Higher order effects in epsilon′/epsilon

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Outline

Motivation

Theoretical Framework

NNLO QCD corrections

Phenomenology

Future improvements

Summary

Direct CPViolation Exists

A non-zero value of Re(ε'/ε) signals that direct CP Violation exists

$$Re\left(\frac{\varepsilon'}{\varepsilon}\right) = \frac{1}{6}\left(1 - \left|\frac{\eta_{00}}{\eta_{+-}}\right|^2\right)$$

The measured quantity is the double ratio of the decay widths

$$R = \left|\frac{\eta_{00}}{\eta_{+-}}\right|^2 = \frac{\Gamma(K_L \to \pi^0\pi^0)\Gamma(K_S \to \pi^+\pi^-)}{\Gamma(K_L \to \pi^+\pi^-)\Gamma(K_S \to \pi^0\pi^0)}$$

(a long series of precision counting experiments)

From NA48 and KTeV collaborations,

$$\left(\frac{\varepsilon'}{\varepsilon}\right)_{\text{exp}} = (16.6 \pm 2.3) \times 10^{-4}$$

ε'/ε Current Situation

Matrix elements can now be determined on the Lattice [Blum et. al., Bai et. al. `15]

$$\left(\frac{\varepsilon'}{\varepsilon}\right)_{\text{SM}} = (1.9 \pm 4.5) \times 10^{-4}$$

$$\left(\frac{\varepsilon'}{\varepsilon}\right)_{\text{exp}} = (16.6 \pm 2.3) \times 10^{-4}$$

2.9σ

[Buras, Gorbahn, Jäger, Jamin '15] **(using input from Lattice results)**

Tension between the theoretical prediction and the experimental data

NEW ANOMALY???

2.9σ What is the reason?

Is **New Physics** there ?

But, this could not be the whole story…

Missing SM ?
QCD corrections

$\langle Q_i \rangle$ off ?

Missing SM ?
EW corrections

Missing ?
QED corrections

Indeed, deeper Understanding of the SM is crucial.

Error budget for ε'/ε

B6= 0.57(19) & B8= 0.76(5)

Cancellation between QCD and EW penguin operators

$$\varepsilon'/\varepsilon = 10^{-4}\left[\frac{\mathrm{Im}\lambda_t}{1.4\times10^{-4}}\right]\left[a(1-\hat{\Omega}_{\mathrm{eff}})(-4.1(8)+24.7\,B_6^{(1/2)})+1.2(1)-10.4\,B_8^{(3/2)}\right]$$

I=0 (V-A)x(V-A) | I=0 (V-A)x(V+A) | I=2 (V-A)x(V-A) | I=2 (V-A)x(V+A)

Perturbative calculation.

Parametrise hadronic matrix elements.

[Blum et. al., Bai et. al. `15]

Isospin breaking.

Quantity	Error on ε'/ε	Quantity	Error on ε'/ε
$B_6^{(1/2)}$	4.1	$m_d(m_c)$	0.2
NNLO	1.6	q	0.2
$\hat{\Omega}_{\mathrm{eff}}$	0.7	$B_8^{(1/2)}$	0.1
p_3	0.6	$\mathrm{Im}\lambda_t$	0.1
$B_8^{(3/2)}$	0.5	p_{72}	0.1
p_5	0.4	p_{70}	0.1
$m_s(m_c)$	0.3	$\alpha_s(M_Z)$	0.1
$m_t(mt)$	0.3		

All in unites of 10^-4

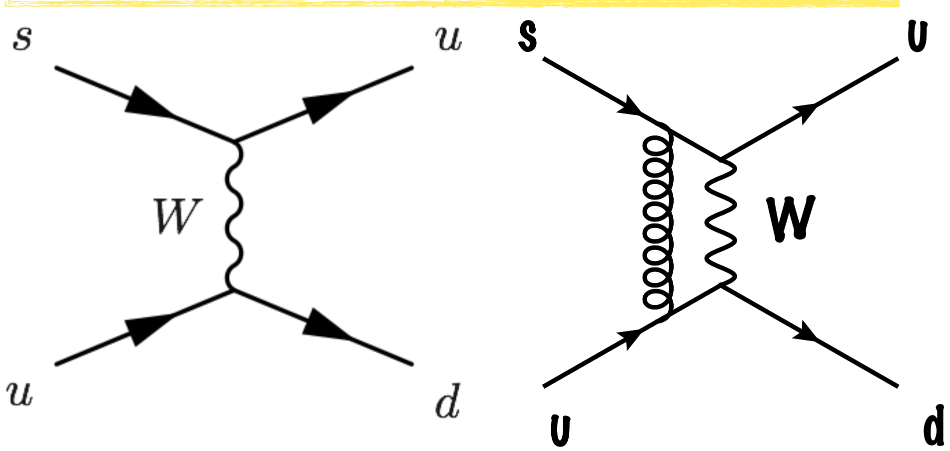
The error is completely dominated by the non-perturbative sector.

Lattice progress & perturbative error are only estimates

CPV in Kaon Decays

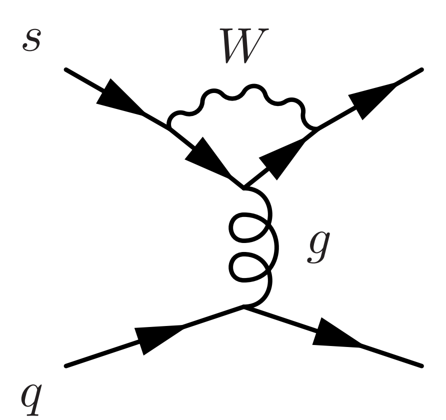
[Buras et. al., Ciuchini et. al. `92 `93]

Current-Current



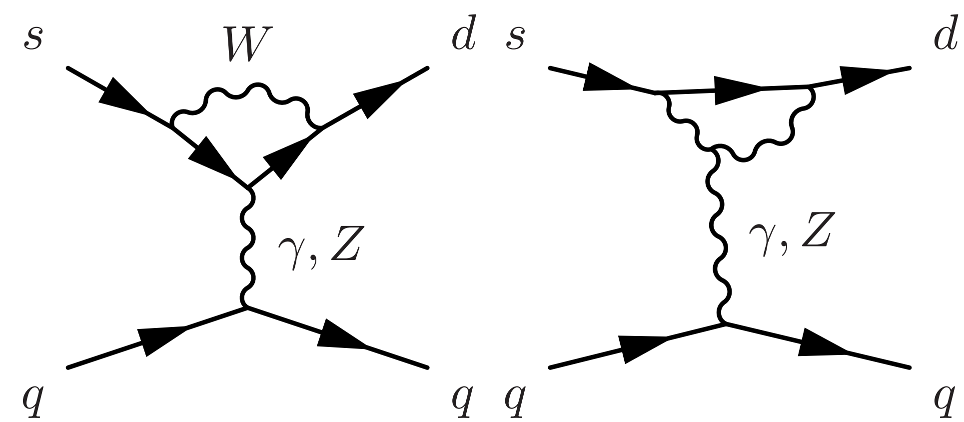
$\mathcal{O}_1, \mathcal{O}_2$

QCD-Penguins



$\mathcal{O}_3 - \mathcal{O}_6$

EW-Penguins



$\mathcal{O}_7 - \mathcal{O}_{10}$

$$V_{ij}^{\mathrm{CKM}} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \sim \mathcal{O}\begin{pmatrix} 1 & \lambda & \lambda^3 \\ \lambda & 1 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix}$$

$$\lambda = \mathcal{O}(0.2)$$

$$\mathbf{s} \rightarrow \mathbf{d} \quad \lambda^5 \sim 10^{-4}$$

The CP violation is small because of flavour suppression

Effective Field Theories

$$H_{eff} = V_{CKM} \sum C_i(\mu)\, O_i$$

short-distance   Long-distance

Weak Effective Theory

Effective Hamiltonian at μ < m_c

$$\mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}} V_{ud} V^*_{us} \sum_{i=1}^{10} (z_i(\mu) + \tau y_i(\mu)) Q_i$$

$$\tau \equiv -\frac{V_{td}V^*_{ts}}{V_{ud}V_{us}}$$

perturbative Wilson coeffs.

Only the Imaginary part of τ is responsible for CPV

(everything else is pure-real)

Current-Current $Q_{1,2/\pm} = (\bar{s}_i u_j)_{\text{V-A}} (\bar{u}_k d_l)_{\text{V-A}}$

QCD-Penguins $Q_{3,\ldots,6} = (\bar{s}_i d_j)_{\text{V-A}} \sum_{q=u,d,s} (\bar{q}_k q_l)_{\text{V}\pm\text{A}}$

EW-Penguins $Q_{7,\ldots,10} = (\bar{s}_i d_j)_{\text{V-A}} \sum_{q=u,d,s} e_q (\bar{q}_k q_l)_{\text{V}\pm\text{A}}$

Long Distance

[Blum et. al., Bai et. al. `15]

Lattice QCD calculation of the Matrix Elements

$$\langle (\pi\pi)_I | Q_i | K \rangle = \langle Q_i \rangle_I$$

by RBC-UKQCD.

They are renormalised non-perturbatively in the RI-SMOM scheme.

Match to the traditional operator basis in the continuum MS-bar renormalisation scheme using NDR:

$$\langle Q_i \rangle^{\overline{\text{MS}}}(\mu_L) = [T^{(0)} + \frac{\alpha_s(\mu_L)}{4\pi} T^{(1)}(\mu_L)]_{ij} \langle Q_j \rangle^{\text{RI-SMOM}}$$

[Sturm, Lehrer '11]

The scheme change to the MS-bar scheme is only known at NLO

Short Distance

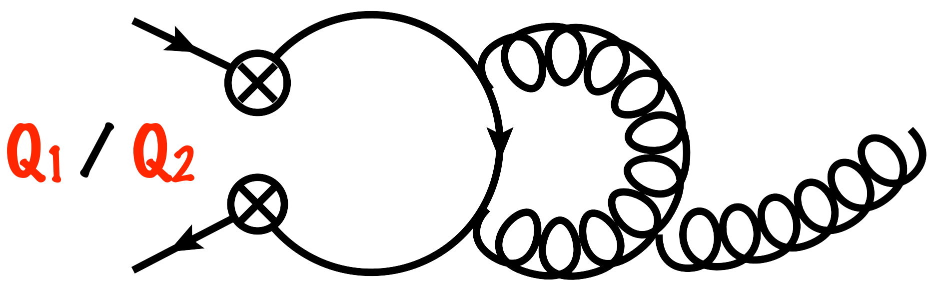
i) [Misiak, Bobeth, Urban]
ii) [Gambino,Buras, U.H]
iii) [Gorbahn, Haisch]
iv) [Gorbahn, Brod]
v) [Buras, M.Jamin, M.E.L]
vi) [Blum et. al., Bai et. al. ‘15]

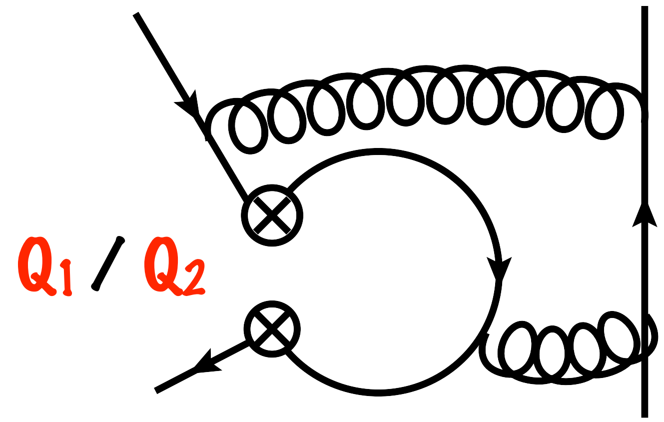
Traditional Basis **Modern Basis**

	Energy scale	Fields	Effective Theory	
Matching at μ_W	M_W	γ,g, H τ, μ, e, ν_i b, c, s, d, u	A5	i) NNLO QCD + Current-Current ii) NNLO EW penguins iii) NNLO in QCD ADM/RGE
Matching at μ_b	m_b	γ,g τ, μ, e, ν_i c, s, d, u	A4	iv) NNLO QCD + Current-Current iii) NNLO in QCD ADM/RGE
Matching at μ_c	m_c	γ,g μ, e, ν_i s, d, u	A3	v) NLO All operators iii) NNLO in QCD ADM/RGE
Lattice QCD	$M_{Lattice}$	u, s, d	Lattice QCD	v) NLO for all operators except Q8g

Charm Matching at NNLO

Q_1 & Q_2 have the largest Wilson Coefficients





The traditional basis requires the calculation of traces with γ5

$$Q_5 = (\bar{s}_i d_j)_{\mathrm{V-A}} \sum_{u,d,s} (\bar{q}_k q_l)_{\mathrm{V+A}}$$

→ **Issues with the treatment of the γ5 in D dimensions**

Higher order calculations can be significantly simplified if we use a different operator basis: Modern basis

$$\mathcal{O}_5 = (\bar{s}\gamma_\mu\gamma_\nu\gamma_\sigma P_L d) \sum_{u,d,s} (\bar{q}\gamma^\mu\gamma^\nu\gamma^\sigma q)$$

[Misiak et al.]

Wilson coefficients at μ$_L$=1.3 GeV

The perturbative results have the following structure

$$\vec{C}^{(3)}(\mu_L) = U^{(3)}(\mu_L, \mu_c) M^{(34)}(\mu_c) U^{(4)}(\mu_c,\mu_b) M^{45}(\mu_b) U^5(\mu_b, \mu_W) \vec{C}^{(5)}(\mu_W)$$

Observables do not depend on μ-scale

$$\sum_i \vec{C}_i^{(3)}(\mu_L)\langle Q_i\rangle(\mu_L)$$

Alternatively, it can be also factorised in terms of scheme and scale independent quantities:

$$\langle\vec{Q}\rangle^{(3)}(\mu_L)\vec{C}^{(3)}(\mu_L) = \langle\hat{\vec{Q}}\rangle^{(3)} \hat{M}^{(34)} \hat{M}^{(45)} \hat{\vec{C}}^{(5)}$$

$$\langle\vec{Q}\rangle^{(3)}(\mu_L) u^{(3)}(\mu_L)$$

$$u^{(3)-1}(\mu_c) M^{(34)}(\mu_c) u^{(4)}(\mu_c)$$

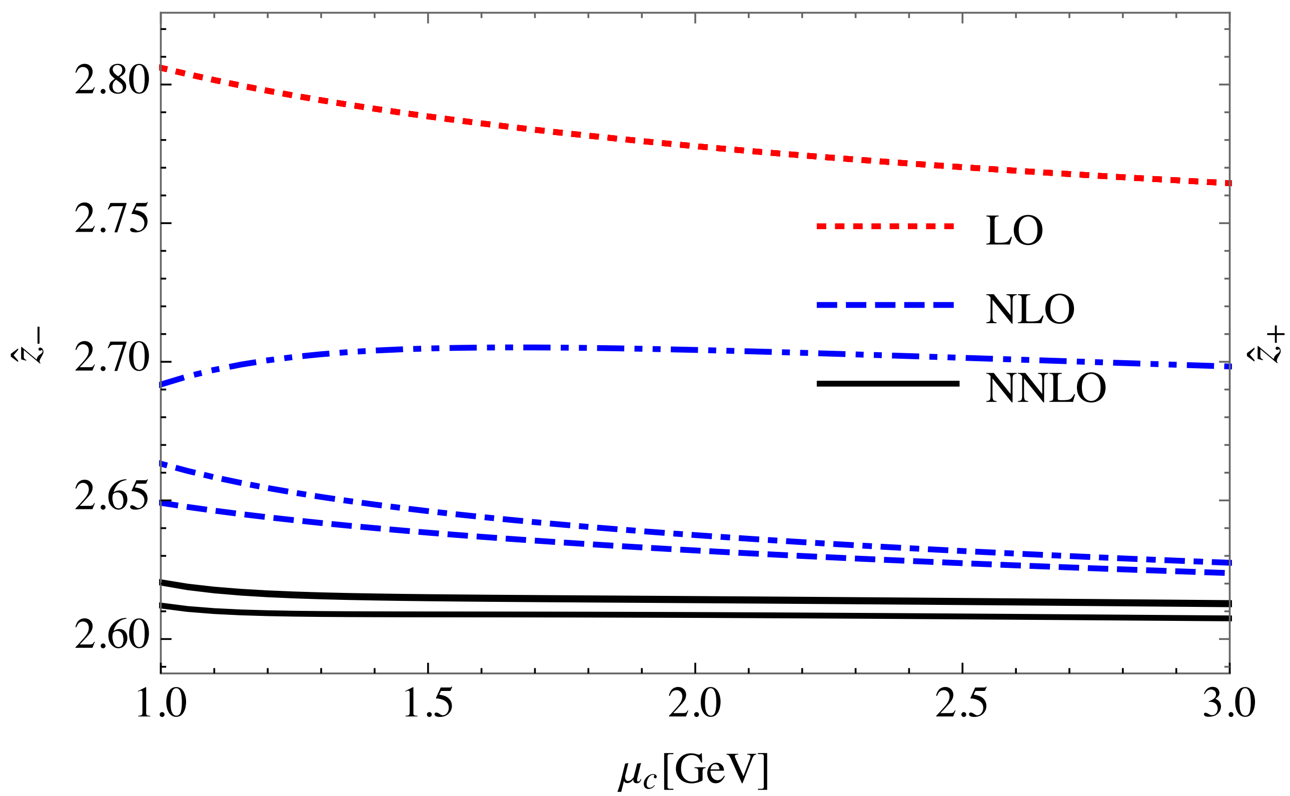
$$u^{(4)-1}(\mu_c) M^{(45)}(\mu_b) u^{(5)}(\mu_b)$$

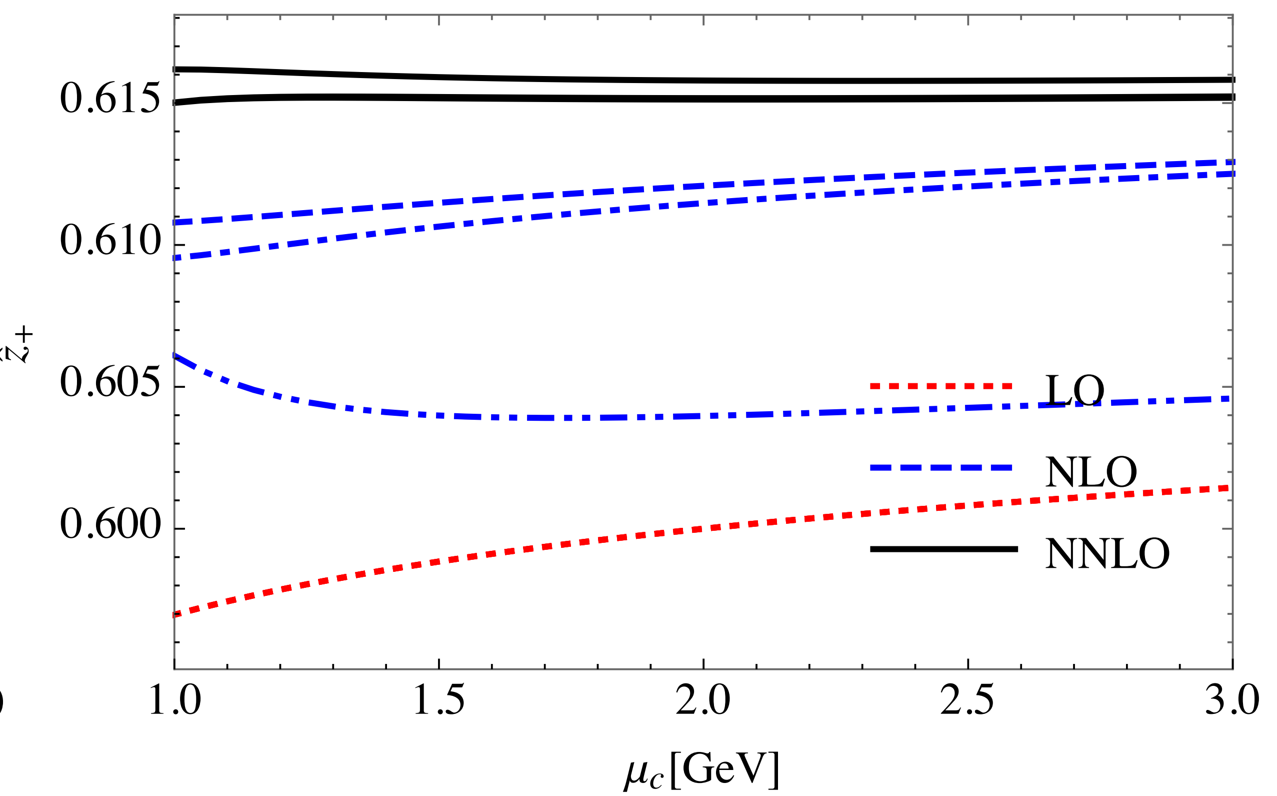
$$u^{(5)}(\mu_W)\vec{C}^{(5)}(\mu_W)$$

Current- Current $\hat{z}_i$

[Cerdà-Sevilla, Gorbahn, Jäger, Kokulu]

The largest coefficients correspond to the current-current terms $\hat{z}_+$ and $\hat{z}_-$





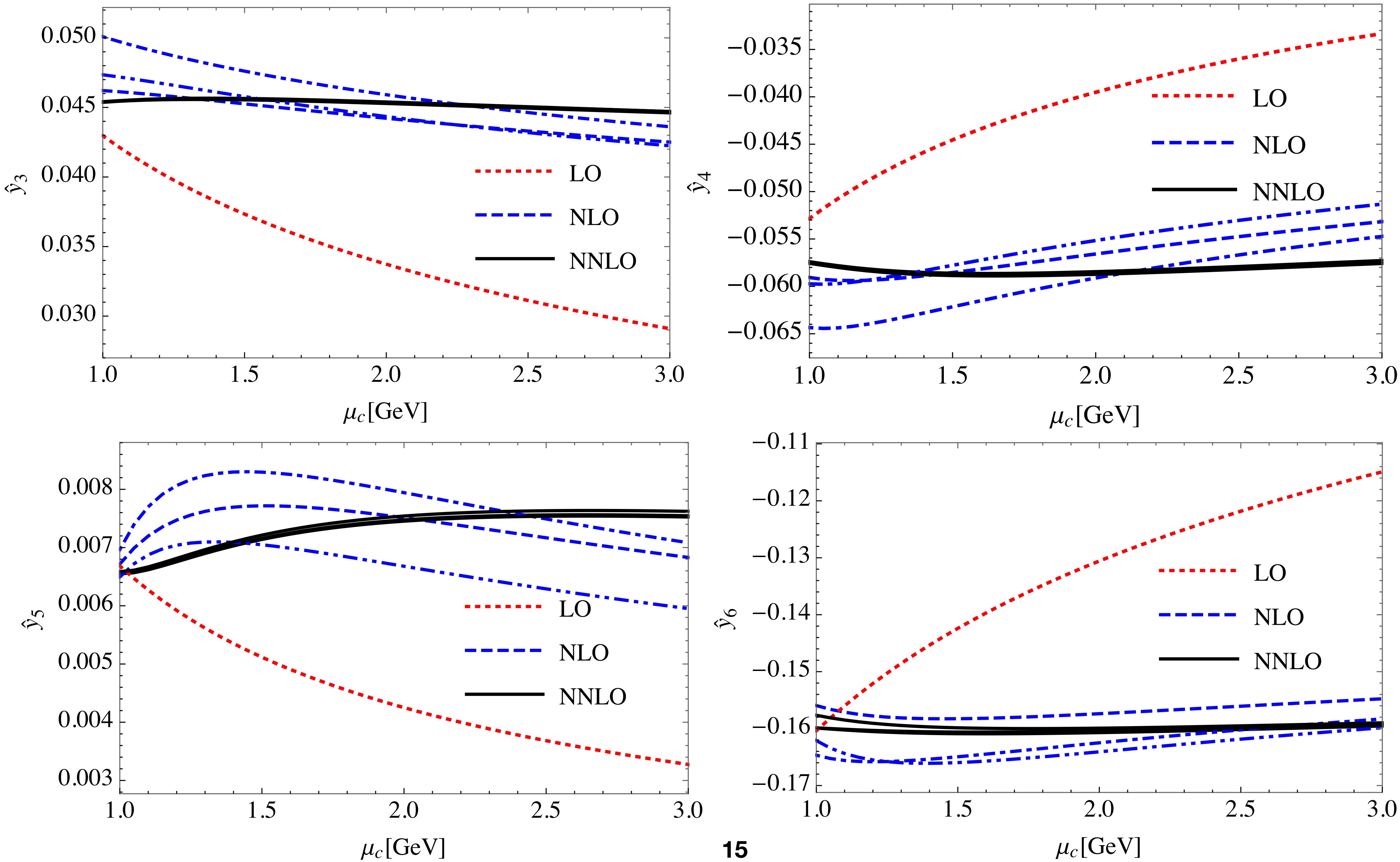
The residual μ_c scale dependence reduces order by order.

No significant shift appears between the different orders.

At NLO there is still a dependence on the implementation of α_s Running.

$\hat{y}_i$

[Cerdà-Sevilla, Gorbahn, Jäger, Kokulu]



Phenomenology

εpsilon'/εpsilon in the SM

$$\langle \pi^0\pi^0|\mathcal{H}_{\rm eff}|K^0\rangle = A_0 e^{i\delta_0} + A_2 e^{i\delta_2}/\sqrt{2}$$
$$\langle \pi^+\pi^-|\mathcal{H}_{\rm eff}|K^0\rangle = A_0 e^{i\delta_0} - A_2 e^{i\delta_2}/\sqrt{2}$$
$$\langle \pi^+\pi^0|\mathcal{H}_{\rm eff}|K^0\rangle = 3A_2^+ e^{i\delta_2^+}/2$$

A_0 & A_2 : Isospin amplitudes for isospin conservation

A_0, A_2 & A_2^+ from experiment
[Cirigliano, et. al. `11]

Normalise to K^+ decay (ω_+, a)
and ε_K
expand in A_2/A_0 and CP violation

Lattice QCD gives us:

$$A_I = \sum_j f(V_{\rm CKM})\mathcal{C}_i \langle (\pi\pi)_I|\mathcal{O}_j|K\rangle$$

The CPV is parametrised as,

Adjusted to keep EW in Im(A0)

$$\varepsilon'/\varepsilon = -i\frac{\omega_+}{\sqrt{2}|\varepsilon_K|}e^{i(\delta_2-\delta_0-\phi_{\varepsilon_K})}\left[\frac{\mathrm{ImA_0}}{\mathrm{ReA_0}}(1-\hat{\Omega}_{\rm eff}) - \frac{1}{a}\frac{\mathrm{ImA_2}}{\mathrm{ReA_2}}\right]$$

[Buras, Gorbahn, Jäger, Jamin `15]** **[Cirigliano, et.al. `11]

Minimizing non-perturbative input

Better control over
$\langle Q_i \rangle_2$
on Lattice

Operators identities

only 7 operators

$\mathrm{Re}A_i$

CP-conserving Data

Electroweak penguin
are suppressed
in I=0 (α_e/α_s)

QCD penguins
cannot
create I=2

Colour hierarchies
Between
$\langle Q_i \rangle \quad \hat{z}_i \quad \hat{y}_i$

QCD penguins
dominate ImA0

ImA2
due to
EW penguins

Broken by QED and $m_u \neq m_d$

estimated separately through

$\mathrm{ImA_0/ReA_0}$

The QCD penguin operators give the dominant contribution to

$$\mathrm{ImA_0}$$

Fierz relations for (V-A)x(V-A) give, e.g.: **dominated by short distance**

$$\left(\frac{\mathrm{ImA_0}}{\mathrm{ReA_0}}\right)_{\text{V-A}} = \mathrm{Im}\tau\,\frac{(2\hat{y}_4 - b[3\hat{y}_9 - \hat{y}_{10}])}{(1+\hat{q})\hat{z}_-} + \mathrm{Im}\tau\;b\,\frac{3[\hat{y}_9+\hat{y}_{10}]\hat{q}}{2(1+\hat{q})\hat{z}_+}$$

is only a function of the Wc's and the ratio

$$\hat{q} = \hat{z}_+\langle\hat{Q}_+\rangle_0/\hat{z}_-\langle\hat{Q}_-\rangle_0$$

For (V-A)x(V+A) operators: **dominated by long distance**

$$\left(\frac{\mathrm{ImA_0}}{\mathrm{ReA_0}}\right)_{\text{V+A}} = -\frac{\mathrm{G_F}}{\sqrt{2}}\mathrm{Im}\lambda_t\hat{y}_6\,\frac{\langle\hat{Q}_6\rangle_0}{\mathrm{ReA_0}}$$

CP-conserving Data

$\mathrm{ImA_2/ReA_2}$

The EW penguin operators give the dominant contribution to

$$\mathrm{ImA}_2$$

For (V-A)x(V-A) structure

Free from hadronic uncertainties.

$$\left(\frac{\mathrm{ImA}_2}{\mathrm{ReA}_2}\right)_{\mathrm{V\text{-}A}} = \mathrm{Im}\tau\,\frac{3(\hat{y}_9+\hat{y}_{10})}{2\hat{z}_+}$$

For (V-A)x(V+A) operators:

Small effects of ME Q7 I=2.

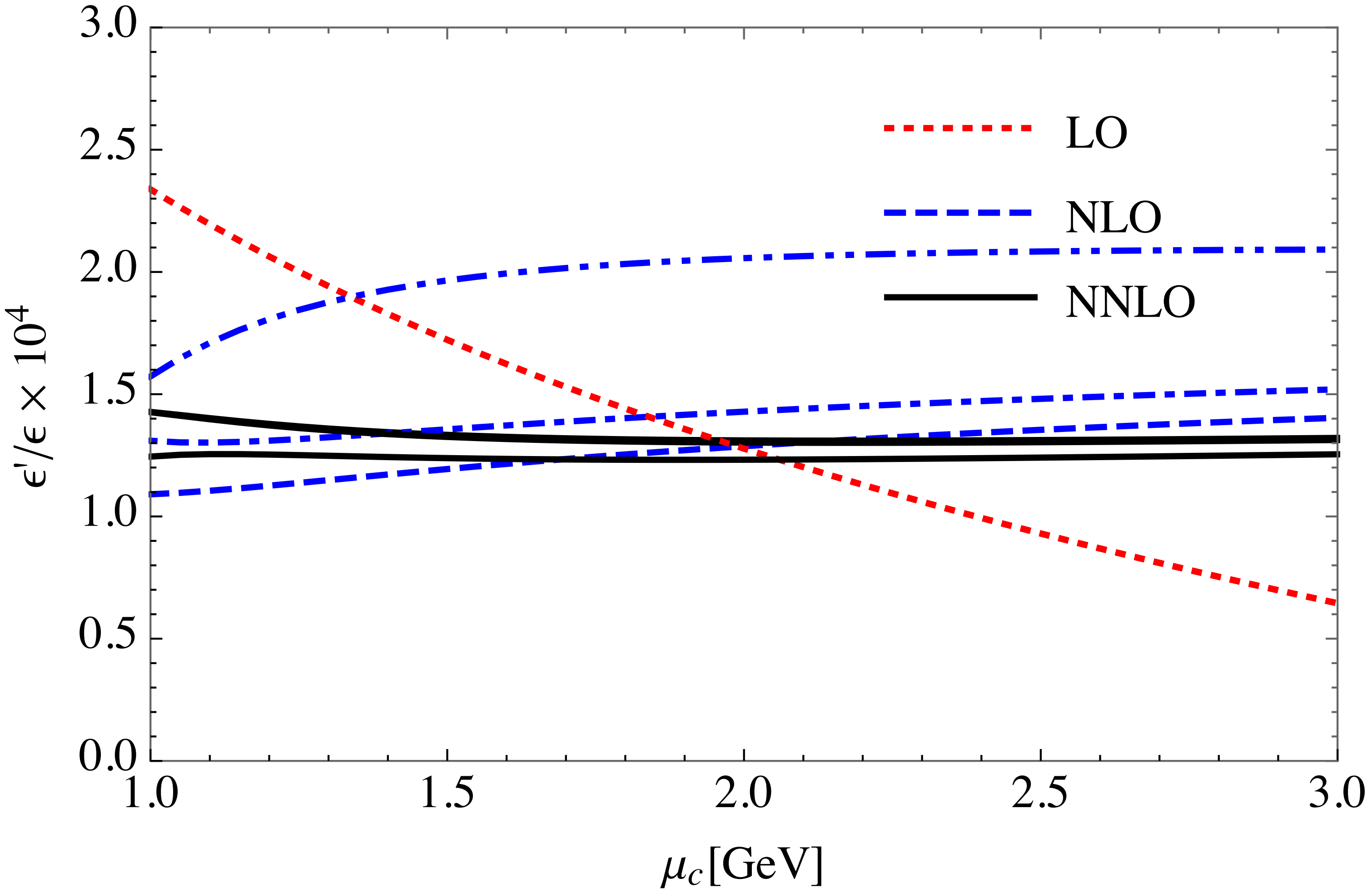
$$\left(\frac{\mathrm{ImA}_2}{\mathrm{ReA}_2}\right)_{\mathrm{V+A}} = -\frac{\mathrm{G}_F}{\sqrt{2}}\mathrm{Im}\lambda_t\;\hat{y}_8^{\mathrm{eff}}\,\frac{\langle\hat{Q}_8\rangle_2}{\mathrm{ReA}_2}$$

CP-conserving Data

ε'/ε & μ_c

[Cerdà-Sevilla, Gorbahn, Jäger, Kokulu]

Residual μ_c scale dependence



- Uncertainty is significantly reduced by going to NNLO

- There are still improvements: e.g. better as implementation & better incorporation of sub-leading corrections.

Future Improvements

Dynamical charm

No evidence for a failure of perturbation theory at the charm scale.

Non-perturbative virtual-charm effects ?

Lattice simulations with dynamical charm are becoming feasible.

From our computed threshold corrections we can provide an estimation of the four-flavour matrix elements:

$$\langle \hat{Q}_i^{(3)} \rangle \hat{C}_i^{(3)} = \langle \hat{Q}_i^{(3)} \rangle \hat{M}_{ij}^{(4)} \hat{C}_j^{(4)} = \langle \hat{Q}_j^{(4)} \rangle \hat{C}_j^{(4)}$$

The formula for ε'/ε has to be modified at the four-flavour theory.

Isospin Breaking effects

. The isospin limit is not very good: O(10%) corrections (Ω_{eff} & a).

Pion are not exact I=1 states

&

electromagnetic effects cannot be neglected

. The $\delta_{0,2}$ are still defined in the isospin limit.

Watson's theorem is only valid when isospin is conserved.

. One matches a QCDxQED evolution to a pure QCD lattice calculation.

Electromagnetism into $\langle Q_i \rangle$

IR-problem

Summary

- ε'/ε at NLO perturbation theory with RBC-UKQCD matrix elements shows a tension with the data.

- New NNLO calculation of the non-EW-penguin part of the weak Hamiltonian does not move the central value.

- ε'/ε can be expressed in terms of RGI objects, to achieve a fuller factorisation between perturbative and non-perturbative pieces.

- Formalism can be extended to $n_f=4$ dynamical quarks.

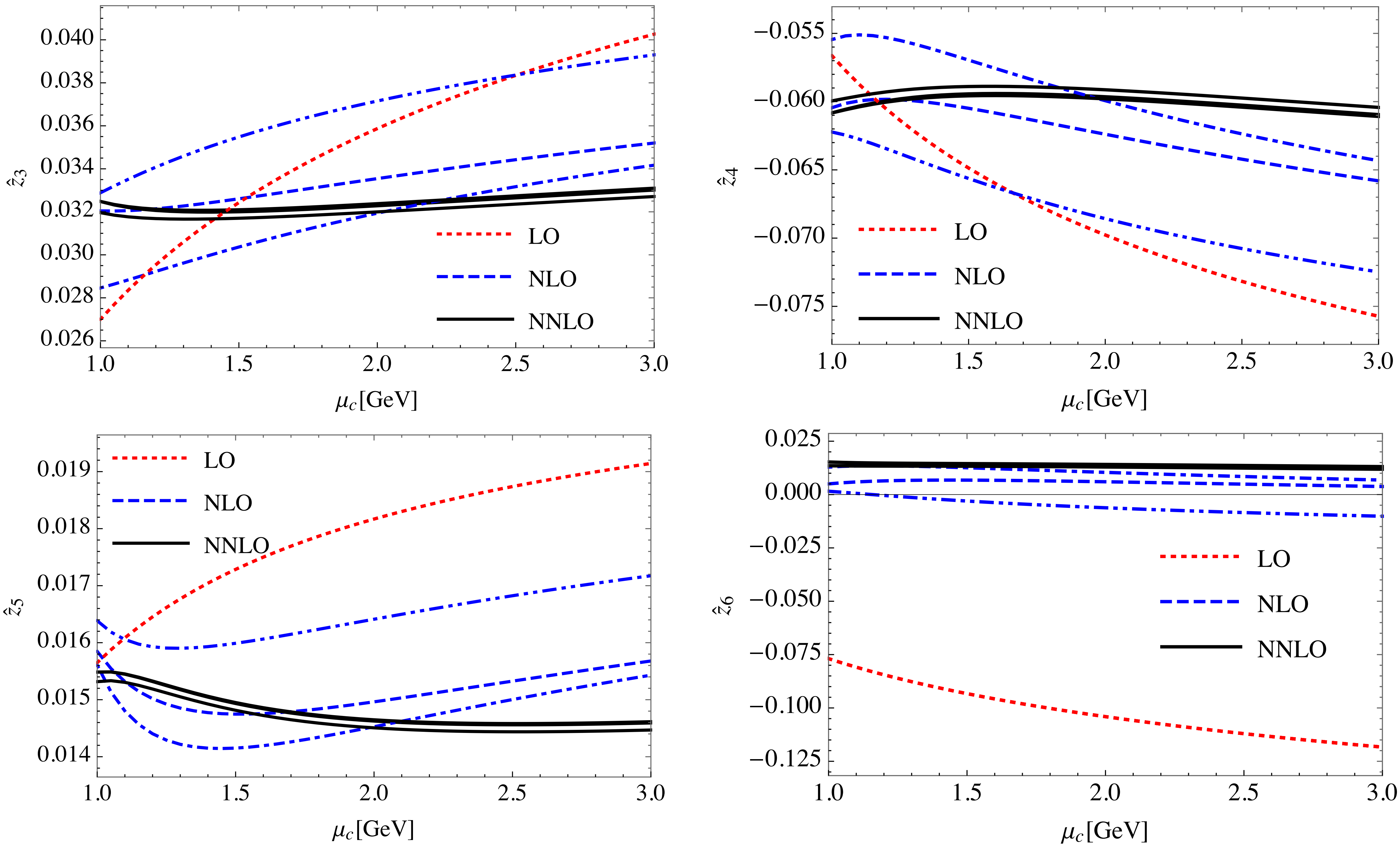
- EW NNLO including systematic treatment of O(α_e) (as well as $m_u \neq m_d$) about the isospin limit are next steps on perturbative side.

Backups

Input for the numerics

	value range
M_W	80.403 GeV
M_Z	91.1876 GeV
$\alpha_s(M_Z)$	0.1181 ± 0.0011
$m_t(m_t)$	(163.4 ± 2.0) GeV
$m_b(m_b)$	(4.18 ± 0.3) GeV
$m_c(m_c)$	(1.280 ± 0.025) GeV
μ_t	120 GeV
μ_b	5 GeV
μ_c	1.5 GeV
$\mathrm{Im}\lambda_t$	$(1.4 \pm 0.1) \times 10^{-4}$
G_F	1.1663787×10^{-5} GeV^{-2}
V_{us}	0.2248(6)
$\hat{\Omega}_{\text{eff}}$	$(14.8 \pm 8.0) \times 10^{-2}$

QCD-penguins $\hat{z}_i$



Fierz-Identities

In general, this identities are violated in dimensional regularisation.

3-flavour theory

O(α_s) corrections

$$0 = -Q_4 + Q_3 + 2\,Q_- \,,$$

$$0 = -Q_9 + \frac{3}{2}(Q_+ - Q_-) - \frac{1}{2}Q_3 \,,$$

$$0 = -Q_{10} + \frac{3}{2}Q_+ + \frac{1}{2}(Q_- - Q_3) \,.$$

But, they are satisfied by the RGI-operators at all orders in perturbation theory.

Real Parts

CP-conserving data

$$\mathrm{ReA}_2 = 1.48 \times 10^{-8}\mathrm{GeV}$$

$$\mathrm{ReA}_0 = 33.2 \times 10^{-8}\mathrm{GeV}$$

ReA_0 & ReA_2 at tree-level within the SM

ReA_0 & ReA_2 only marginally affected by NP.

The current-current operators give the dominant contribution to ReA_i

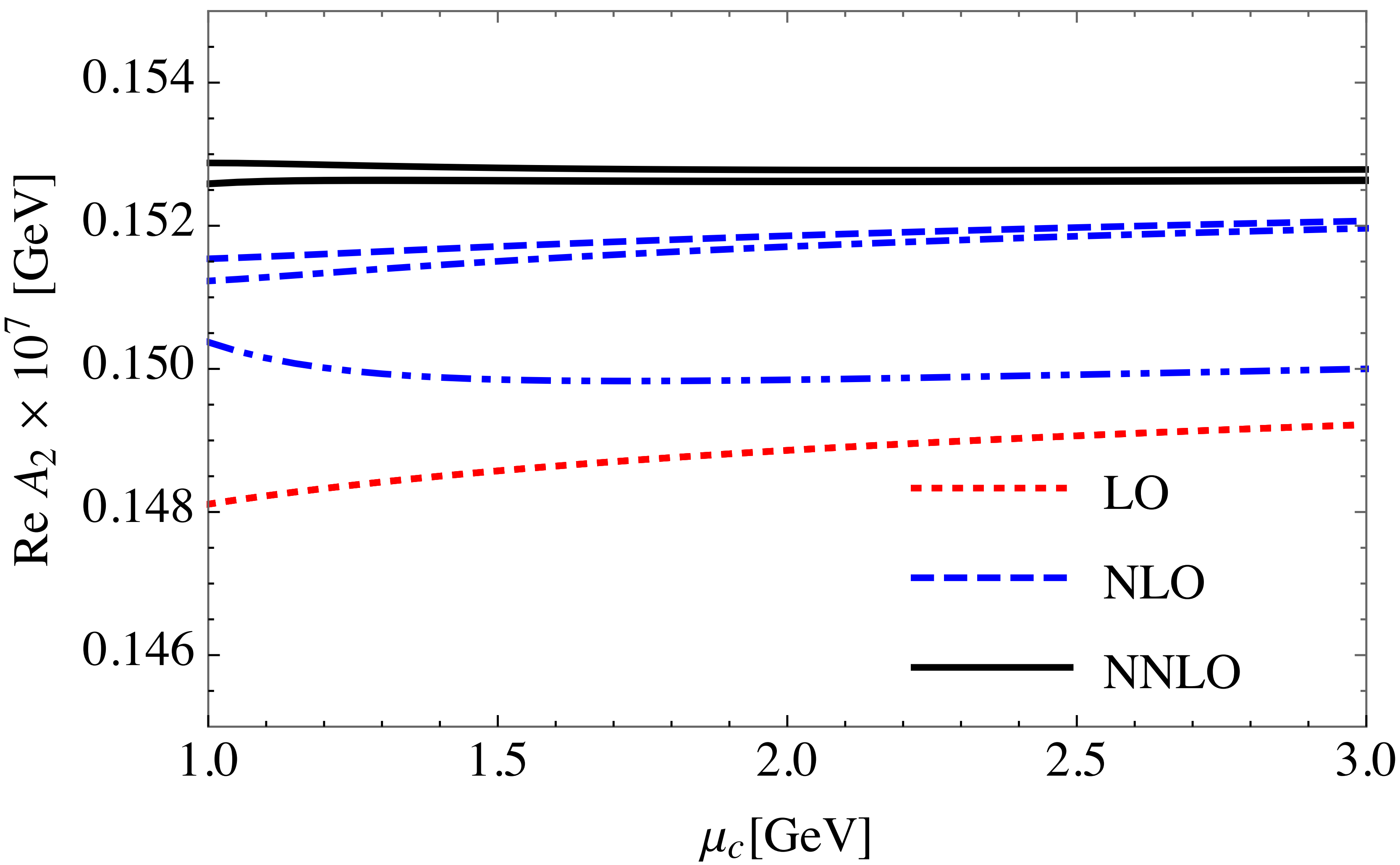
$Q_- \; Q_3 \; Q_5 \; Q_6$ **are pure I=1/2** $\longrightarrow$ $\langle Q_-\rangle_2 = \langle Q_3\rangle_2 = \langle Q_5\rangle_2 = \langle Q_6\rangle_2 = 0$

$$\mathrm{ReA}_2 = \frac{\mathrm{G_F}}{\sqrt{2}}\lambda_u \hat{z}_+ \langle \hat{Q}_+\rangle_2$$

$$\mathrm{ReA}_0 = \frac{\mathrm{G_F}}{\sqrt{2}}\lambda_u (\hat{z}_+ \langle \hat{Q}_+\rangle_0 + \hat{z}_- \langle \hat{Q}_-\rangle_0)$$

ReA$_2$

Residual μ_c scale dependence



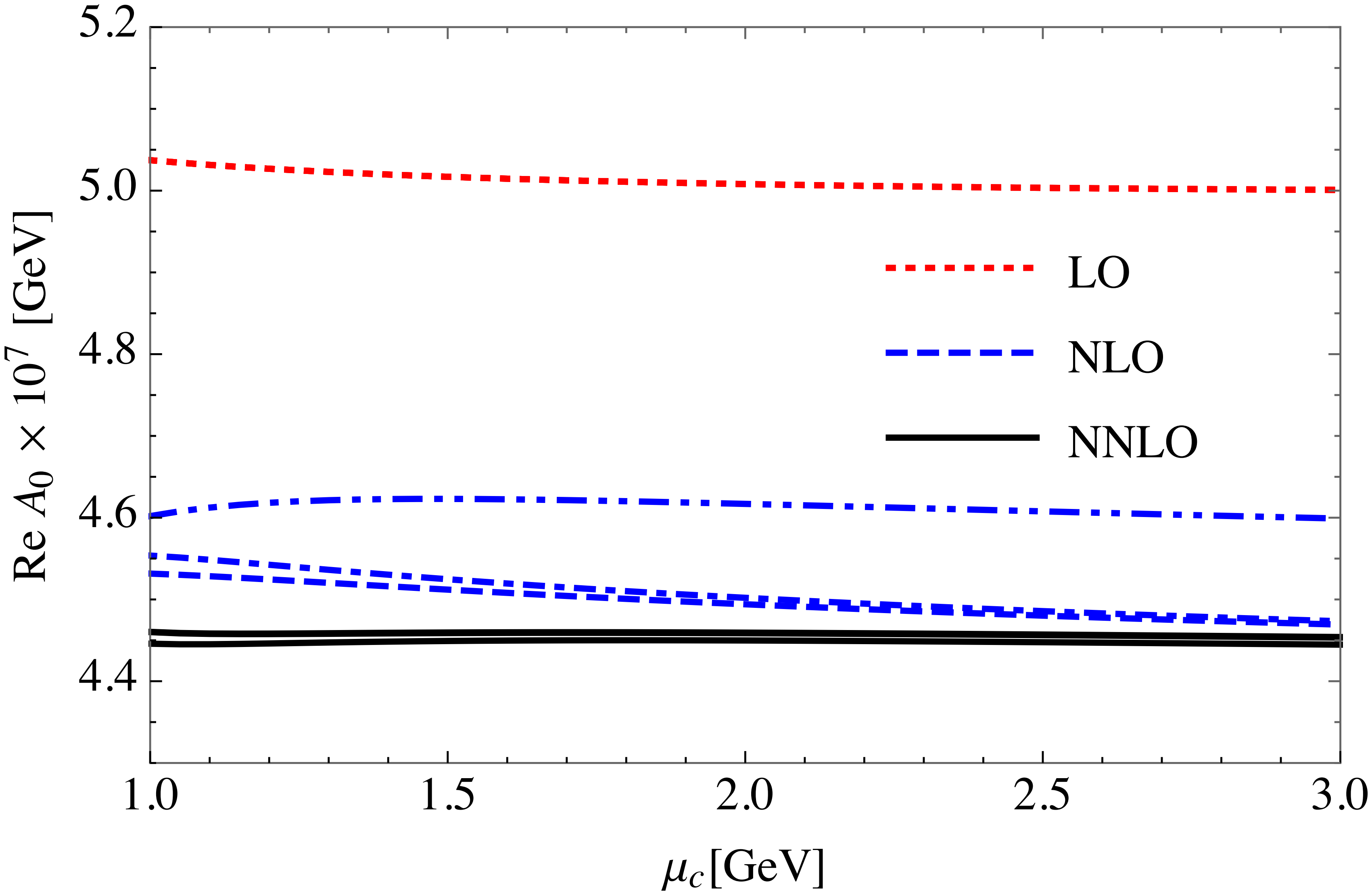
$$\mathrm{ReA}_2|_{\mathrm{SM}} = 1.526(87)(17) \times 10^{-8}\mathrm{GeV}$$

Non-perturbative error

NNNLO Corrections

ReA0

Residual μ_c scale dependence



Lattice input has still 20%-25% stat/sys. uncertainty

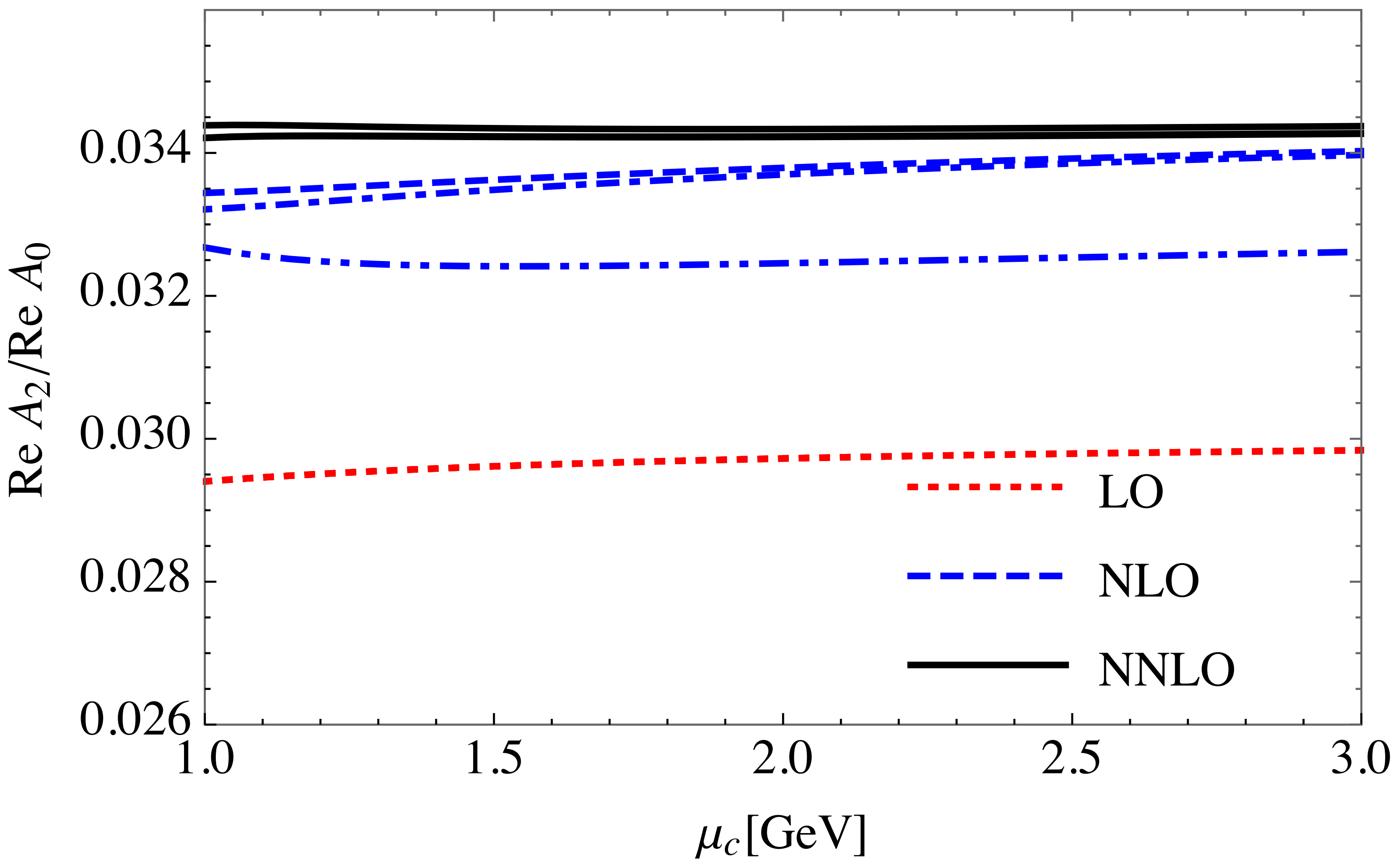
$$\mathrm{ReA}_0|_{\mathrm{SM}} = 44.4(11.0)(1.0) \times 10^{-8}\mathrm{GeV}$$

Non-perturbative error

NNNLO Corrections

ΔI=1/2 Rule

Residual μ_c scale dependence



$$\left.\frac{\mathrm{ReA}_0}{\mathrm{ReA}_2}\right|_{\mathrm{SM}} = 29.2(7.4)(1.0)$$