$K \rightarrow \pi\pi$ decay, epsilon' and the RBC-UKQCD kaon physics program

KAON2019

University of Perugia (Italy)

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N.H. Christ

RBC/UKQCD Collaboration

Outline

- RBC-UKQCD kaon program:
  - ΔM_K, $(\varepsilon_K)_{LD}$, $(K^+ \to \pi^+ \nu \bar{\nu})_{LD}$, $K^+ \to \pi^+ l^+ l^-$, $(K_L \to \mu^+ \mu^+)_{\gamma\gamma}$
  - E&M corrections to $\pi \to \mu\nu$ and $K \to \mu/e\,\nu$
- $K \to \pi\pi$ decay and ε'
  - Quick review of standard model ~~CP~~
  - Overview of 2015 calculation
  - Overview of 2019 calculation
  - Multi-operator results for $\pi\pi$ scattering
  - $K \to \pi\pi$ decay amplitudes (no new result yet for ε')
- Conclusion

The RBC & UKQCD collaborations

BNL and BNL/RBRC

Yasumichi Aoki (KEK)
Taku Izubuchi
Yong-Chull Jang
Chulwoo Jung
Meifeng Lin
Aaron Meyer
Hiroshi Ohki
Shigemi Ohta (KEK)
Amarjit Soni

UC Boulder

Oliver Witzel

CERN

Mattia Bruno

Columbia University

Ryan Abbot
Norman Christ
Duo Guo
Christopher Kelly
Bob Mawhinney
Masaaki Tomii
Jiqun Tu
Bigeng Wang
Tianle Wang
Yidi Zhao

University of Connecticut

Tom Blum
Dan Hoying (BNL)
Luchang Jin (RBRC)
Cheng Tu

Edinburgh University

Peter Boyle
Luigi Del Debbio
Felix Erben
Vera Gülpers
Tadeusz Janowski
Julia Kettle
Michael Marshall
Fionn Ó hÓgáin
Antonin Portelli
Tobias Tsang
Andrew Yong
Azusa Yamaguchi

University of Liverpool

Nicolas Garron

MIT

David Murphy

Peking University

Xu Feng

University of Regensburg

Christoph Lehner (BNL)

University of Southampton

Nils Asmussen
Jonathan Flynn
Ryan Hill
Andreas Jüttner
James Richings
Chris Sachrajda

Stony Brook University

Jun-Sik Yoo
Sergey Syritsyn (RBRC)

RBC-UKQCD kaon program

- ΔM_K: 153 configs., 1/a=2.38 GeV, 64^3 x 128, all masses physical (Bigeng Wang).
- $(\varepsilon_K)_{LD}$: exploratory calculation done (Ziyuan Bai)
- $(K^+ \to \pi^+ \nu \bar{\nu})_{LD}$: 37 configs., 1/a=2.38 GeV 64^3 x 128, all masses physical. (Xu Feng)
- $K^+ \to \pi^+ l^+ l^-$: Fionn Ó hÓgáin's talk on Thurs.
- $(K_L \to \mu^+ \mu^-)_{\gamma\gamma}$: under study, $\pi \to e^+ e^-$ done (Y. Zhao)
- E&M corrections:
  - $\pi \to \mu \nu$
  - $K \to \mu/e\, \nu$
  - $K \to \mu/e\, \nu\, \gamma$

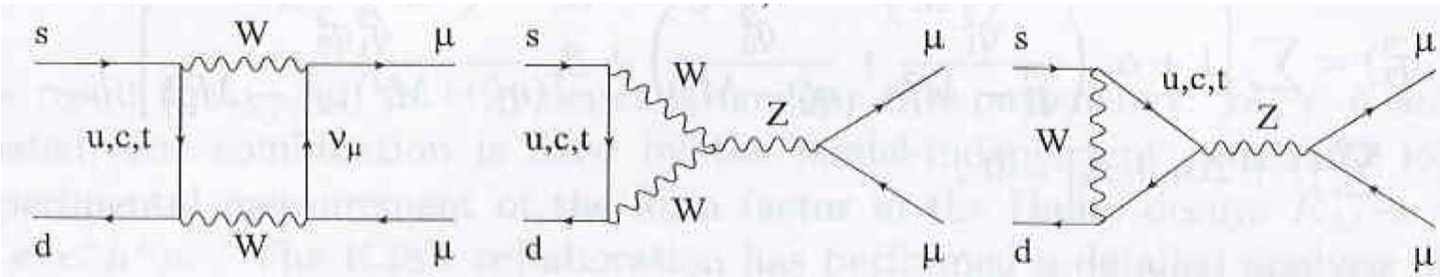
(for $\pi \to \mu\nu$ and $K \to \mu/e\,\nu$:)

- QED_L or
- new IVR method without power-law corrections, (Feng & Jin: arXiv:1812)

$K \rightarrow \mu^+ \mu^-$

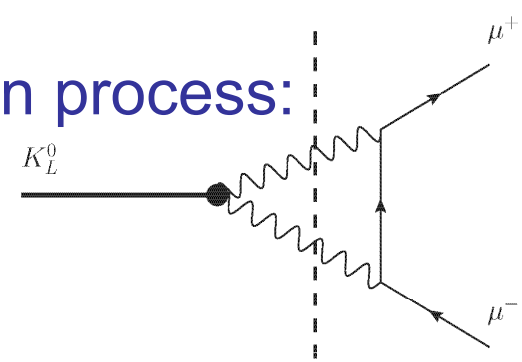
Physics of $K_L \rightarrow \mu^+ \mu^-$

- A second order weak, ``strangeness changing neutral current''



(Cirigliano, *et al.* , Rev. Mod. Phys., **84**, 2012)

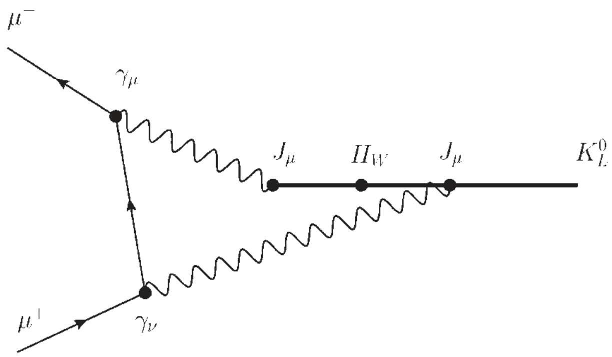
- $K_L \rightarrow \mu^+ \mu^-$ decay rate is known:
  - BR($K_L \rightarrow \mu^+ \mu^-$) = (6.84 ± 0.11) x 10^{-9}
- Large ``background'' from two-photon process:
  - Third-order electroweak amplitude
  - Optical theorem gives imaginary part.
  - $K_L \rightarrow \gamma\gamma$ decay rate is known



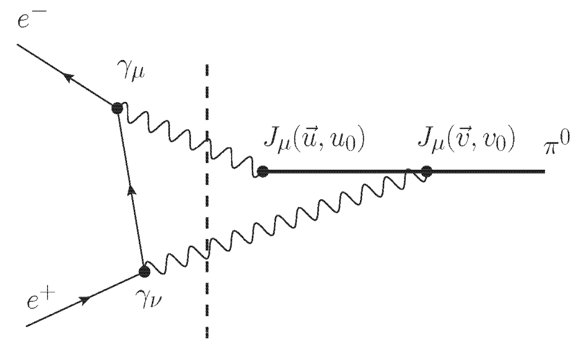
Physics of $K_L \to \mu^+ \mu^-$ (con't)

- Define: $$\frac{\Gamma(K_L \to \mu^+\mu^-)}{\Gamma(K_L \to \gamma\gamma)} = 2\beta_\mu \left(\frac{\alpha}{\pi}\frac{m_\mu}{M_K}\right)^2 \left(|F_{\rm imag}|^2 + |F_{\rm real}|^2\right)$$
- Optical theorem determines:

  $$|F_{\rm real}| = |(F_{\rm real})_{\rm E\&M} + (F_{\rm real})_{\rm Weak}| = 1.167 \pm 0.094$$
- Standard model: $(F_{\rm real})_{\rm Weak} = -1.82 \pm 0.04$
- A 10% lattice calculation of $(F_{\rm real})_{\rm E\&M}$ would allow a test of $(F_{\rm real})_{\rm Weak}$ with 6 – 17% accuracy
- Lattice calculation more difficult than ΔM_K
  - 5 vertices, 60 time orders
  - many states $|n\rangle$ with $E_n < M_K$
- First try simpler $\pi^0 \to e^+ e^-$

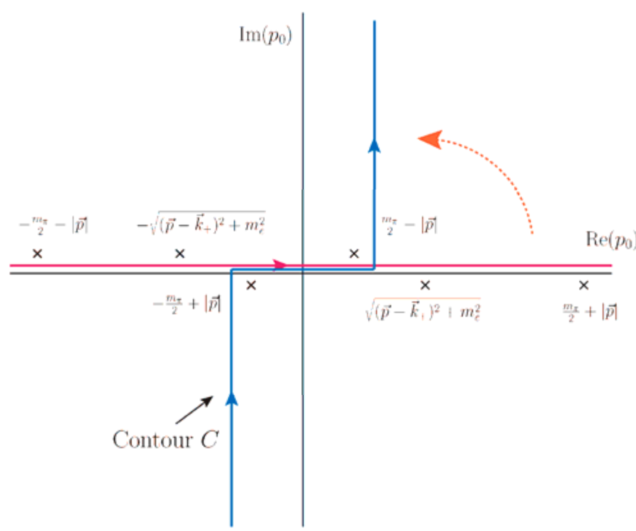


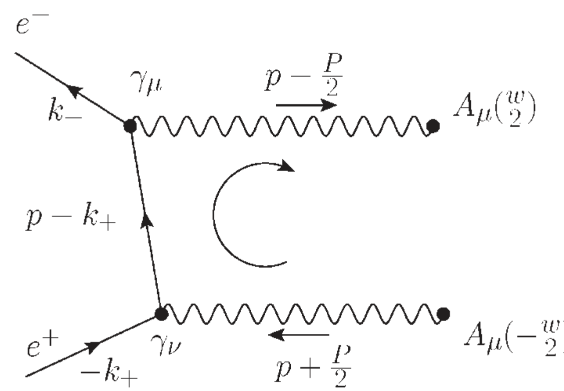
Consider simpler $\pi^0 \to e^+ e^-$



- Euclidean non-covariant P.T. difficult:
  - 12 time orders,
  - $E_{\gamma\gamma} < M_{\pi 0}$
- Try something different:
  - Evaluate in Minkowski space
  - Wick rotate internal time integral:

$$\mathcal{A}_{\pi^0\to e^+e^-} \to \int d^4w\, \widetilde{L}(k_-, k_+, w)_{\mu\nu}\langle 0|T\left\{J_\mu(\frac{w}{2})J_\nu(-\frac{w}{2})\right\}|\pi^0(\vec{P}=0)\rangle$$



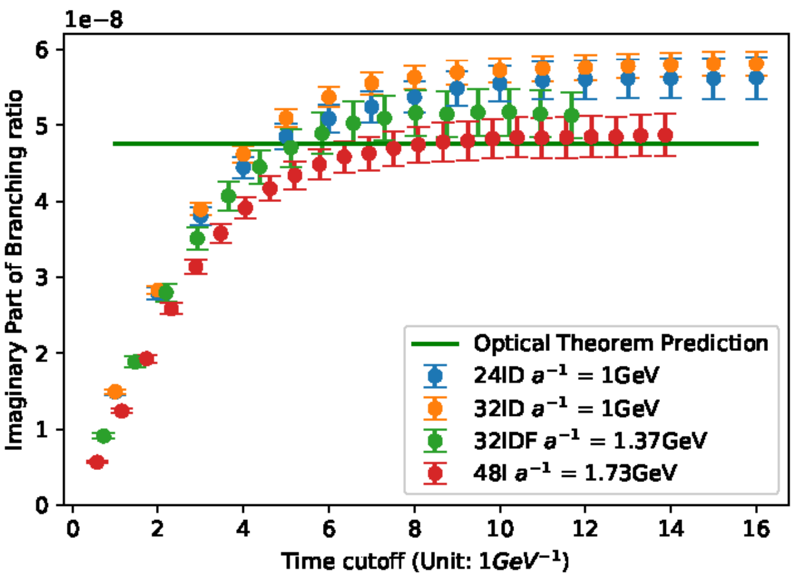


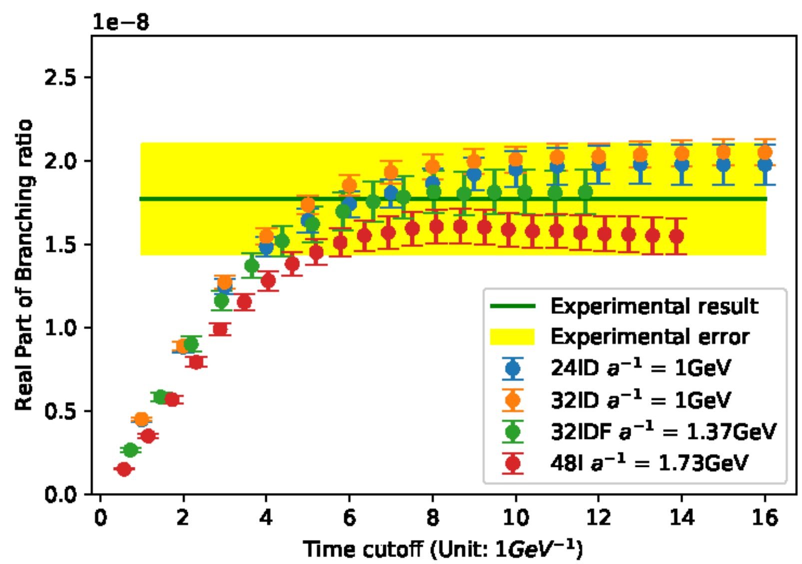
Lattice Results

(Yidi Zhao)

$$\mathcal{A}_{\pi^0\to e^+e^-} \to \int d^4w\, \widetilde{L}(k_-,k_+,w)_{\mu\nu}\langle 0|T\left\{J_\mu(\frac{w}{2})J_\nu(-\frac{w}{2})\right\}|\pi^0(\vec{P}=0)\rangle$$

- Lattice result is complex:
  - Exponentially small FV corrections
  - Physical kinematics, $1/a \le 1.73$ GeV :
    - Im(A) = 35.94(1.01)(1.09) [Expt: 35.07(37)]
    - Re(A) = 20.39(72)(70). [Expt: 21.51(2.02)]





$K \to \pi\pi$ decay and ε'

Cabibbo-Kobayashi-Maskawa mixing

- $W^{\pm}$ emission scrambles the quark flavors

$$\begin{pmatrix} u \\ c \\ t \end{pmatrix} \overset{W^{\pm}}{\longleftrightarrow} \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

CP violation!

$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} 1-\frac{\lambda^2}{2} & \lambda & A\lambda^3(\overline{\rho}-i\overline{\eta}) \\ -\lambda & 1-\frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1-\overline{\rho}-i\overline{\eta}) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4)$$

$$\lambda = 0.22535 \pm 0.00065\,, \qquad A = 0.811^{+0.022}_{-0.012}\,,$$

$$\bar{\rho} = 0.131^{+0.026}_{-0.013}\,, \qquad \bar{\eta} = 0.345^{+0.013}_{-0.014}\,.$$

$K^0 - \overline{K^0}$ mixing

- ΔS=1 weak decays allow K^0 and $\overline{K^0}$ to decay to the same $\pi\pi$ state.
- Resulting mixing described by Wigner-Weisskopf

$$i\frac{d}{dt}\begin{pmatrix} K^0 \\ \overline{K}^0 \end{pmatrix} = \left\{ \begin{pmatrix} M_{00} & M_{0\bar{0}} \\ M_{\bar{0}0} & M_{\bar{0}\bar{0}} \end{pmatrix} - \frac{i}{2}\begin{pmatrix} \Gamma_{00} & \Gamma_{0\bar{0}} \\ \Gamma_{\bar{0}0} & \Gamma_{\bar{0}\bar{0}} \end{pmatrix} \right\} \begin{pmatrix} K^0 \\ \overline{K}^0 \end{pmatrix}$$

- Decaying states are mixtures of K^0 and $\overline{K^0}$

$$|K_S\rangle = \frac{K_+ + \overline{\epsilon} K_-}{\sqrt{1+|\overline{\epsilon}|^2}}$$

$$|K_L\rangle = \frac{K_- + \overline{\epsilon} K_+}{\sqrt{1+|\overline{\epsilon}|^2}}$$

$$\overline{\epsilon} = \frac{i}{2}\left\{ \frac{\mathrm{Im} M_{0\bar{0}} - \frac{i}{2}\mathrm{Im}\Gamma_{0\bar{0}}}{\mathrm{Re} M_{0\bar{0}} - \frac{i}{2}\mathrm{Re}\Gamma_{0\bar{0}}} \right\}$$

Indirect CP violation

CP violation

- CP violating, experimental amplitudes:

$$\eta_{+-} \equiv \frac{\langle \pi^+\pi^- | H_w | K_L \rangle}{\langle \pi^+\pi^- | H_w | K_S \rangle} = \epsilon + \epsilon'$$

$$\eta_{00} \equiv \frac{\langle \pi^0\pi^0 | H_w | K_L \rangle}{\langle \pi^0\pi^0 | H_w | K_S \rangle} = \epsilon - 2\epsilon'$$

- Where: $\epsilon = \overline{\epsilon} + i \frac{\mathrm{Im} A_0}{\mathrm{Re} A_0}$

Indirect: $|\varepsilon| = (2.228 \pm 0.011) \times 10^{-3}$

Direct: $\mathrm{Re}(\varepsilon'/\varepsilon) = (1.66 \pm 0.23) \times 10^{-3}$

$K \rightarrow \pi\,\pi$ and CP violation

- Final $\pi\pi$ states can have I = 0 or 2.

$$\langle \pi\,\pi(I=2)|H_w|K^0\rangle = A_2 e^{i\delta_2} \qquad \Delta I = 3/2$$
$$\langle \pi\,\pi(I=0)|H_w|K^0\rangle = A_0 e^{i\delta_0} \qquad \Delta I = 1/2$$

- CP symmetry requires A_0 and A_2 be real.
- Direct CP violation in this decay is characterized by:

$$\epsilon' = \frac{i\,e^{\delta_2-\delta_0}}{\sqrt{2}}\left|\frac{A_2}{A_0}\right|\left(\frac{\mathrm{Im}A_2}{\mathrm{Re}A_2}-\frac{\mathrm{Im}A_0}{\mathrm{Re}A_0}\right)$$

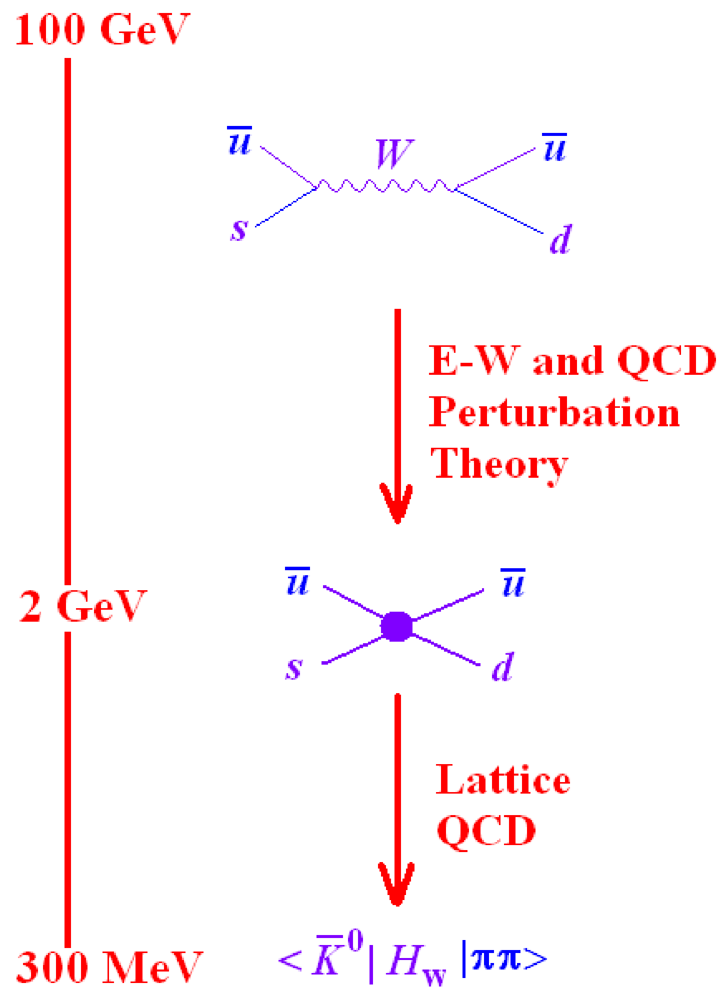
Direct CP violation

Low Energy Effective Theory

- Represent weak interactions by local four-quark Lagrangian

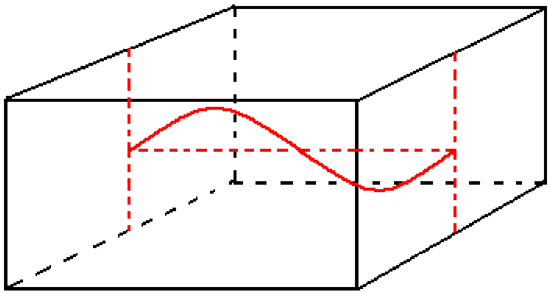
$$\mathcal{H}^{\Delta S=1} = \frac{G_F}{\sqrt{2}} V_{ud} V_{us}^* \left\{ \sum_{i=1}^{10} \Big[z_i(\mu) + \tau y_i(\mu) \Big] Q_i \right\}$$

- $\tau = -\frac{V_{td} V_{ts}^*}{V_{ud} V_{us}^*} = (1.543 + 0.635i) \times 10^{-3}$
- $V_{qq'}$ – CKM matrix elements
- z_i and y_i – Wilson Coefficients
- Q_i – four-quark operators



Lattice calculation of $<\pi\pi|H_W|K>$

- The operator product $\overline{d}(x)s(x)$ easily creates a kaon.
- Use finite-volume energy quantization (Lellouch-Luscher) and adjust L so that n^{th} excited state obeys: $E_{\pi\pi}^{(n)} = M_K$



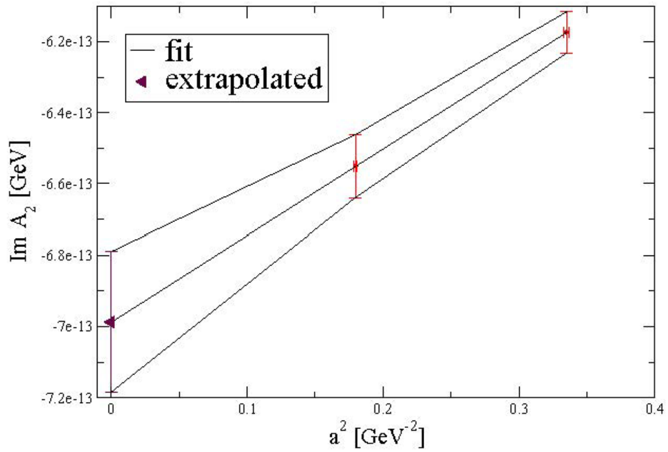
$$\langle \pi^+\pi^- | H_W | K^0 \rangle \quad \propto \quad \langle \overline{d}u(t_{\pi_1})\overline{u}d(t_{\pi_2})\ H_W(t_{\rm op})\ \overline{d}u(t_K) \rangle$$

- Use boundary conditions on the quarks: $E_{\pi\pi}^{(\rm gnd)} = M_K$
- For $(\pi\pi)_{I=2}$ make d anti-periodic
- For $(\pi\pi)_{I=0}$ use G-parity boundary conditions: **arXiv:1908.08**

Calculation of A_2

$\Delta I = 3/2$ – Continuum Results

(M. Lightman, E. Goode T. Janowski)



- Use two large ensembles to remove a^2 error (m_π=135 MeV, L=5.4 fm)
  - 48^3 x 96, 1/a=1.73 GeV
  - 64^3 x 128, 1/a=2.28 GeV
- Continuum results:
  - Re(A_2) = 1.50(0.04$_{stat}$) (0.14)$_{syst}$ × 10^{-8} GeV
  - Im(A_2) = - 6.99(0.20)$_{stat}$ (0.84)$_{syst}$ × 10^{-13} GeV
- Experiment: Re(A_2) = 1.479(4) 10^{-8} GeV
- $E_{\pi\pi}$ → δ_2 = −11.6(2.5)(1.2)^{o}
- [Phys.Rev. **D91**, 074502 (2015)]

Calculation of A_0 and ε'

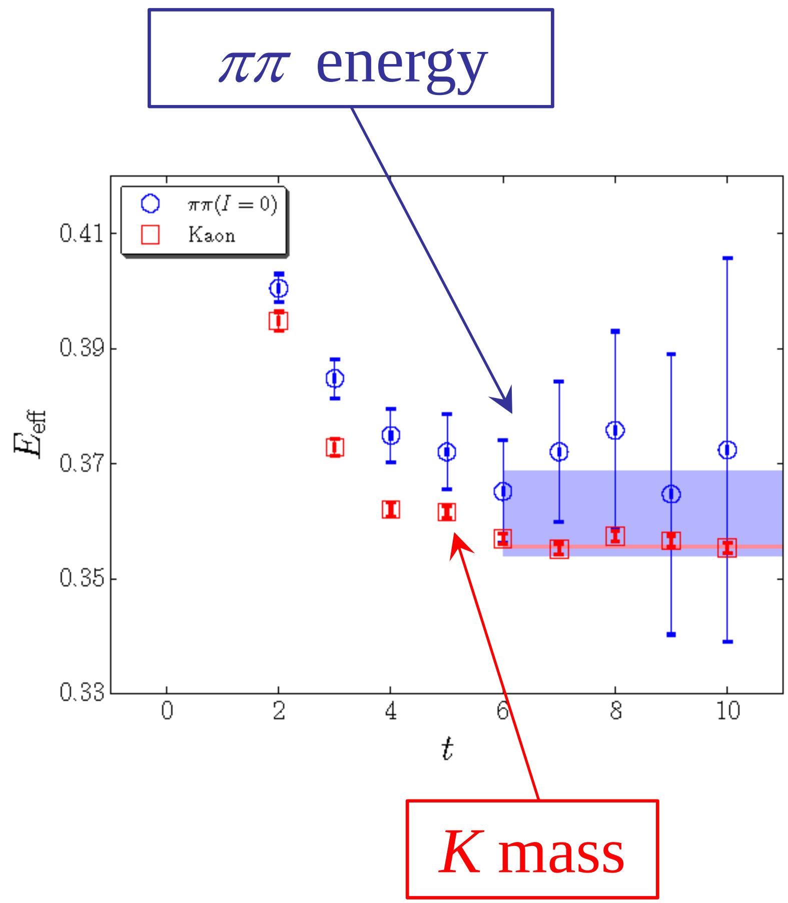
Overview of 2015 calculation

(Chris Kelly and Daiqian Zhang)

- Use 32^3 x 64 ensemble
  - $1/a$ = 1.3784(68) GeV, L = 4.53 fm.
  - G-parity boundary condition in 3 directions
  - 216 configurations separated by 4 time units
- Achieve essentially physical kinematics:
  - M_π = 143.1(2.0)
  - M_K = 490.6(2.2) MeV
  - $E_{\pi\pi}$ = 498(11) MeV

$I = 0$, $\pi\pi - \pi\pi$ correlator

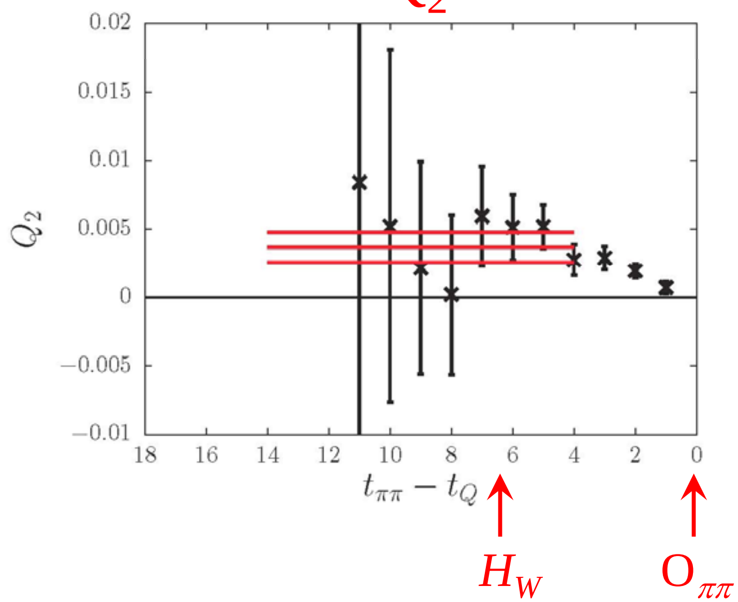
- Determine normalization of $\pi\pi$ interpolating operator
- Determine energy of finite volume, $I = 0$, $\pi\pi$ state: $E_{\pi\pi} =$ 498(11) MeV
- Obtained consistent results from a one-state fit with t_{min}=6 or a two-state fit with t_{min}=4.



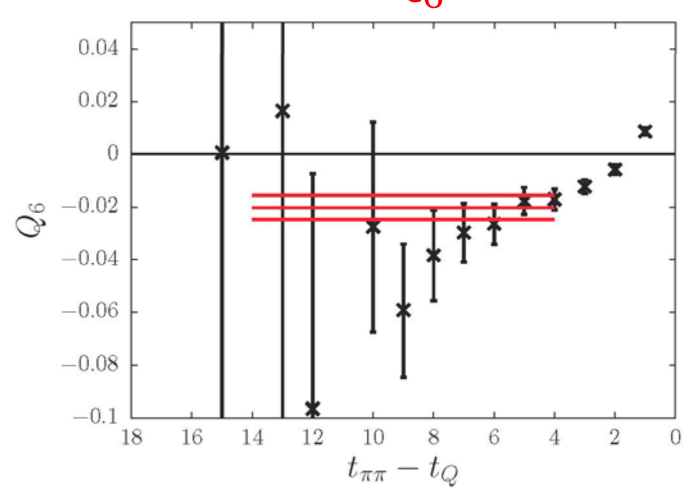
$I = 0$ $K \rightarrow \pi\pi$ matrix elements

- Vary time separation between H_W and $\pi\pi$ operator.
- Show data for all $K - H_W$ separations $t_Q - t_K \geq 6$ and $t_{\pi\pi} - t_K = 10, 12, 14, 16$ and 18.
- Fit correlators with $t_{\pi\pi} - t_Q \geq 4$
- Obtain consistent results for $t_{\pi\pi} - t_Q \geq 3$ or 5

Q_2



Q_6



Systematic errors

Description	Error
Operator renormalization	15%
Wilson coefficients	12%
Finite lattice spacing	12%
Lellouch-Luscher factor	11%
Finite volume	7%
Parametric errors	5%
Excited states	5%
Unphysical kinematics	3%
Total	27%

2015 Results

[Phys. Rev. Lett. 115 (2015) 212001]

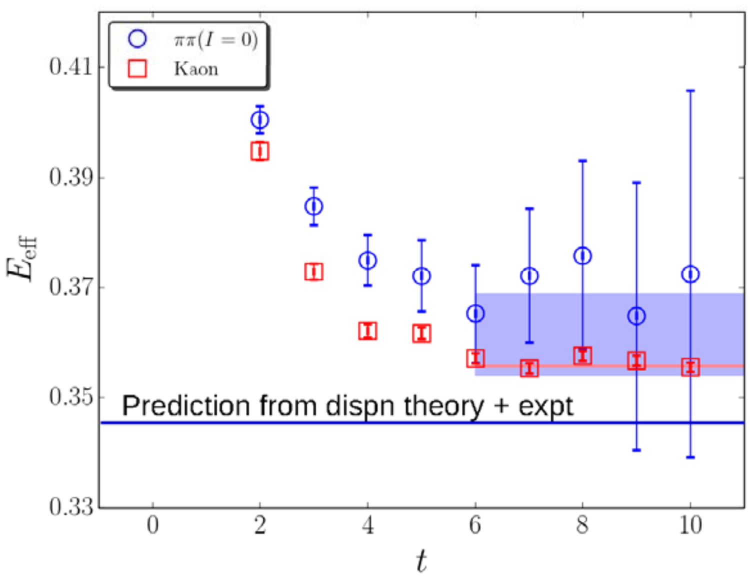
- $E_{\pi\pi}$(499 MeV) determines δ_0 :
  - $I = 0$ $\pi\pi$ phase shift: $\delta_0 = 23.8(4.9)(2.2)^o$
  - Dispersion theory result: $\delta_0 = 34^o$ [G. Colangelo, *et al.*]
- Re(ε'/ε) = (1.38 ± 5.15$_{stat}$ ± 4.59$_{sys}$) x 10^{-4}
  - Expt.: (16.6 ± 2.3) x 10^{-4}
  - 2.1 σ difference
- Unanswered questions:
  - Is this 2.1 σ difference real? → Reduce errors
  - Why is δ_0 so different from the dispersive result? → Introduce more $\pi\pi$ operators to distinguish excited states

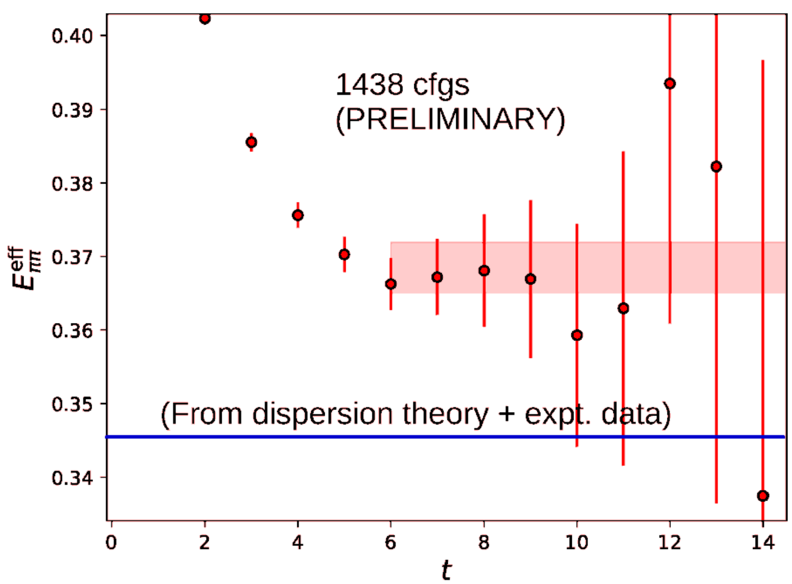
Extend and improve calculation
(Chris Kelly and Tianle Wang)

- ✓- Increase statistics: 216 → 1438 configs.
  - – Reduce statistical errors
  - – Allow in depth study of systematic errors
- ✓- Study operators neglected in our NPR implementation
- ✓- Use step-scaling to allow perturbative matching at a higher energy
- ✓- Use an expanded set of $\pi\pi$ operators
- Use X-space NPR to cross charm threshold (Masaaki Tomii).

Adding more statistics

- Increasing statistics: 216 → 1438 configs.
  - $\pi\pi - \pi\pi$ correlator well-described by a single $\pi\pi$ state
  - δ_0 = 23.8(4.9)(2.2)^o → 19.1(2.5)(1.2)^o χ^2 / DoF = 1.6





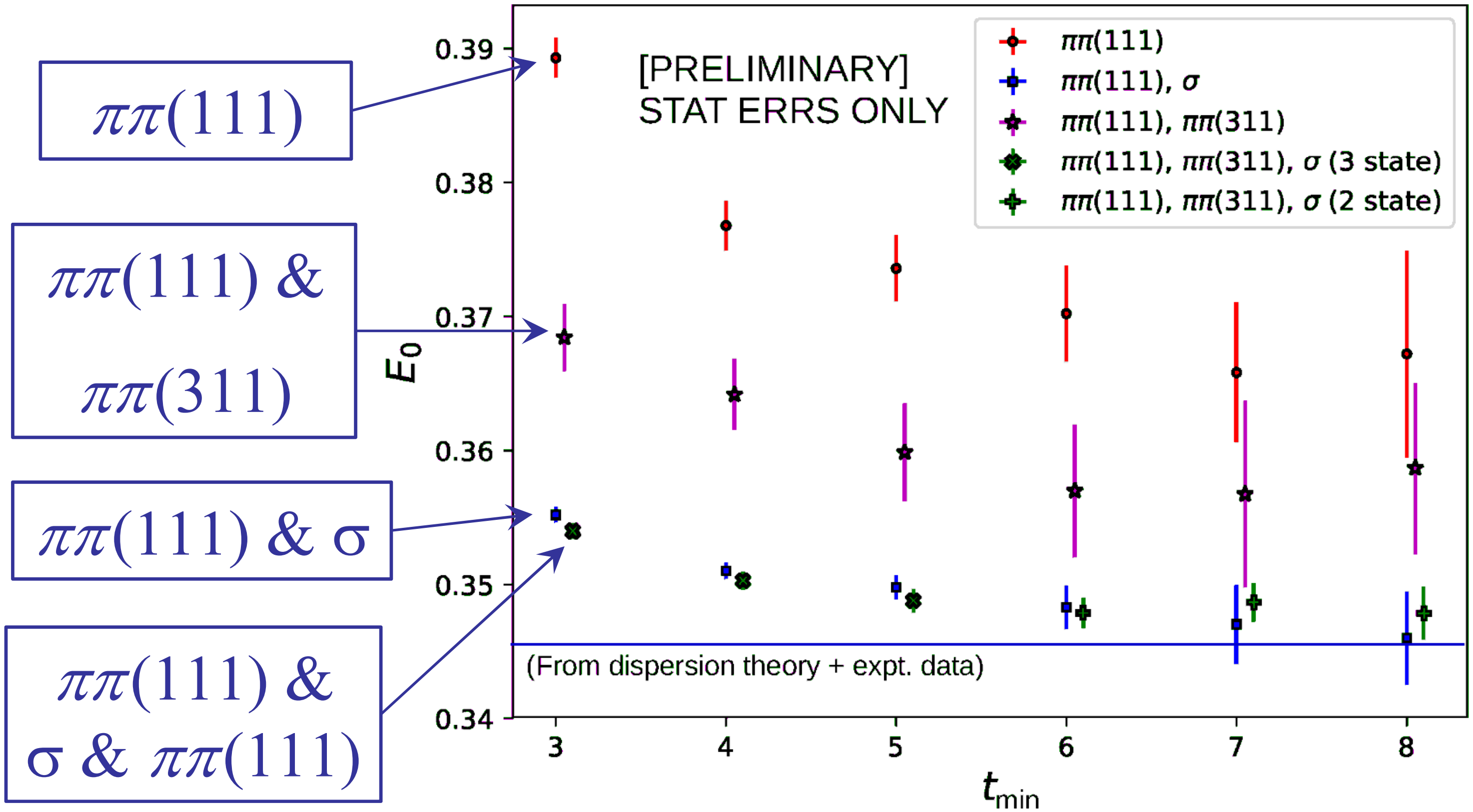
Adding more $\pi\pi$ operators

- Adding a second σ-like ($\bar{u}u+\bar{d}d$) operator reveals a second state!
- If only one state, 2 x 2 correlator matrix will have determinant = 0. For $t_f - t_i = 5$:

$$\det\begin{pmatrix} \langle \pi\pi(t_f)\pi\pi(t_i)\rangle & \langle \pi\pi(t_f)\sigma(t_i)\rangle \\ \langle \sigma(t_f)\pi\pi(t_i)\rangle & \langle \sigma(t_f)\sigma(t_i)\rangle \end{pmatrix} = 0.439(50)$$

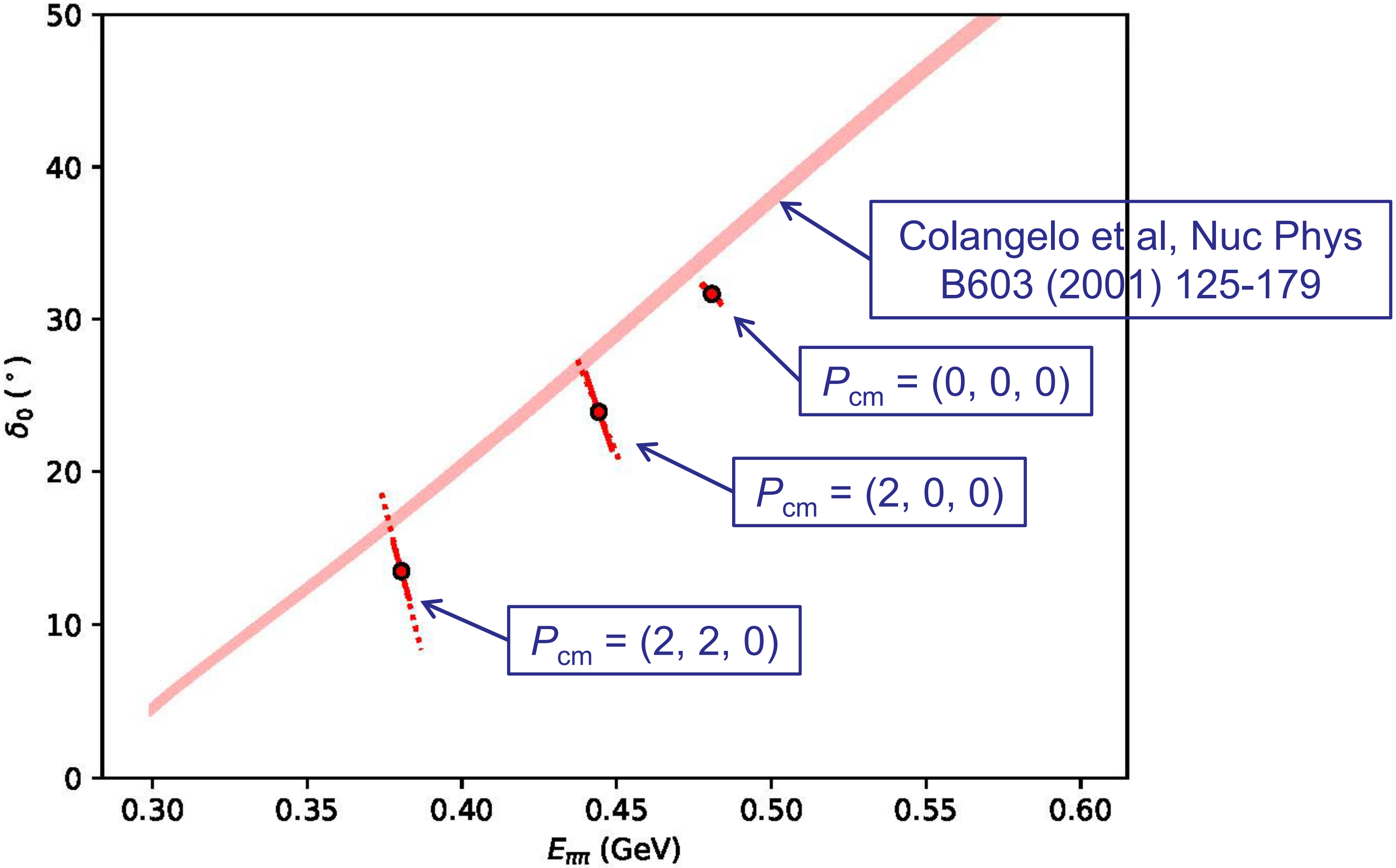
- Add a third operator giving each pion a larger momentum: $p = \pm\,(3,1,1)\,\pi/L$
- Label operators as $\pi\pi(111)$, σ, $\pi\pi(311)$
- Only 741 configurations with new operators

I = 0 $\pi\pi$ scattering with three operators



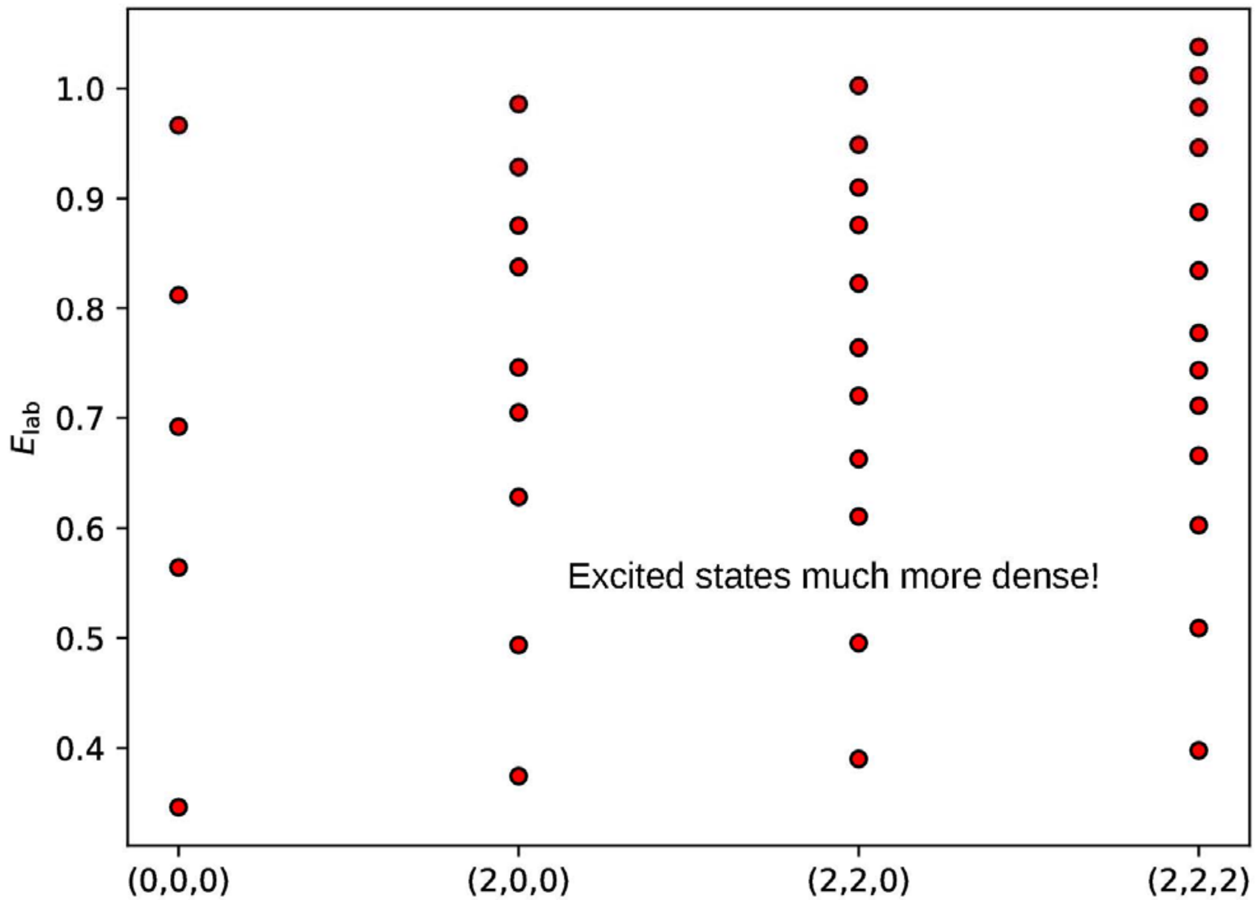
- Third $\pi\pi(311)$ operator not important.
- δ_0 = 31.7(6)° vs 34° prediction (5-15 fit, statistical errors only).

$I = 0$ $\pi\pi$ scattering with $P_{cm} \geq 0$ (*preliminary*)



$I = 0$ $\pi\pi$ scattering with $P_{cm} \geq 0$

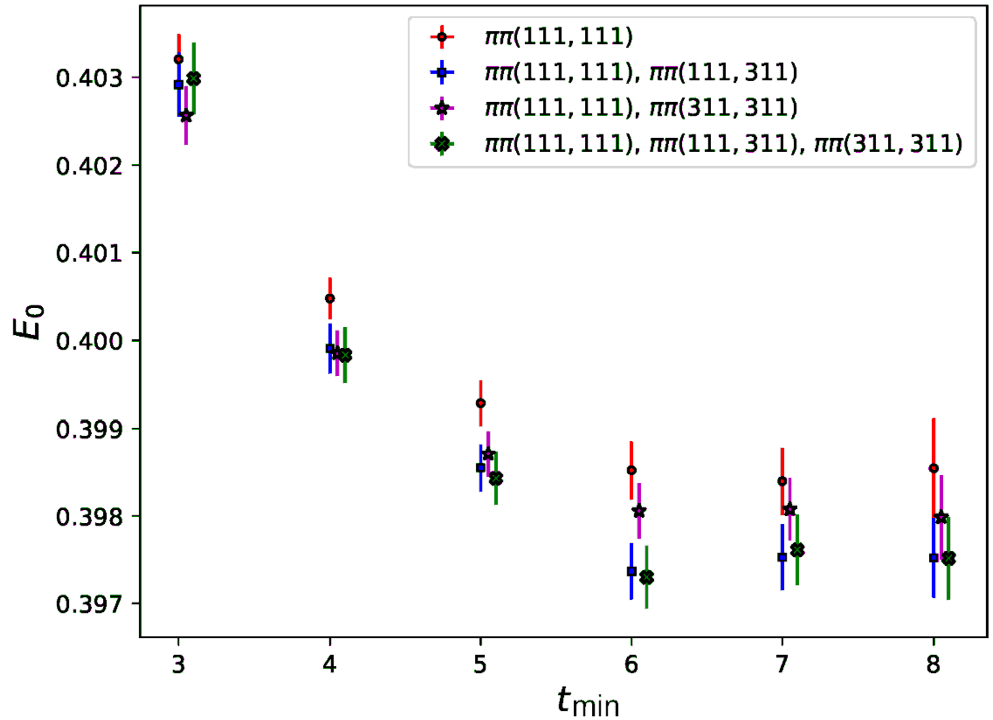
- Expect increased difficulty separating excited states for $P_{cm} \geq 0$.



$I = 0$ $\pi\pi$ scattering with $P_{cm} \geq 0$

- Failure of 3-operator fit easy to recognize:

$P_{cm} = (222)\pi/L$



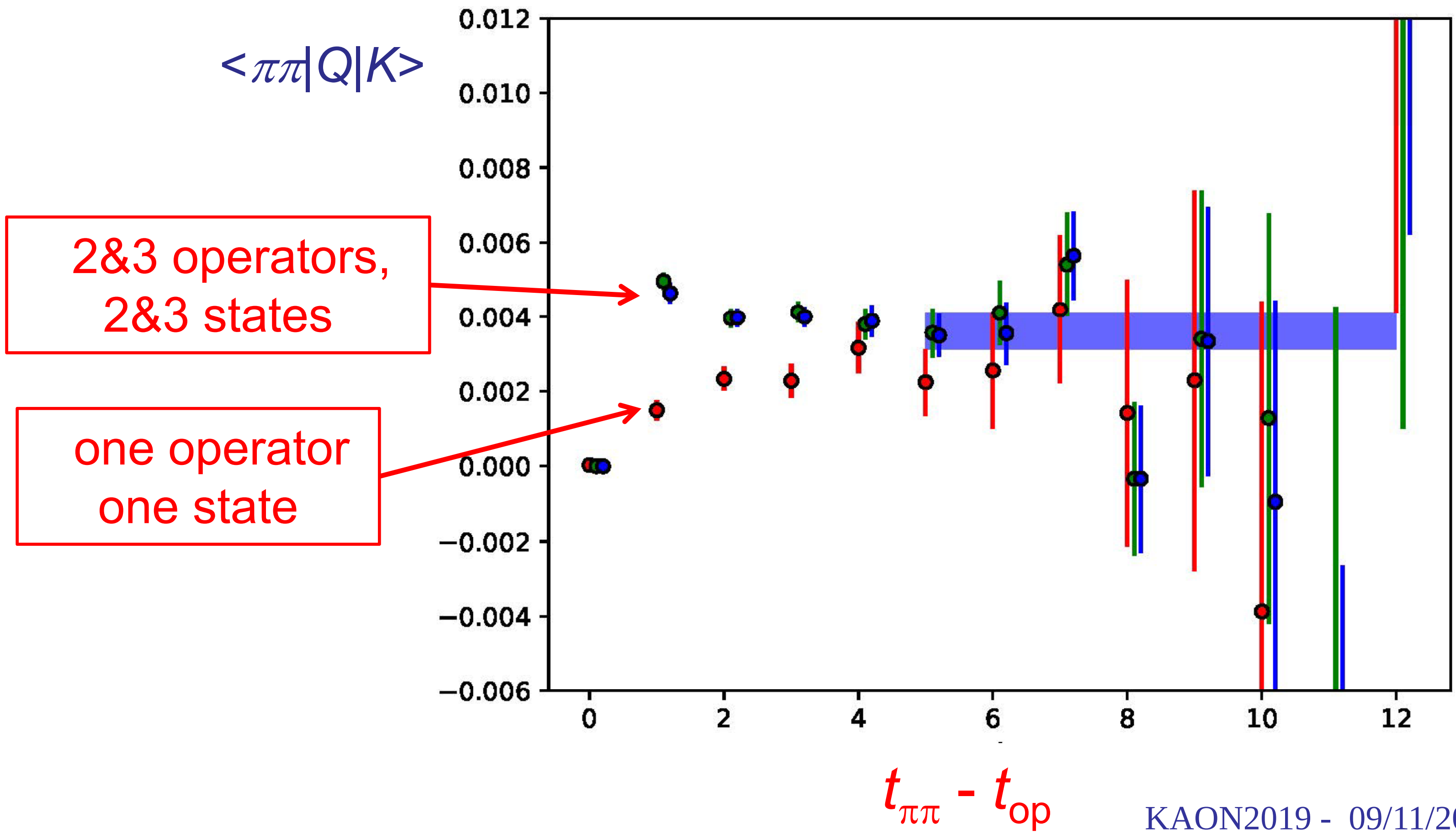
- Plateau does not extend to smaller *t* when extra operators are added.

$I = 0$ $\pi\pi$ scattering with $P_{cm} \geq 0$

- Plateau does not extend to smaller t when extra operators are added.
- The matrix of amplitudes $A_{|a>,O_b}$ is largely diagonal.
- The fit to each operator is effectively a single-state fit with the same problems as those in 2015.
- Perhaps the result having no moving σ operator implemented (yet)?

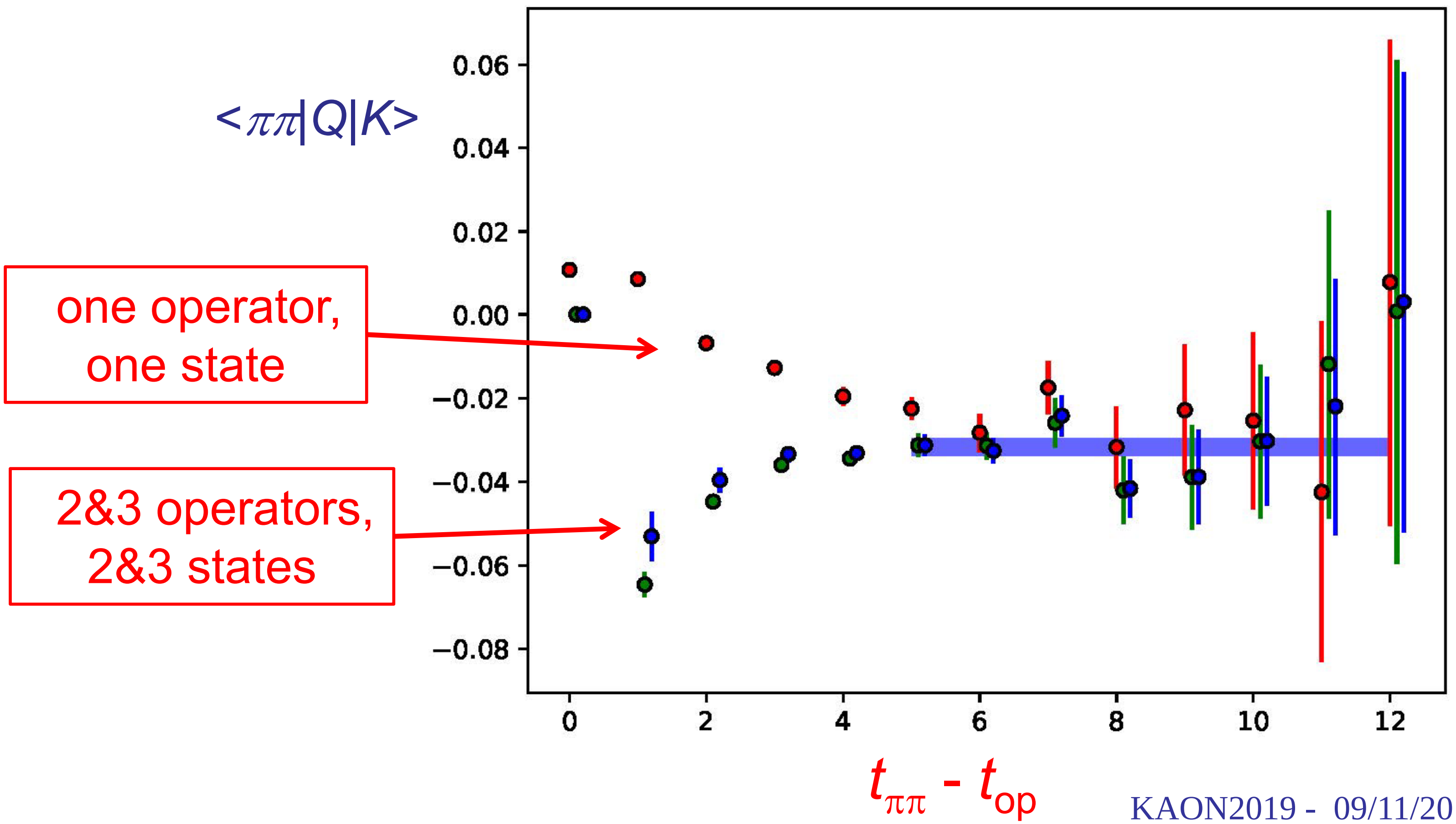
K → ππ from 3-operator fits (case I)

- Fit using up to 3 operators and 3 states with energies and amplitudes from ππ scattering:



K → $\pi\pi$ from 3-operator fits (case II)

- Fit using up to 3 operators and 3 states with energies and amplitudes from $\pi\pi$ scattering:



Two data analysis challenges

- Auto-correlations – we must be careful that our errors are accurate
- We need estimates of goodness of fit (*p*-values)
  - Demonstrate that our fits describe the data.
  - Decide if alternative fits used to estimate systematic errors are plausible.
  - However, our lattice QCD *p*-values are traditionally unreasonably small!

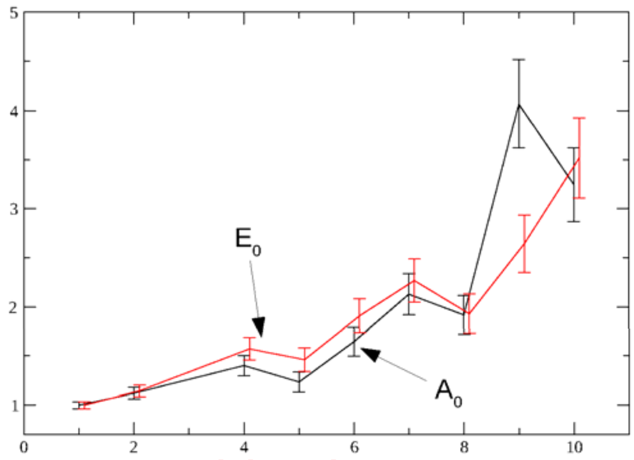
Auto-correlations

- Our measurements are made every 4 MD time units and are mildly correlated.
- While we have N=741 configurations, the covariance matrix for three operators and t = 5-15 time slices is 66 x 66!
- Noise grows as we bin the data and have fewer samples to measure the fluctuations.
- Solved by the *blocked jackknife* method:
  - Identify *N*/*B* blocks of size *B.*
  - Sequentially remove each block and analyze the remaining N-B (not N/B-1) samples

I=0 $\pi\pi$ two-point function errors

Binned data errors

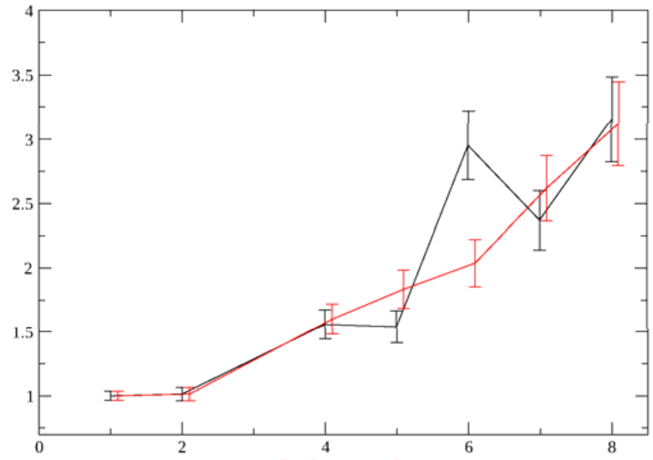
relative error



bin size

Binned scrambled data errors

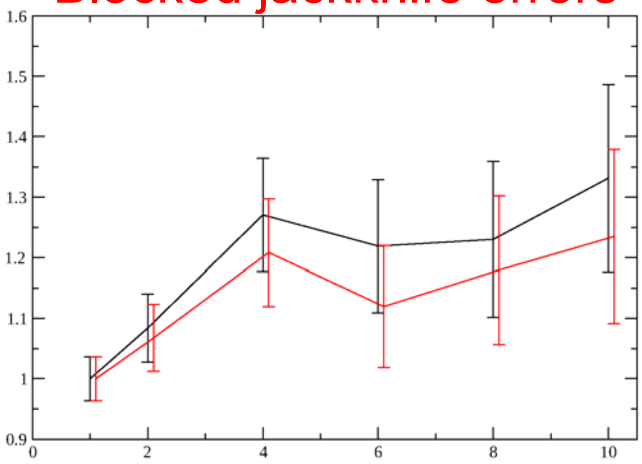
relative error



bin size

Blocked jackknife errors

relative error



block size

Poor *p*-values

- We obtain *p*-values of 0.1– 0.2 for most "best fits"!
- Last spring, Tanmoy Bhattacharya pointed out that this is often caused by ignoring fluctuations in the covariance matrix.
- This broadens the χ^2 distribution into the Hotelling T^2 distribution (related to F distrib.).

Hotelling T^2 is *insufficient*

- Hotelling assumes that the data (not their averages) are Gaussian and uncorrelated.
- Not true for our case.
- Use a bootstrap analysis to determine the correct generalized χ^2 distribution from the data. (C. Kelly)
- Use this correct χ^2 distribution to determine the *p*-value for the fit.

Conclusions

- Calculation of $K \rightarrow \pi\pi$ decay substantially improved over 2015 result.
- 216 → 741 configurations.
- Three $\pi\pi$ interpolating operators: discriminate between ground and excited states → $\delta_0\,(E=M_K) = 31.7(6)°$
- Errors reduced by using correlated fits.
- Auto-correlations are taken into account.
- Bootstrap-determined χ^2 distribution gives correct p-values. [p=0.261(BS) vs 0.037(χ^2)]
- Final results available very soon.