# From B-anomalies to Kaon physics 



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## Outline

- Introduction on B-physics anomalies and EFT interpretations
- Implications of $\mathrm{R}\left(\mathrm{D}^{(*)}\right)$ : $\mathrm{U}(2)^{5}$ flavor symmetry \& $\mathrm{K} \rightarrow \pi \nu \vee$
- Implications of $\mathrm{R}\left(\mathrm{K}^{(*)}\right)$ :

1. Rank-One Flavour Violation (ROFV) assumption
2. Constraints from $K_{L, S} \rightarrow \mu \mu$ and $K_{L} \rightarrow \pi^{0} \mu \mu$

- Summary


## Charged-current anomalies


$b \rightarrow c \tau v$ vs. $b \rightarrow c \ell v$

Tree-level SM process with $\mathrm{V}_{\text {cb }}$ suppression.

$$
\begin{array}{r}
R\left(D^{(*)}\right) \equiv \frac{\mathcal{B}\left(B^{0} \rightarrow D^{(*)+} \tau \nu\right)}{\mathcal{B}\left(B^{0} \rightarrow D^{(*)+\ell \nu)}\right.} \\
\ell=\mu, e
\end{array}
$$

All measurements since 2012 consistently above the SM predictions



## Charged-current anomalies


$b \rightarrow c \tau v$ vs. $b \rightarrow c \ell v$

Tree-level SM process with $\mathrm{V}_{\mathrm{cb}}$ suppression.

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R\left(D^{(*)}\right) \equiv \frac{\mathcal{B}\left(B^{0} \rightarrow D^{(*)+} \tau \nu\right)}{\mathcal{B}\left(B^{0} \rightarrow D^{(*)+} \ell \nu\right)}, \\
\quad \ell=\mu, e
\end{array}
$$

Assuming $R(D)=R\left(D^{*}\right): \quad R\left(D^{(*)}\right) / R\left(D^{(*)}\right)_{\mathrm{SM}}=1.142 \pm 0.038$

~14\% enhancement from the SM
~3 $\mathbf{\sigma}$ from the SM (3.7 $\sigma$ when combined)

While $\mu$ /e universality well tested

$$
R(D)^{\mu / e}=0.995 \pm 0.045
$$

Belle - [1510.03657]

## Neutral-Current B-anomalies

 $b \rightarrow \boldsymbol{s} \mu^{+} \mu^{-}$
## Lepton Flavor Universality ratios

$$
R\left(K^{(*)}\right)=\frac{\mathcal{B}\left(B \rightarrow K^{(*)} \mu^{+} \mu^{-}\right)}{\mathcal{B}\left(B \rightarrow K^{(*)} e^{+} e^{-}\right)}
$$

Clean SM prediction: $1 \pm O(1 \%)$
Bordone, Isidori, Pattori 2016

## Angular distributions

$$
B \rightarrow \boldsymbol{K}^{*}(\rightarrow \boldsymbol{K} \pi) \boldsymbol{\mu}^{+} \boldsymbol{\mu}^{-}
$$




Differential branching fractions in $q_{\mu \mu^{2}}{ }^{2}$ in several channels.


## Low-energy interpretations

$b \longrightarrow \mathcal{c} \tau v$
$\mathcal{L}_{\mathrm{BSM}}=\frac{2 c}{\Lambda^{2}}\left(\bar{c}_{L} \gamma_{\mu} b_{L}\right)\left(\bar{\tau}_{L} \gamma^{\mu} \nu_{\tau}\right)+$ h.c.
if $c=1 \quad \rightarrow \quad \Lambda_{\mathbf{R}(\mathrm{D})} \sim 4.5 \mathrm{TeV}$
Freytsis et al. 2015, Angelescu et al. 1808.08179, Shi et al. 1905.08498,
Murgui et al. 1904.09311, Bardhan, Ghosh 1904.10432, ..
$\boldsymbol{b} \rightarrow \boldsymbol{s} \boldsymbol{\mu}^{+} \boldsymbol{\mu}^{-}$
$\mathcal{L}_{\text {eff }} \supset \frac{e^{i \alpha_{b s}}}{\Lambda_{b s}^{2}}\left(\bar{s}_{L} \gamma^{\mu} b_{L}\right)\left(\bar{\mu}_{L} \gamma_{\mu} \mu_{L}\right)+$ h.c.
(if $\alpha_{b s}=0$ ) $\quad \Lambda_{R(K)} \sim 34 \mathrm{TeV}$
D’Amico et al. 1704.05438, Algueró et al. 1903.09578, Alok et al.
1903.09617, Ciuchini et al. 1903.09632, Aebischer et al 1903.10434,

$$
\begin{aligned}
\frac{e^{i \alpha_{b s}}}{\Lambda_{b s}^{2}} & =\frac{G_{F} \alpha}{\sqrt{2} \pi} V_{t b} V_{t s}^{*}\left(\Delta C_{9}^{\mu}-\Delta C_{10}^{\mu}\right) \\
\Lambda_{b s}^{\mathrm{SM}} & \approx 12 \mathrm{TeV}
\end{aligned}
$$

$\mathcal{H}_{\mathrm{SM}}=\frac{4 G_{F}}{\sqrt{2}} V_{c b}\left(\bar{c}_{L} \gamma_{\mu} b_{L}\right)\left(\bar{\tau}_{L} \gamma^{\mu} \nu_{\tau}\right)+$ h.c.


Takeaway:

1) $\boldsymbol{\Lambda R}_{\mathbf{R}(\mathrm{K})}>\underset{\text { i.e. }}{\gg} \boldsymbol{\Lambda}(\mathrm{D})$

Coupling to $\mu \ll$ Coupling to $\tau$
2) Coupling to LH fields required

## Combined Fit of B anomalies (SMEFT)

Buttazzo, Greljo, Isidori, DM 1706.07808
Adding SM SU(2)L gauge invariance:

$$
\begin{gathered}
\mathcal{L}_{\text {SMEFT }}=\lambda_{i j}^{q} \lambda_{\alpha \beta}^{\ell}\left[C_{T}\left(\bar{Q}_{L}^{i} \gamma_{\mu} \sigma^{a} Q_{L}^{j}\right)\left(\bar{L}_{L}^{\alpha} \gamma^{\mu} \sigma^{a} L_{L}^{\beta}\right)+C_{S}\left(\bar{Q}_{L}^{i} \gamma_{\mu} Q_{L}^{j}\right)\left(\bar{L}_{L}^{\alpha} \gamma^{\mu} L_{L}^{\beta}\right)\right] \\
\text { triplet operator } \\
\text { singlet operator }
\end{gathered}
$$

## Very good fit!

## Flavour Structure:

$\lambda^{9} \sim\left(\begin{array}{ccc}0 & \lambda q_{s d} \lambda_{b s} V_{b s} \\ \lambda q_{s d} & \lambda_{s s} & \lambda_{b s} \\ 2 v_{s s} & \lambda_{b s} & \lambda_{b s}\end{array}\right) \begin{aligned} & \lambda_{b s} \sim O\left(V_{s s}\right) \\ & \lambda_{s s} \sim 0\left(\lambda_{s s}^{*}\right)\end{aligned}$
$\lambda^{l} \sim\left(\begin{array}{ccc}0 & 0 & 0 \\ 0 & \lambda_{\mu \mu} & \lambda_{z_{\mu}} \\ 0 & \lambda_{z \mu} & 1\end{array}\right) \lambda_{\mu \mu} \sim 0\left(\lambda_{v_{\varphi}}^{2}\right)$

These values are compatible with a minimally-broken $\mathbf{S U ( 2 )} \mathbf{q}_{\mathbf{q}} \times \mathbf{S U ( 2 ) \ell}$
flavour symmetry

$$
\boldsymbol{C}_{\boldsymbol{T}} \sim \boldsymbol{C}_{S} \sim(1.7 \mathrm{TeV})^{-2}
$$

$$
\begin{gathered}
\frac{\lambda q_{b s} \approx 3 V_{t s}}{\lambda \ell_{\mu \mu} \sim 10^{-2}} \\
\lambda^{\ell}{ }_{\tau \mu} \sim 10^{-1}
\end{gathered}
$$



Small $C_{T, S}$ to evade EWPT,
Large b-s coupling to fit $\mathrm{R}\left(\mathrm{D}^{(*)}\right)$,
$C_{T} \sim C_{S}$ to evade $B \rightarrow K^{*} \nu v$.

## Combined Fit of B anomalies (SMEFT)

Buttazzo, Greljo, Isidori, DM 1706.07808
Adding SM SU(2)เ gauge invariance:

$$
\mathcal{L}_{\mathrm{SMEFT}}=\lambda_{i j}^{q} \lambda_{\alpha \beta}^{\ell}\left[C_{T}\left(\bar{Q}_{L}^{i} \gamma_{\mu} \sigma^{a} Q_{L}^{j}\right)\left(\bar{L}_{L}^{\alpha} \gamma^{\mu} \sigma^{a} L_{L}^{\beta}\right)+C_{S}\left(\bar{Q}_{L}^{i} \gamma_{\mu} Q_{L}^{j}\right)\left(\bar{L}_{L}^{\alpha} \gamma^{\mu} L_{L}^{\beta}\right)\right]
$$

## Flavour Structure:


$\lambda^{l} \sim\left(\begin{array}{ccc}0 & 0 & 0 \\ 0 & \lambda_{\mu \mu} & \lambda_{z_{\mu}} \\ 0 & \lambda_{r \mu} & 1\end{array}\right) \lambda_{\text {rp }} \sim 0\left(\lambda_{t \varphi}^{2}\right)$

B-anomalies are driven by the 3-3 and 3-2 entries. $\lambda q$ bs

Kaon physics depends instead on the 1-2 entry $\lambda q_{s d}$

1) To correlate $B$ and $K$ physics, a flavor assumption is needed.
2) Given the low scale, explicit UV models are required and affect this EFT picture (e.g. additional RH couplings)

## $\mathrm{U}(2)^{5}$ flavour symmetry

Keeping only the third-generation Yukawa couplings, the SM enjoys an approximate $U(2)^{5}$ flavor symmetry
$U(2)^{5} \equiv U(2)_{q} \times U(2)_{\ell} \times U(2)_{u} \times U(2)_{d} \times U(2)_{e} \quad \psi_{i}=\left(\psi_{1} \psi_{2} \psi_{3}\right)$
Assume this is minimally broken

$$
\begin{aligned}
\Delta Y_{u} & =(\mathbf{2}, \overline{\mathbf{2}}, \mathbf{1}, \mathbf{1}, \mathbf{1}), & \Delta Y_{d} & =(\mathbf{2}, \mathbf{1}, \overline{\mathbf{2}}, \mathbf{1}, \mathbf{1}), \quad \Delta Y_{e}=(\mathbf{1}, \mathbf{1}, \mathbf{1}, \mathbf{2}, \overline{\mathbf{2}}) \\
V_{q} & =(\mathbf{2}, \mathbf{1}, \mathbf{1}, \mathbf{1}, \mathbf{1}), & V_{l} & =(\mathbf{1}, \mathbf{1}, \mathbf{1}, \mathbf{2}, \mathbf{1})
\end{aligned}
$$

by the spurions:

The Yukawa matrices get this structure:

$$
y_{u} \sim y_{t}\left(\begin{array}{cc}
\Delta Y_{u} & V_{q} \\
0 & 1
\end{array}\right), \quad y_{d} \sim y_{b}\left(\begin{array}{cc}
\Delta Y_{d} & V_{q} \\
0 & 1
\end{array}\right), \quad y_{e} \sim y_{\tau}\left(\begin{array}{cc}
\Delta Y_{e} & V_{l} \\
0 & 1
\end{array}\right)
$$

The doublet spurions regulate the mixing of the third generation with the lighter ones:
Quark flavor matrix:

In the down-quark mass basis:

$$
V_{q}=a_{q}\binom{V_{t d}^{*}}{V_{t s}^{*}}
$$

Directly related to CKM
See e.g. [1909.02519]

$$
\lambda^{q} \sim\left(\begin{array}{c:c}
V_{q} V_{q}^{+} & V_{q} \\
\hdashline V_{q} & 1
\end{array}\right)
$$

$$
V_{q^{\alpha}}\binom{V_{t d}^{+}}{V_{t s}^{t}}
$$

$$
\begin{aligned}
& \lambda_{s d}^{q} \sim V_{t s}^{*} V_{t d} \\
& \lambda_{b s}^{q} \sim V_{t s}
\end{aligned}
$$

b-s and s-d are correlated!

All is up to unknown $O(1)$ factors!

## Kaon Physics and $\mathrm{R}\left(\mathrm{D}^{(*)}\right)$

> The flavor symmetry predicts larger NP effects in 3rd gen. leptons
$>$ In Kaon physics the largest effects involve tau-neutrinos: $\mathrm{K} \rightarrow \pi \nu \nu$
$>$ The main correlation is with $\mathrm{R}\left(\mathrm{D}^{(*)}\right)$
For possible connections with $R(K)$ see [Fajfer et al. 1802.00786] For connections between B-anomalies and $\varepsilon$ ' see [Bobeth, Buras 1712.01295]

Contribution to $\mathrm{b} \rightarrow \mathrm{c} \mathrm{\tau v}$ :

$$
\mathcal{L}_{R\left(D^{(*)}\right)}^{\mathrm{NP}}=2 C_{R\left(D^{(*)}\right)} \lambda_{\tau \tau}^{\ell}\left(\bar{c}_{L} \gamma_{\mu} b_{L}\right)\left(\bar{\tau}_{L} \gamma_{\mu} \nu_{\tau}\right)+\text { h.c. }
$$

$$
C_{R\left(D^{(*)}\right)} \approx C_{T} \lambda_{b s}^{q} \quad \lambda_{b s}^{q} \sim V_{t s}
$$

Contribution to $s \rightarrow d v v:$

$$
\begin{array}{r}
\mathcal{L}_{s \rightarrow d \nu \nu}^{\mathrm{NP}}=C_{s d \nu \nu}\left[\lambda_{\tau \tau}^{\ell}\left(\bar{s}_{L} \gamma_{\mu} d_{L}\right)\left(\bar{\nu}_{\tau} \gamma_{\mu} \nu_{\tau}\right)+\frac{\left.\lambda_{\mu \mu}^{\ell}\left(\bar{s}_{L} \nu_{\mu} d_{t}\right)\left(\overline{\bar{p}_{\mu} \gamma_{\mu} \nu_{\mu}}\right)\right]}{}+h_{\mu . c .}^{\ell} \ll \lambda_{\tau \tau}^{\ell}=1\right. \\
C_{s d \nu \nu}=\left(C_{S}-C_{T}\right) \lambda_{s d}^{q} \quad \lambda_{s d}^{q} \sim V_{t s}^{*} V_{t d}
\end{array}
$$

Present status

| Observable | Experimental value/bound | SM prediction |
| :---: | :---: | :---: |
| $\operatorname{Br}\left(K^{+} \rightarrow \pi^{+} \nu_{\mu} \overline{\bar{\mu}}\right)$ | $\left(17.3_{-10.5}^{+11.5}\right) \times 10^{-11}$ | $(8.4 \pm 1.0) \times 10^{-11}$ |
| $\operatorname{Br}\left(K_{L} \rightarrow \pi^{0} \nu_{\mu} \overline{\nu_{\mu}}\right)$ | $<3.0 \times 10^{-9}(90 \% \mathrm{CL})$ | $(3.4 \pm 0.6) \times 10^{-11}$ |

E949 '08, Buras et al. 1503.02693
KOTO '18, Buras et al. 1503.02693

$$
\operatorname{Br}\left(K^{+} \rightarrow \pi^{+} v \bar{v}\right)<2.44 \times 10^{-10} \text { NA62 } 2017 \text { (preliminary) }
$$

| $\operatorname{Br}\left(K_{L} \rightarrow \pi^{0} \nu \nu\right)$ | $\sim 1.8 \times 10^{-10}$ | KOTO phase-I $^{7}$ |
| :---: | :---: | :--- |
|  | $20 \%$ | KOTO phase-II $^{7}$ |
|  | $20 \%$ | KLEV for SM rate |
| $\operatorname{Br}\left(K^{+} \rightarrow \pi^{+} \nu \nu\right)$ | $10 \%$ | NA62 goal |

## Kaon Physics and $\mathrm{R}\left(\mathrm{D}^{(*)}\right)$

> The flavor symmetry predicts larger NP effects in 3rd gen. leptons
> In Kaon physics the only chance is with tau-neutrino in $\mathrm{K} \rightarrow \pi \nu v$
$>$ The main correlation is with $\mathrm{R}\left(\mathrm{D}^{(*)}\right)$
For the connection with $R(K)$ see [Fajfer et al. 1802.00786]

Connection in the SMEFT, assuming U(2) ${ }^{5}$ structure
[Bordone, Buttazzo, Isidori, Monnard 1705.10729]


While the precise correlation depends on the details of the model, it is clear that a future measurements by NA62, KOTO, and KLEVER will cover most of the parameter space.

For a complete analysis it is necessary to take into account the bounds from $\mathrm{B} \rightarrow \mathrm{K}^{(*)} v v, \Delta \mathrm{~F}=2$, LFV , LEP data, and direct searches.

Need a full UV model which can address the anomalies.

## $S_{1}+S_{3}$ model

$\mathcal{L}_{S_{1}+S_{3}}=\left(\bar{q}^{c} \lambda^{1 L} \epsilon \ell+\bar{u}^{c} \lambda^{1 R} e\right) S_{1}+\bar{q}^{c} \lambda^{3 L} \epsilon \sigma^{I} \ell S_{3}^{I}+h . c$.
A very good fit of all data (including $\Delta \mathrm{F}=2$ ) can be achieved in this model.
work in progress with V . Gherardi and E. Venturini
The contributions to $R\left(D^{( }\right)$) arise via a combination of (V-A) + (scalar) + (tensor) operators, uncorrelated with electroweak precision tests or $\mathrm{B}_{\mathrm{s}}$-mixing.

$$
\mathcal{O}_{V_{L}}^{\tau}=\left(\bar{c}_{L} \gamma_{\mu} b_{L}\right)\left(\bar{\tau}_{L} \gamma^{\mu} \nu_{\tau}\right), \quad \mathcal{O}_{T}^{\tau}=\left(\bar{c}_{R} \sigma_{\mu \nu} b_{L}\right)\left(\bar{\tau}_{R} \sigma^{\mu \nu} \nu_{\tau}\right), \quad \mathcal{O}_{S_{L}}^{\tau}=\left(\bar{c}_{R} b_{L}\right)\left(\bar{\tau}_{R} \nu_{\tau}\right)
$$

The coupling ( $\mathrm{S}_{1} \mathrm{c}_{\mathrm{R}} \tau_{\mathrm{R}}$ ) is a non-minimal breaking of the $\mathrm{U}(2)^{5}$ flavor symmetry.
The correlation between B and Kaon physics is unchanged.

Since the model is fully renormalisable, all loop-generated observables can be computed and included in the fit.

A full NLO matching to the SMEFT and NLO analysis is in progress.

## $S_{1}+S_{3}$ model

Scalar Leptoquarks
$S_{1}=(\overline{\mathbf{3}}, \mathbf{1}, 1 / 3)$,
Crivellin et al. 1703.09226; Buttazzo, Greljo, Isidori, DM 1706.07808;
D.M. 1803.10972; work in progress with V. Gherardi and E. Venturini
$S_{3}=(\overline{3}, \mathbf{3}, 1 / 3)$,

$$
\mathcal{L}_{S_{1}+S_{3}}=\left(\bar{q}^{c} \lambda^{1 L} \epsilon \ell+\bar{u}^{c} \lambda^{1 R} e\right) S_{1}+\bar{q}^{c} \lambda^{3 L} \epsilon \sigma^{I} \ell S_{3}^{I}+h . c .
$$

A very good fit of all data (including $\Delta F=2$ ) can be achieved in this model.
work in progress with V. Gherardi and E. Venturini


This are $\sim 3 k$ points from a parameter scan, each is within the $95 \% \mathrm{CL}$ interval of the fit (B-anomalies and all relevant constraints).

## Implications for $\mathrm{K} \rightarrow \pi \nu \nu$

Work in progress with V. Gherardi and E. Venturini

$$
\mathcal{L}_{S_{1}+S_{3}}=\left(\bar{q}^{c} \lambda^{1 L} \epsilon \ell+\bar{u}^{c} \lambda^{1 R} e\right) S_{1}+\bar{q}^{c} \lambda^{3 L} \epsilon \sigma^{I} \ell S_{3}^{I}+h . c .
$$

Under $\mathrm{U}(2)^{5}$ flavor symmetry assumption, the LQ coupling to 1st ten is correlated with the one to $2 n d$ gen:

$$
\lambda_{d_{L} \tau_{L}}=\lambda_{s_{L} \tau_{L}} \frac{V_{t d}^{*}}{V_{t s}^{*}}
$$

We can obtain a set of predictions for $\boldsymbol{K}^{+} \rightarrow \boldsymbol{\pi}^{+} \boldsymbol{\nu} \boldsymbol{\nu}$ and $\boldsymbol{K}_{L} \rightarrow \boldsymbol{\pi}^{0} \boldsymbol{\nu} \boldsymbol{\nu}$.


The two are very correlated in this framework because there is only one overall free phase.

Here we chose $\lambda_{s_{L} \tau_{L}}$ to be real (larger effect in R(D))

Many points can already be excluded by Kaon physics.


## Kaon physics and $\mathrm{R}\left(\mathrm{K}^{(*)}\right)$ ?

Under the $U(2)^{5}$ flavor symmetry: very small effect in kaon observables with muons.

$$
\Lambda_{\mathrm{R}(\mathrm{~K})} \sim 34 \mathrm{TeV} \quad \lambda_{\mu \mu}^{\ell} \ll \lambda_{\tau \tau}^{\ell}=1 \quad \& \quad \lambda_{s d}^{q} \sim V_{t s}^{*} V_{t d}
$$

To see an effect we need a more general flavor structure, allowing for larger NP contributions in light quark generations.

The operator(s) responsible for the anomalies are part of an EFT involving all three families

$$
\mathcal{L}_{\mathrm{NP}}^{\mathrm{EFT}}=C_{i j}\left(\bar{d}_{L}^{i} \gamma_{\mu} d_{L}^{i}\right)\left(\bar{\mu}_{L} \gamma^{\mu} \mu_{L}\right) \longrightarrow \mathcal{C}=\left(\begin{array}{ccc}
\mathcal{C}_{d d} & \overline{\mathcal{C}_{d s}} & \mathcal{C}_{d b} \\
\hline \mathcal{C}_{d s}^{*} & \mathcal{C}_{s s} & \mathcal{C}_{s b} \\
\mathcal{C}_{d b}^{*} & \mathcal{C}_{s b}^{*} & \mathcal{C}_{b b}
\end{array}\right)
$$

We need another motivated ansatz for the flavor structure of this matrix.

## Directions in $\operatorname{SU}(3)_{q}$ space

| We can parametrise directions in $\mathrm{SU}(3)_{\mathrm{q}}$ as: Via $a(1)_{1}$ s phase redefinition we can always set $\hat{n}_{3}>0$ | $\hat{n}=\left(\begin{array}{c}\sin \theta \cos \phi e^{i \alpha_{b d}} \\ \sin \theta \sin \phi e^{i \alpha_{b s}} \\ \cos \theta\end{array}\right)$ |
| :---: | :---: |

$$
\theta \in\left[0, \frac{\pi}{2}\right], \quad \phi \in[0,2 \pi), \quad a_{b t} \in\left[-\frac{\pi}{2}, \frac{\pi}{2}\right], \quad \alpha_{b s t} \in\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]
$$

In the mass eigenstate basis of down-quarks: $\quad q_{L}^{i}=\binom{V_{j i}^{*} u_{L}^{i}}{d_{L}^{i}}$

$\left\{q_{L}^{i}\right\}$ space, neglecting phases

| quark | $\hat{n}$ | $\phi$ | $\theta$ | $\alpha_{b d}$ | $\alpha_{b s}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| down | $(1,0,0)$ | 0 | $\pi / 2$ | 0 | 0 |
| strange | $(0,1,0)$ | $\pi / 2$ | $\pi / 2$ | 0 | 0 |
| bottom | $(0,0,1)$ | 0 | 0 | 0 | 0 |
| up | $e^{i \arg \left(V_{u b}\right)}\left(V_{u d}^{*}, V_{u s}^{*}, V_{u b}^{*}\right)$ | 0.23 | 1.57 | -1.17 | -1.17 |
| charm | $e^{i \arg \left(V_{c b}\right)}\left(V_{c d}^{*}, V_{c s}^{*}, V_{c b}^{*}\right)$ | 1.80 | 1.53 | $-6.2 \times 10^{-4}$ | $-3.3 \times 10^{-5}$ |
| top | $e^{i \arg \left(V_{t b}\right)}\left(V_{t d}^{*}, V_{t s}^{*}, V_{t b}^{*}\right)$ | 4.92 | 0.042 | -0.018 | 0.39 |

The misalignment between down- and up-quarks is described by the CKM matrix.

## Rank-One Flavor Violation

Valerio Gherardi, D.M., Marco Nardecchia, Andrea Romanino [1903.10954]

$$
\mathcal{L}_{\mathrm{NP}}^{\mathrm{EFT}}=C_{i j}\left(\bar{d}_{L}^{i} \gamma_{\mu} d_{L}^{i}\right)\left(\bar{\mu}_{L} \gamma^{\mu} \mu_{L}\right)
$$

We assume that the flavor matrix of the semi-leptonic couplings to muons is of rank-one:

$$
C_{i j}=C \hat{n}_{i} \hat{n}_{j}^{*}
$$

$\hat{n}$ is some (arbitrary) unitary vector in flavour space $\mathrm{SU}(3)_{q}$. It selects a direction in that space.

We aim to answer the following question


> Assuming B-anomalies are reproduced, what are the experimentally allowed directions for $\hat{n} ?$

## Comment on UV realisations

This rank-1 condition is automatically realised $\quad \mathcal{L}=\lambda_{i} \bar{q}_{L}^{i} \mathcal{O}_{\mathrm{NP}}+$ h.c.
in many UV scenarios

Single leptoquark models $\mathcal{L} \supset g_{i \mu} \bar{q}_{L}^{i} \gamma_{\mu} \ell_{L}^{2} U_{1}^{\mu}+h . c$.

$$
C_{i j}=C \hat{n}_{i} \hat{n}_{j}^{*}
$$

Single vector-like quark mixing

$$
\begin{gathered}
\mathcal{L} \supset M_{i} \bar{q}_{L}^{i} \Psi_{Q} \\
\hat{n}_{i} \propto M_{i}
\end{gathered}
$$

Loop models with 1 set of mediators $\quad \mathcal{L} \supset \lambda_{i Q} \bar{q}_{L}^{i} \Psi_{Q} \Phi+$ h.c. See e.g. talk by M. Fedele and references therein

## Constraints in ROFV

1) Fix a direction $\hat{n}$.

$$
\text { We fix the phases } \alpha_{b s}, \alpha_{b d} \text { and plot } \theta, \varphi \text {. }
$$

2) Solve for $\mathbf{C}$ by imposing $\mathbf{R}\left(\mathbf{K}^{(*)}\right)$ (from the fit)

$$
\begin{gathered}
\mathcal{L}_{\mathrm{NP}}^{\mathrm{EFT}}=C \hat{n}_{i} \hat{n}_{j}^{*}\left(\bar{d}_{L}^{i} \gamma_{\mu} d_{L}^{i}\right)\left(\bar{\mu}_{L} \gamma^{\mu} \mu_{L}\right) \\
\hat{n}=\left(\begin{array}{c}
\sin \theta \cos \phi e^{i \alpha_{b d}} \\
\sin \theta \sin \phi e^{\alpha_{b_{s}}} \\
\cos \theta
\end{array}\right)
\end{gathered}
$$

$$
\begin{array}{r}
\boldsymbol{b} \longrightarrow \boldsymbol{d} \boldsymbol{\mu}^{+} \boldsymbol{\mu}^{-} \quad C_{s b}=C \sin \theta \cos \theta \sin \phi e^{i \alpha_{b s}}=\frac{e^{i \alpha_{b s}}}{\Lambda_{b s}^{2}}= \\
C=C_{s b}^{\mathrm{fit} R\left(K^{(*)}\right)} e^{-i \alpha_{b s}}(\sin \theta \cos \theta \sin \phi)^{-1}
\end{array}
$$

3) Compute NP contribution for other flavor transitions:
$\boldsymbol{b} \rightarrow \boldsymbol{d} \boldsymbol{\mu}^{+} \boldsymbol{\mu}^{-} C_{d b}=C \sin \theta \cos \theta \cos \phi e^{i \alpha_{b d}}$

$\boldsymbol{S} \longrightarrow \boldsymbol{d} \boldsymbol{\mu}^{+} \boldsymbol{\mu}^{-} C_{d s}=C \sin ^{2} \theta \sin \phi \cos \phi e^{i\left(\alpha_{b d}-\alpha_{b s}\right)}$
4) Check if experimentally excluded or not.

## General correlations (LH)

Direct correlations with other $d_{i} d_{j} \mu \mu$ observables $\quad \mathcal{L}_{\mathrm{NP}}^{\mathrm{EFT}}=C \hat{n}_{i} \hat{n}_{j}^{*}\left(\bar{d}_{L}^{i} \gamma_{\mu} d_{L}^{i}\right)\left(\bar{\mu}_{L} \gamma^{\mu} \mu_{L}\right)$

|  | Observable | Experimental value/bound | SM prediction |
| ---: | :---: | :---: | :---: |
| $C_{d b}$ | $\operatorname{Br}\left(B_{d}^{0} \rightarrow \mu^{+} \mu^{-}\right)$ | $<2.1 \times 10^{-10}(95 \% \mathrm{CL})$ | $(1.06 \pm 0.09) \times 10^{-10}$ |
|  | $\operatorname{Br}\left(B^{+} \rightarrow \pi^{+} \mu^{+} \mu^{-}\right)_{[1,6]}$ | $\left(4.55_{-1.00}^{+1.05} \pm 0.15\right) \times 10^{-9}$ | $(6.55 \pm 1.25) \times 10^{-9}$ |
| $\operatorname{Im}\left(C_{d s}\right)$ | $\operatorname{Br}\left(K_{S} \rightarrow \mu^{+} \mu^{-}\right)$ | $<1.0 \times 10^{-9}(95 \% \mathrm{CL})$ | $(5.0 \pm 1.5) \times 10^{-12}$ |
| $\operatorname{Re}\left(C_{d s}\right)$ | $\operatorname{Br}\left(K_{L} \rightarrow \mu^{+} \mu^{-}\right)_{\mathrm{SD}}$ | $<2.5 \times 10^{-9}$ | LHCb |
| $\operatorname{Im}\left(C_{d s}\right)$ | $\operatorname{Br}\left(K_{L} \rightarrow \pi^{0} \mu^{+} \mu^{-}\right)$ | $<3.8 \times 10^{-10}(90 \% \mathrm{CL})$ | $1.41_{-0.26}^{+0.28}\left(0.95_{-0.21}^{+0.22}\right) \times 10^{-11}$ |
|  |  |  |  |

D'Ambrosio et al '98, Buchalla et al '03,


$$
\mathrm{LH} \text { - General correlations }\left(\alpha_{b s}=0, \alpha_{\mathrm{bd}}=0\right)
$$


$|\mathrm{C}|^{-1 / 2}[\mathrm{TeV}]$


Each colored region is excluded by the respective observable

## General correlations (LH)

Direct correlations with other $d_{i} d_{j} \mu \mu$ observables $\quad \mathcal{L}_{\mathrm{NP}}^{\mathrm{EFT}}=C \hat{n}_{i} \hat{n}_{j}^{*}\left(\bar{d}_{L}^{i} \gamma_{\mu} d_{L}^{i}\right)\left(\bar{\mu}_{L} \gamma^{\mu} \mu_{L}\right)$

|  | Observable | Experimental value/bound | SM prediction |
| :---: | :---: | :---: | :---: |
| $C_{d b}$ | $\operatorname{Br}\left(B_{d}^{0} \rightarrow \mu^{+} \mu^{-}\right)$ | $<2.1 \times 10^{-10}(95 \% \mathrm{CL})$ | $(1.06 \pm 0.09) \times 10^{-10}$ |
| $\operatorname{Im}\left(C_{d s}\right)$ | $\operatorname{Br}\left(B^{+} \rightarrow \pi^{+} \mu^{+} \mu^{-}\right)_{[1,6]}$ | $\left(4.55_{-1.00}^{+1.05} \pm 0.15\right) \times 10^{-9}$ | $(6.55 \pm 1.25) \times 10^{-9}$ |
| $\operatorname{Re}\left(C_{d s}\right)$ | $\operatorname{Br}\left(K_{S} \rightarrow \mu^{+} \mu^{-}\right)$ | $<1.0 \times 10^{-9}(95 \% \mathrm{CL})$ | $(5.0 \pm 1.5) \times 10^{-12}$ |
| $\operatorname{Im}\left(C_{d s}\right)$ | $\operatorname{Br}\left(K_{L} \rightarrow \mu^{+} \mu^{-}\right)_{\mathrm{SD}}$ | $<2.5 \times 10^{-9}$ | $\approx 0.9 \times 10^{-9}$ |
|  | $\operatorname{Br}\left(K_{L} \rightarrow \pi^{0} \mu^{+} \mu^{-}\right)$ | $<3.8 \times 10^{-10}(90 \% \mathrm{CL})$ | $1.41_{-0.26}^{+0.28}\left(0.95_{-0.21}^{+0.22}\right) \times 10^{-11}$ |

Fix the phases and plot on the angles $\varphi, \theta$ (it's a semi-sphere in $\left.\mathrm{SU}(3)_{q}\right)$
LH - General correlations ( $\alpha_{\text {bs }}=0, \alpha_{\text {bd }}=0$ )


Region suggested by $\mathrm{U}(2)^{5}$ flavour symmetry or partial compositeness (close to third generation).

$$
\hat{n}=\left(\mathcal{O}\left(V_{t d}\right), \mathcal{O}\left(V_{t s}\right), \mathcal{O}(1)\right)
$$

Each colored region is excluded by $|\mathrm{C}|^{-1 / 2}[\mathrm{TeV}]$
 the respective observable

## General correlations (LH)

Direct correlations with other $d_{i} d_{j} \mu \mu$ observables $\quad \mathcal{L}_{\mathrm{NP}}^{\mathrm{EFT}}=C \hat{n}_{i} \hat{n}_{j}^{*}\left(\bar{d}_{L}^{i} \gamma_{\mu} d_{L}^{i}\right)\left(\bar{\mu}_{L} \gamma^{\mu} \mu_{L}\right)$

|  | Observable | Experimental value/bound | SM prediction |
| :---: | :---: | :---: | :---: |
| $C_{d b}$ | $\operatorname{Br}\left(B_{d}^{0} \rightarrow \mu^{+} \mu^{-}\right)$ | $<2.1 \times 10^{-10}(95 \% \mathrm{CL})$ | $(1.06 \pm 0.09) \times 10^{-10}$ |
| $\operatorname{Ir}\left(C_{d s}\right)$ | $\operatorname{Br}\left(B^{+} \rightarrow \pi^{+} \mu^{+} \mu^{-}\right)_{[1,6]}$ | $\left(4.55_{-1.00}^{+1.05} \pm 0.15\right) \times 10^{-9}$ | $(6.55 \pm 1.25) \times 10^{-9}$ |
| $\operatorname{Br}\left(K_{S} \rightarrow \mu^{+} \mu^{-}\right)$ | $<1.0 \times 10^{-9}(95 \% \mathrm{CL})$ | $(5.0 \pm 1.5) \times 10^{-12}$ |  |
| $\operatorname{Re}\left(C_{d s}\right)$ | $\operatorname{Br}\left(K_{L} \rightarrow \mu^{+} \mu^{-}\right)_{\mathrm{SD}}$ | $<2.5 \times 10^{-9}$ | $\approx 0.9 \times 10^{-9}$ |
| $\operatorname{Im}\left(C_{d s}\right)$ | $\operatorname{Br}\left(K_{L} \rightarrow \pi^{0} \mu^{+} \mu^{-}\right)$ | $<3.8 \times 10^{-10}(90 \% \mathrm{CL})$ | $1.41_{-0.26}^{+0.28}\left(0.95_{-0.21}^{+0.22}\right) \times 10^{-11}$ |
|  |  |  |  |

$\hat{n}=\left(\begin{array}{c}\sin \theta \cos \phi e^{i \alpha_{b d}} \\ \sin \theta \sin \phi e^{i \alpha_{b s}} \\ \cos \theta\end{array}\right)$

For complex coefficients, $K_{L} \rightarrow \pi^{0} \mu \mu$ and $K_{S} \rightarrow \mu \mu$ become important


## Prospects

| Observable | Expected sensitivity | Experiment |
| :---: | :---: | :--- |
| $R_{K}$ | $0.7(1.7) \%$ | LHCb 300 (50) fb-1 |
|  | $3.6(11) \%$ | Belle II 50 (5) $\mathrm{ab}^{-1}$ |
| $R_{K^{*}}$ | $0.8(2.0) \%$ | LHCb 300 (50) $\mathrm{fb}^{-1}$ |
|  | $3.2(10) \%$ | Belle II 50 (5) $\mathrm{ab}^{-1}$ |
| $R_{\pi}$ | $4.7(11.7) \%$ | LHCb 300 (50) fb ${ }^{-1}$ |
| $\operatorname{Br}\left(B_{s}^{0} \rightarrow \mu^{+} \mu^{-}\right)$ | $4.4(8.2) \%$ | LHCb 300 (23) fb-1 |
|  | $7(12) \%$ | CMS 3 (0.3) ab ${ }^{-1}$ |
| $\operatorname{Br}\left(B_{d}^{0} \rightarrow \mu^{+} \mu^{-}\right)$ | $9.4(33) \%$ | LHCb 300 (23) fb-1 |
|  | $16(46) \%$ | CMS 3 (0.3) ab ${ }^{-1}$ |
| $\operatorname{Br}\left(K_{S} \rightarrow \mu^{+} \mu^{-}\right)$ | $\sim 10^{-11}$ | LHCb 300fb-1 |
|  | $\sim 1.8 \times 10^{-10}$ | KOTO phase-I ${ }^{6}$ |
| $\operatorname{Br}\left(K_{L} \rightarrow \pi^{0} \nu \nu\right)$ | $20 \%$ | KOTO phase-II ${ }^{6}$ |
|  | $20 \%$ | KLEVER |
| $\operatorname{Br}\left(K^{+} \rightarrow \pi^{+} \nu \nu\right)$ | $10 \%$ | NA62 goal |

Future improvements in the measurements of these observables will allow to cover the majority of the parameter space


## Summary

The B-physics anomalies are one of the few experimental hints for NP at TeV scales. If confirmed, understanding the flavor structure of this new breaking of the SM flavor symmetries will be crucial.

Specific flavor structures imply correlated effects in Kaon physics.

In $\mathrm{U}(2)^{5}$ flavor symmetry, $\mathrm{R}\left(\mathrm{D}^{(*)}\right)$ is correlated with $K \rightarrow \pi \nu \nu$ : large effects possible.

The Rank-One Flavor Violation assumption, realised in several UV completions, allows to correlate $\mathrm{R}\left(\mathrm{K}^{(*)}\right)$ with other Kaon observables,
e.g. $K_{L, S} \rightarrow \mu \mu$ and $K_{L} \rightarrow \pi^{0} \mu \mu$, but also $K \rightarrow \pi \nu \nu$.

Already now a sizeable part of parameter space is tested and future measurements will cover the majority of the framework.

## Grazie!

## Backup

## Simplified* fit of clean observables

$$
\begin{aligned}
\mathcal{L}_{\mathrm{eff}}^{\mathrm{NP}} & \supset \frac{G_{F} \alpha}{\sqrt{2} \pi} V_{t b} V_{t s}^{*}\left[\Delta C_{9}^{\mu}\left(\bar{s}_{L} \gamma^{\mu} b_{L}\right)\left(\bar{\mu} \gamma_{\mu} \mu\right)+\Delta C_{10}^{\mu}\left(\bar{s}_{L} \gamma^{\mu} b_{L}\right)\left(\bar{\mu} \gamma_{\mu} \gamma_{5} \mu\right)\right]+h . c . \\
\mathcal{L}_{\mathrm{eff}} & \supset \frac{e^{i \alpha_{b s}}}{\Lambda_{b s}^{2}}\left(\bar{s}_{L} \gamma^{\mu} b_{L}\right)\left(\bar{\mu}_{L} \gamma_{\mu} \mu_{L}\right)+h . c .
\end{aligned}
$$


*Simplified = no theory uncertainties considered. Agrees well "enough" with full fits.

## SMEFT case \& mediators

$$
\mathcal{L}_{\mathrm{NP}}^{\mathrm{SMEFT}}=C_{S}^{i j}\left(\bar{q}_{L}^{i} \gamma_{\mu} q_{L}^{j}\right)\left(\bar{\ell}_{L}^{2} \gamma^{\mu} \ell_{L}^{2}\right)+C_{T}^{i j}\left(\bar{q}_{L}^{i} \gamma_{\mu} \sigma^{a} q_{L}^{j}\right)\left(\bar{\ell}_{L}^{2} \gamma^{\mu} \sigma^{a} \ell_{L}^{2}\right)+C_{R}^{i j}\left(\bar{q}_{L}^{i} \gamma_{\mu} q_{L}^{j}\right)\left(\mu_{R} \gamma^{\mu} \mu_{R}\right)^{\left(V_{j i}^{*} u_{L}^{j}, d_{L}^{i}\right)^{t}}
$$

The ROFV assumption is

$$
C_{S, T, R}^{i j}=C_{S, T, R} \hat{n}_{i} \hat{n}_{j}^{*}
$$

Channel Coefficient dependencies

Three overall coefficients

| $d_{i} \rightarrow d_{j} \mu^{+} \mu^{-}$ | $C_{S}+C_{T}, C_{R}$ |
| :---: | :---: |
|  |  |
| Different processes depend <br> on different combinations of |  |
| $u_{i} \rightarrow u_{j} \overline{\nu_{\mu}} \nu_{\mu}$ | $C_{S}+C_{T}$ |
| $u_{i} \rightarrow u_{j} \mu^{+} \mu^{-}$ | $C_{S}-C_{T}, C_{R}$ |
| the three overall coefficients |  |

Assuming a LH solution $\left(C_{R}=0\right)$ :

$$
\begin{array}{ll}
C_{+} \equiv C_{S}+C_{T} & \begin{array}{l}
\text { This combination is fixed by the anomaly. } \\
d_{i} d_{j} \mu \mu \text { transitions, are directly correlated with } b s \mu \mu
\end{array} \\
C_{-} \equiv C_{S}-C_{T} & \begin{array}{l}
\text { In general this is an independent parameter. } \\
\text { Must be fixed e.g. by assuming a specific mediator. }
\end{array}
\end{array}
$$

## SMEFT case \& mediators

$$
\mathcal{L}_{\mathrm{NP}}^{\mathrm{SMEFT}}=C_{S}^{i j}\left(\bar{q}_{L}^{i} \gamma_{\mu} q_{L}^{j}\right)\left(\bar{\ell}_{L}^{2} \gamma^{\mu} \ell_{L}^{2}\right)+C_{T}^{i j}\left(\bar{q}_{L}^{i} \gamma_{\mu} \sigma^{a} q_{L}^{j}\right)\left(\bar{\ell}_{L}^{2} \gamma^{\mu} \sigma^{a} \ell_{L}^{2}\right)+C_{R}^{i j}\left(\bar{q}_{L}^{i} \gamma_{\mu} q_{L}^{j}\right)\left(\mu_{R} \gamma^{\mu} \mu_{R}\right)
$$

The ROFV assumption is

$$
C_{S, T, R}^{i j}=C_{S, T, R} \hat{n}_{i} \hat{n}_{j}^{*}
$$

Three overall coefficients

Channel Coefficient dependencies

| $d_{i} \rightarrow d_{j} \mu^{+} \mu^{-}$ | $C_{S}+C_{T}, C_{R}$ |
| :---: | :---: |
| $u_{i} \rightarrow u_{j} \overline{\nu_{\mu}} \nu_{\mu}$ | $C_{S}+C_{T}$ |
| $u_{i} \rightarrow u_{j} \mu^{+} \mu^{-}$ | $C_{S}-C_{T}, C_{R}$ |
| $d_{i} \rightarrow d_{j} \overline{\nu_{\mu}} \nu_{\mu}$ | $C_{S}-C_{T}$ |
| $u_{i} \rightarrow d_{j} \mu^{+} \nu_{\mu}$ | $C_{T}$ |

Different processes depend on different combinations of the three overall coefficients
$K^{+} \rightarrow \pi^{+} \boldsymbol{\nu} \boldsymbol{\nu}$ is important

We can ask what are the possible tree-level mediators which generate these operators.
Different ones generate different combinations of $C_{S, T, R}$.

| Simplified model | Spin | SM irrep | $\left(c_{S}, c_{T}, c_{R}\right)$ |
| :---: | :---: | :---: | :---: |
| $S_{3}$ | 0 | $(\overline{3}, 3,1 / 3)$ | $(3 / 4,1 / 4,0)$ |
| $U_{1}$ | 1 | $(3,1,2 / 3)$ | $(1 / 2,1 / 2,0)$ |
| $U_{3}$ | 1 | $(3,3,2 / 3)$ | $(3 / 2,-1 / 2,0)$ |
| $V^{\prime}$ | 1 | $(1,3,0)$ | $(0,1,0)$ |
| $Z_{(L)}^{\prime}$ | 1 | $(1,1,0)$ | $(1,0,0)$ |
| $Z_{(V)}^{\prime}$ | 1 | $(1,1,0)$ | $(1,0,1)$ |

As representative examples, we study:
$S_{3}$
$U_{1} \quad$ Z'v
(backup slides)

## $\mathrm{S}_{3}$ scalar leptoquark $\quad \mathrm{S}_{3}=(\overline{3}, 3,1 / 3)$

$$
\mathcal{L}_{\mathrm{NP}} \supset \beta_{3, i \mu}\left(\bar{q}_{L}^{c i} \epsilon \sigma^{a} \ell_{L}^{2}\right) S_{3}^{a}+\mathrm{h.c.} \longrightarrow C_{S}^{i j}=\frac{3}{4} \frac{\beta_{3, i \mu}^{*} \beta_{3, j \mu}}{M_{S_{3}}^{2}}, \quad C_{T}^{i j}=\frac{1}{4} \frac{\beta_{3, i \mu}^{*} \beta_{3, j \mu}}{M_{S_{3}}^{2}}, \quad C_{R}^{i j}=0
$$

$$
\beta_{3, i \mu}^{*} \equiv \beta_{3}^{*} \hat{n}_{i}
$$


$\mathrm{U}(2)$-like $\quad\left|C_{+}\right|^{-1 / 2}[\mathrm{TeV}] \quad M_{S_{3}}^{\max }[\mathrm{TeV}]$
LHC dimuon searches are relevant only for small $\theta$, i.e. very close to the 3rd generation.

Still far from testing $U(2)$ hypothesis [Greljo, D.M. 1704.09015]


At 1-loop it generates $\Delta F=2$ operators
$\Delta \mathcal{L}_{\Delta F=2}=-\frac{5\left|\beta_{3}\right|^{4}}{128 \pi^{2} M_{S_{3}}^{2}}\left[\left(\hat{n}_{i} \hat{n}_{j}^{*} \bar{d}_{L}^{i} \gamma^{\alpha} d_{L}^{j}\right)^{2}+\left(V_{i k} \hat{n}_{k} \hat{n}_{l}^{*} V_{j l}^{*} u_{L}^{i} \gamma^{\alpha} u_{L}^{j}\right)^{2}\right]$
Limits on $D-\bar{D}, K-\bar{K}, B_{d}-\bar{B}_{d}, B_{s}-\bar{B}_{s}$ give an upper limit on the leptoquark mass

## $\mathrm{U}_{1}$ vector leptoquark

$$
\mathcal{L}_{\mathrm{NP}} \supset \beta_{1, i \mu}\left(\bar{q}_{L}^{i} \gamma_{\alpha} \ell_{L}^{2}\right) U_{1}^{\alpha}+\text { h.c. }
$$




$B_{B^{+} \rightarrow \pi^{+} \mu \mu} B_{B^{0} \rightarrow \mu \mu} \quad K_{K_{L} \rightarrow \mu \mu} \quad \underset{K_{S} \rightarrow \mu \mu}{ } \quad K_{K_{L} \rightarrow \pi^{0} \mu \mu} \underset{\mathrm{pp} \rightarrow \mu \mu}{\mathrm{U}(2)-- \text { like }^{\left|C_{+}\right|^{-1 / 2}[\mathrm{TeV}]}} C=-\left|\beta_{1}\right|^{2} / M_{U_{1}}^{2}<0$
$\Delta \mathrm{F}=2$ loops are divergent, need a UV completion.

## Z' \& vector-like couplings to $\mu$

For example see the gauged $U(1)$ Lu-Lt model with 1 vector-like quark.
[Altmannshofer, Gori, et al 1403.1269, 1609.04026]
$\mathcal{L} \supset M_{i} \bar{q}_{L}^{i} \Psi_{Q}$
$\hat{n}_{i} \propto M_{i}$

$$
\mathcal{L}_{\mathrm{NP}} \supset\left[g_{q} \hat{n}_{i} \hat{n}_{j}^{*}\left(\bar{q}_{L}^{i} \gamma^{\alpha} q_{L}^{j}\right)+g_{\mu}\left(\bar{l}_{L}^{2} \gamma^{\alpha} \ell_{L}^{2}+\bar{\mu}_{R} \gamma^{\alpha} \mu_{R}\right)\right] Z_{\alpha}^{\prime} \quad \longrightarrow C_{S}^{i j}=-\frac{g_{q} g_{\mu}}{M_{Z^{\prime}}^{2}} \hat{n}_{i} \hat{n}_{j}^{*}, \quad C_{T}^{i j}=0, \quad C_{R}^{i j}=-\frac{g_{q} g_{\mu}}{M_{Z^{\prime}}^{2}} \hat{n}_{i} \hat{n}_{j}^{*}
$$



$$
C_{+}=-g_{q} g_{\mu} /\left(M_{Z^{\prime}}^{2}\right)
$$


$\Delta \mathrm{F}=2$ operators are generated at the tree level.

$$
\Delta \mathcal{L}_{\Delta F=2}=-\frac{g_{q}^{2}}{2 M_{Z^{\prime}}^{2}}\left[\left(\hat{n}_{i} \hat{n}_{j}^{*} \bar{d}_{L}^{i} \gamma^{\alpha} d_{L}^{j}\right)^{2}+\left(V_{i k} \hat{n}_{k} \hat{n}_{l}^{*} V_{j l}^{*} \bar{u}_{L}^{i} \gamma^{\alpha} u_{L}^{j}\right)^{2}\right]
$$

We can put upper limits on $\mathrm{r}_{\mathrm{q} \mu}=g_{q} / g_{\mu}$, or for a given maximum $g_{\mu}$, an upper limit on the $Z$ ' mass

$$
M_{Z^{\prime}}^{\lim }=\sqrt{\frac{r_{q \mu}^{\lim }}{4|C|}}\left|g_{\mu}^{\max }\right|
$$

## $\Delta F=2$ observables (and $\varepsilon^{\prime} / \varepsilon$ )

| Limits on $\Delta F=2$ coefficients $\left[\mathrm{GeV}^{-2}\right]$ |
| :---: |
| $\operatorname{Re} C_{K}^{1} \in[-6.8,7.7] \times 10^{-13}, \operatorname{Im} C_{K}^{1} \in[-1.2,2.4] \times 10^{-15}$ |
| $\operatorname{Re} C_{D}^{1} \in[-2.5,3.1] \times 10^{-13}, \operatorname{Im} C_{D}^{1} \in[-9.4,8.9] \times 10^{-15}$ |
| $\left\|C_{B_{d}}^{1}\right\|<9.5 \times 10^{-13}$ |
| $\left\|C_{B_{s}}^{1}\right\|<1.9 \times 10^{-11}$ |

$$
\mathcal{L}_{\Delta \mathrm{F}=2}^{\mathrm{NP}}=C_{i j}\left(\bar{q}_{L}^{i} \gamma_{\mu} q_{L}^{j}\right)^{2}
$$

[UTfit 0707.0636, update by L. Silvestrini @ La Thuile '18]
For example, the $Z^{\prime}$ contribution is: $\quad \Delta \mathcal{L}_{\Delta F=2}=-\frac{g_{q}^{2}}{2 M_{Z^{\prime}}^{2}}\left[\left(\hat{n}_{i} \hat{n}_{j}^{*} \overline{d_{i L}} \gamma^{\alpha} d_{j L}\right)^{2}+\left(V_{i k} \hat{n}_{k} \hat{n}_{l}^{*} V_{j l}^{*} \overline{u_{i L}} \gamma^{\alpha} u_{j L}\right)^{2}\right]$

Also $\varepsilon^{\prime} / \varepsilon$ provides a potential constrain on the coefficient of

$$
\begin{aligned}
&\left(\bar{s} \gamma_{\mu} P_{L} d\right)\left(\bar{q} \gamma^{\mu} P_{L} q\right) \\
& q=u, d, s, c
\end{aligned}
$$

[Aebisher et al. 1807.02520, 1808.00466]
$\left(\frac{\varepsilon^{\prime}}{\varepsilon}\right)_{\mathrm{BSM}}=\sum_{i} P_{i}\left(\mu_{\mathrm{ew}}\right) \operatorname{Im}\left[C_{i}\left(\mu_{\mathrm{ew}}\right)-C_{i}^{\prime}\left(\mu_{\mathrm{ew}}\right)\right] \leqslant 10 \times 10^{-4}$
In this framework, this constraint is not competitive with $\Delta \mathrm{F}=2$

