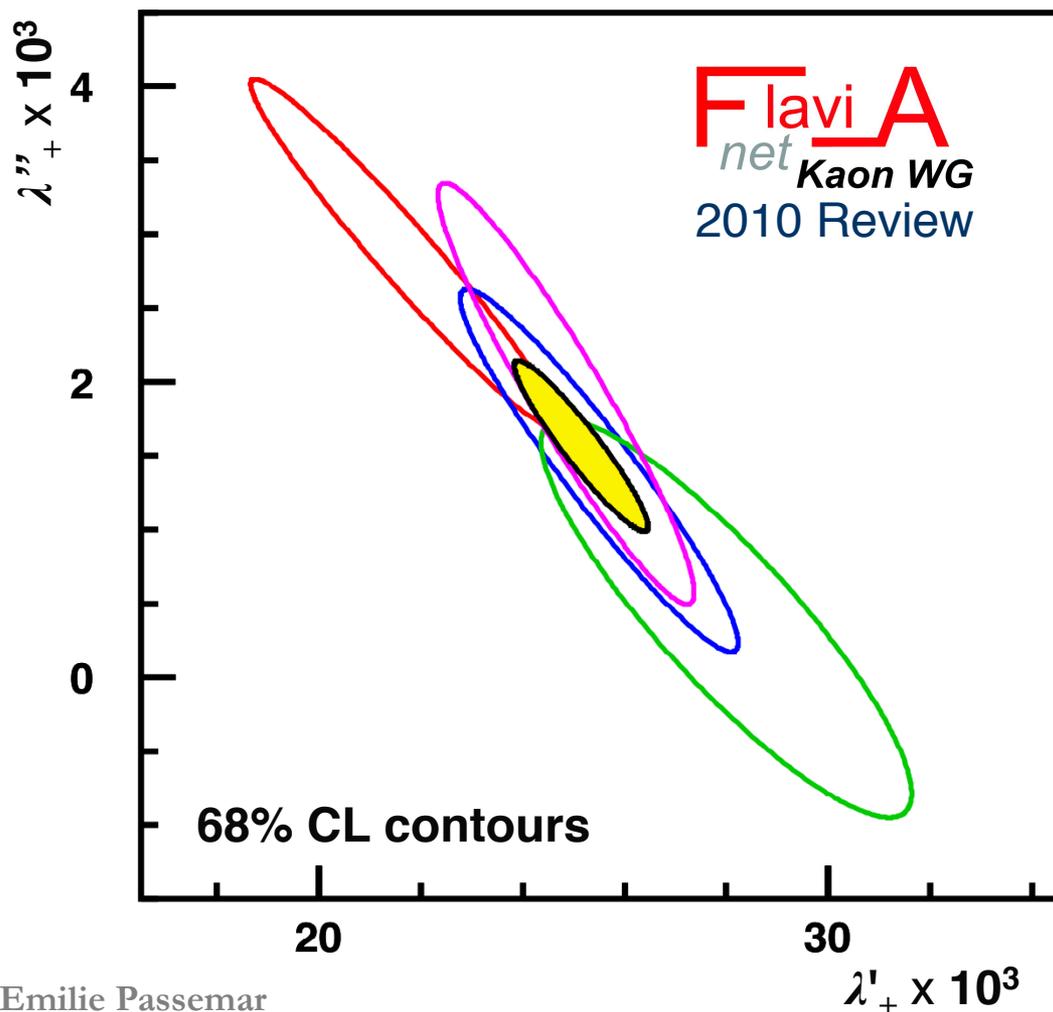
Not yet included in updated K_{e3} fit



See talk by [Gianluca Lamanna](#)

Fit to K_{e3} form-factor slopes: 2010

Slopes from **KTeV** **KLOE** **ISTRA+** **NA48** **2010 fit**



Slope parameters $\times 10^3$

$$\lambda'_+ = 25.15 \pm 0.87$$

$$\lambda''_+ = 1.57 \pm 0.38$$

$$\rho(\lambda'_+, \lambda''_+) = -0.941$$

$$\chi^2/\text{ndf} = 5.3/6 \text{ (51\%)}$$

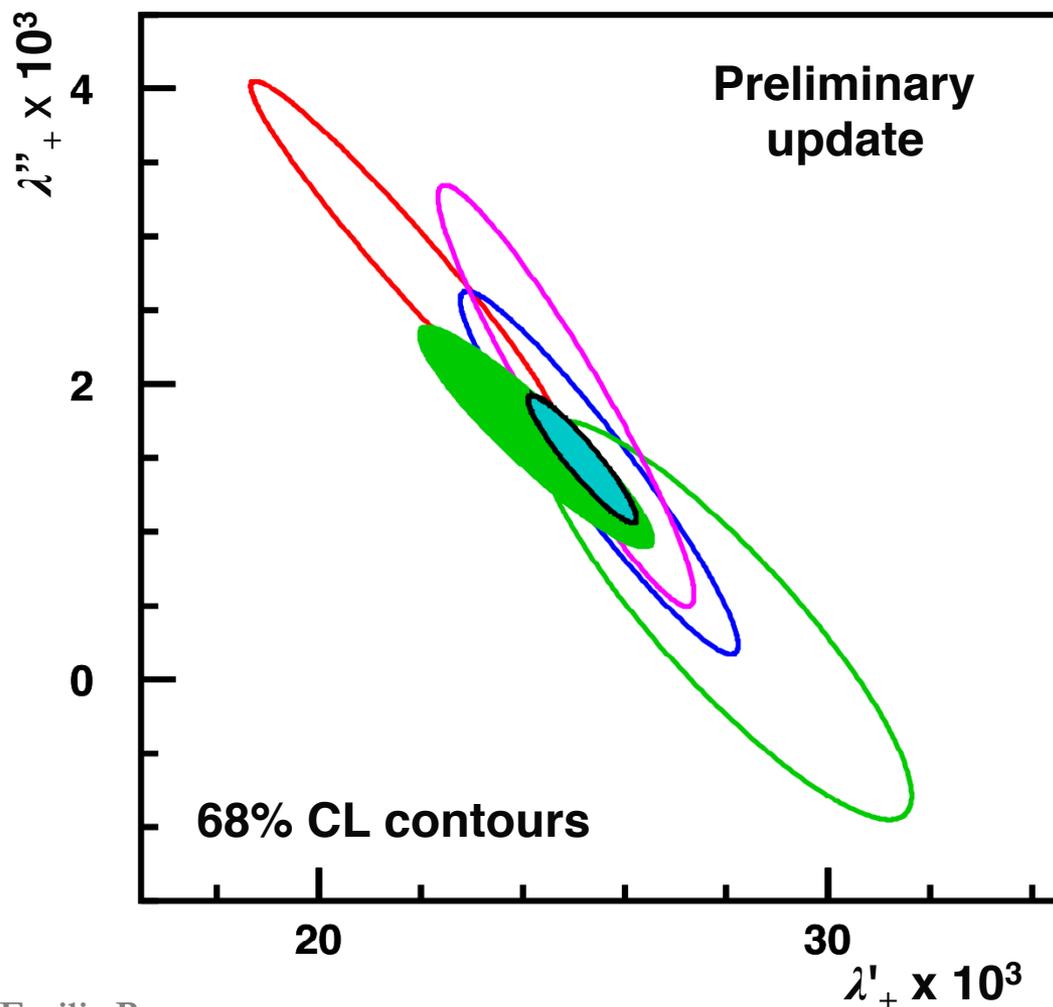
Excellent compatibility
Significance of $\lambda''_+ > 4\sigma$

$$I(K^0_{e3}) = 0.15463(21)$$

$$I(K^+_{e3}) = 0.15900(22)$$

Fit to K_{e3} form-factor slopes: Update

Slopes from **KTeV** **KLOE** **ISTRA+** **NA48** **NA48/2** **Update**



Slope parameters $\times 10^3$

$$\lambda'_+ = 25.17 \pm 0.70$$

$$\lambda''_+ = 1.49 \pm 0.29$$

$$\rho(\lambda'_+, \lambda''_+) = -0.929$$

$$\chi^2/\text{ndf} = 6.4/10 \text{ (61\%)}$$

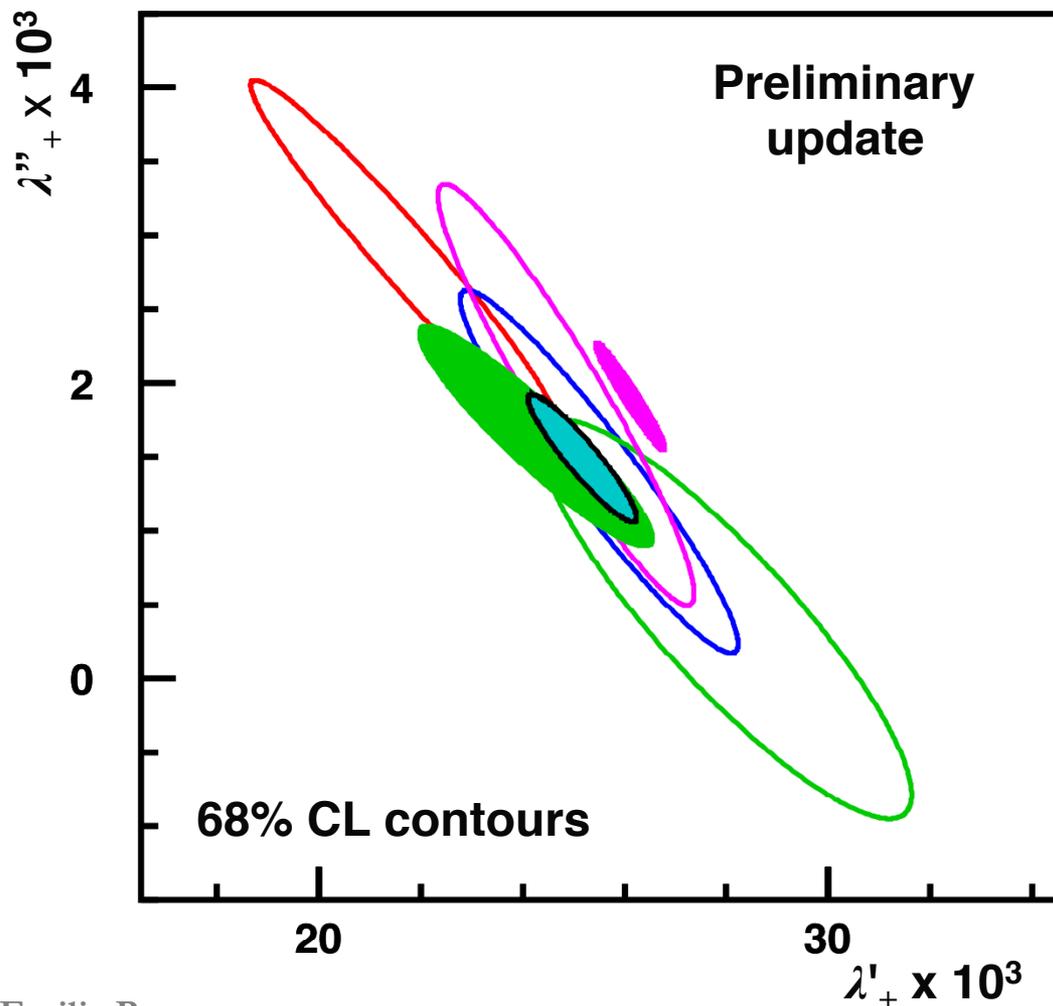
Excellent compatibility
Very small change in λ'_+

$$I(K^0_{e3}) = 0.15463(21)$$

$$I(K^+_{e3}) = 0.15900(22)$$

Fit to K_{e3} form-factor slopes: Update

Slopes from **KTeV** **KLOE** **ISTRA+** **NA48** **NA48/2** **Update**



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OKA

JETPL 107 (2018)

Not included in the fit

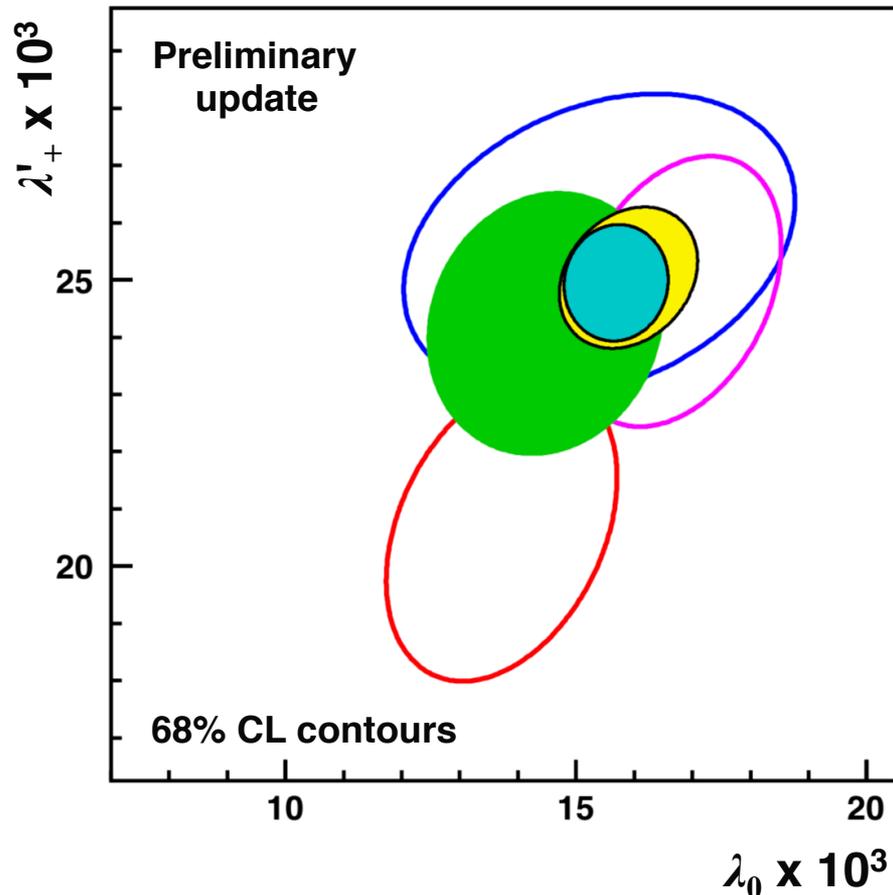
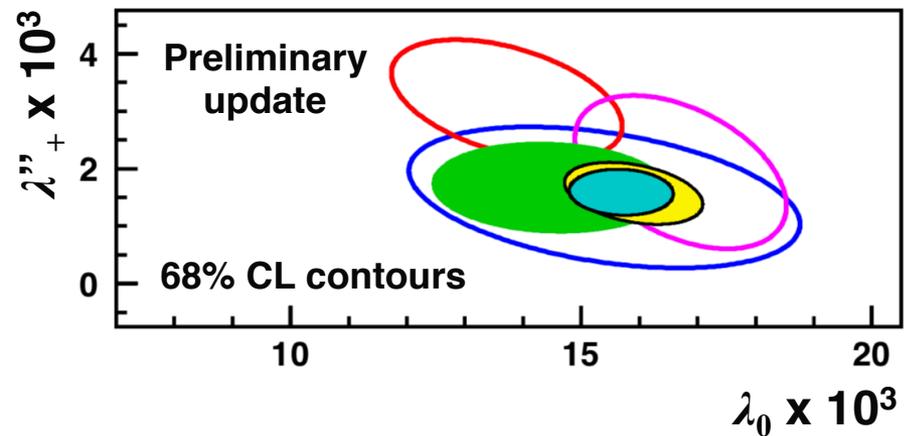
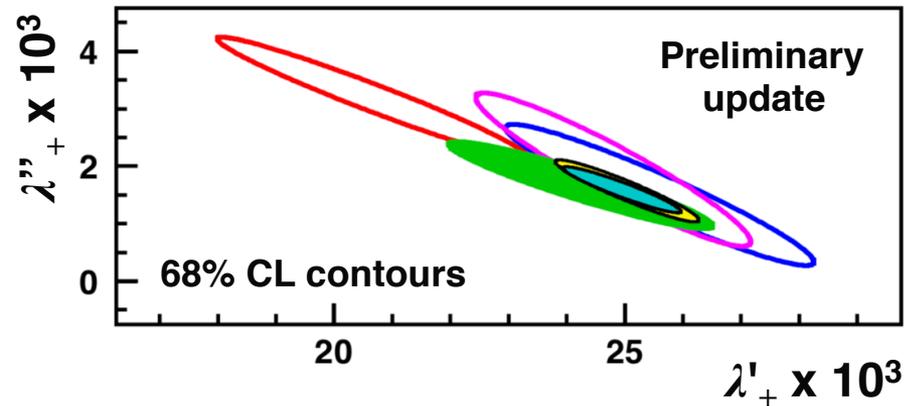
- If included: $\chi^2/\text{ndf} \rightarrow 45/10$
($P \sim 10^{-6}$)

Fits to $K_{e3} + K_{\mu3}$ form-factor slopes: Update

KTeV **KLOE** **ISTRA+** **NA48/2**

2010 fit **Update**

NA48 K_{e3} data included in fits but not shown



2010: $\chi^2 = 12.1/8$ ($P = 14.5\%$)

Update: $\chi^2 = 13.4/11$ ($P = 26.8\%$)

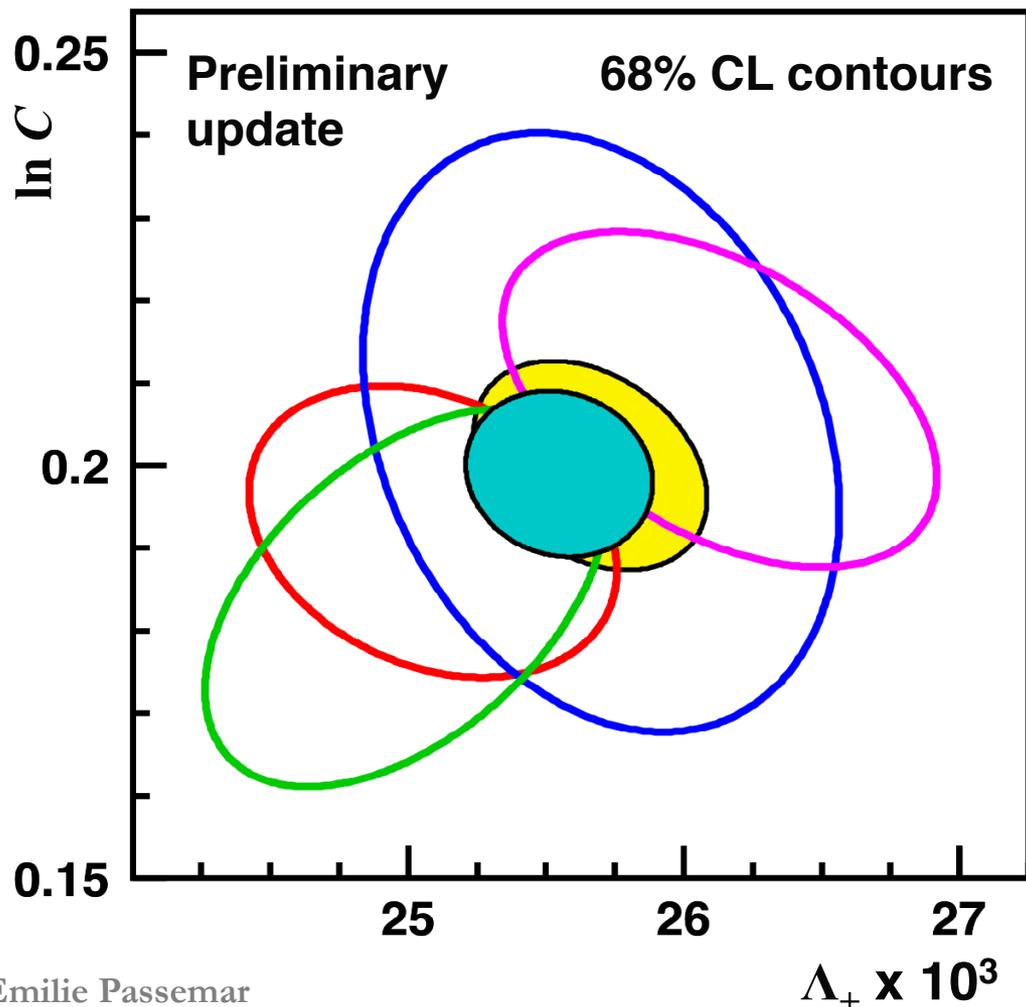
Dispersive parameters for $K_{\ell 3}$ form factors

$K_{\ell 3}$ avgs from **KTeV** **KLOE** **ISTRA+** **NA48/2**

NA48 K_{e3} data included in fits but not shown

2010 fit

Update



$$\Lambda_+ \times 10^3 = 25.55 \pm 0.38$$

$$\ln C = 0.1992(78)$$

$$\rho(\Lambda_+, \ln C) = -0.110$$

$$\chi^2/\text{ndf} = 7.5/7 \text{ (38\%)}$$

Integrals

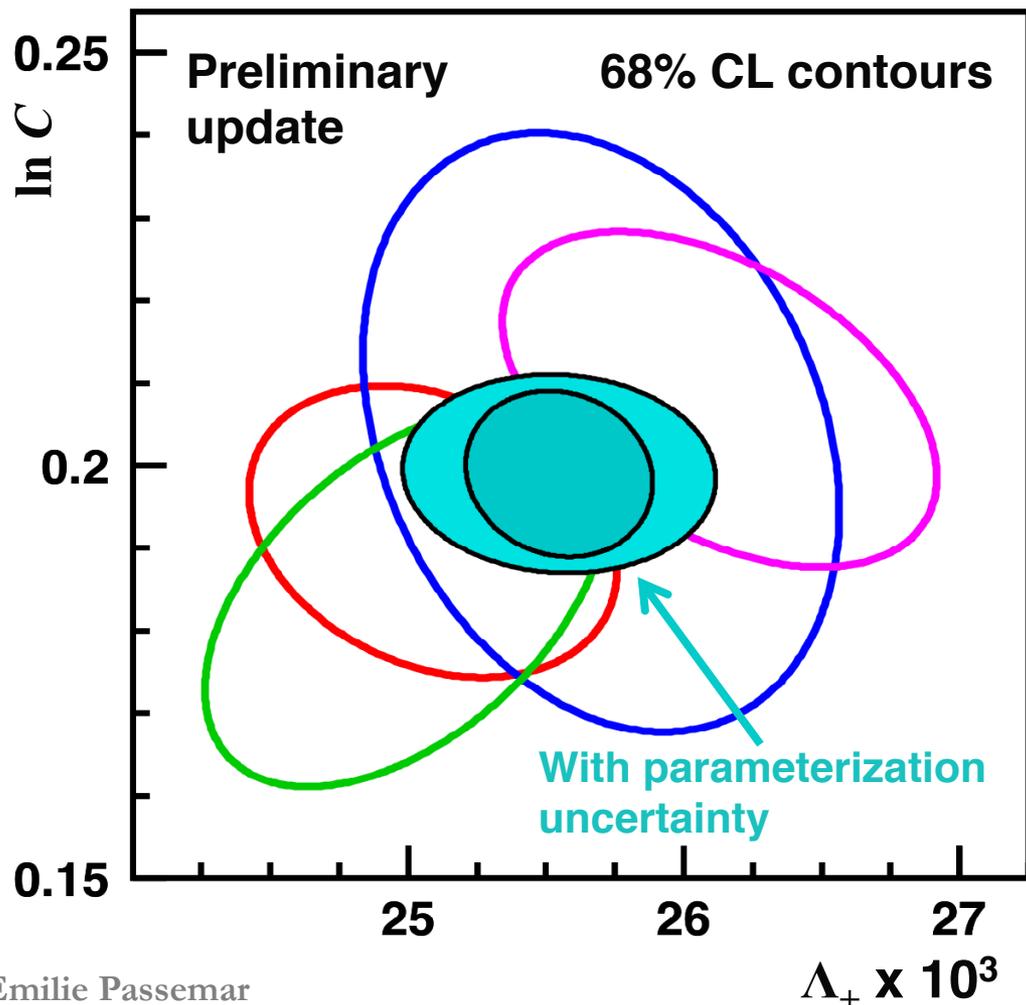
Mode	Update	2010
K^0_{e3}	0.15470(15)	0.15476(18)
K^+_{e3}	0.15915(15)	0.15922(18)
$K^0_{\mu 3}$	0.10247(15)	0.10253(16)
$K^+_{\mu 3}$	0.10553(16)	0.10559(17)

Only tiny changes in central values

Dispersive parameters for $K_{\ell 3}$ form factors

$K_{\ell 3}$ avgs from **KTeV** **KLOE** **ISTRA+** **NA48/2**
 NA48 K_{e3} data included in fits but not shown

2010 fit **Update**



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Fit results include common uncertainty from $H(t)$, $G(t)$:

$$\sigma_{\text{param}}(\Lambda_+) = 0.3 \times 10^{-3}$$

$$\sigma_{\text{param}}(\ln C) = 0.0040$$

KTeV, Bernard et al.'09

Confidence ellipses shown **without** common uncertainty (except as indicated)

2.4 Electromagnetic and isospin breaking corrections

- Master formula for

$$\Gamma(K \rightarrow \pi l \nu [\gamma]) = \frac{Br(K_{l3})}{\tau} C_K^2 \frac{G_F^2 m_K^5}{192\pi^3} S_{EW}^K |V_{us}|^2 |f_+^{K^0\pi^-}(0)|^2 I_{KI} \left(1 + 2\Delta_{EM}^{KI} + 2\Delta_{SU(2)}^{K\pi}\right)$$

- Short distance electroweak correction *Sirlin'82*

$$S_{ew} = 1 + \frac{2\alpha}{\pi} \left(1 + \frac{\alpha_s}{4\pi}\right) \log \frac{m_Z}{m_\rho} + O\left(\frac{\alpha\alpha_s}{\pi^2}\right) \Rightarrow S_{ew} = 1.0232(3)$$

Cirigliano, Giannotti, Neufeld'08

- Long distance EM corrections: $\Delta_{EM}^{K\ell}$ Computed in ChPT at $O(p^2e^2)$

- Isospin breaking : $\Delta_{SU(2)}^{K\pi} = \frac{f_+^{K^+\pi^0}(0)}{f_+^{K^0\pi^-}(0)} - 1$ *Gasser & Leutwyler'85* $\left[\hat{m} \equiv \frac{m_d + m_u}{2}\right]$

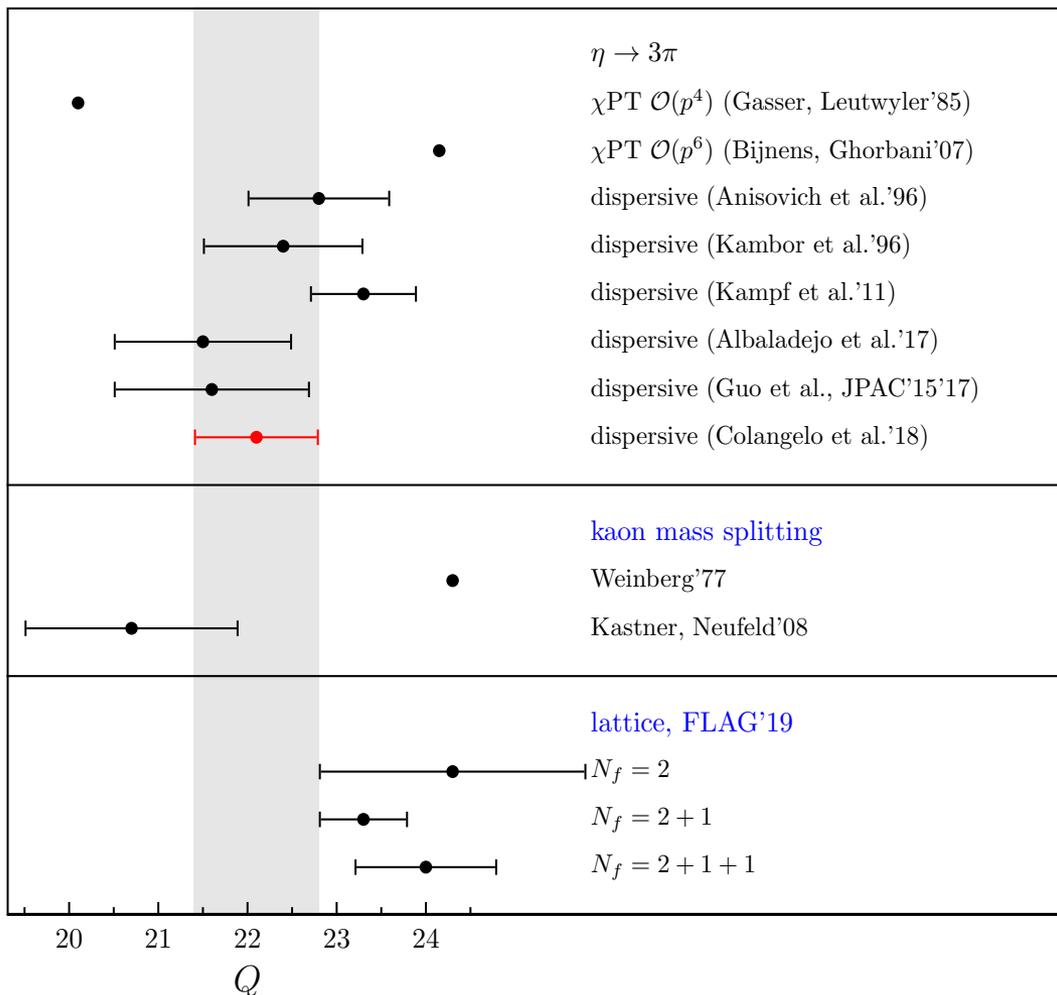
Computed in ChPT at $O(p^4)$: $\Delta_{SU(2)}^{K\pi} = \frac{3}{4} \frac{1}{Q^2} \left[\frac{\overline{M}_K^2}{\overline{M}_\pi^2} + \frac{\chi_{p^4}}{2} \left(1 + \frac{m_s}{\hat{m}}\right) \right] = 2.61(17)\%$

Inputs from lattice QCD and from $\eta \rightarrow 3\pi$ analysis for Q

$$Q^2 \equiv \frac{m_s^2 - \hat{m}^2}{m_d^2 - m_u^2}$$

Input on Q

Previous to new results on Q , uncertainty on $\Delta^{SU(2)}$ leading contributor to uncertainty on V_{us} from K^\pm decays — **can it be reduced?**



Continuing progress + systematic review of existing results for light-quark masses may help

Recent dispersion relation analyses of $\eta \rightarrow 3\pi$ Dalitz plot

e.g. Colangelo, Lanz, Leutwyler, E.P'18

1.6 fb⁻¹ KLOE '04 -'05 data

Continuing progress on lattice

E.g. **BMW '16** PRL 117

$N_f = 2+1$ QCD, 5sp, m_π phys
Partially quenched QED

$Q = 23.4(4)_{\text{st}}(3)_{\text{sy}}(4)_{\text{QED}}$

but some tension

2.4 Electromagnetic and isospin breaking corrections

- Master formula for

$$\Gamma(K \rightarrow \pi l \nu [\gamma]) = \frac{Br(K_{l3})}{\tau} C_K^2 \frac{G_F^2 m_K^5}{192\pi^3} S_{EW}^K |V_{us}|^2 |f_+^{K^0\pi^-}(0)|^2 I_{KI} \left(1 + 2\Delta_{EM}^{KI} + 2\Delta_{SU(2)}^{K\pi}\right)$$

- Short distance electroweak correction $\Rightarrow S_{ew} = 1.0232(3)$ *Sirlin'82*
- Long distance EM corrections: Computed in ChPT at $O(p^2e^2)$

- Isospin breaking : $\Delta_{SU(2)}^{K\pi} = \frac{f_+^{K^+\pi^0}(0)}{f_+^{K^0\pi^-}(0)} - 1$ *Cirigliano, Giannotti, Neufeld'08*

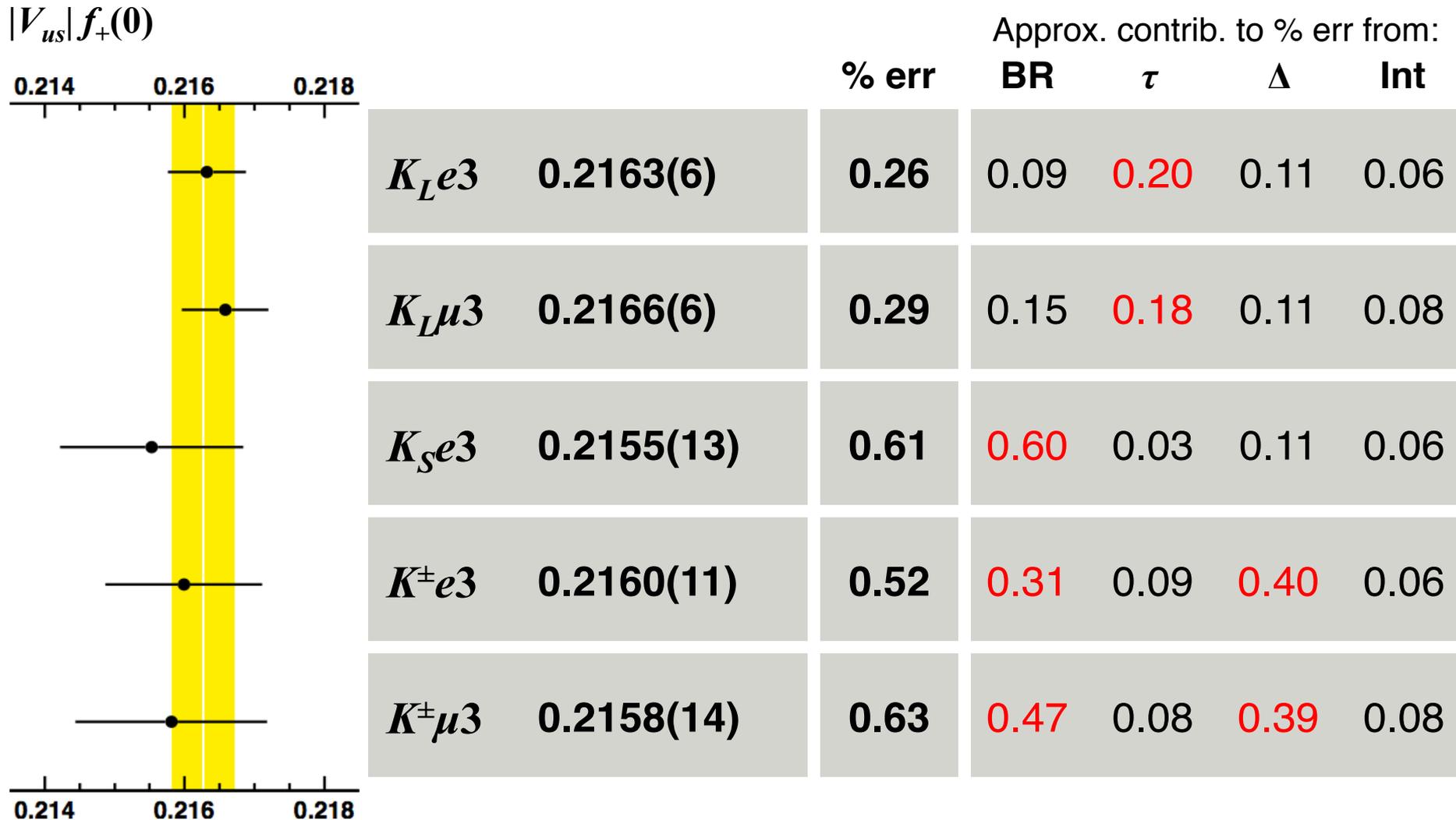
Computed in ChPT at $O(p^4)$: $\Delta_{SU(2)}^{K\pi} = \frac{3}{4} \frac{1}{Q^2} \left[\frac{\overline{M}_K^2}{\overline{M}_\pi^2} + \frac{\chi_{p^4}}{2} \left(1 + \frac{m_s}{m}\right) \right] = 2.61(17)\%$

Inputs from lattice QCD and from $\eta \rightarrow 3\pi$ analysis for Q

Gasser & Leutwyler'85

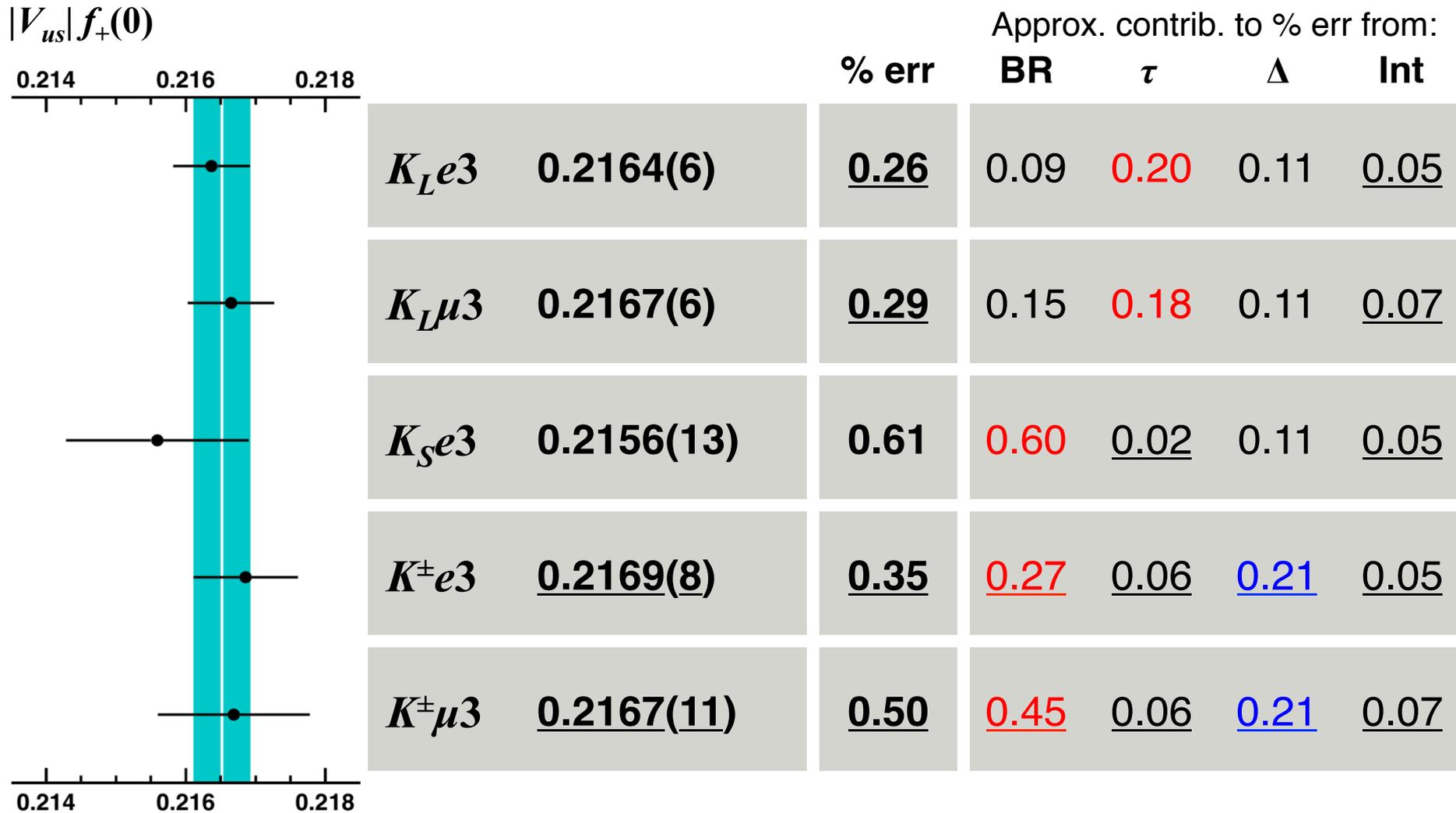
Test by evaluating V_{us} from K^\pm and K^0 data with **no** corrections:
Equality of V_{us} values would require $\Delta^{SU(2)} = 2.82(38)\%$

2.5 $|V_{us}|f_+(0)$ from world data: 2010



Average: $|V_{us}|f_+(0) = 0.2163(5)$ $\chi^2/\text{ndf} = 0.77/4$ (94%)

2.5 $|V_{us}|f_+(0)$ from world data: Update



Average: $|V_{us}|f_+(0) = 0.21652(41)$ $\chi^2/\text{ndf} = 0.98/4$ (91%)

2.6 Determination of $f_+(0)$

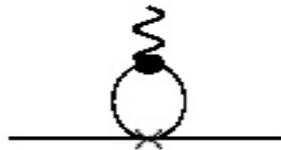
- SU(3) breaking in $f_+(0)$
 - CVC + Ademollo-Gatto theorem: $f_+^{K^0\pi^-}(0) - 1 = O((m_s - m_u)^2)$

$$f_+^{K^0\pi^-}(0) = 1 + \underbrace{f_{p^4}}_{O(m_q)} + \underbrace{f_{p^6}}_{O(m_q^2)} + \dots$$

chiral expansion

- f_{p^4} :

→ One loop graph :



Gasser & Leutwyler'85

→ First order in m_q , 2nd order in $(m_s - m_u)$ $\Rightarrow f_{p^4} \sim \frac{(m_s - m_u)^2}{m_s}$

→ No local operators, UV finite, free of uncertainties



$$f_{p^4} = -0.0227$$

2.6 Determination of $f_+(0)$

- SU(3) breaking in $f_+(0)$

– CVC + Ademollo-Gatto theorem: $f_+^{K^0\pi^-}(0) - 1 = O((m_s - m_u)^2)$

$$f_+^{K^0\pi^-}(0) = 1 + f_{p^4} + f_{p^6} + \dots$$

$O(m_q)$ $O(m_q^2)$

chiral expansion

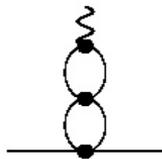
Bijnens & Talavera'02

– f_{p^6} :

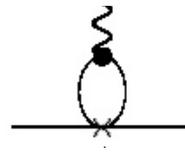
$$f_{p^6} = f_{p^6}^{2\text{-loops}}(\mu) + f_{p^6}^{L_i \times \text{loop}}(\mu) + f_{p^6}^{\text{tree}}(\mu)$$

$$f_{p^6}^{2\text{-loops}}(M_\rho) = 0.0113$$

$$f_{p^6}^{L_i \times \text{loop}}(M_\rho) = -0.0020$$



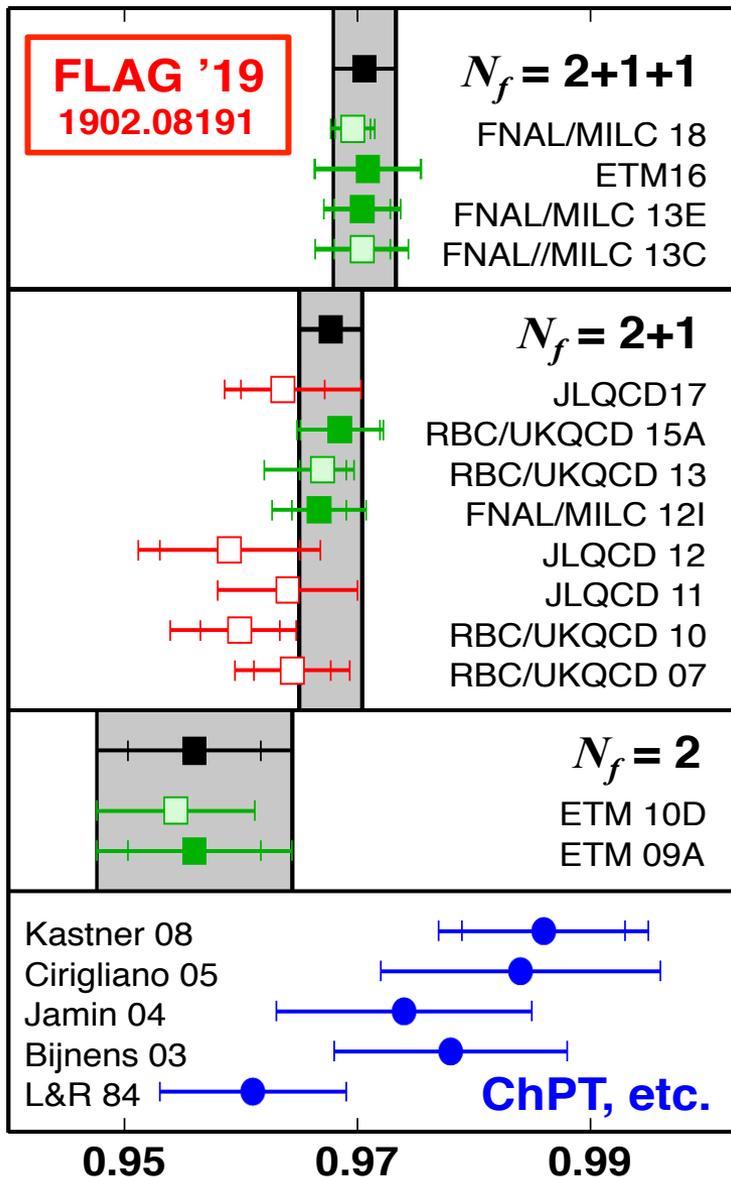
Large positive
chiral loop cont.



$$8 \frac{(M_K^2 - M_\pi^2)^2}{F_\pi^2} \left[\frac{(L_5^r(M_\rho))^2}{F_\pi^2} - C_{12}^r(M_\rho) - C_{34}^r(M_\rho) \right]$$

LECs not fixed by chiral symmetry:
quark model, large-Nc estimates, **LQCD**

2.6 Determination of $f_+(0)$



FLAG '19 averages:

$N_f = 2+1$ $f_+(0) = 0.9677(27)$
Uncorrelated average of:
RBC/UKQCD 15A: DWF, $m_\pi \rightarrow 139$ MeV
FNAL/MILC 12I: HISQ, $m_\pi \sim 300$ MeV

$N_f = 2+1+1$ $f_+(0) = 0.9704(32)$
FNAL/MILC 13E: HISQ, $m_\pi \rightarrow 135$ MeV

Recent updates:

$N_f = 2+1$ $f_+(0) = 0.9636(+62_{-65})$ PRD96 (2017)
JLQCD: Overlap, $m_\pi \rightarrow 300$ MeV
Exact chiral symmetry, one lattice spacing

$N_f = 2+1+1$ $f_+(0) = 0.9709(44)(9)(11)$ PRD 93 (2016)
ETM 16: TwMW, 3sp, $m_\pi \rightarrow 210$ MeV
Full q^2 dependence of f_+, f_0

$f_+(0) = 0.9696(15)(11)$ PRD 99 (2019)
FNAL/MILC update 13E

ChPT:

$N_f = 2+1$ $f_+(0) = 0.970(8)$ Chiral Dynamics 15
Ecker 15: According to Bijmens 03
New LECs from Bijmens, Ecker 14

2019 averages for $f_+(0)$

$$N_f = 2+1+1$$

$$f_+(0) = 0.9698(17)$$

FNAL/MILC18 preliminary replaces FNAL/MILC13E in FLAG average

$$\text{ETM16} \quad 0.9709(44)(9)(11)_{\text{ext}}$$

$$\text{FNAL/MILC18} \quad 0.9696(15)(11)$$

$$N_f = 2+1$$

$$f_+(0) = 0.9677(27)$$

FLAG average, Nov 2016 update

JLQCD17 not included because only 1 lattice spacing used

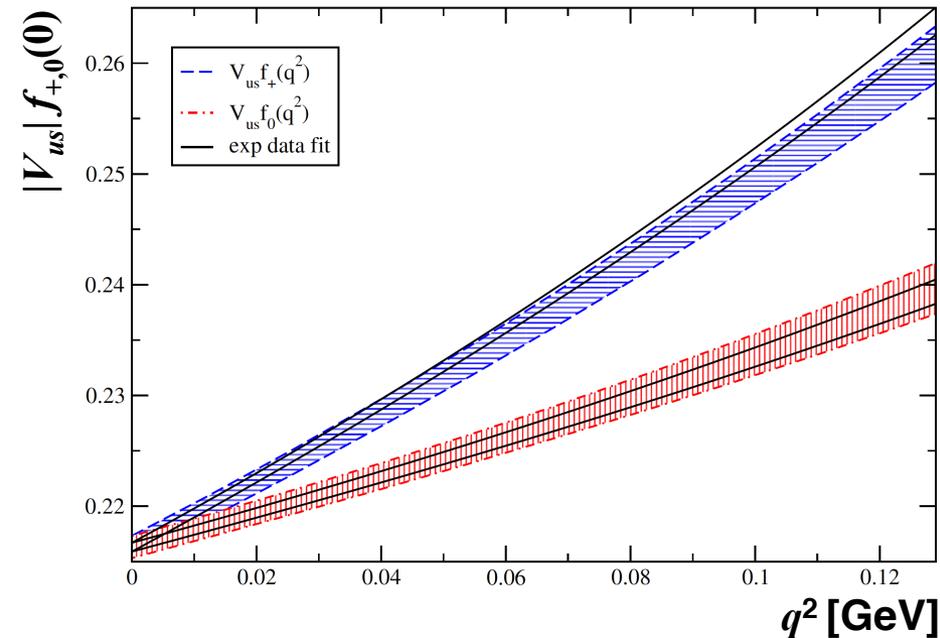
$$\text{FNAL/MILC12I} \quad 0.9667(23)(33)$$

$$\text{RBC/UKQCD15A} \quad 0.9685(34)(14)$$

2.7 q^2 dependence of $K\pi$ form factors

ETM
PRD 93 (2016)

$N_f = 2+1+1$ Twisted-mass Wilson fermions
3 lattice spacings, smallest $m_\pi \rightarrow 210$ MeV
Results for full q^2 dependence of f_+, f_0



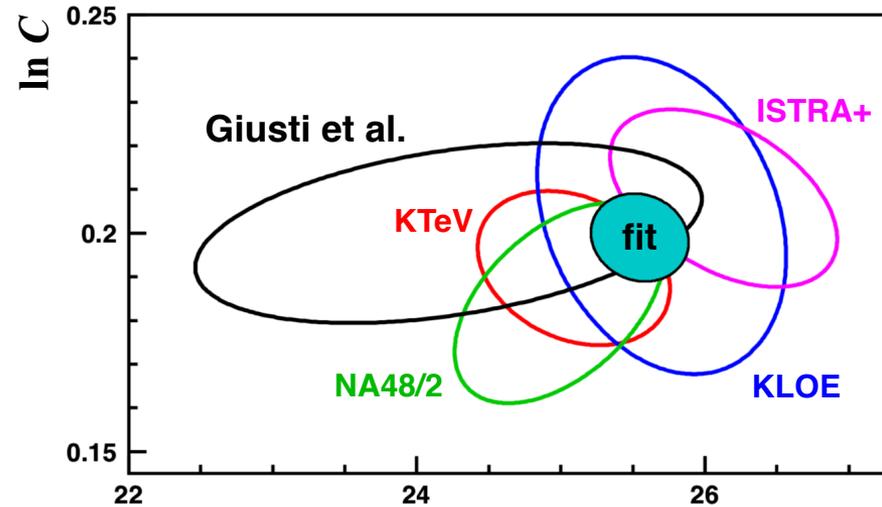
Fit synthetic data points with
dispersive parameterization

$$\Lambda_+ = 24.22(1.16) \times 10^{-3} \quad \rho(\Lambda_+, f_+(0)) = -0.228$$

$$\ln C = 0.1998(138) \quad \rho(\ln C, f_+(0)) = -0.719$$

$$\rho(\Lambda_+, \ln C) = +0.376$$

$$f_+(0) = 0.9709(44)_{\text{st}}(9)_{\text{sy}}(11)_{\text{ext}}$$



- Basic agreement with experimental results
- Confirms basic correctness of lattice calculations for $f_+(0)$
- In the near future FF parameters will be obtained on lattice?

2.8 Extraction of V_{us}

- Decay rate master formula

$$\Gamma(K \rightarrow \pi l \nu [\gamma]) = Br(K_{l3}) / \tau = C_K^2 \frac{G_F^2 m_K^5}{192 \pi^3} S_{EW}^K |V_{us}|^2 |f_+^{K^0 \pi^-}(0)|^2 I_{KI} (1 + 2\Delta_{EM}^{KI} + 2\Delta_{SU(2)}^{K\pi})$$

- $N_f = 2+1$

$$f_+(0) = 0.9677(27)$$

FLAG'19
 $N_f = 2+1$

$$f_+(0) |V_{us}| = 0.21652 \pm 0.00041_{\text{exp}}$$



$$|V_{us}| = 0.22375 \pm 0.00043_{\text{exp}} \pm 0.00062_{\text{lat}}$$

-1.6 σ with unitarity

- $N_f = 2+1+1$

$$f_+(0) = 0.9698(17)$$

FLAG'19
 $N_f = 2+1+1$

$$f_+(0) |V_{us}| = 0.21652 \pm 0.00041_{\text{exp}}$$



$$|V_{us}| = 0.22326 \pm 0.00043_{\text{exp}} \pm 0.00039_{\text{lat}}$$

-2.2 σ with unitarity

3. V_{us}/V_{ud} from $K_{\ell 2}/\pi_{\ell 2}$ decays

3.1 Master formula for V_{us}/V_{ud} from $K_{\ell 2}/\pi_{\ell 2}$ decays

- From $K_{\ell 2}/\pi_{\ell 2}$:

$$\frac{|V_{us}|}{|V_{ud}|} \frac{f_K}{f_\pi} = \left(\frac{\Gamma_{K_{\mu 2}(\gamma)} m_{\pi^\pm}}{\Gamma_{\pi_{\mu 2}(\gamma)} m_{K^\pm}} \right)^{1/2} \frac{1 - m_\mu^2/m_{\pi^\pm}^2}{1 - m_\mu^2/m_{K^\pm}^2} \left(1 - \frac{1}{2} \delta_{EM} - \frac{1}{2} \delta_{SU(2)} \right)$$

Inputs from theory:

Cirigliano, Neufeld '11

$$\delta_{EM} = -0.0069(17)$$

Long-distance EM corrections

$$\delta_{SU(2)} = -0.0043(5)(11)$$

Strong isospin breaking

$$f_K/f_\pi \rightarrow f_{K^\pm}/f_{\pi^\pm}$$

Lattice: f_K/f_π

Cancellation of lattice-scale uncertainties from ratio

NB: Most lattice results already

corrected for $SU(2)$ -breaking: f_{K^\pm}/f_{π^\pm}

Inputs from experiment:

Updated K^\pm BR fit:

$$\text{BR}(K_{\mu 2}^\pm) = 0.6358(11)$$

$$\tau_{K^\pm} = 12.384(15) \text{ ns}$$

PDG:

$$\text{BR}(\pi_{\mu 2}^\pm) = 0.9999$$

$$\tau_{\pi^\pm} = 26.033(5) \text{ ns}$$

$$|V_{us}/V_{ud}| \times f_{K^\pm}/f_{\pi^\pm} = 0.27599(37)$$

No $SU(2)$ -breaking correction

3.2 Electromagnetic corrections

Giusti et al.
PRL 120 (2018)

First lattice calculation of EM corrections to P_{l2} decays

- Ensembles from ETM
- $N_f = 2+1+1$ Twisted-mass Wilson fermions

$$\delta_{SU(2)} + \delta_{EM} = -0.0122(16)$$

- Uncertainty from quenched QED included (0.0006)

Compare to ChPT result from Cirigliano, Neufeld '11:

$$\delta_{SU(2)} + \delta_{EM} = -0.0112(21)$$

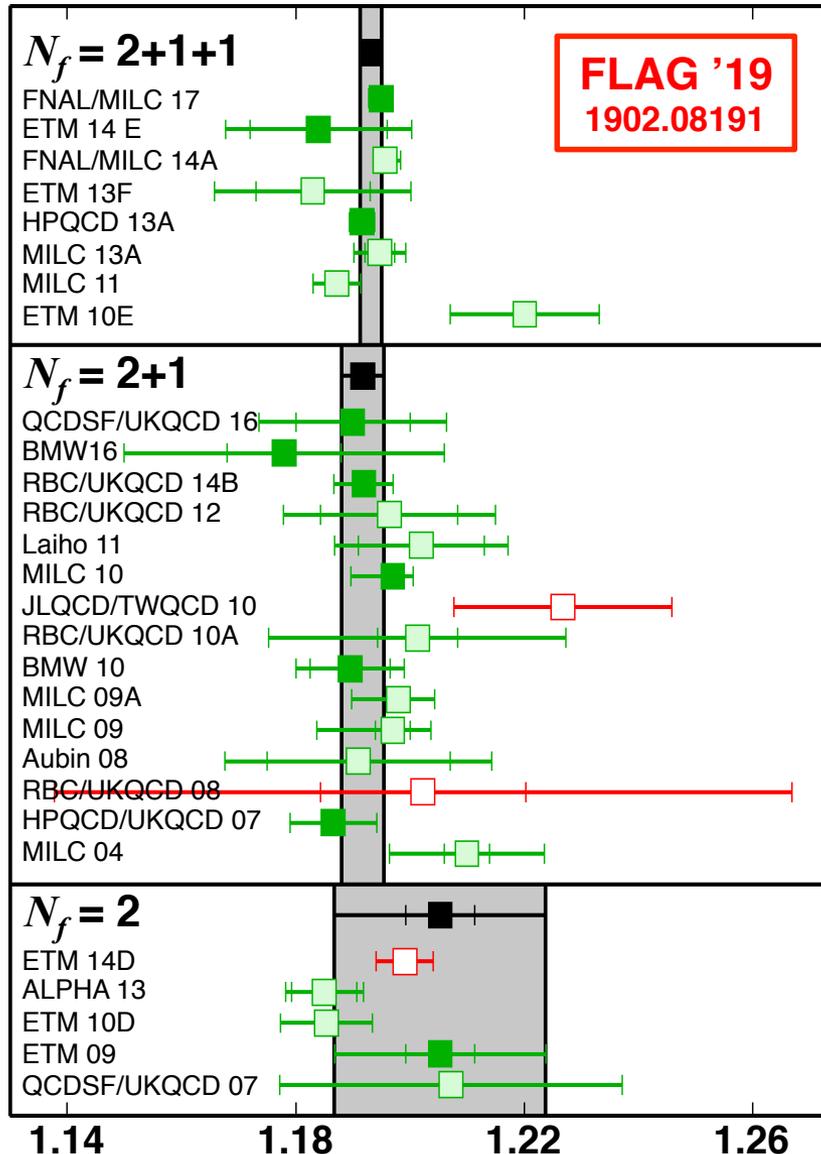
Di Carlo et al.
1904.08731

Update, extended description, and systematics of Giusti et al.

$$\delta_{SU(2)} + \delta_{EM} = -0.0126(14)$$

$$|V_{us}/V_{ud}| \times f_K/f_\pi = 0.27679(28)_{\text{BR}}(20)_{\text{corr}}$$

3.3 Lattice results for f_K/f_π



FLAG '19 averages:

$N_f = 2+1+1$ $f_{K^\pm}/f_{\pi^\pm} = 1.1932(19)$

Includes:

FNAL/MILC 17:

HISQ, 4sp, m_π phys

Updates MILC 13A, FNAL/MILC 14A

HPQCD 13A

HISQ, 3sp, m_π phys,

Same ensembles as FNAL/MILC 17

ETM 14E

TwM, 3sp, $m_\pi = 210-450$ MeV

$N_f = 2+1$ $f_{K^\pm}/f_{\pi^\pm} = 1.1917(37)$

Recent measurements:

QCDSF/UKQCD 16:

Clover, 4sp, $m_\pi \rightarrow 220$ MeV

BMW16:

Clover, 5sp, $m_\pi \rightarrow 139$ MeV

RBC/UKQCD 14B:

DWF, $m_\pi = 139$ MeV

f_K and f_π separately (isospin limit)

3.3 Lattice results for f_K/f_π

Recalculate FLAG averages for results without $SU(2)$ -breaking
Isospin-limit results as reported in original papers

$N_f = 2+1+1$

$f_K/f_\pi = 1.1967(18)$

FNAL/MILC17 1.1980($^{+13}_{-19}$)

HPQCD13A 1.1948(15)(18)

ETM14E 1.188(15)

} Correlated uncertainties
← Uncorrelated uncertainty

$N_f = 2+1$

$f_K/f_\pi = 1.1946(34)^*$

QCDSF/UKQCD17 1.192(10)(13)

BMW16 1.182(10)(26)

RBC/UKQCD14B 1.1945(45)

BMW10 1.192(7)(6)

HPQCD/UKQCD07 1.198(2)(7)

*MILC10 omitted from average because unpublished

3.4 V_{us} from $K_{12} + V_{ud}$ ($0^+ \rightarrow 0^+$)

$$|V_{us}/V_{ud}| \times f_K/f_\pi = 0.27679(34)$$

$$\delta_{SU(2)} + \delta_{EM} = -0.0126(14) \text{ from Di Carlo et al. '19}$$

$$N_f = 2+1$$

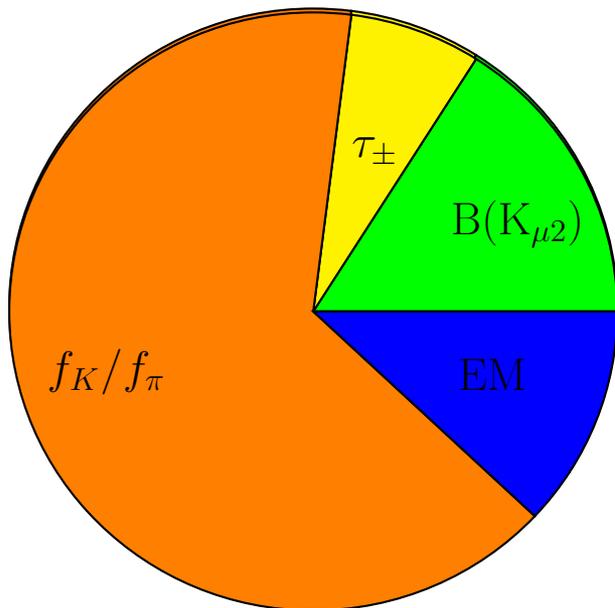
$$f_K/f_\pi = 1.1946(34)$$

$$V_{us}/V_{ud} = 0.23170(29)_{\text{exp}}(66)_{\text{lat}}$$

$$N_f = 2+1+1$$

$$f_K/f_\pi = 1.1967(18)$$

$$V_{us}/V_{ud} = 0.23129(29)_{\text{exp}}(35)_{\text{lat}}$$



$$\frac{|V_{us}| f_K}{|V_{ud}| f_\pi} = \left(\frac{\Gamma_{K_{\mu 2}(\gamma)} m_{\pi^\pm}}{\Gamma_{\pi_{\mu 2}(\gamma)} m_{K^\pm}} \right)^{1/2} \frac{1 - m_\mu^2/m_{\pi^\pm}^2}{1 - m_\mu^2/m_{K^\pm}^2} \left(1 - \frac{1}{2} \delta_{EM} - \frac{1}{2} \delta_{SU(2)} \right)$$

3.4 V_{us} from $K_{12} + V_{ud}$ ($0^+ \rightarrow 0^+$)

$$|V_{us}/V_{ud}| \times f_{K^\pm}/f_{\pi^\pm} = 0.27679(34) \text{ and } |V_{ud}| = 0.97418 \text{ (21)}$$

$$\delta_{SU(2)} + \delta_{EM} = -0.0126(14) \text{ from Di Carlo et al. '19}$$

$N_f = 2+1$ $f_K/f_\pi = 1.1946(34)$	$V_{us} = 0.22572(28)_{\text{exp}}(64)_{\text{lat}}(05)_{ud}$ $\Delta_{\text{CKM}} = +0.00003(13)_{\text{exp}}(29)_{\text{lat}}(43)_{ud} = +0.1\sigma$
$N_f = 2+1+1$ $f_K/f_\pi = 1.1967(18)$	$V_{us} = 0.22532(28)_{\text{exp}}(34)_{\text{lat}}(05)_{ud}$ $\Delta_{\text{CKM}} = -0.00015(13)_{\text{exp}}(15)_{\text{lat}}(43)_{ud} = -0.3\sigma$

$K_{\ell 2}$ results give better agreement with unitarity via V_{ud} than $K_{\ell 3}$ results (-2σ)

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1 + \Delta_{\text{CKM}}$$

4. V_{us} and Unitarity of the CKM matrix

4.1 V_{us} and CKM unitarity: All data, $N_f=2+1$

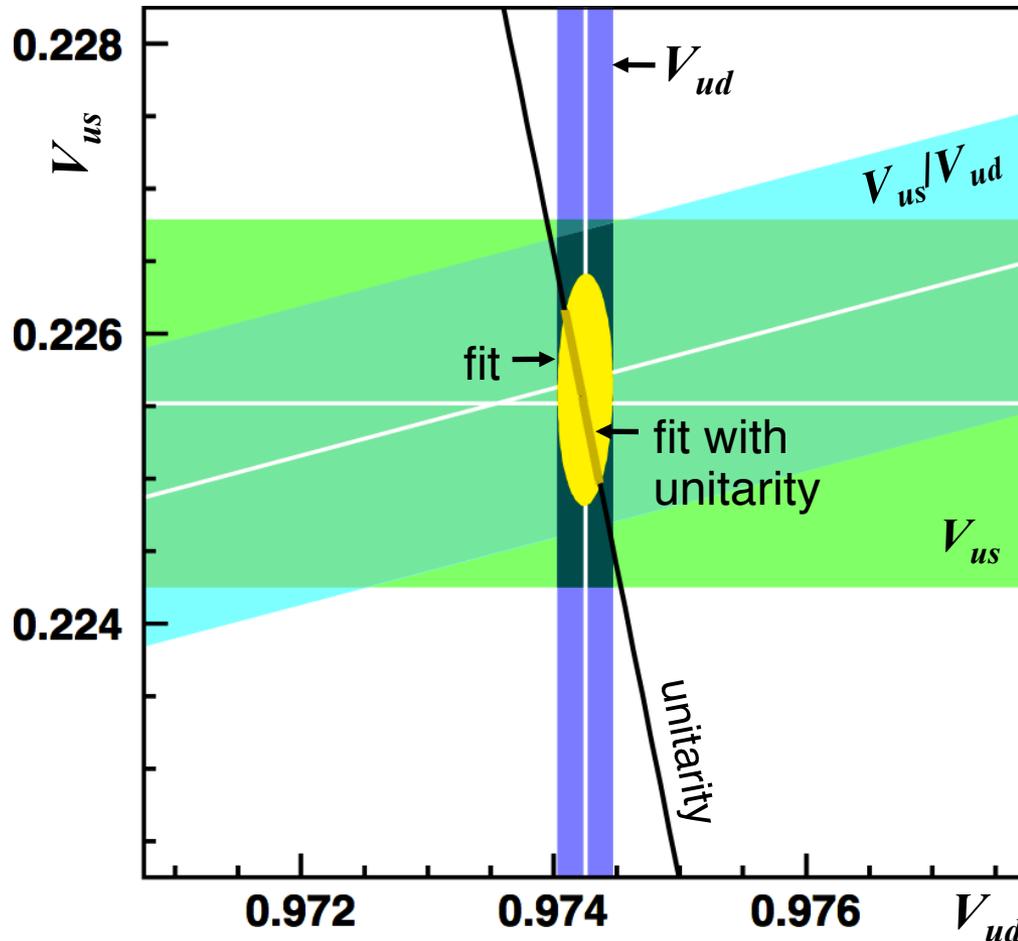
Use $f_+(0) = 0.959(5)$ and $f_K/f_\pi = 1.193(5)$
 Fit to results for $|V_{ud}|$, $|V_{us}|$, $|V_{us}|/|V_{ud}|$



$$|V_{ud}| = 0.97425(22)$$

$$|V_{us}| = 0.2255(13)$$

$$|V_{us}|/|V_{ud}| = 0.2317(11)$$



Fit results, no constraint

$$V_{ud} = 0.97425(22)$$

$$V_{us} = 0.2256(8)$$

$$\chi^2/\text{ndf} = 0.01/1 \text{ (92\%)}$$

$$\Delta_{\text{CKM}} = +0.0001(6)$$

Fit results, unitarity constraint

$$V_{ud} = 0.97423(14)$$

$$V_{us} = 0.2255(6)$$

$$\chi^2/\text{ndf} = 0.03/2 \text{ (99\%)}$$

Status in 2010

4.2 V_{us} and CKM unitarity: All data, “Conv.” V_{ud}

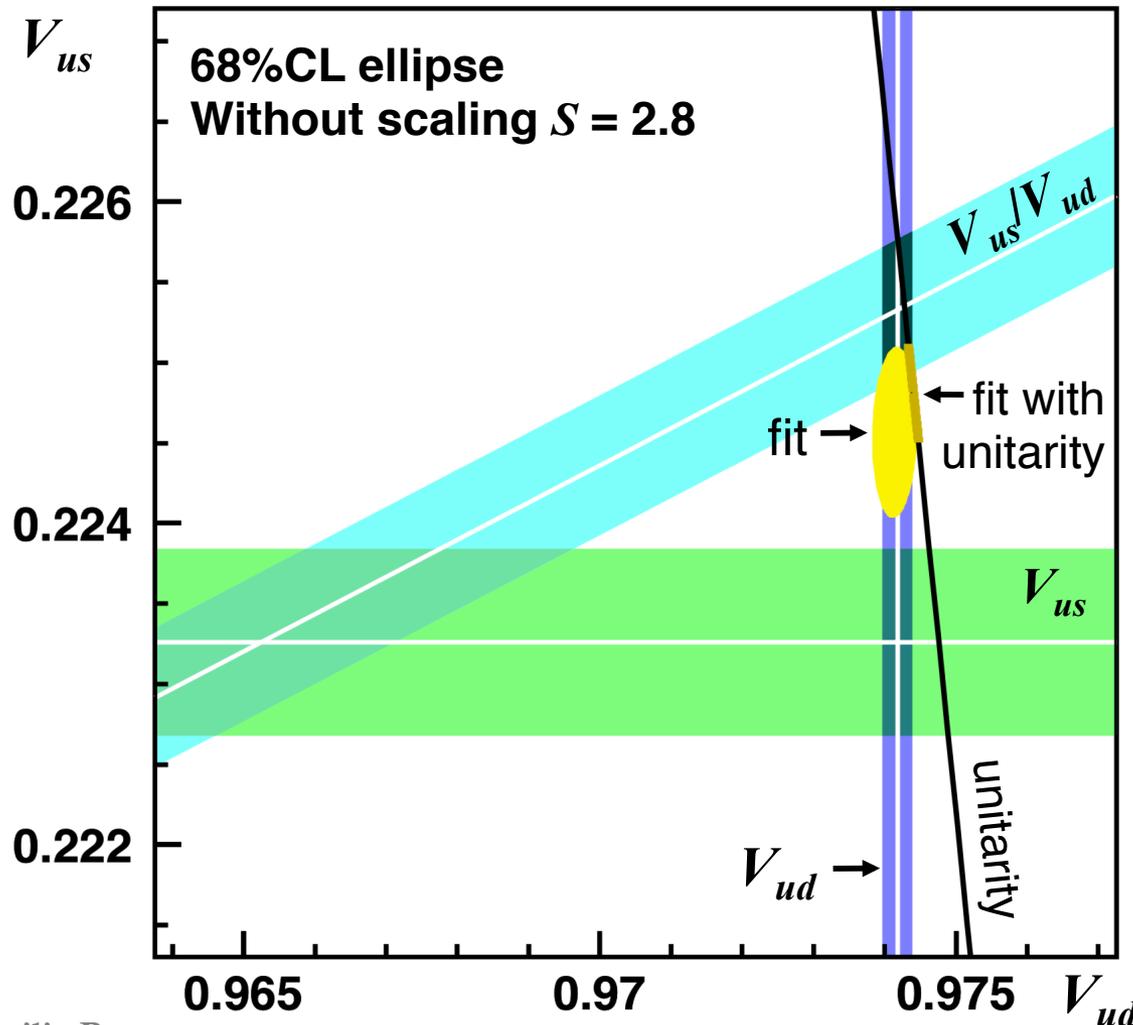
$N_f = 2+1+1$: Fit to results for $|V_{ud}|$, $|V_{us}|$, $|V_{us}|/|V_{ud}|$
 $f_+(0) = 0.9698(17)$, $f_K/f_\pi = 1.1967(18)$



$$|V_{ud}| = 0.97418(21)$$

$$|V_{us}| = 0.2233(6)$$

$$|V_{us}|/|V_{ud}| = 0.2313(5)$$



Fit results, no constraint

$$V_{ud} = 0.97414(21)$$

$$V_{us} = 0.22456(35)$$

$$\chi^2/\text{ndf} = 8.0/1 \text{ (0.5\%)}$$

$$\Delta_{\text{CKM}} = -0.00062(45)$$

$$-1.4\sigma$$

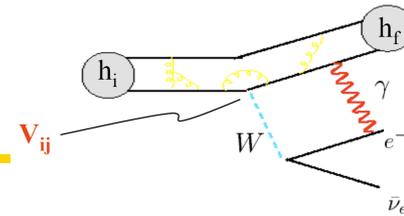
With scale factor $S = 2.8$

$$V_{ud} = 0.97414(60)$$

$$V_{us} = 0.2246(10)$$

Update

4.3 V_{ud} from $0^+ \rightarrow 0^+$



$$\frac{1}{t} = \frac{G_\mu^2 |V_{ud}|^2 m_e^5}{\pi^3 \log 2} f(Q) (1 + RC) \longrightarrow ft (1 + RC) = \frac{2984.48(5) s}{|V_{ud}|^2}$$

$$(1 + RC) = (1 - \delta_C) (1 + \delta_R) (1 + \Delta_C)$$

$\langle f | \tau_+ | i \rangle = \sqrt{2} (1 - \delta_C/2)$
Coulomb distortion
of wave-functions

$$\delta_C \sim 0.5\%$$

Towner-Hardy
Ormand-Brown

Nucleus-dependent
rad. corr.
(Z, E^{\max} , nuclear structure)

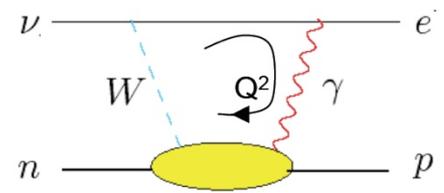
$$\delta_R \sim 1.5\%$$

Sirlin-Zucchini '86
Jaus-Rasche '87

Nucleus-independent
short distance rad. corr.

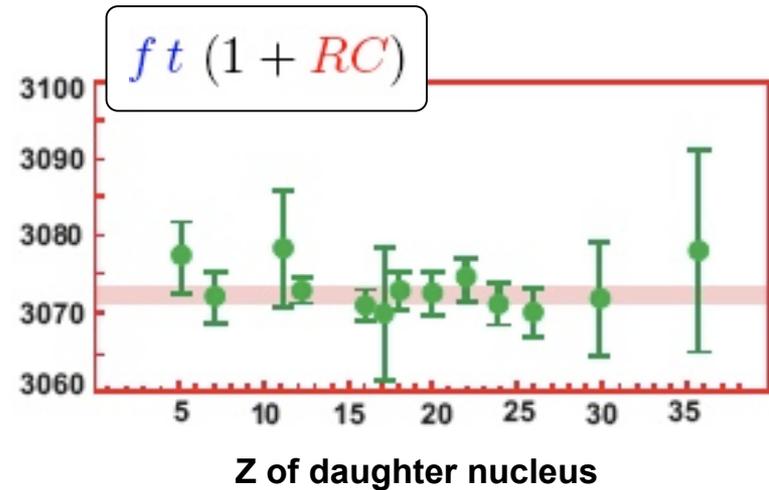
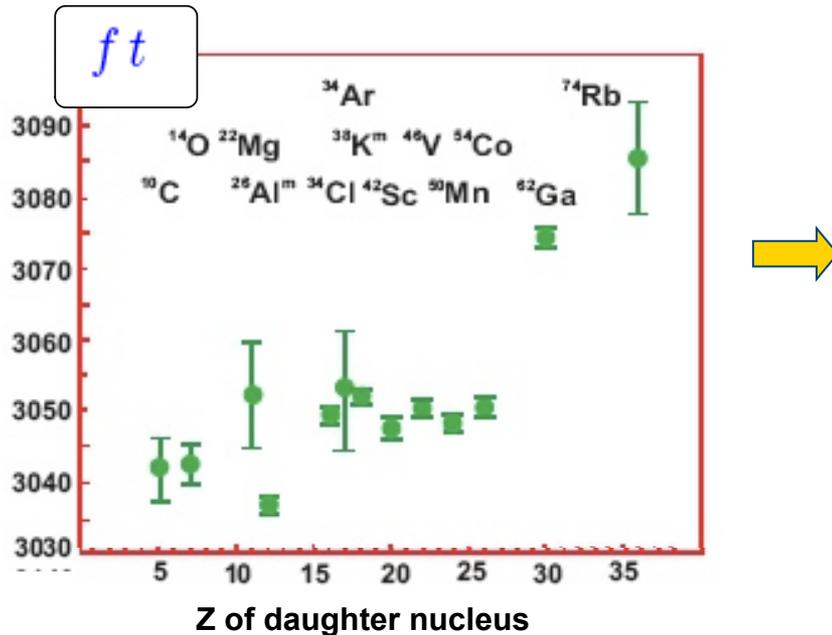
$$\Delta_R \sim 2.4\%$$

Marciano-Sirlin '06



4.3 V_{ud} from $0^+ \rightarrow 0^+$

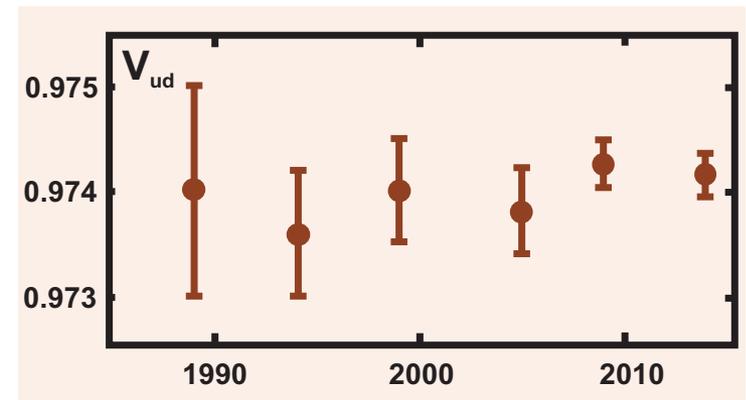
Hardy@Amherst'19



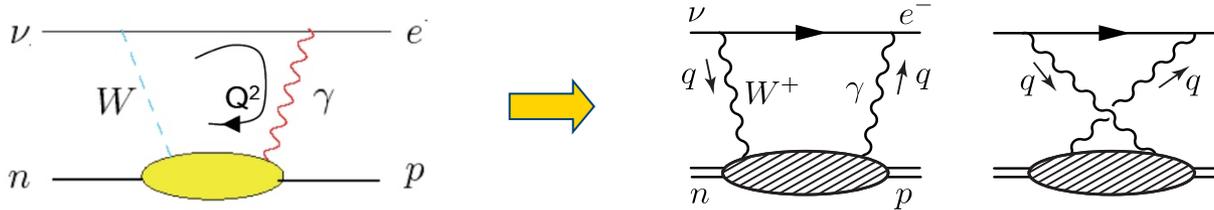
$|V_{ud}| = 0.97418(21)$

Improvements over years :

- Survey of 150 measurements of 13 different $0^+ \rightarrow 0^+$ β decays
- 27 new ft measurements including Penning-trap measurements for QEC
- Improved EW radiative corrections [Marciano & Sirlin'06](#)
- New $SU(2)$ -breaking corrections [Towner & Hardy'08](#)



4.4 New Radiative Corrections for $0^+ \rightarrow 0^+$



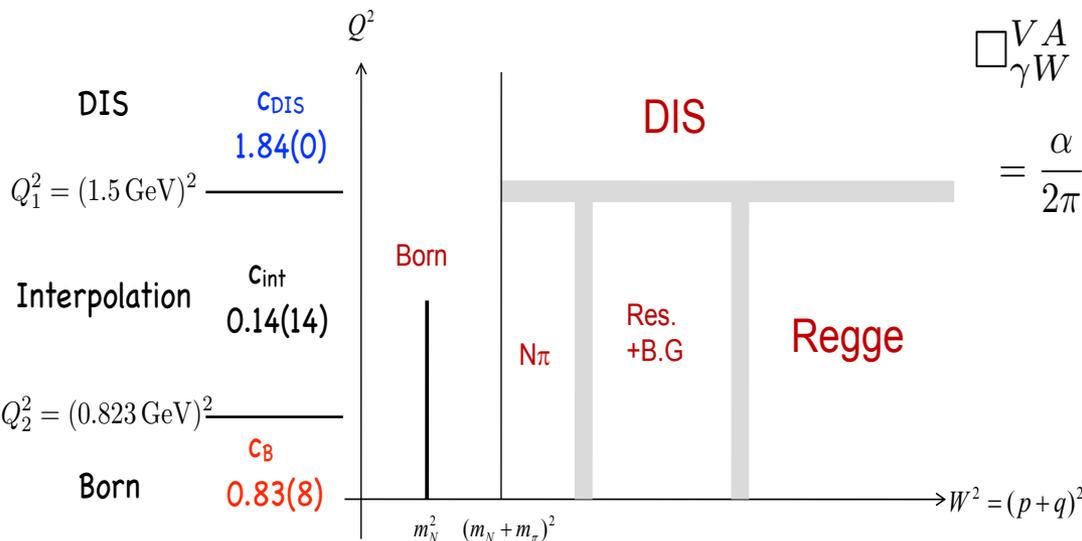
Gorchtein@CIPANP'18
Amherst'19

Marciano & Sirlin'06

$$\square_{\gamma W}^{VA} = \frac{\alpha}{2\pi} [c_B + c_{int} + c_{DIS}] = \frac{\alpha}{2\pi} [0.83(8) + 0.14(14) + 1.84(0)]$$

$$\square_{\gamma W}^{MS} = \frac{\alpha}{2\pi} 2.79(17) = 3.24(20) \times 10^{-3}$$

New evaluation: Seng, Gorchtein, Patel & Ramsey-Musolf'18



$$\square_{\gamma W}^{VA} = \frac{\alpha}{2\pi} [c_B + c_{piN} + c_{Res} + c_{Regge} + c_{DIS}]$$

$$= \frac{\alpha}{2\pi} [0.91(5) + 0.044(5) + 0.01(1) + 0.238(14) + 1.84(0)]$$

$$\square_{\gamma W}^{New} = \frac{\alpha}{2\pi} 3.03(5) = 3.51(6) \times 10^{-3}$$

4.4 New Radiative Corrections for $0^+ \rightarrow 0^+$

$$|V_{ud}|^2 = \frac{2984.432(3) \text{ s}}{\mathcal{F}t(1 + \Delta_R^V)}$$

- Conventional calculation:

$$\Delta_R^V = 0.02361(38)$$

Marciano & Sirlin'06

- Dispersion Relations:

$$\Delta_R^V = 0.02467(22)$$

Seng, Gorchtein, Patel & Ramsey-Musolf'18



$$|V_{ud}| = 0.97418(10)_{\mathcal{F}t} (18)_{\Delta_R^V}$$

$$|V_{ud}| = 0.97370(10)_{\mathcal{F}t} (11)_{\Delta_R^V}$$

~1.8 σ smaller

4.5 V_{us} and CKM unitarity: All data, New V_{ud}

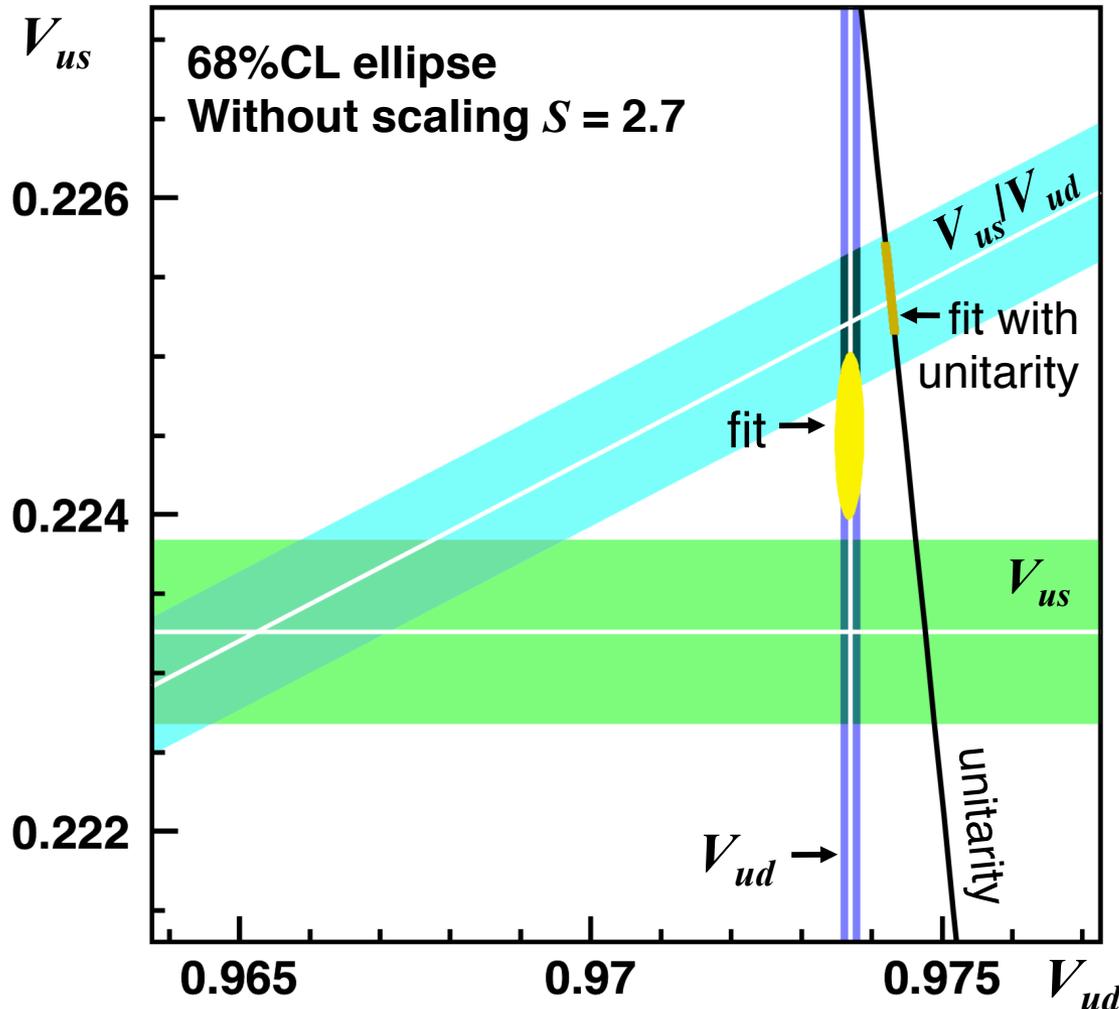
$N_f = 2+1+1$: Fit to results for $|V_{ud}|$, $|V_{us}|$, $|V_{us}|/|V_{ud}|$
 $f_+(0) = 0.9698(17)$, $f_K/f_\pi = 1.1967(18)$



$$|V_{ud}| = 0.97370(14)$$

$$|V_{us}| = 0.2233(6)$$

$$|V_{us}|/|V_{ud}| = 0.2313(5)$$



Fit results, no constraint

$$V_{ud} = 0.97368(14)$$

$$V_{us} = 0.22450(35)$$

$$\chi^2/\text{ndf} = 7.2/1 \text{ (0.7\%)}$$

$$\Delta_{\text{CKM}} = -0.00154(32)$$

$$-4.8\sigma$$

With scale factor $S = 2.7$

$$V_{ud} = 0.97368(38)$$

$$V_{us} = 0.2245(9)$$

New Radiative Corrections for $0^+ \rightarrow 0^+$

$$|V_{ud}|^2 = \frac{2984.432(3) \text{ s}}{\mathcal{F}t(1 + \Delta_R^V)}$$

- Conventional calculation:

$$\Delta_R^V = 0.02361(38)$$

Marciano & Sirlin'06

- Dispersion Relations:

$$\Delta_R^V = 0.02467(22)$$

Seng, Gorchtein, Patel & Ramsey-Musolf'18



$$|V_{ud}| = 0.97418(10)_{\mathcal{F}t} (18)_{\Delta_R^V}$$

$$|V_{ud}| = 0.97370(10)_{\mathcal{F}t} (11)_{\Delta_R^V}$$

~1.8 σ smaller



New Analysis

$$|V_{ud}| = 0.97389(19)$$

Czarnecki, Marciano, Sirlin'19

4.5 V_{us} and CKM unitarity: All data, New V_{ud}

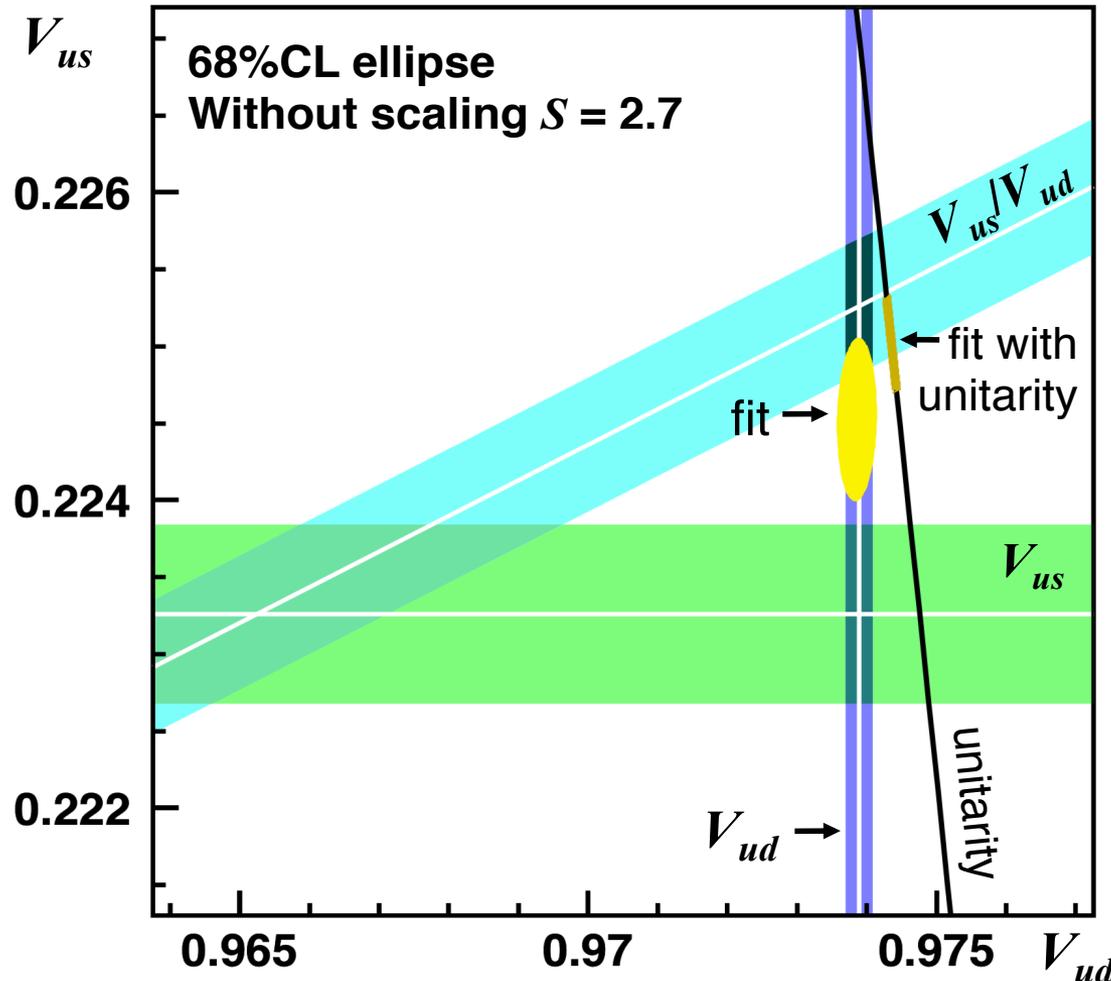
$N_f = 2+1+1$: Fit to results for $|V_{ud}|$, $|V_{us}|$, $|V_{us}|/|V_{ud}|$
 $f_+(0) = 0.9698(17)$, $f_K/f_\pi = 1.1967(18)$



$$|V_{ud}| = 0.97389(19)$$

$$|V_{us}| = 0.2233(6)$$

$$|V_{us}|/|V_{ud}| = 0.2313(5)$$



Fit results, no constraint

$$V_{ud} = 0.97386(19)$$

$$V_{us} = 0.22452(35)$$

$$\chi^2/\text{ndf} = 7.5/1 \text{ (0.6\%)}$$

$$\Delta_{\text{CKM}} = -0.00119(41)$$

$$-2.9\sigma$$

With scale factor $S = 2.7$

$$V_{ud} = 0.97368(53)$$

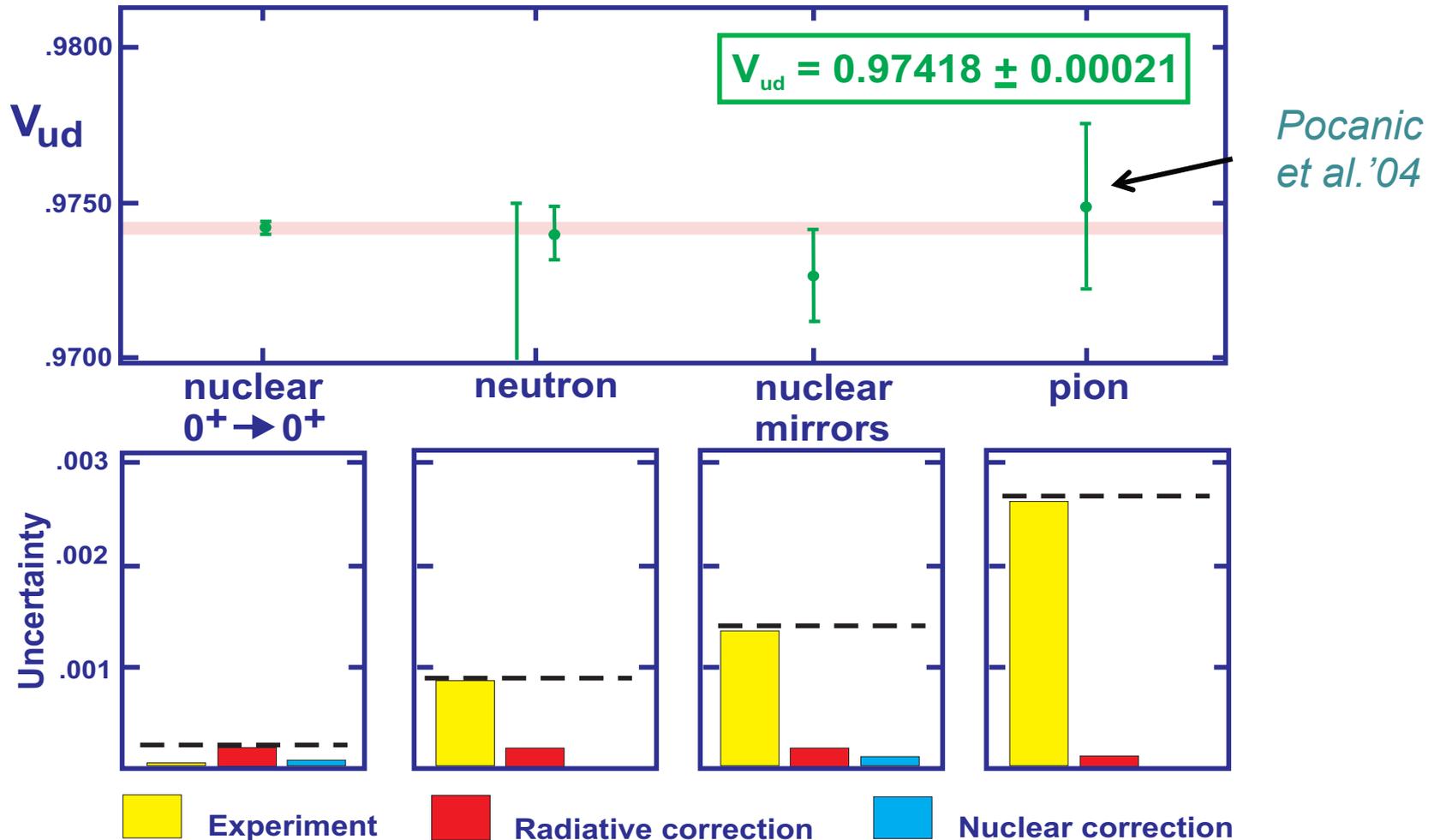
$$V_{us} = 0.2245(10)$$

4.6 V_{us} and CKM unitarity: Hint of New Physics?

- With new calculation of radiative corrections: $2.9\sigma - 4.8\sigma$ discrepancy with unitarity  calculations need to be checked and model dependence understood
- Can we extract V_{ud} differently?

Extraction of V_{ud} : summary

Hardy@Amherst'19



4.6 V_{us} and CKM unitarity: Hint of New Physics?

- With new calculation of radiative corrections: $2.9\sigma - 4.8\sigma$ discrepancy with unitarity  calculations need to be checked and model dependence understood
- Can we extract V_{ud} differently?
Yes but for the neutron decay same *radiative correction* to evaluate!
And also life time experiments (bottle vs. beam) do not agree!

4.6 V_{us} and CKM unitarity: Hint of New Physics?

- With new calculation of radiative corrections: $2.9\sigma - 4.8\sigma$ discrepancy with unitarity \rightarrow calculations need to be checked and model dependence understood

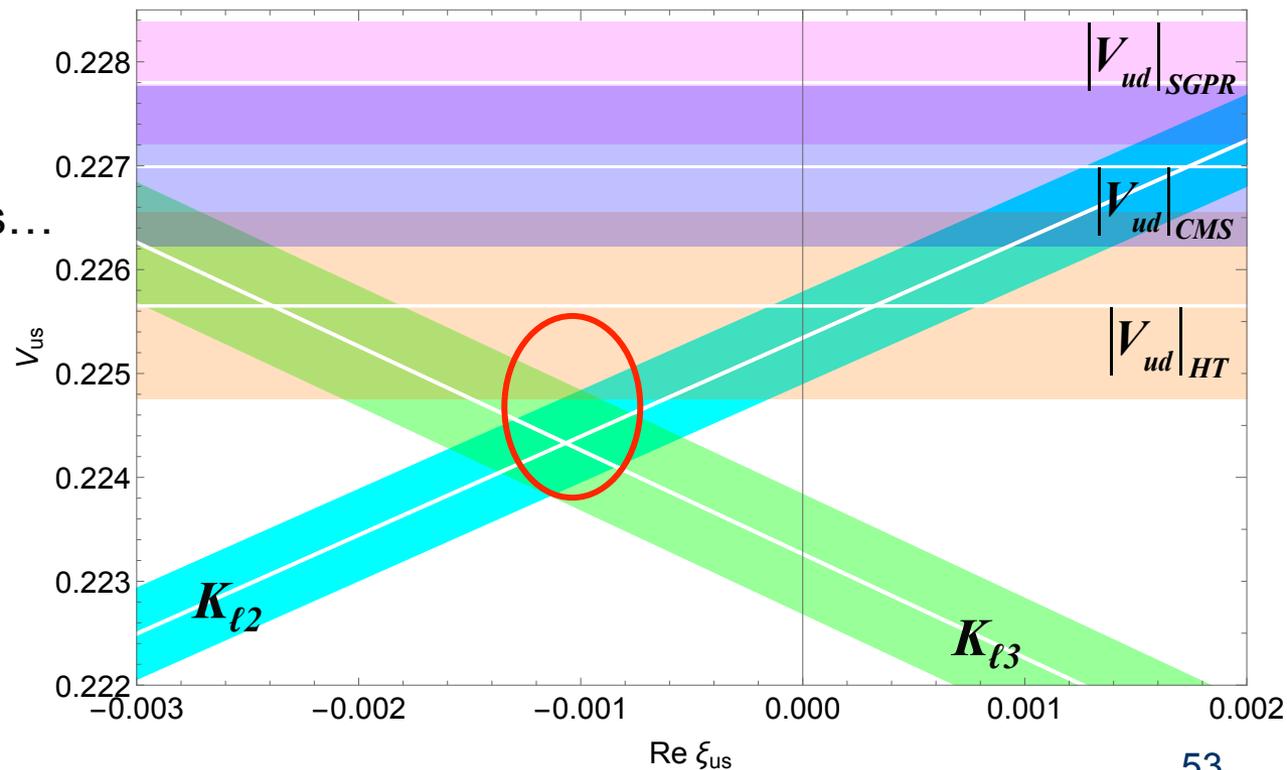
- If this disagreement correct, what does it tell us?

Several studies:

Crivellin, Hoferichter, Kitahara, E.P. in progress

- 4th quark b'
- gauge horizontal family symmetry
- Right handed currents...

Belfatto, Beradze, Berezhiani'19



5. Conclusion and Outlook

5.1 Conclusion

- We have reviewed the different extractions of V_{us} and V_{ud}
  We have entered a precision era
- On experimental side many new measurements on kaon decays from **KLOE**, **NA48**, **KTeV**, **ISTRA+** and more to come **NA62**, **OKA**, **KLOE II**, **LHCb**, **TREK E36**
- On theoretical side: small effects start to matter: radiative corrections, e.m. effects, isospin breaking etc
 Many of these effects can now be computed on the lattice in addition to matrix elements
- With new lattice data: tension towards a discrepancy in unitarity of the first row of CKM matrix
- On V_{ud} side: an incredible precision has been achieved on extracting V_{ud} from nuclear beta decays but new radiative corrections
- Experimentalists working hard to extract V_{ud} from neutron side

5.2 Outlook

- Tensions in unitarity of 1st row of CKM matrix have reappeared!
- We need to work hard to understand where they come from:
 - On experimental side:
 - For V_{us} , new measurements in kaons but mainly in tau decays, hyperons
 - For V_{ud} , understand the situation of the proton lifetime, beta decay of pion?
 - On theory side:
 - Calculate very precisely radiative corrections, isospin breaking effects and matrix elements
 - Be sure the uncertainties are under control
- If these tensions are confirmed  what do they tell us?
- Interesting time ahead of us!

6. Back up

V_{ud} from neutron decay

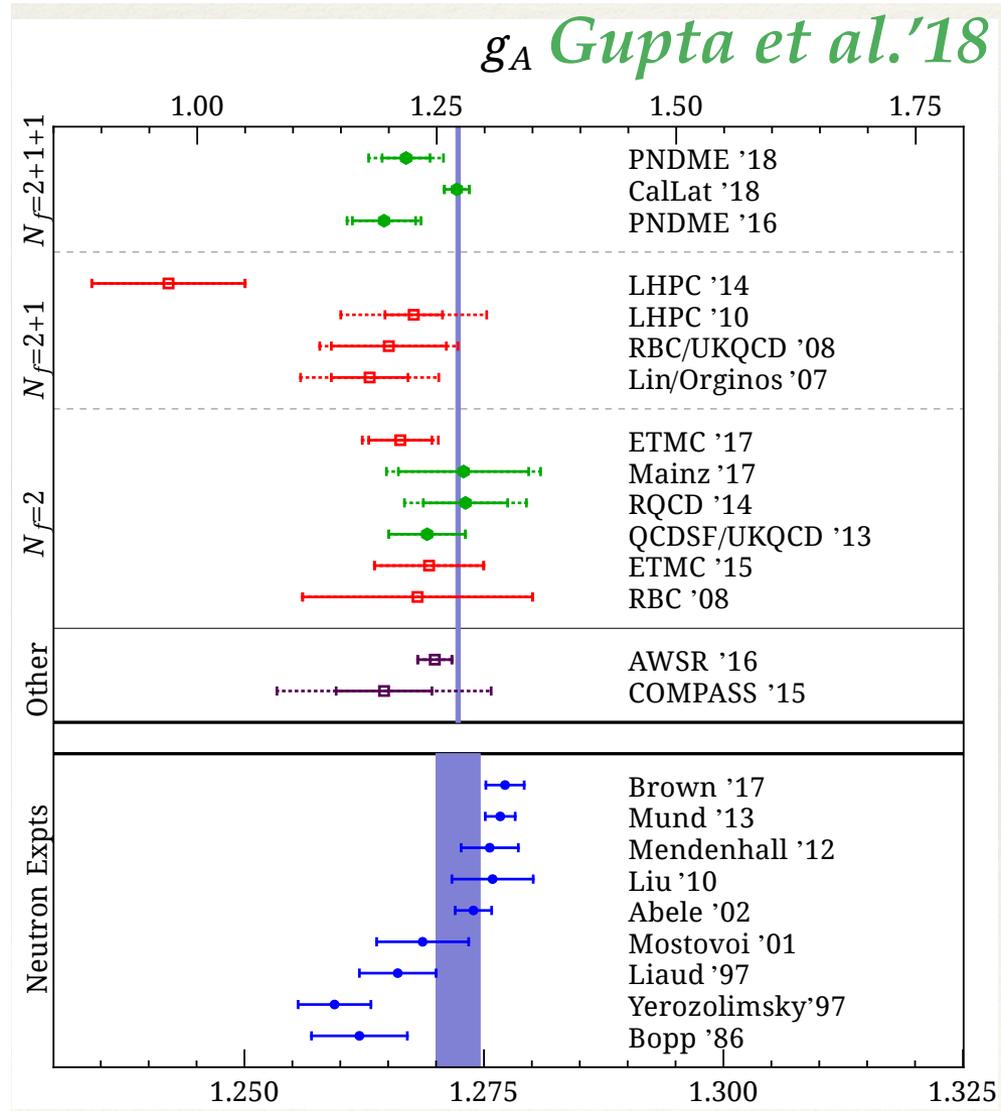
$$\bar{V}_{ud} = \left[\frac{4908.6(1.9) \text{ s}}{\tau_n (1 + 3\bar{g}_A^2)} \right]^{1/2}$$

Marciano, Sirlin'06

- Need to measure neutron lifetime τ_n and
- $\lambda = g_A/g_V$: determined through measurement of the neutron β -decay asymmetry parameter A which defines the correlation between the spin of the neutron and the momentum of the emitted electron

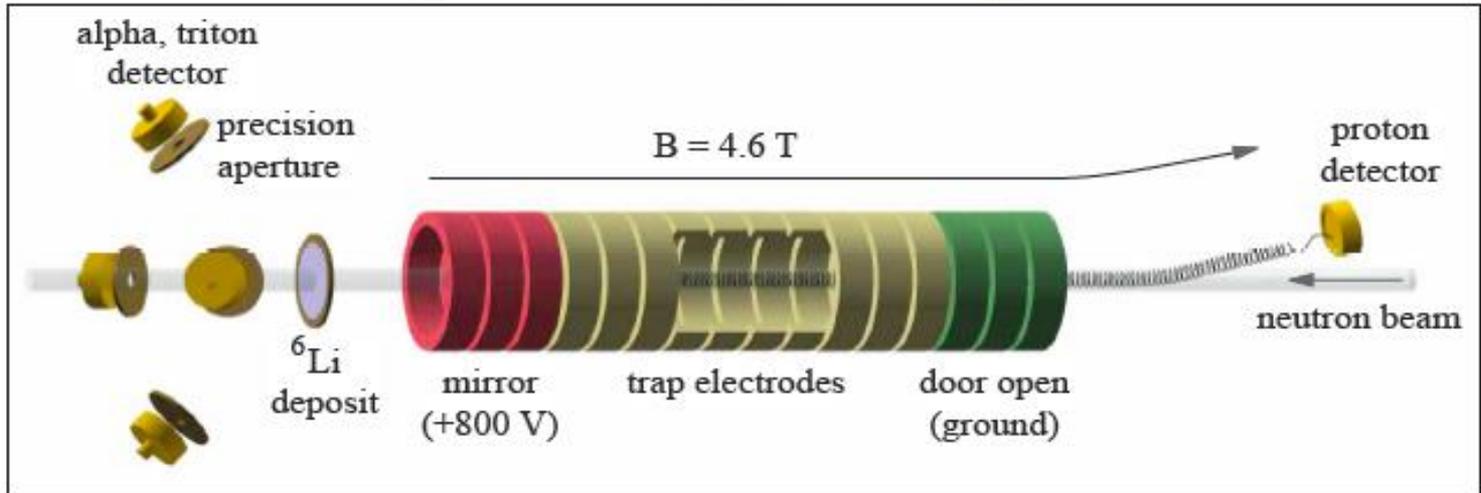
V_{ud} from neutron decay

- Or take g_A from lattice QCD determination:

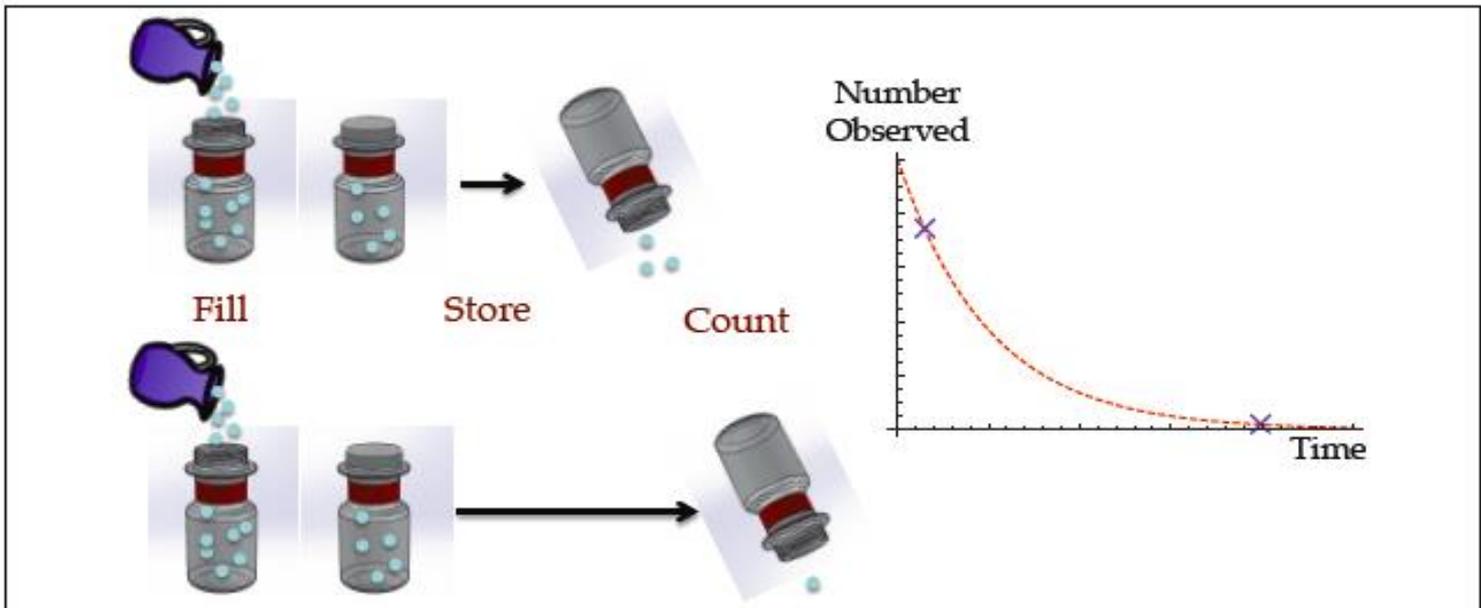


Neutron lifetime measurements : two techniques

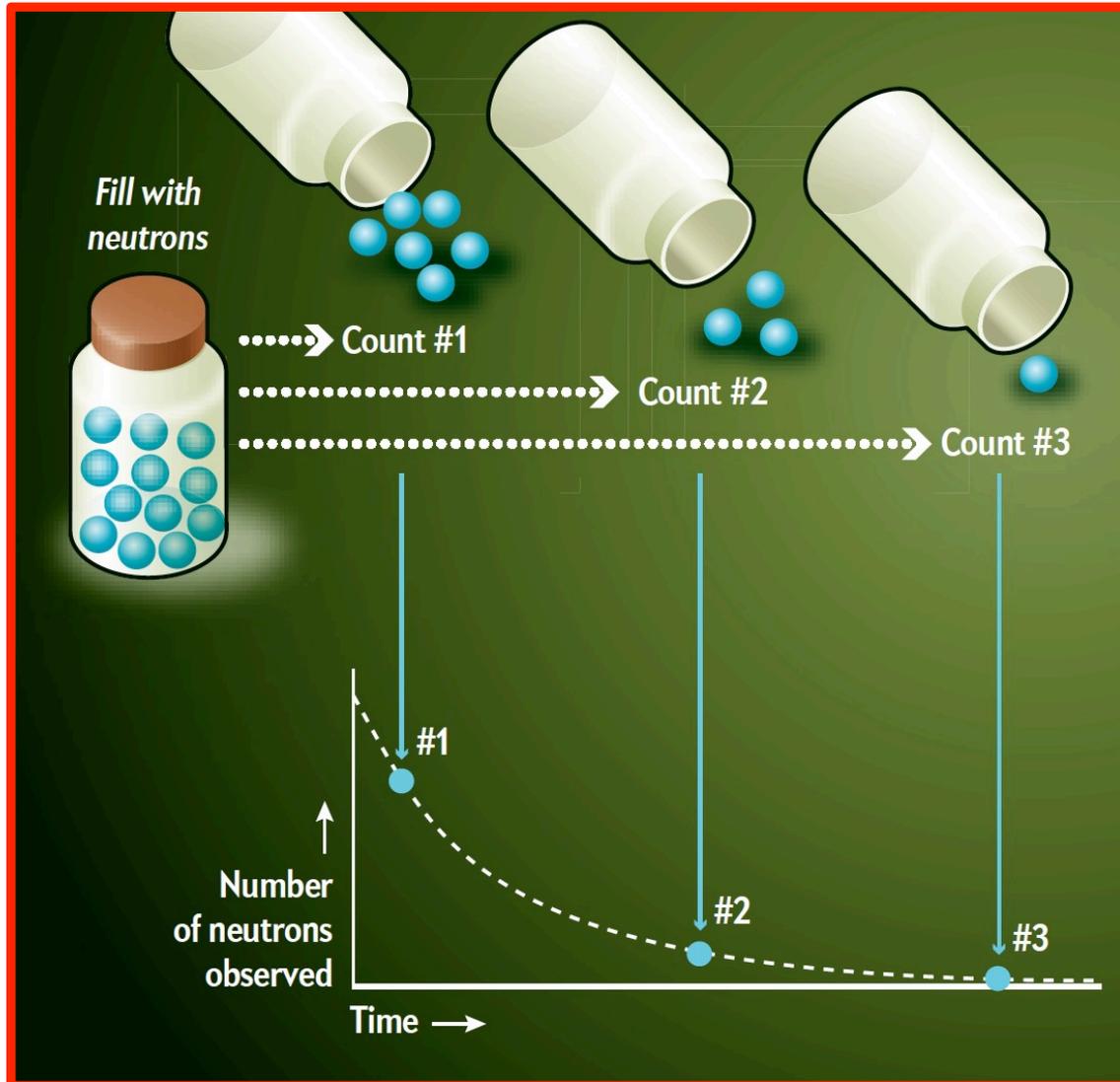
Cold Neutron Beam



Ultracold Neutron (UCN) Bottle



Bottle Experiment



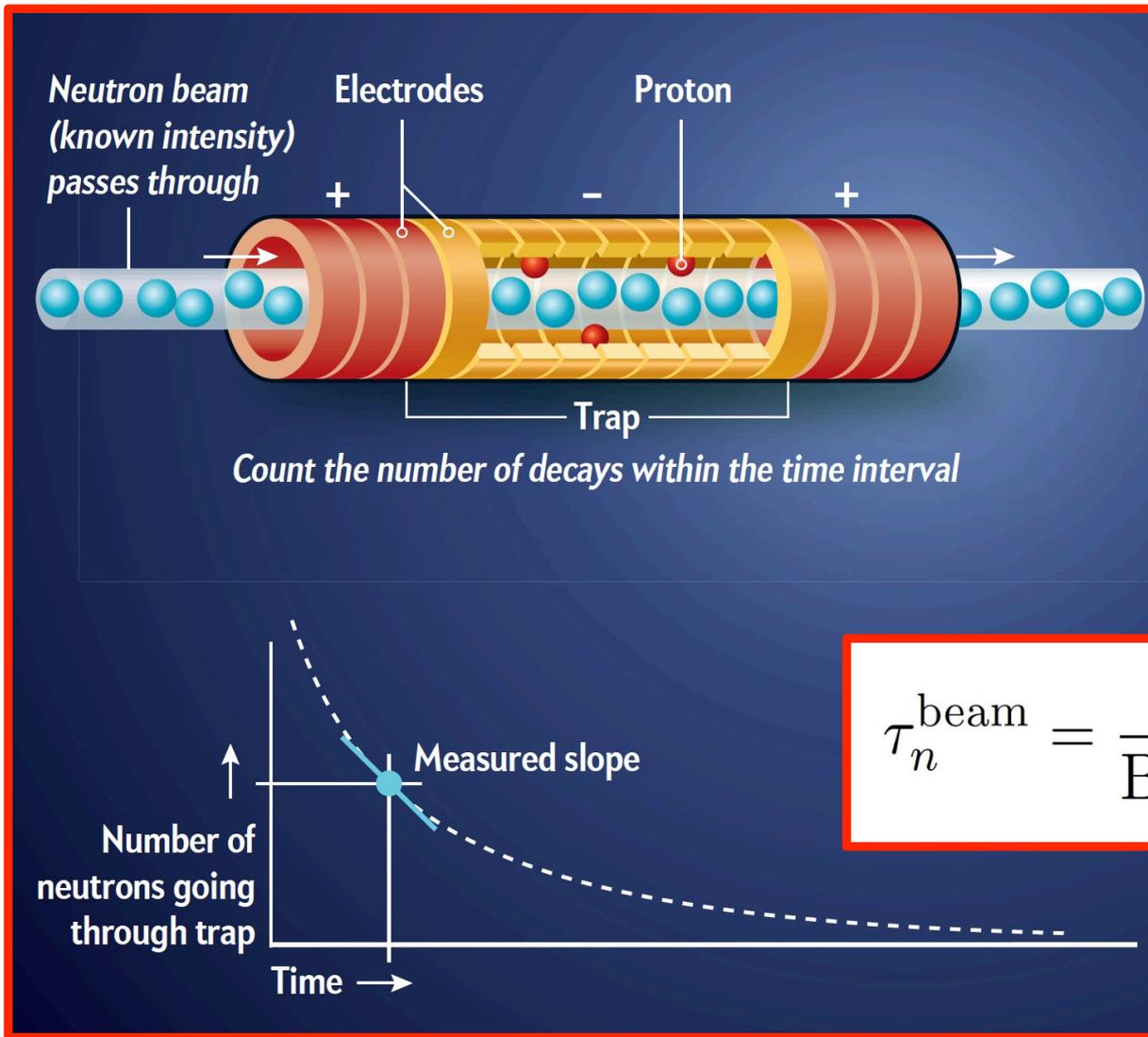
- Data points fit to an exponential decay

$$N = N_0 e^{-\lambda t}$$

- Lifetime

$$\tau = \frac{1}{\lambda}$$

Beam experiment

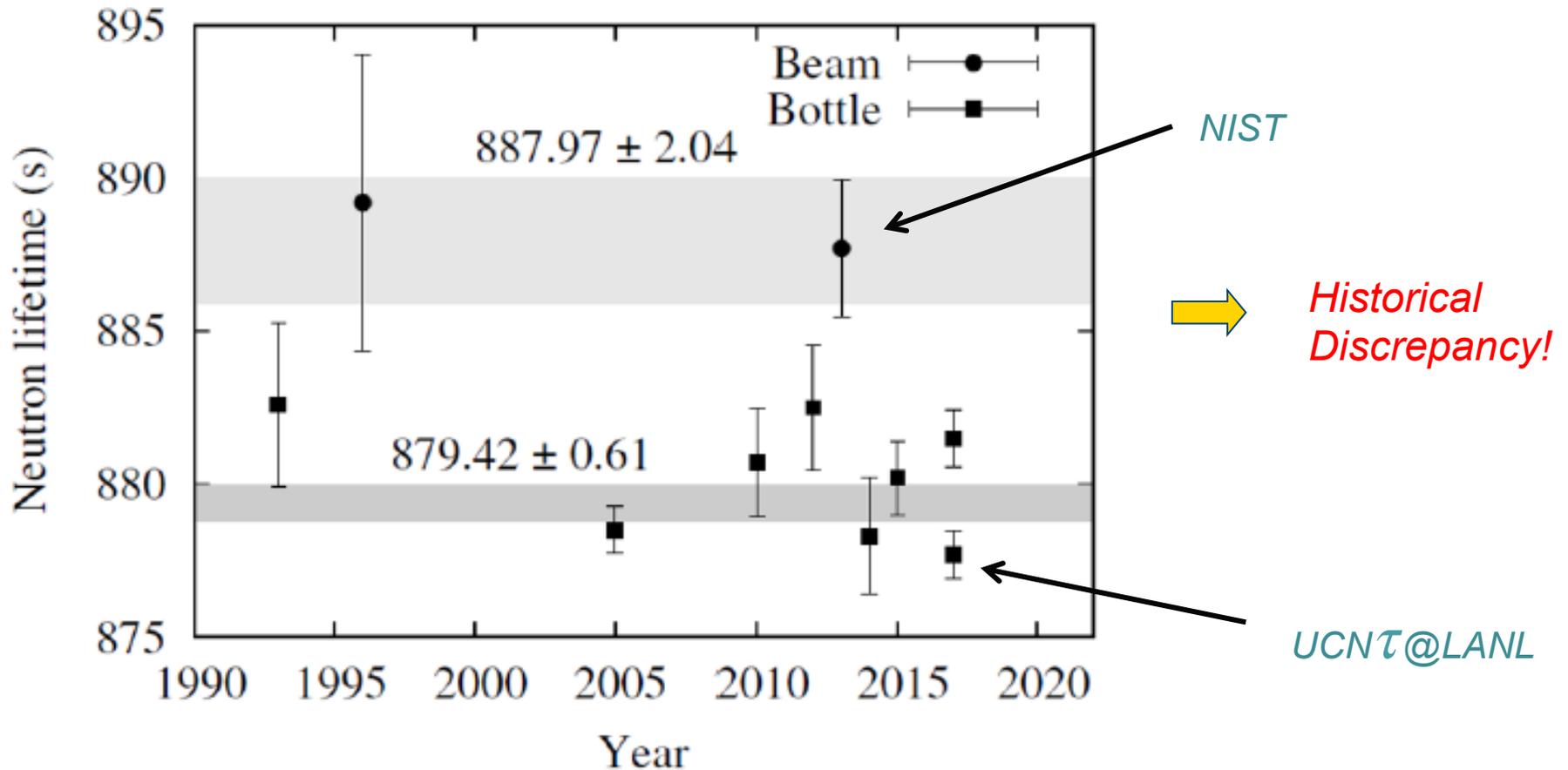


Only the decay rate to protons is measured

$$\frac{dN}{dt} = -\lambda N$$

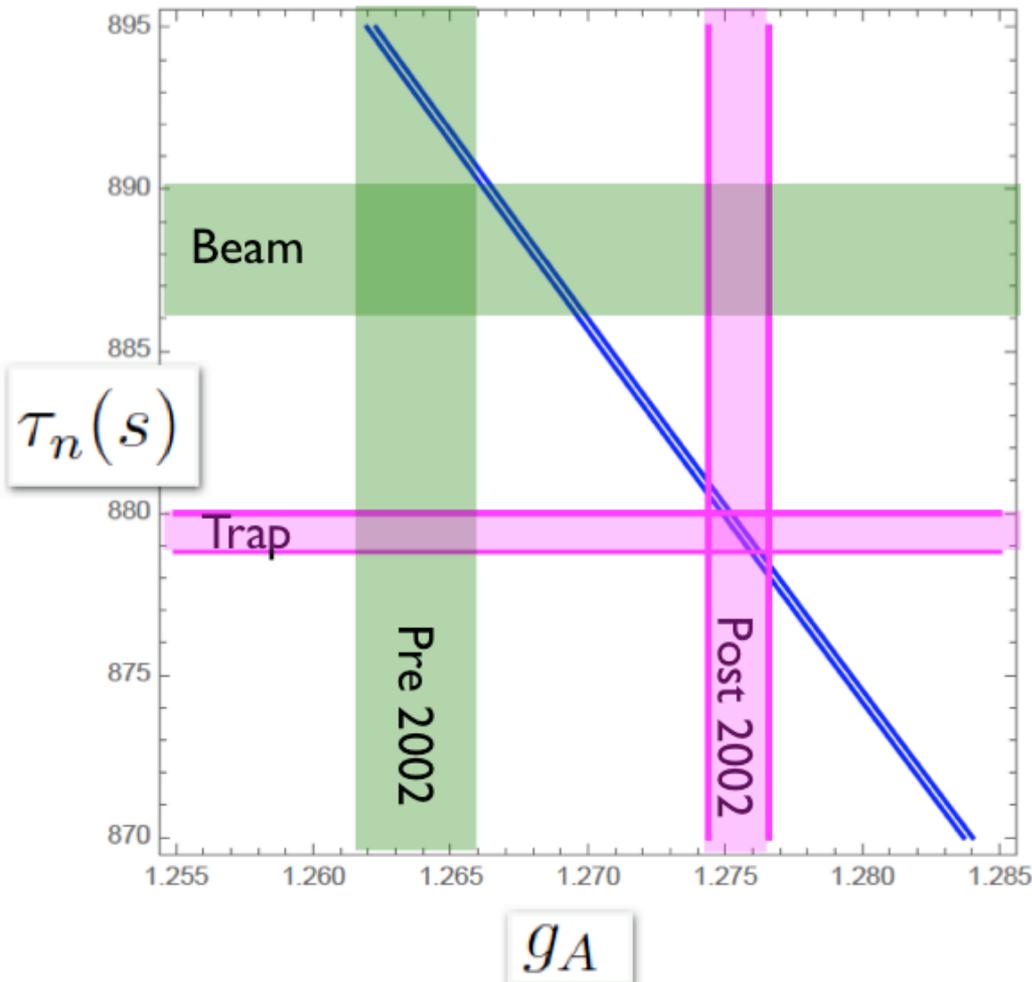
$$\tau_n^{\text{beam}} = \frac{\tau_n}{\text{Br}(n \rightarrow p + \text{anything})}$$

Neutron lifetime measurements : results



Neutron lifetime measurements : Interpretation

Czarnecki, Marciano, Sirlin'18



$$\bar{V}_{ud} = \left[\frac{4908.6(1.9) s}{\tau_n (1 + 3\bar{g}_A^2)} \right]^{1/2}$$

- Input V_{ud} from $0^+ \rightarrow 0^+$:
Blue line
- UCN lifetime and post-2002 g_A consistent with SM

Neutron lifetime measurements : Interpretation

$$\text{Br}(n \rightarrow p + \text{anything}) \approx 99\%$$

Fornal & Grinstein'18

- Remaining 1% :

$n \rightarrow \text{SM particles (other than } p)$



$n \rightarrow \text{dark particle(s) + SM particle(s)}$



$n \rightarrow \text{dark particles}$



Neutron lifetime measurements : Interpretation

$$\text{Br}(n \rightarrow p + \text{anything}) \approx 99\%$$

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- Remaining 1% :

$n \rightarrow \text{SM particles (other than } p)$



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$n \rightarrow \text{dark particles}$



Problem with neutron stars

Baym, Beck, Shelton, Geltenbort'18

McKeen, Nelson, Reddy, Zhou'18

Motta, Guichon, & A.W. Thomas'18

V_{ud} from neutron decay : prospects to reach 0.02%

Czarnecki, Marciano,
Sirlin'18
Cirigliano@Amherst'18

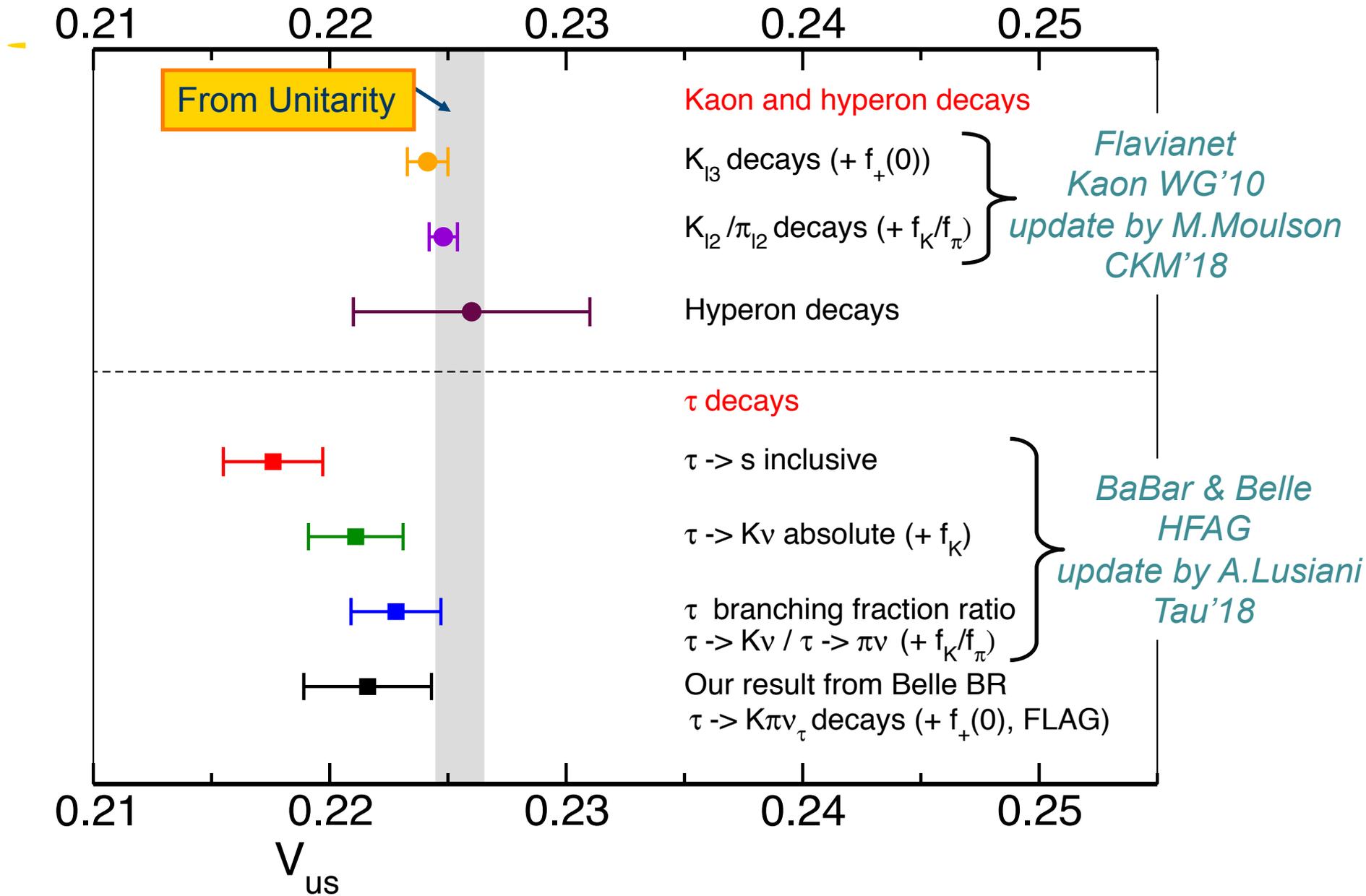
$$\bar{V}_{ud} = \left[\frac{4908.6(1.9) \text{ s}}{\tau_n (1 + 3\bar{g}_A^2)} \right]^{1/2}$$

$$\begin{aligned} \delta\tau_n &\sim 0.35 \text{ s} \\ \delta\tau_n/\tau_n &\sim 0.04 \% \end{aligned}$$

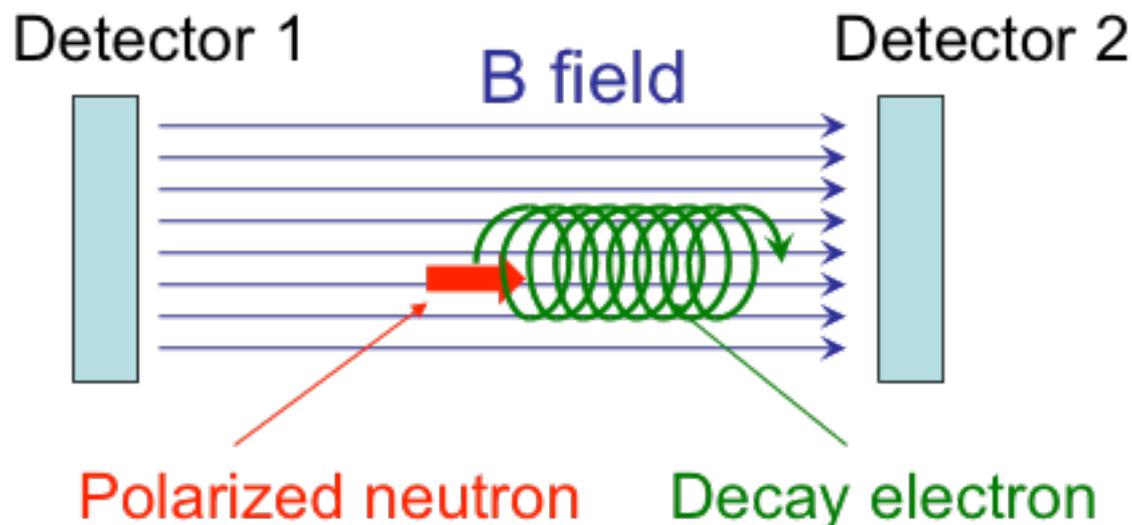
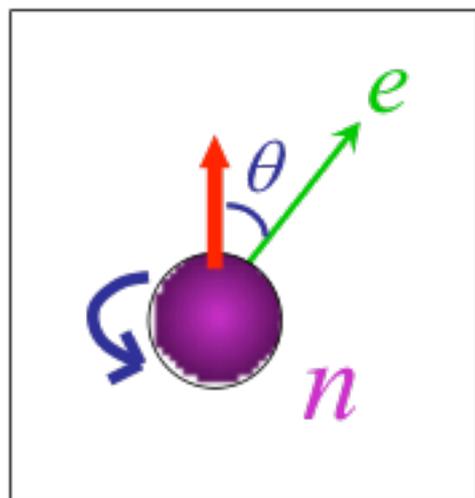
$$\begin{aligned} \delta g_A/g_A &\sim 0.15\% \rightarrow 0.03\% \\ (\delta a/a, \delta A/A &\sim 0.14\%) \end{aligned}$$

UCNT @ LANL [$\tau_n \sim 877.7(7)(3)\text{s}$]
is almost there, will reach $\delta\tau_n \sim 0.2 \text{ s}$
1707.01817

$\delta A/A < 0.2\%$ can be reached
by PERC, UCNA+
 $\delta a/a \sim 0.1\%$ at Nab



Principle of the A -coefficient Measurement (and B and C as well)

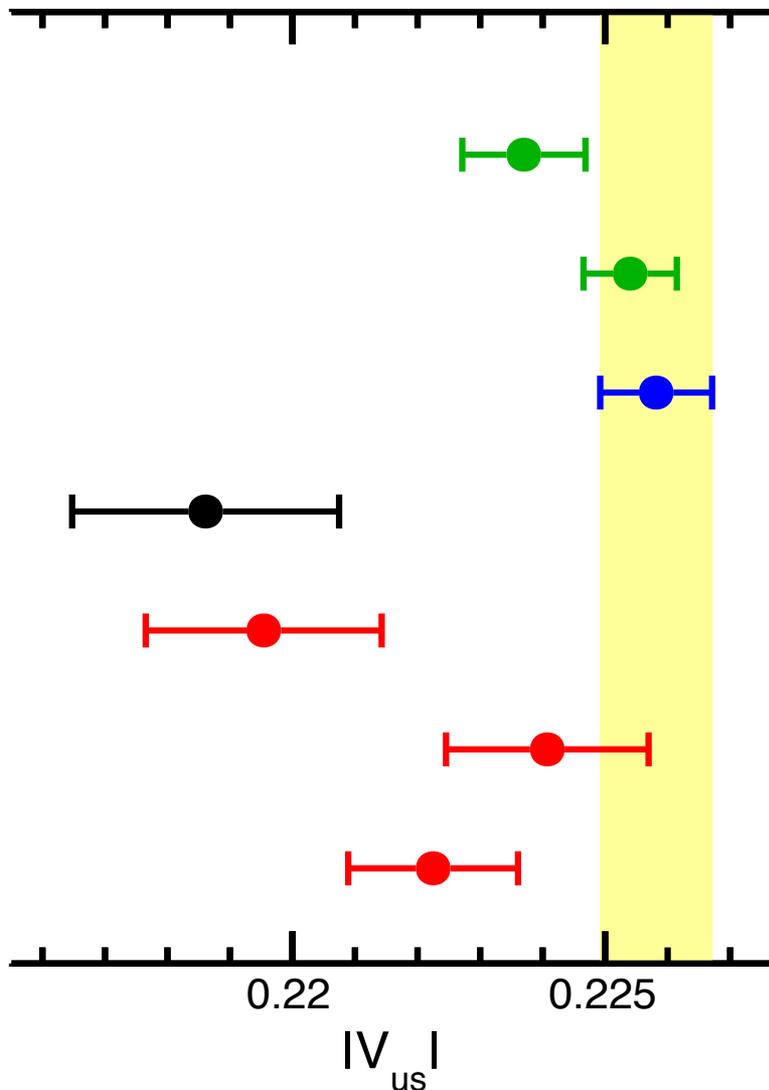


$$dW = [1 + \beta P A \cos \theta] d\Gamma(E)$$

$$A_{\text{exp}}(E) = \frac{N_1(E) - N_2(E)}{N_1(E) + N_2(E)} \approx \langle P \rangle A \beta \langle \cos \theta \rangle$$

(End point energy = 782 keV)

Determination of V_{us}



K_{l3} , PDG 2016
 0.2237 ± 0.0010

K_{l2} , PDG 2016
 0.2254 ± 0.0007

CKM unitarity, PDG 2016
 0.2258 ± 0.0009

$\tau \rightarrow s$ incl., HFLAV Spring 2017
 0.2186 ± 0.0021

$\tau \rightarrow s$ incl.
 0.2195 ± 0.0019

$\tau \rightarrow K\nu / \tau \rightarrow \pi\nu$
 0.2241 ± 0.0016

τ average
 0.2222 ± 0.0014

A.L. elab.

CKM 2018

Callan-Treiman Low Energy Theorem

- Callan-Treiman theorem:

Bernard, Oertel, E.P., Stern'06, '08

$$C = \bar{f}_0(\Delta_{K\pi}) = \frac{F_K}{F_\pi f_+(0)} + \Delta_{CT} = \underbrace{\frac{F_K |V^{us}|}{F_\pi |V^{ud}|} \frac{1}{f_+(0) |V^{us}|} |V^{ud}|}_r + \Delta_{CT}$$

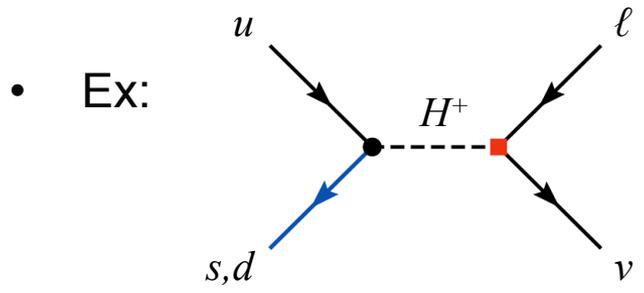
Very precisely known from $\text{Br}(Kl2/\pi l2)$, $\Gamma(\text{Ke}3)$ and $|V_{ud}|$

$$B_{\text{exp}} = 1.2446(41)$$

- In the Standard Model : $r = 1$ $(\ln C_{SM} = 0.2141(73))$ $\Delta_{CT} = (-3.5 \pm 8) \cdot 10^{-3}$

NLO value + large error bars in agreement with *Bijnens&Ghorbani'07* *Kastner & Neufeld'08*

- In presence of new physics, new couplings : $r \neq 1$



4.2 Looking for New Physics with K_{12} and K_{13}

- Callan-Treiman theorem:

Bernard, Oertel, E.P., Stern'06, '08

$$C = \frac{\bar{f}_0(\Delta_{K\pi})}{m_K^2 - m_\pi^2} = \frac{F_K}{F_\pi f_+(0)} + \Delta_{CT} = \underbrace{\frac{F_K |V^{us}|}{F_\pi |V^{ud}|} \frac{1}{f_+(0) |V^{us}|} |V^{ud}|}_B + \Delta_{CT}$$

Very precisely known from $\text{Br}(K_{12}/\pi_{12})$, $\Gamma(\text{Ke}3)$ and $|V_{ud}|$

$$B_{\text{exp}} = 1.2446(41)$$

- In the Standard Model : $r = 1$ ($\ln C_{SM} = 0.2141(73)$) $\Delta_{CT} = (-3.5 \pm 8) \cdot 10^{-3}$
- In presence of new physics, new couplings : $r \neq 1$

NLO value + large error bars in agreement with

*Bijnens&Ghorbani'07
Kastner & Neufeld'08*

Experiment $K_{e3}+K_{\mu 3}$	$\ln C$
NA48'07 ($K_{\mu 3}$ alone)	0.144(14)
KLOE'08	0.204(25)
KTeV'10	0.192(12)
NA48 (preliminary)	?

V_{us} and CKM unitarity: All data, $N_f=2+1$

$N_f = 2+1$: Fit to results for $|V_{ud}|$, $|V_{us}|$, $|V_{us}|/|V_{ud}|$
 $f_+(0) = 0.9677(27)$, $f_K/f_\pi = 1.1946(34)$



$$|V_{ud}| = 0.97420(21)$$

$$|V_{us}| = 0.2238(8)$$

$$|V_{us}|/|V_{ud}| = 0.2317(7)$$

Fit results, no constraint

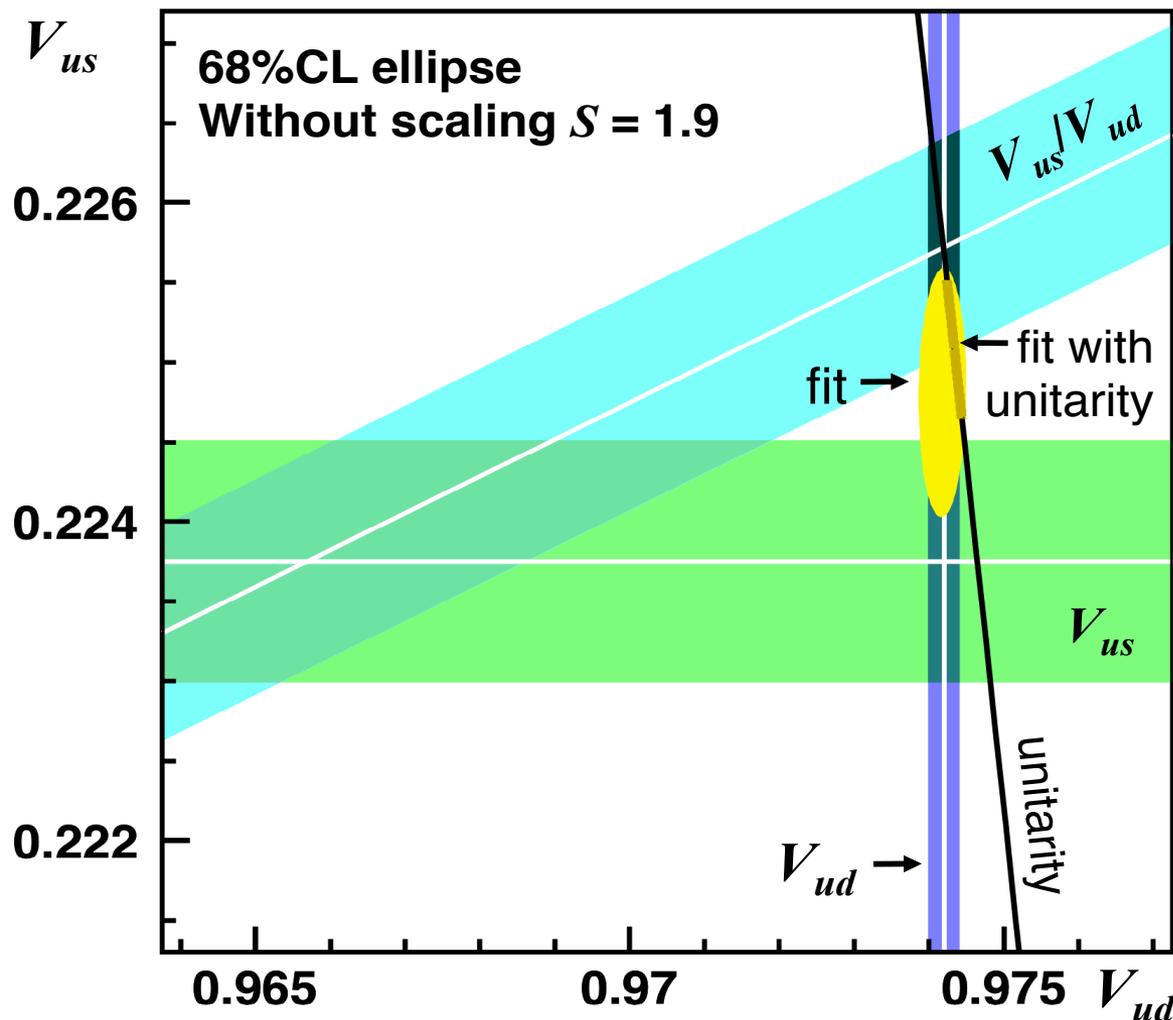
$$V_{ud} = 0.97418(21)$$

$$V_{us} = 0.2248(5)$$

$$\chi^2/\text{ndf} = 3.7/1 \text{ (6\%)}$$

$$\Delta_{\text{CKM}} = -0.0004(5)$$

$$-0.9\sigma$$



5.5 V_{us} and CKM unitarity: All data, $N_f=2+1$

$N_f = 2+1$: Fit to results for $|V_{ud}|$, $|V_{us}|$, $|V_{us}|/|V_{ud}|$
 $f_+(0) = 0.9677(27)$, $f_K/f_\pi = 1.1946(34)$



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$$V_{ud} = 0.97418(21)$$

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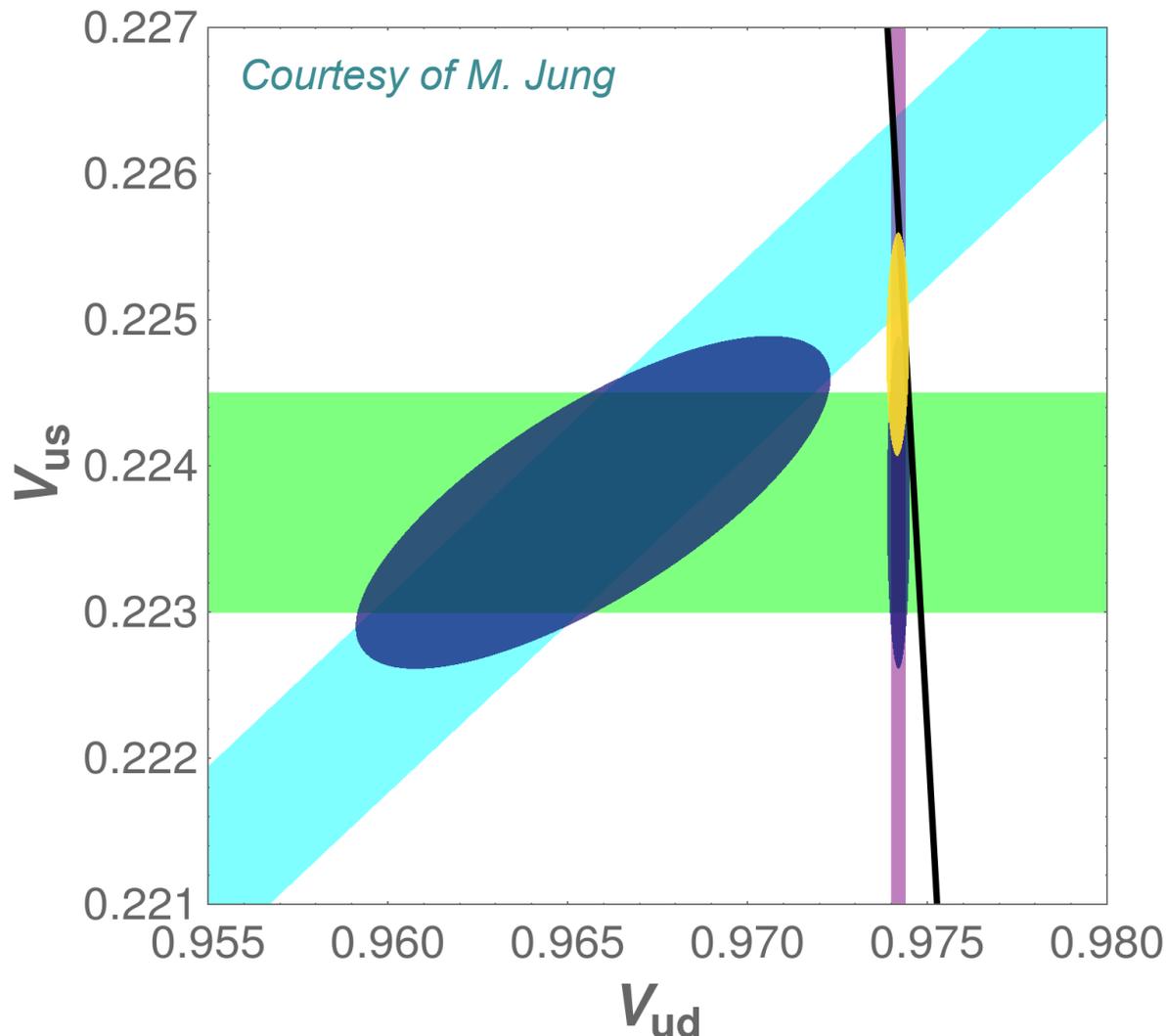
$$\Delta_{\text{CKM}} = -0.0004(5)$$

$$-0.9\sigma$$

Only kaons:

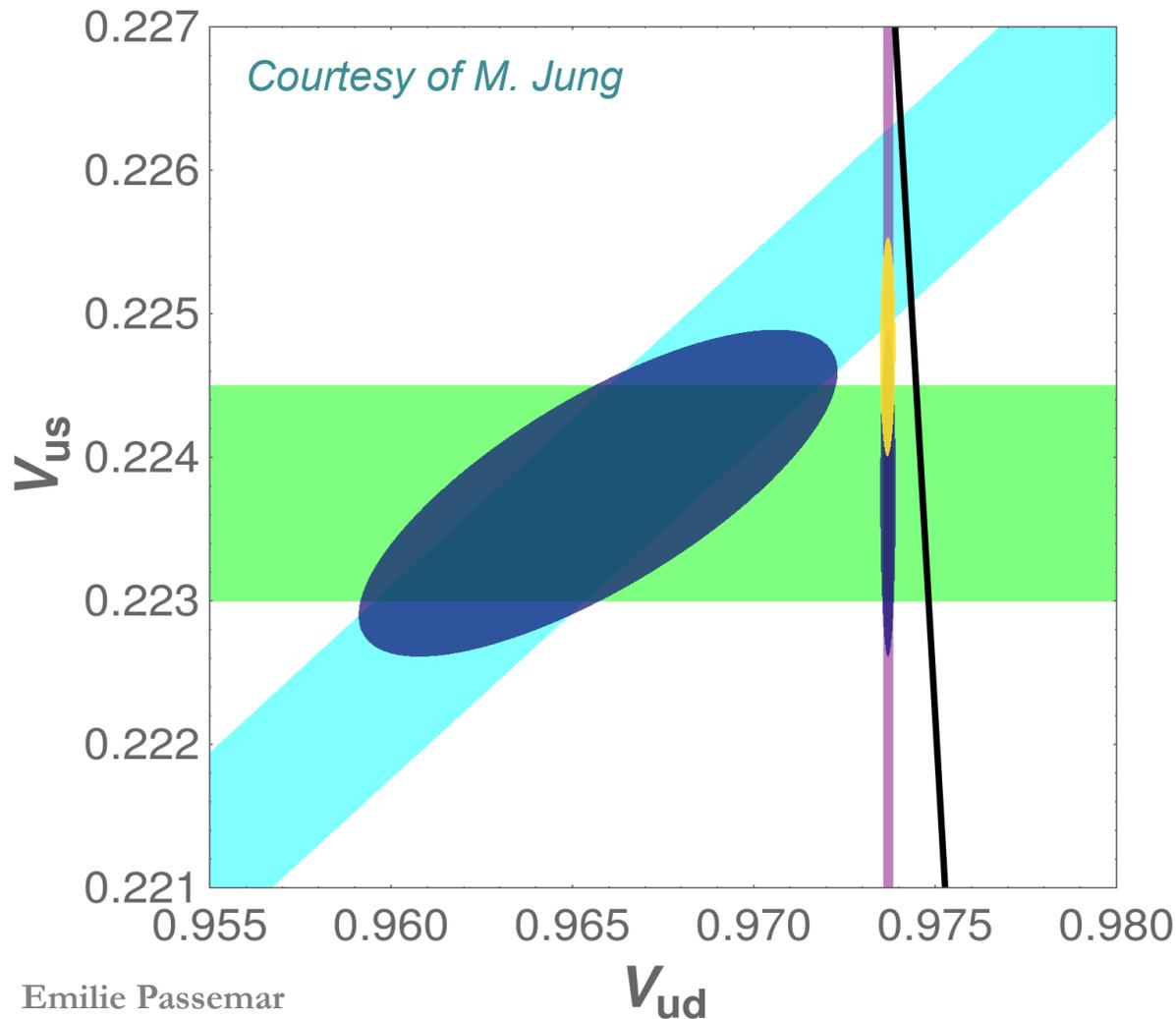
$$V_{ud} = 0.9657(44)$$

$$V_{us} = 0.2238(8)$$



V_{us} and CKM unitarity: All data, $N_f=2+1$, New V_{ud}

$N_f = 2+1$: Fit to results for $|V_{ud}|$, $|V_{us}|$, $|V_{us}|/|V_{ud}|$
 $f_+(0) = 0.9677(27)$, $f_K/f_\pi = 1.1946(34)$



$$\Delta_{CKM} = -0.0014(4)$$

-3.9σ

V_{us} and CKM unitarity: All data, $N_f=2+1+1$

$N_f = 2+1+1$: Fit to results for $|V_{ud}|$, $|V_{us}|$, $|V_{us}|/|V_{ud}|$
 $f_+(0) = 0.9698(17)$, $f_K/f_\pi = 1.1967(18)$



$$|V_{ud}| = 0.97420(21)$$

$$|V_{us}| = 0.2233(6)$$

$$|V_{us}|/|V_{ud}| = 0.2313(5)$$

Fit results, no constraint

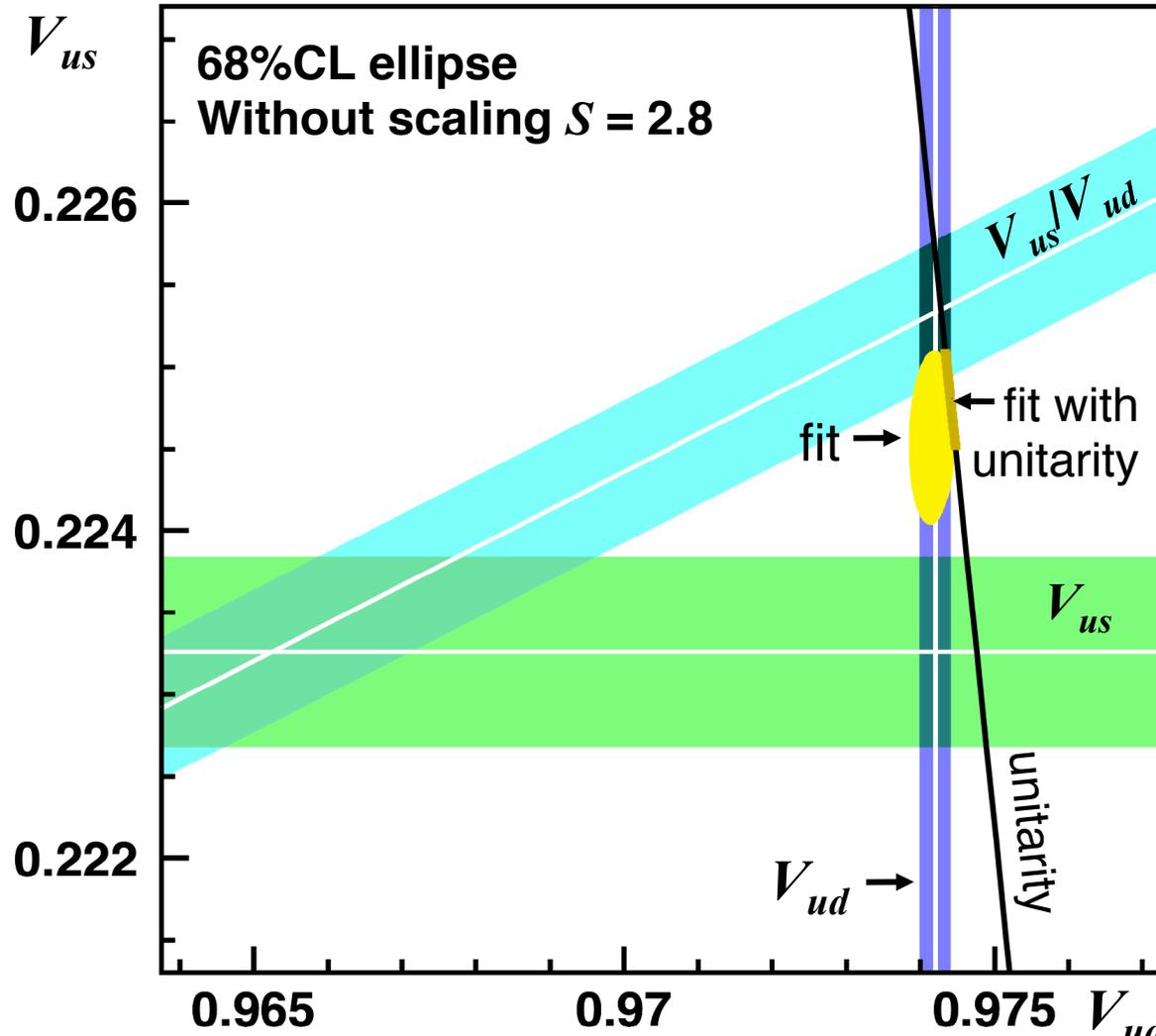
$$V_{ud} = 0.97416(21)$$

$$V_{us} = 0.22457(35)$$

$$\chi^2/\text{ndf} = 8.0/1 \text{ (0.5\%)}$$

$$\Delta_{\text{CKM}} = -0.0005(4)$$

$$-1.3\sigma$$



V_{us} and CKM unitarity: All data, $N_f=2+1+1$

$N_f = 2+1+1$: Fit to results for $|V_{ud}|$, $|V_{us}|$, $|V_{us}|/|V_{ud}|$
 $f_+(0) = 0.9698(17)$, $f_K/f_\pi = 1.1967(18)$



$$|V_{ud}| = 0.97420(21)$$

$$|V_{us}| = 0.2233(6)$$

$$|V_{us}|/|V_{ud}| = 0.2313(5)$$

Fit results, no constraint

$$V_{ud} = 0.97416(21)$$

$$V_{us} = 0.22457(35)$$

$$\chi^2/\text{ndf} = 8.0/1 \text{ (0.5\%)}$$

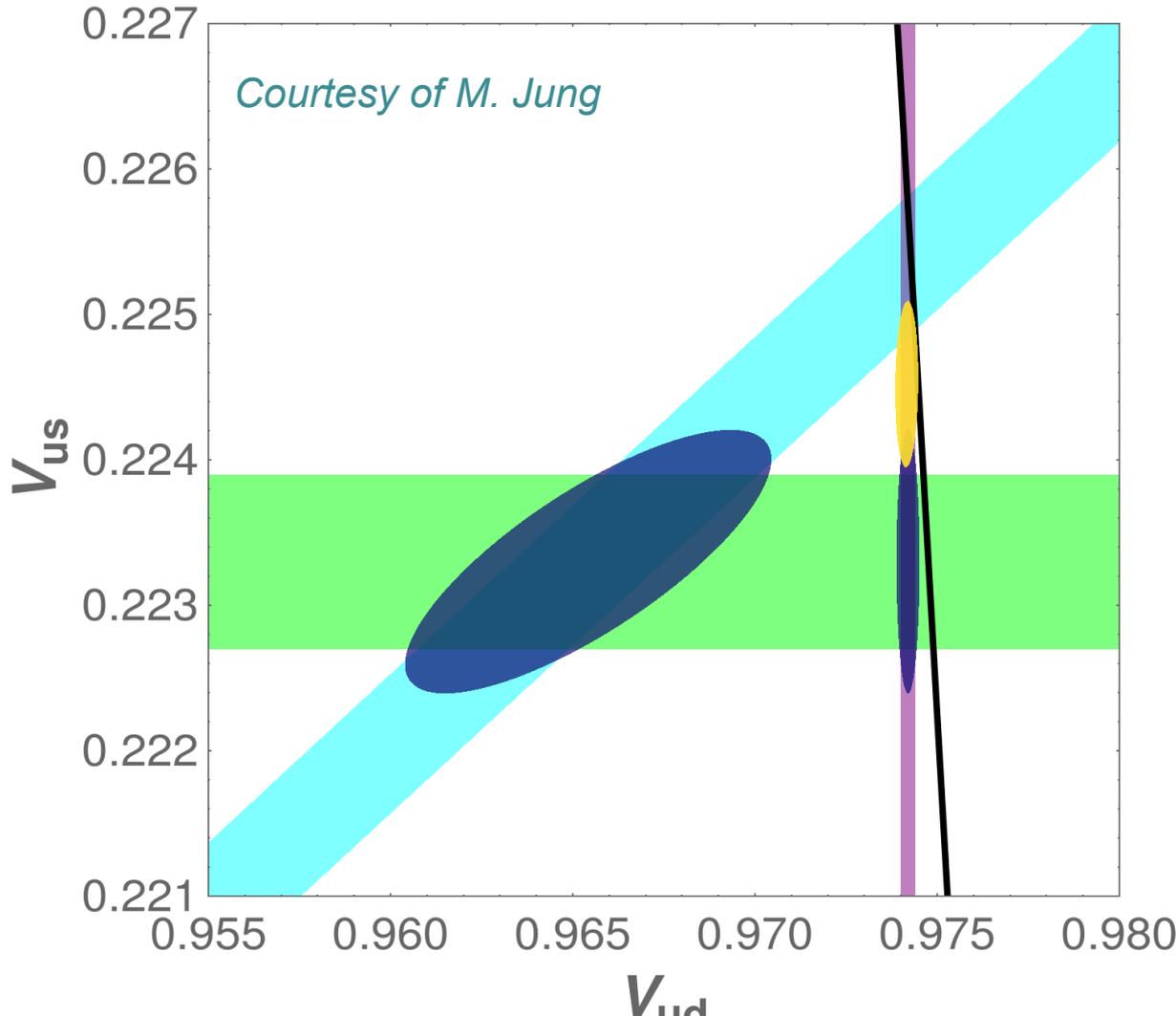
$$\Delta_{\text{CKM}} = -0.0005(4)$$

$$-1.3\sigma$$

Only kaons:

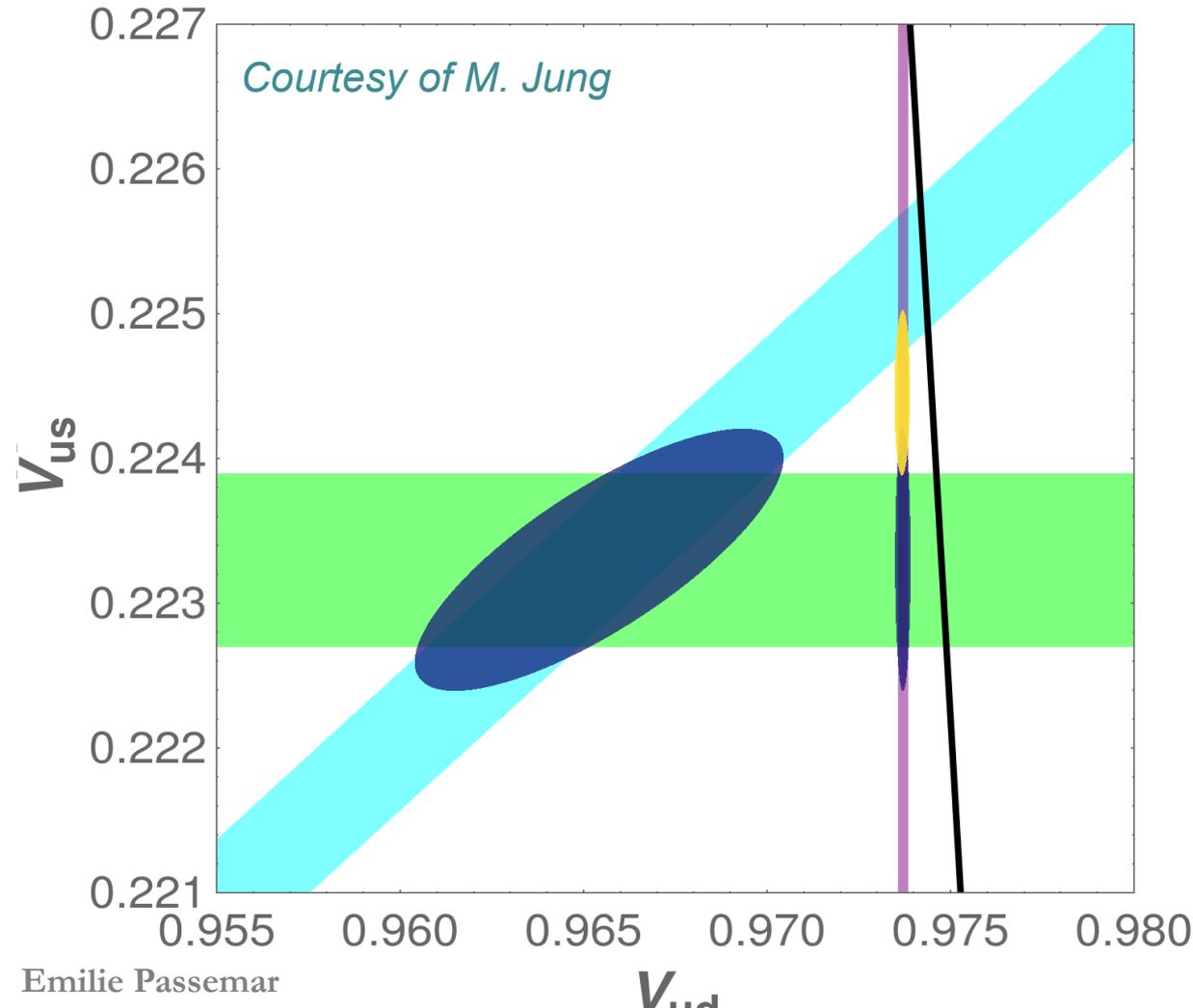
$$V_{ud} = 0.9657(36)$$

$$V_{us} = 0.2233(6)$$



V_{us} and CKM unitarity: All data, $N_f=2+1+1$, New V_{ud}

$N_f = 2+1+1$: Fit to results for $|V_{ud}|$, $|V_{us}|$, $|V_{us}|/|V_{ud}|$
 $f_+(0) = 0.9698(17)$, $f_K/f_\pi = 1.1967(18)$

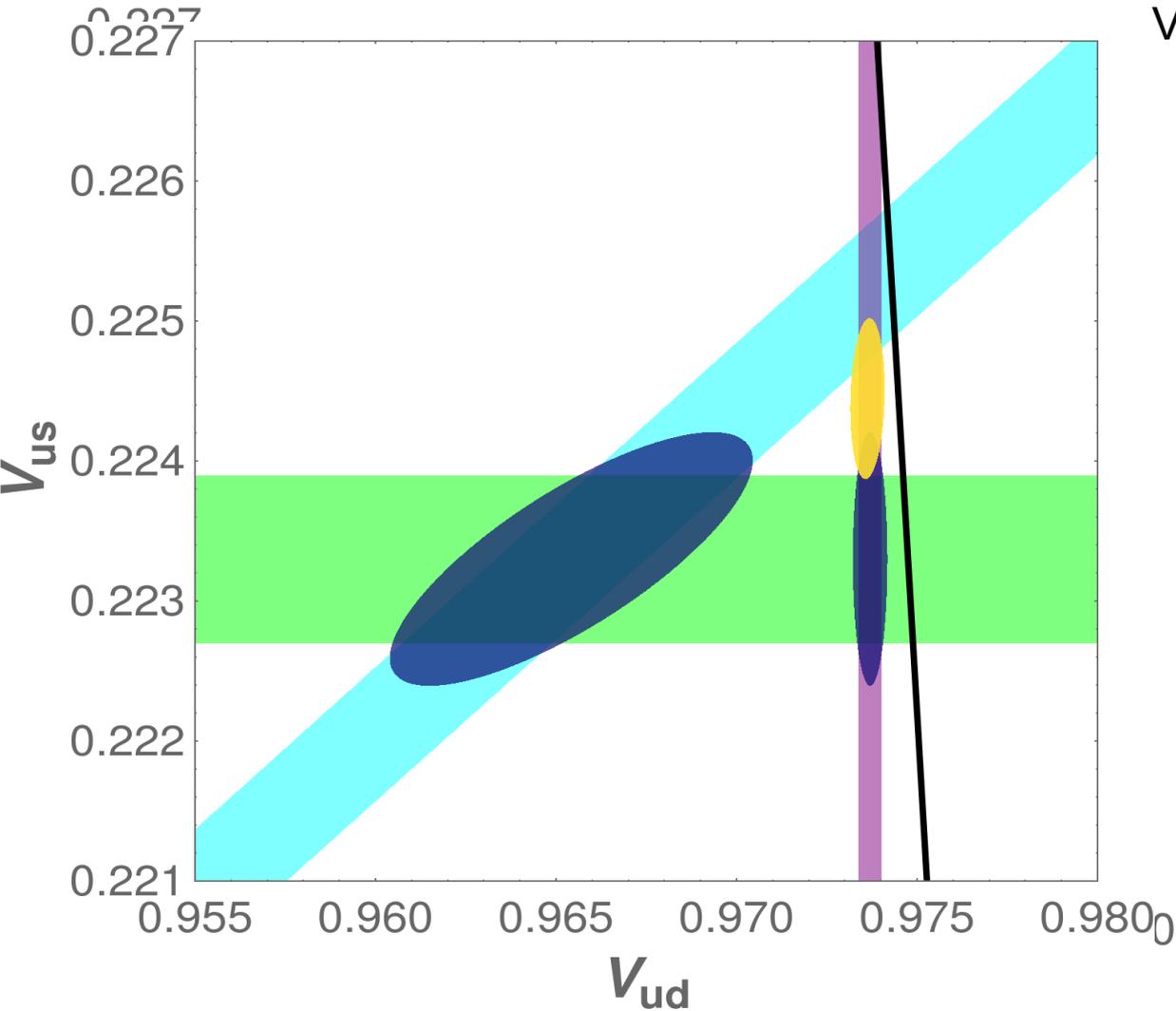


$$\Delta_{CKM} = -0.0016(3)$$

-4.3σ

V_{us} and CKM unitarity: All data, $N_f=2+1+1$

$N_f = 2+1+1$: Fit to results for $|V_{ud}|$, $|V_{us}|$, $|V_{us}|/|V_{ud}|$
 $f_+(0) = 0.9698(17)$, $f_K/f_\pi = 1.1967(18)$



V_{ud} from Marciano & Sirlin'19

Test of the Standard Model: V_{us} and CKM unitarity

- Extraction of the Cabibbo-Kobayashi-Maskawa matrix element V_{us}
 - Fundamental parameter of the Standard Model

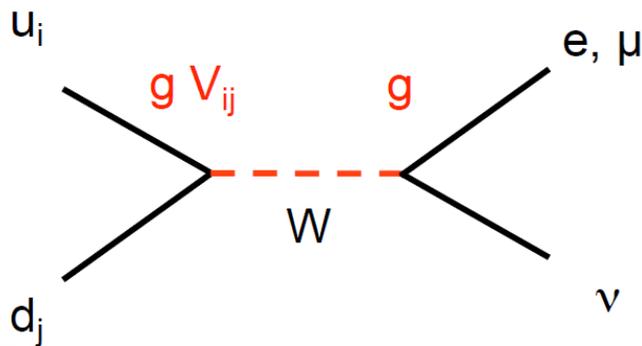
Description of the **weak interactions**:

$$\mathcal{L}_{EW} = \frac{g}{\sqrt{2}} W_{\alpha}^{+} \left(\bar{D}_L V_{CKM} \gamma^{\alpha} U_L + \bar{e}_L \gamma^{\alpha} \nu_{e_L} + \bar{\mu}_L \gamma^{\alpha} \nu_{\mu_L} + \bar{\tau}_L \gamma^{\alpha} \nu_{\tau_L} \right) + \text{h.c.}$$

Gauge coupling

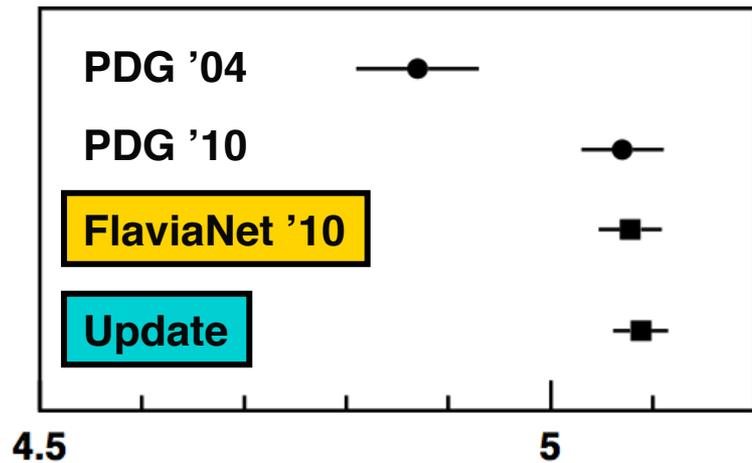
- Universality: Is G_F from μ decay equals to G_F from π , K, nuclear β decay?

$$G_{\mu}^2 = (g_{\mu} g_e)^2 / M_W^4 \stackrel{?}{=} G_{CKM}^2 = (g_q g_l)^2 (|V_{ud}|^2 + |V_{us}|^2) / M_W^4$$

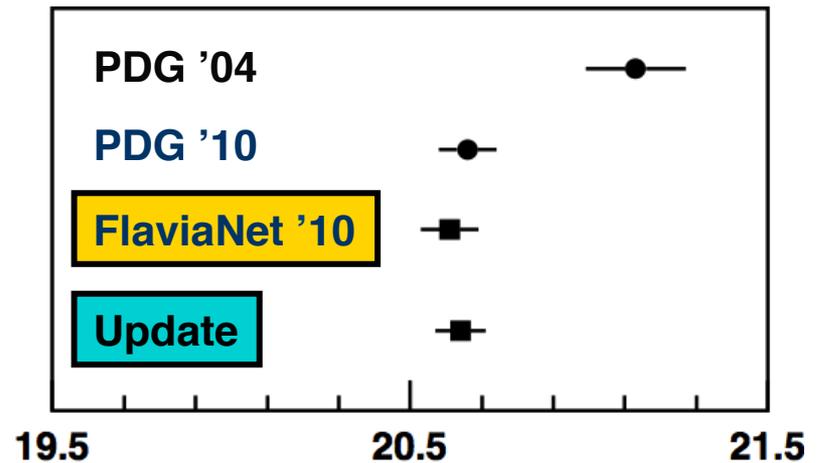


Updates: K^\pm BRs and lifetimes

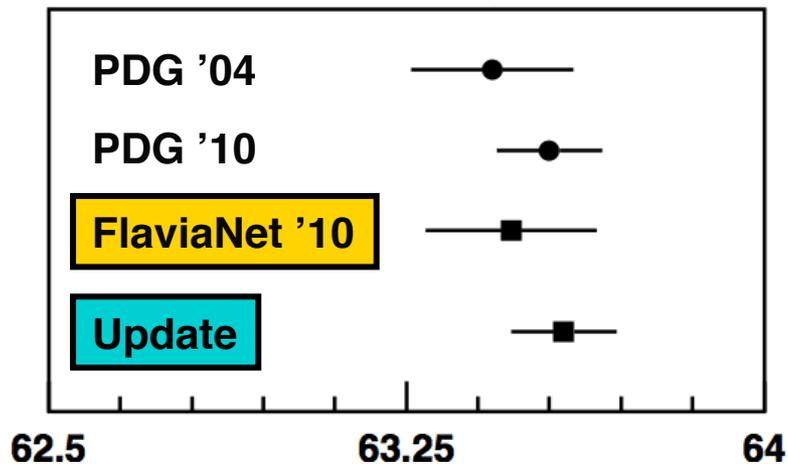
$BR(K^\pm \rightarrow \pi^0 e \nu)$



$BR(K^\pm \rightarrow \pi \pi^0)$



$BR(K^\pm \rightarrow \mu \nu)$



$BR(K^\pm \rightarrow \pi \pi \pi)$

