The SM prediction of ε'/ε

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CP Violation in $K \to \pi\pi$

$$\eta_{00} \equiv \frac{\mathcal{M}(K_L^0 \to \pi^0\pi^0)}{\mathcal{M}(K_S^0 \to \pi^0\pi^0)} \equiv \varepsilon - 2\varepsilon' \qquad , \qquad \eta_{+-} \equiv \frac{\mathcal{M}(K_L^0 \to \pi^+\pi^-)}{\mathcal{M}(K_S^0 \to \pi^+\pi^-)} \equiv \varepsilon + \varepsilon'$$

- **Indirect $\not{CP}$:** $|\varepsilon| = \frac{1}{3}\,|\eta_{00} + 2\,\eta_{+-}| = (2.228 \pm 0.011) \cdot 10^{-3}$
- **Direct $\not{CP}$:** $\mathrm{Re}(\varepsilon'/\varepsilon) = \frac{1}{3}\left(1 - \left|\frac{\eta_{00}}{\eta_{+-}}\right|\right) = (16.6 \pm 2.3) \cdot 10^{-4}$

First evidence in 1988 by NA31

Time evolution of ε'/ε predictions: 10^{-3} units

- 1983	SD (Q_6), LO	~ 2	Gilman-Hagelin
- 1990-2000	SD, large m_t (Q_8), NLO	$\sim \text{few} \cdot 10^{-1}$	Munich, Rome
	+ models of LD contributions	$\sim \mathcal{O}(1)$	Dortmund, Trieste
- 1999-2001	SD + LD (χPT) at NLO	1.7 ± 0.9	Scimemi-Pallante-Pich
- 2000-2003	models of LD contributions	$\sim \mathcal{O}(1)$	Lund, Marseille
- 2003	Isospin breaking in χPT	1.9 ± 1.0	Cirigliano-Ecker-Neufeld-Pich
- 2015	Lattice	0.14 ± 0.70	RBC-UKQCD
- 2015-2017	Dual QCD, Lattice input	0.19 ± 0.45	Munich
- 2017	χPT re-analysis	1.5 ± 0.7	Gisbert-Pich
- 2019	χPT re-analysis of IB	$1.3\,^{+\,0.6}_{-\,0.7}$	Cirigliano-Gisbert-Pich-Rodríguez

$K \to 2\pi$ Isospin Amplitudes

$$A[K^0 \to \pi^+\pi^-] \equiv A_0\, e^{i\chi_0} + \frac{1}{\sqrt{2}} A_2\, e^{i\chi_2}$$

$$A[K^0 \to \pi^0\pi^0] \equiv A_0\, e^{i\chi_0} - \sqrt{2}\, A_2\, e^{i\chi_2}$$

$$A[K^+ \to \pi^+\pi^0] \equiv \frac{3}{2} A_2^+\, e^{i\chi_2^+}$$

1) $\Delta I = 1/2$ Rule: $\omega \equiv \frac{\mathrm{Re}\,(A_2)}{\mathrm{Re}\,(A_0)} \approx \frac{1}{22}$

2) Strong Final State Interactions: $\chi_0 - \chi_2 \approx \delta_0 - \delta_2 \approx 45^\circ$

$$\varepsilon'_K = \frac{-i}{\sqrt{2}}\, e^{i(\chi_2-\chi_0)}\, \omega \left\{ \frac{\mathrm{Im}\,(A_0)}{\mathrm{Re}\,(A_0)} - \frac{\mathrm{Im}\,(A_2)}{\mathrm{Re}\,(A_2)} \right\}$$

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$$A[K^+ \to \pi^+\pi^0] \;\equiv\; \frac{3}{2} A_2^+\, e^{i\chi_2^+}$$

$$\begin{aligned} A_0\, e^{i\chi_0} &= \mathcal{A}_{1/2} \\ A_2\, e^{i\chi_2} &= \mathcal{A}_{3/2} + \mathcal{A}_{5/2} \\ A_2^+\, e^{i\chi_2^+} &= \mathcal{A}_{3/2} - \frac{2}{3}\mathcal{A}_{5/2} \end{aligned}$$

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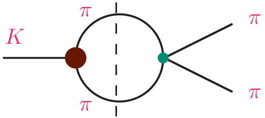
Implications of a Large Phase Shift

$$\mathcal{A}_I \equiv A_I \, e^{i\delta_I} = \mathrm{Dis}\,(\mathcal{A}_I) + i\,\mathrm{Abs}\,(\mathcal{A}_I)$$

❶ **Unitarity:** $\delta_0(M_K) = (39.2 \pm 1.5)^\circ$ ➡ $A_0 \approx 1.3 \times \mathrm{Dis}\,(\mathcal{A}_0)$



$$\tan\delta_I = \frac{\mathrm{Abs}\,(\mathcal{A}_I)}{\mathrm{Dis}\,(\mathcal{A}_I)}$$

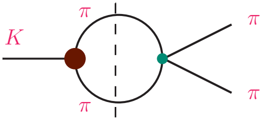
$$A_I = \mathrm{Dis}\,(\mathcal{A}_I)\,\sqrt{1+\tan^2\delta_I}$$

❷ **Analyticity:** $\Delta\,\mathrm{Dis}\,(\mathcal{A}_I)[s] = \frac{1}{\pi}\int dt\,\frac{\mathrm{Abs}\,(\mathcal{A}_I)[t]}{t-s-i\epsilon} + \text{subtractions}$

Large δ_0 ➡ Large $\mathrm{Abs}\,(\mathcal{A}_0)$ ➡ Large correction to $\mathrm{Dis}\,(\mathcal{A}_0)$

Absorptive amplitude: on-shell intermediate $\pi\pi$ state

$$\mathcal{A}_I \equiv A_I \, e^{i\delta_I} = \mathrm{Dis}\,(\mathcal{A}_I) + i\,\mathrm{Abs}\,(\mathcal{A}_I) \qquad\qquad \sigma_\pi \equiv \sqrt{1 - 4M_\pi^2/M_K^2}$$



$$\Delta_L \mathcal{A}_0/\mathcal{A}_0^{\mathrm{tree}} = (2M_K^2 - M_\pi^2)\; B_{\mathrm{loop}} + \cdots$$

$$\Delta_L \mathcal{A}_2/\mathcal{A}_2^{\mathrm{tree}} = -(M_K^2 - 2M_\pi^2)\; B_{\mathrm{loop}} + \cdots$$

$$B_{\mathrm{loop}} = \frac{1}{32\pi^2 F_\pi^2} \left\{ \sigma_\pi \left[\log\left(\frac{1-\sigma_\pi}{1+\sigma_\pi}\right) + i\pi \right] + \log\left(\frac{\nu_\chi^2}{M_\pi^2}\right) + 1 \right\}$$

- **Finite 1-loop absorptive amplitude** (model independent)
- **Universal correction** (only depends on $\pi\pi$ quantum numbers):

$$\mathrm{Abs}\,(\mathcal{A}_0)/\mathcal{A}_0^{\mathrm{tree}} = 0.47 \qquad , \qquad \mathrm{Abs}\,(\mathcal{A}_2)/\mathcal{A}_2^{\mathrm{tree}} = -0.21$$

- **Any (SM or NP) short-distance contribution** leads to $\Delta\mathcal{A}_I^{\mathrm{tree}} \sim g_I^{\mathrm{SD}}\,\mathcal{O}_I$
- **Same correction** for $\mathrm{Re}\,(g_I^{\mathrm{SD}})$ ($\mathcal{CP}$ conserving) and $\mathrm{Im}\,(g_I^{\mathrm{SD}})$ ($\not{\mathcal{C}}\not{\mathcal{P}}$)

2015 Lattice Results

Isospin limit:

RBC-UKQCD 1505.07863, 1502.00263

$$\sqrt{\frac{3}{2}}\,\mathrm{Re}\,A_2 \;=\; (1.50 \pm 0.04 \pm 0.14)\cdot 10^{-8}\ \mathrm{GeV} \qquad \mathrm{exp}:\ 1.482\,(2)\cdot 10^{-8}\ \mathrm{GeV} \quad 0.1\,\sigma$$

$$\sqrt{\frac{3}{2}}\,\mathrm{Im}\,A_2 \;=\; -(6.99 \pm 0.20 \pm 0.84)\cdot 10^{-13}\ \mathrm{GeV}$$

$$\sqrt{\frac{3}{2}}\,\mathrm{Re}\,A_0 \;=\; (4.66 \pm 1.00 \pm 1.26)\cdot 10^{-7}\ \mathrm{GeV} \qquad \mathrm{exp}:\ 3.112\,(1)\cdot 10^{-7}\ \mathrm{GeV} \quad 1.0\,\sigma$$

$$\sqrt{\frac{3}{2}}\,\mathrm{Im}\,A_0 \;=\; -(1.90 \pm 1.23 \pm 1.08)\cdot 10^{-11}\ \mathrm{GeV}$$

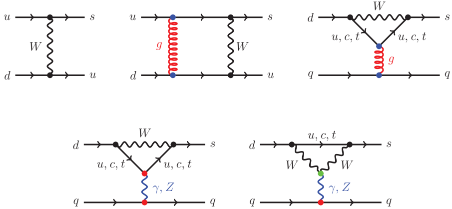
$$\mathrm{Re}\,(\varepsilon'/\varepsilon) \;=\; (1.38 \pm 5.15 \pm 4.59)\cdot 10^{-4} \qquad \mathrm{exp}:\ (16.8 \pm 1.4)\cdot 10^{-4} \quad 2.2\,\sigma$$

$$\delta_0 \;=\; (23.8 \pm 4.9 \pm 1.2)^\circ \qquad \mathrm{exp}:\ (39.2 \pm 1.5)^\circ \quad 2.9\,\sigma$$

$$\delta_2 \;=\; -(11.6 \pm 2.5 \pm 1.2)^\circ \qquad \mathrm{exp}:\ -(8.5 \pm 1.5)^\circ \quad 1.0\,\sigma$$

Physics contributions from different scales

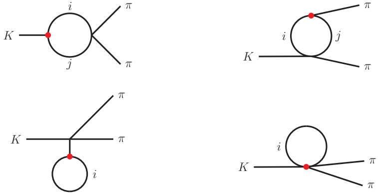
Short-Distance



Top quark, GIM, $\not{\mathcal{C}}\not{\mathcal{P}}$

New Physics ?

Long-Distance



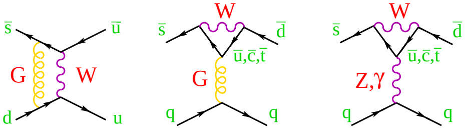
Chiral Dynamics

Multi-Scale Problem:

$\log(M/\mu)$ **OPE** , $\log(\nu_\chi/M_\pi)$ χ**PT**

Energy Scale	Fields	Effective Theory
M_W	W, Z, γ, g τ, μ, e, ν_i t, b, c, s, d, u	Standard Model
	$\Downarrow$ OPE	
$\lesssim m_c$	$\gamma, g \;;\; \mu, e, \nu_i$ s, d, u	$\mathcal{L}_{\mathrm{QCD}}^{(n_f=3)}, \mathcal{L}_{\mathrm{eff}}^{\Delta S=1,2}$
	$\Downarrow$ $N_C \to \infty$	
M_K	$\gamma \;;\; \mu, e, \nu_i$ π, K, η	χPT

$\Delta S = 1$ TRANSITIONS



$$\mathcal{L}_{\mathrm{eff}}^{\Delta S=1} = -\frac{G_F}{\sqrt{2}} V_{ud} V_{us}^* \sum_j C_j(\mu)\, Q_j(\mu)$$

$$Q_1 = (\bar{s}_\alpha u_\beta)_{\mathrm{V-A}} (\bar{u}_\beta d_\alpha)_{\mathrm{V-A}} \qquad Q_2 = (\bar{s}u)_{\mathrm{V-A}} (\bar{u}d)_{\mathrm{V-A}}$$

$$Q_{3,5} = (\bar{s}d)_{\mathrm{V-A}} \sum_q (\bar{q}q)_{\mathrm{V\mp A}} \qquad Q_4 = (\bar{s}_\alpha d_\beta)_{\mathrm{V-A}} \sum_q (\bar{q}_\beta q_\alpha)_{\mathrm{V-A}}$$

$$Q_{7,9} = \tfrac{3}{2} (\bar{s}d)_{\mathrm{V-A}} \sum_q e_q (\bar{q}q)_{\mathrm{V\pm A}} \qquad Q_{10} = \tfrac{3}{2} (\bar{s}_\alpha d_\beta)_{\mathrm{V-A}} \sum_q e_q (\bar{q}_\beta q_\alpha)_{\mathrm{V-A}}$$

$$Q_6 = -8 \sum_q (\bar{s}_L q_R)(\bar{q}_R d_L) \qquad Q_8 = -12 \sum_q e_q (\bar{s}_L q_R)(\bar{q}_R d_L)$$

- $q > \mu$: $\quad C_j(\mu) = z_j(\mu) - y_j(\mu)\, (V_{td} V_{ts}^* / V_{ud} V_{us}^*)$

  NLO: $\mathcal{O}(\alpha_s^n t^n)$, $\mathcal{O}(\alpha_s^{n+1} t^n)$ $\quad [t \equiv \log(M/m)]$ Munich / Rome, 1992-1993

  NNLO: Ongoing calculation M. Cerdà-Sevilla et al

- $q < \mu$: $\quad \langle \pi\pi | Q_j(\mu) | K \rangle$? **Physics does not depend on μ**

Simplified Estimate

1. **CP violation** ➡ **Penguin operators**

2. **Chirality** ➡ **Enhanced $(V-A)\otimes(V+A)$ operators**

$$Q_6 \;=\; -8\sum_q (\bar s_L q_R)(\bar q_R d_L) \qquad , \qquad Q_8 \;=\; -12\sum_q e_q\,(\bar s_L q_R)(\bar q_R d_L)$$

3. **Large-N_C:** $\langle J\cdot J\rangle \;=\; \langle J\rangle\,\langle J\rangle\;\{1+\mathcal{O}(1/N_C)\}$

$$\mathcal{M}_{LL} \;\equiv\; \langle\pi^+\pi^-|(\bar s_L\gamma^\mu u_L)(\bar u_L\gamma_\mu d_L)|K^0\rangle \;=\; \langle\pi^+|\bar u_L\gamma_\mu d_L|0\rangle\,\langle\pi^-|\bar s_L\gamma^\mu u_L|K^0\rangle \;=\; \frac{i\sqrt2}{4}\,F_\pi\,(M_K^2-M_\pi^2)$$

$$\mathcal{M}_{LR}(\mu) \;\equiv\; \langle\pi^+\pi^-|(\bar s_L u_R)(\bar u_R d_L)|K^0\rangle \;=\; \langle\pi^+|\bar u_R d_L|0\rangle\,\langle\pi^-|\bar s_L u_R|K^0\rangle \;=\; \frac{i\sqrt2}{4}\,F_\pi\left[\frac{M_K^2}{m_d(\mu)+m_s(\mu)}\right]^2$$

At $\mu=1$ GeV, $\qquad \mathcal{M}_{LR}(\mu)/\mathcal{M}_{LL} \;\sim\; M_K^2/[m_s(\mu)+m_d(\mu)]^2 \;\sim\; 14$

➡ $$\mathrm{Re}(\varepsilon'/\varepsilon) \;\approx\; 2.2\cdot 10^{-3}\;\left\{B_6^{(1/2)}\,(1-\Omega_{\mathrm{eff}}) - 0.48\;B_8^{(3/2)}\right\}$$

Strong Cancellation

$B_6^{(1/2)} = B_8^{(3/2)} = 1\,,\qquad \Omega_{\mathrm{eff}} = 0.12$ ➡ $\mathrm{Re}(\varepsilon'/\varepsilon)\approx 0.9\cdot 10^{-3}$

Buras et al: $\quad B_6^{(1/2)} = 0.57\,,\quad B_8^{(3/2)} = 0.76\,,\quad \Omega_{\mathrm{eff}} = 0.15$ ➡ $\mathrm{Re}(\varepsilon'/\varepsilon)\approx 2.6\cdot 10^{-4}$

CHIRAL PERTURBATION THEORY (χPT)

- **Expansion in powers of p^2/Λ_χ^2 :** $\mathcal{A} = \sum_n \mathcal{A}^{(n)}$ $(\Lambda_\chi \sim 4\pi F_\pi \sim 1.2 \text{ GeV})$
- **Amplitude structure fixed by chiral symmetry**

$$\mathrm{SU(3)_L} \otimes \mathrm{SU(3)_R} \to \mathrm{SU(3)_V}$$

- **Short-distance dynamics encoded in Low-Energy Couplings**
- **$O(p^2)$ χPT: Goldstone interactions (π, K, η)** $\Phi \equiv \frac{1}{\sqrt{2}} \vec{\lambda}\,\vec{\varphi}$

$$\mathcal{L}_2^{\Delta S=1} = G_8\, F^4\, \mathrm{Tr}(\lambda L_\mu L^\mu) + G_{27}\, F^4 \left(L_{\mu 23} L^\mu_{11} + \frac{2}{3}\, L_{\mu 21} L^\mu_{13} \right)$$

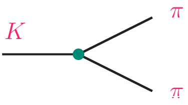
$G_R \equiv -\frac{G_F}{\sqrt{2}}\, V_{ud} V^*_{us}\, g_R$; $L_\mu = -i U^\dagger D_\mu U$; $\lambda \equiv \frac{1}{2}\,\lambda_{6-i7}$; $U \equiv \exp\left\{ i\sqrt{2}\,\Phi/F \right\}$

- **Loop corrections (χPT logarithms) unambiguously predicted**
- **LECs can be determined at $N_C \to \infty$ (matching)**
- **$O(p^2)$ LECs (G_8, G_{27}) can be phenomenologically determined**

$O(p^2)$ χPT

$$\mathcal{L}_2^{\Delta S=1} = G_8\, F^4\, \langle \lambda L_\mu L^\mu \rangle \;+\; G_{27}\, F^4 \left(L_{\mu 23}\, L^\mu_{11} + \frac{2}{3}\, L_{\mu 21}\, L^\mu_{13} \right)$$

$$G_R \equiv -\frac{G_F}{\sqrt{2}}\, V_{ud} V_{us}^*\, g_R \quad ; \quad L_\mu = -i U^\dagger D_\mu U \quad ; \quad \lambda \equiv \tfrac{1}{2}\lambda_{6-i7} \quad ; \quad U \equiv \exp\left\{ i\sqrt{2}\,\Phi/F \right\}$$



$$\mathcal{A}_{1/2} = \sqrt{2}\, F_\pi \left(G_8 + \frac{1}{9}\, G_{27} \right) (M_K^2 - M_\pi^2)$$

$$\mathcal{A}_{3/2} = \frac{10}{9}\, F_\pi\, G_{27}\, (M_K^2 - M_\pi^2)$$

$$\mathcal{A}_{5/2} = 0 \quad ; \quad \delta_0 = \delta_2 = 0$$

$$[\Gamma(K \to 2\pi) + \delta_I]_{\mathrm{Exp}} \quad \Longrightarrow \quad |g_8| \approx 5.0 \quad ; \quad |g_{27}| \approx 0.29$$

$\mathbf{O(p^2, e^2p^0)}$ χPT

$$\mathcal{Q} = \mathrm{diag}\left(\tfrac{2}{3}, -\tfrac{1}{3}, -\tfrac{1}{3}\right)$$

$$\mathcal{L}_2^{\Delta S=1} = G_8\, F^4\, \langle \lambda L_\mu L^\mu \rangle + G_{27}\, F^4 \left(L_{\mu 23}\, L^\mu_{11} + \frac{2}{3}\, L_{\mu 21}\, L^\mu_{13} \right) + e^2\, F^6\, G_8\, g_{ew}\, \langle \lambda U^\dagger \mathcal{Q} U \rangle$$

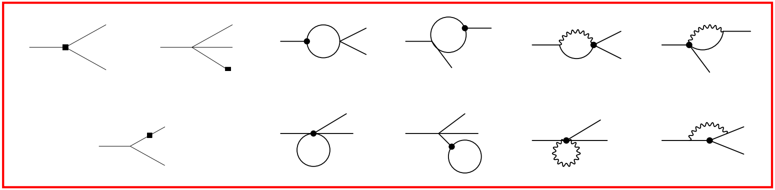
$$\mathcal{A}_{1/2} = \sqrt{2}\, F_\pi \left\{ G_8 \left[(M_K^2 - M_\pi^2) \left(1 - \frac{2}{3\sqrt{3}}\, \varepsilon^{(2)} \right) - \frac{2}{3}\, F_\pi^2\, e^2\, (g_{ew} + 2\, Z) \right] + \frac{1}{9}\, G_{27}\, (M_K^2 - M_\pi^2) \right\}$$

$$\mathcal{A}_{3/2} = \frac{2}{3}\, F_\pi \left\{ \left(\frac{5}{3}\, G_{27} + \frac{2}{\sqrt{3}}\, \varepsilon^{(2)}\, G_8 \right) (M_K^2 - M_\pi^2) - F_\pi^2\, e^2\, G_8\, (g_{ew} + 2\, Z) \right\}$$

$$\mathcal{A}_{5/2} = 0 \qquad ; \qquad \delta_0 = \delta_2 = 0$$

$\varepsilon^{(2)} = (\sqrt{3}/4)\,(m_d - m_u)/(m_s - \hat{m}) \approx 0.011$; $Z \approx (M^2_{\pi^\pm} - M^2_{\pi^0})/(2\, e^2 F_\pi^2) \approx 0.8$

$\mathbf{O}\left[\mathbf{p^4}, (\mathbf{m_u} - \mathbf{m_d})\,\mathbf{p^2}, \mathbf{e^2 p^0}, \mathbf{e^2 p^2}\right]$ χPT



- **Nonleptonic weak Lagrangian:** $\mathcal{O}(G_F\, p^4)$

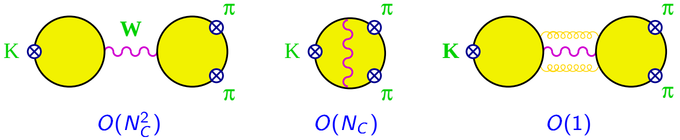
$$\mathcal{L}_{\rm weak}^{(4)} = \sum_i G_8\, N_i\, F^2\, O_i^8 + \sum_i G_{27}\, D_i\, F^2\, O_i^{27} + \text{h.c.}$$

- **Electroweak Lagrangian:** $\mathcal{O}(G_F\, e^2 p^{0,2})$

$$\mathcal{L}_{\rm EW} = e^2 F^6 G_8\, g_{ew}\, \mathrm{Tr}(\lambda U^\dagger \mathcal{Q} U) + e^2 \sum_i G_8\, Z_i\, F^4\, O_i^{EW} + \text{h.c.}$$

- $\mathcal{O}(e^2 p^{0,2})$ **Electromagnetic** $+$ $\mathcal{O}(p^4)$ **Strong:** $Z\,,\ K_i\,,\ L_i$

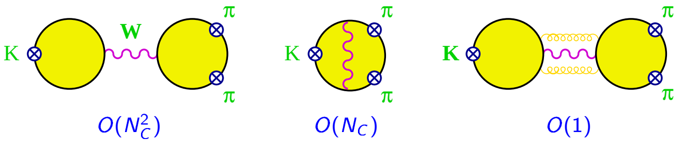
Weak Currents Factorize at Large N_C



$$A[K^0 \to \pi^0\pi^0] = 0 \quad \Rightarrow \quad A_0 = \sqrt{2}\, A_2$$

No $\Delta I = \frac{1}{2}$ enhancement at leading order in $1/N_C$

Weak Currents Factorize at Large N_C



$$A[K^0 \to \pi^0\pi^0] = 0 \quad \Rightarrow \quad A_0 = \sqrt{2}\, A_2$$

No $\Delta I = \frac{1}{2}$ enhancement at leading order in $1/N_C$

- **Multiscale problem:** **OPE** $\frac{1}{N_C} \log\left(\frac{M_W}{\mu}\right) \sim \frac{1}{3} \times 4$

Short-distance logarithms must be summed

- **Large χPT logarithms:** **FSI** $\frac{1}{N_C} \log\left(\frac{\nu_\chi}{M_\pi}\right) \sim \frac{1}{3} \times 2$

Infrared logarithms must also be included $[\delta_I \sim O(1/N_C)\ ,\ \delta_0 - \delta_2 \approx 45^\circ]$

Energy Scale	Fields	Effective Theory
M_W	W, Z, γ, g τ, μ, e, ν_i t, b, c, s, d, u	Standard Model
	$\Downarrow$ OPE	
$\lesssim m_c$	$\gamma, g \; ; \; \mu, e, \nu_i$ s, d, u	$\mathcal{L}_{\mathrm{QCD}}^{(n_f=3)}, \; \mathcal{L}_{\mathrm{eff}}^{\Delta S=1,2}$
	$\Downarrow$ $N_C \to \infty$	
M_K	$\gamma \; ; \; \mu, e, \nu_i$ π, K, η	χPT

Multi-Scale Problem: Summation of logarithms needed

A large $\log(M_1/M_2)$ compensates a $1/N_C$ suppression

❶ **Short-distance:** $\frac{1}{N_C}\log(M_W/\mu)$ — Bardeen-Buras-Gerard

$$\Rightarrow \begin{cases} g_8^\infty = 1.15\,{}^{+0.14}_{-0.17}{}_\mu \pm 0.04_{L_{5,8}} \pm 0.01_{m_s} \\ g_{27}^\infty = 0.46 \pm 0.02_\mu \end{cases}$$

Cirigliano et al, Pallante et al

❷ **Long-distance (χPT):** $\frac{1}{N_C}\log(\mu/m_\pi)$ — Kambor et al, Pallante et al

$$g_8^{\mathrm{LO}} = 5.0 \;\Rightarrow\; g_8^{\mathrm{NLO}} = 3.6$$

$$g_{27}^{\mathrm{LO}} = 0.286 \;\Rightarrow\; g_{27}^{\mathrm{NLO}} = 0.288$$

Cirigliano et al

❸ **Isospin Violation:** $g_{27}^{\mathrm{NLO}} = 0.296$ — Cirigliano et al

$N_C \to \infty$

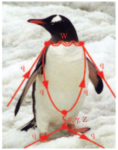
$$g_8 = \left(\frac{3}{5}\,C_2 - \frac{2}{5}\,C_1 + C_4\right) - 16\;L_5\;\left(\frac{\langle\bar q\, q\rangle(\mu)}{F_\pi^3}\right)^2 C_6(\mu)$$

$$g_{27} = \frac{3}{5}\,(C_2 + C_1)$$

$$e^2\, g_8\, g_{ew} = -3\,\left(\frac{\langle\bar q\, q\rangle(\mu)}{F_\pi^3}\right)^2 \left[C_8(\mu) + \frac{16}{9}\,C_6(\mu)\; e^2\,(K_9 - 2\,K_{10})\right]$$

$$\frac{\langle\bar q\, q\rangle(\mu)}{F_\pi^3} = \frac{M_{K^0}^2}{(m_s + m_d)(\mu)\,F_\pi}\left\{1 - \frac{8\,M_{K^0}^2}{F_\pi^2}\,(2L_8 - L_5) + \frac{4\,M_{\pi^0}^2}{F_\pi^2}\,L_5\right\}$$

- **Equivalent to standard calculations of B_i**
- **μ dependence only captured for $Q_{6,8}$**



Anomalous Dimension Matrix

$$
\gamma_s^{(0)} = \begin{pmatrix}
-\frac{3}{N_c^2} & \frac{3}{N_c} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\frac{3}{N_c} & -\frac{3}{N_c^2} & -\frac{1}{3N_c^2} & \frac{1}{3N_c} & -\frac{1}{3N_c^2} & \frac{1}{3N_c} & 0 & 0 & 0 & 0 \\
0 & 0 & -\frac{11}{3N_c^2} & \frac{11}{3N_c} & -\frac{2}{3N_c^2} & \frac{2}{3N_c} & 0 & 0 & 0 & 0 \\
0 & 0 & \frac{3}{N_c} - \frac{n_f}{3N_c^2} & \frac{n_f}{3N_c} - \frac{3}{N_c^2} & -\frac{n_f}{3N_c^2} & \frac{n_f}{3N_c} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{3}{N_c^2} & -\frac{3}{N_c} & 0 & 0 & 0 & 0 \\
0 & 0 & -\frac{n_f}{3N_c^2} & \frac{n_f}{3N_c} & -\frac{n_f}{3N_c^2} & -3 + \frac{n_f}{3N_c} + \frac{3}{N_c^2} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \frac{3}{N_c^2} & -\frac{3}{N_c} & 0 & 0 \\
0 & 0 & \frac{-n_u + \frac{n_d}{2}}{3N_c^2} & \frac{n_u - \frac{n_d}{2}}{3N_c} & \frac{-n_u + \frac{n_d}{2}}{3N_c^2} & \frac{n_u - \frac{n_d}{2}}{3N_c} & 0 & -3 + \frac{3}{N_c^2} & 0 & 0 \\
0 & 0 & \frac{1}{3N_c^2} & -\frac{1}{3N_c} & \frac{1}{3N_c^2} & -\frac{1}{3N_c} & 0 & 0 & -\frac{3}{N_c^2} & 0 \\
0 & 0 & \frac{-n_u + \frac{n_d}{2}}{3N_c^2} & \frac{n_u - \frac{n_d}{2}}{3N_c} & \frac{-n_u + \frac{n_d}{2}}{3N_c^2} & \frac{n_u - \frac{n_d}{2}}{3N_c} & 0 & 0 & 0 & -\frac{3}{N_c^2}
\end{pmatrix}
$$

Only γ_{66} and γ_{88} survive the large–N_C limit

Anatomy of ε'/ε calculation

$$\mathrm{Re}\left(\frac{\varepsilon'}{\varepsilon}\right) = -\frac{\omega_+}{\sqrt{2}|\varepsilon|}\left\{\frac{\mathrm{Im}\, A_0^{(0)}}{\mathrm{Re}\, A_0^{(0)}}\,(1-\Omega_{\rm eff}) - \frac{\mathrm{Im}\, A_2^{\rm emp}}{\mathrm{Re}\, A_2^{(0)}}\right\}$$

$$\mathcal{A}_n^{(X)} = a_n^{(X)}\left[1 + \Delta_L\,\mathcal{A}_n^{(X)} + \Delta_C\,\mathcal{A}_n^{(X)}\right]$$

Cirigliano-Gisbert-Pich-Rodríguez 2019

❶ **O(p^4) χPT Loops: Large correction** (NLO in $1/N_C$) FSI

$$\Delta_L\,\mathcal{A}_{1/2}^{(8)} = 0.27 + 0.47\, i \qquad ; \qquad \Delta_L\,\mathcal{A}_{3/2}^{(g)} = -0.50 - 0.21\, i$$

❷ **O(p^4) LECs fixed at $N_C \to \infty$: Small correction**

$$\Delta_C\,[\mathcal{A}_{1/2}^{(8)}]^- = 0.11 \pm 0.05 \qquad ; \qquad \Delta_C\,[\mathcal{A}_{3/2}^{(g)}]^- = -0.19 \pm 0.19$$

❸ **Isospin Breaking** $O\left[(m_u - m_d)\, p^2, e^2 p^2\right]$: **Sizeable correction**

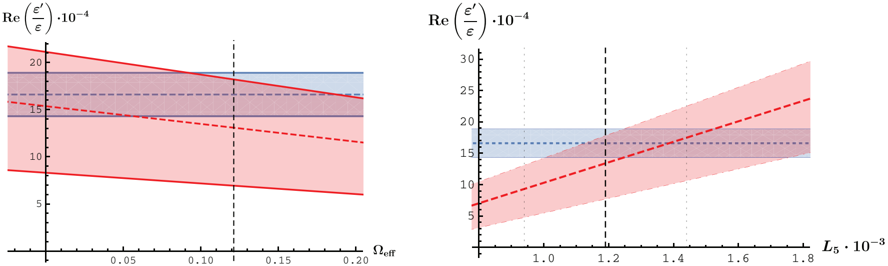
$$\Omega_{\rm eff} = 0.12 \pm 0.09$$

❹ **Re(g$_8$), Re(g$_{27}$), $\chi_0 - \chi_2$ fitted to data**

SM Prediction of ε'/ε

$$\mathrm{Re}\,(\varepsilon'/\varepsilon)_{\rm SM} \;=\; \left(13\,{}^{+\,6}_{-\,7}\right)\cdot 10^{-4}$$

Cirigliano, Gisbert, Pich, Rodríguez-Sánchez



$$\mathrm{Re}\,(\varepsilon'/\varepsilon)_{\rm SM} \;=\; \left(13.1 \pm 0.4_{m_s}\,{}^{+\,2.2}_{-\,4.0}{}_{\mu}\,{}^{+\,3.0}_{-\,3.2}{}_{\nu_\chi} \pm 1.2_{\gamma_5} \pm 4.3_{L_{5,8}} \pm 1.1_{L_7} \pm 0.2_{K_i} \pm 0.3_{X_i}\right)\cdot 10^{-4}$$

Large uncertainty but no anomalies!

Outlook: Needed Improvements

- **Wilson coefficients at NNLO** — Cerda et al
- **Updated value of Ω_{eff}** ✓ — Cirigliano et al
- **$g_8\, g_{\text{ew}}$ at NLO in $1/N_C$** — Rodríguez-Sánchez, A.P.
- **g_8 and higher-order LECs at NLO** — New ideas needed
- **χPT logarithms at NNLO** — Feasible
- **Improved lattice input** — Eagerly expected

Difficult, but worth while enterprise

Best strategy: χPT (amplitudes) + Lattice (LECs)

Backup


International Conference on Kaon Physics 2019
University of Perugia, Italy, 10-13 September 2019

Isospin Breaking in ε'/ε

$$\varepsilon' \sim \omega_+ \left\{ \frac{\text{Im}\, A_0^{(0)}}{\text{Re}\, A_0^{(0)}} \left(1 + \Delta_0 + f_{5/2}\right) - \frac{\text{Im}\, A_2}{\text{Re}\, A_2^{(0)}} \right\}$$

$$\sim \omega_+ \left\{ \frac{\text{Im}\, A_0^{(0)}}{\text{Re}\, A_0^{(0)}} \left(1 - \Omega_{\text{eff}}\right) - \frac{\text{Im}\, A_2^{\text{emp}}}{\text{Re}\, A_2^{(0)}} \right\}$$

$$\omega \equiv \frac{\text{Re}\, A_2}{\text{Re}\, A_0} = \omega_+ \left(1 + f_{5/2}\right) \quad ; \quad \omega_+ \equiv \frac{\text{Re}\, A_2^+}{\text{Re}\, A_0} \quad , \quad \Omega_{IB} = \frac{\text{Re}\, A_0^{(0)}}{\text{Re}\, A_2^{(0)}} \cdot \frac{\text{Im}\, A_2^{\text{non-emp}}}{\text{Im}\, A_0^{(0)}}$$

Cirigliano, Gisbert, Pich, Rodríguez-Sánchez, 2019 (Cirigliano et al 2003)

$\times 10^{-2}$	$\alpha = 0$ LO	$\alpha = 0$ NLO	$\alpha \neq 0$ LO	$\alpha \neq 0$ NLO
Ω_{IB}	13.7	$17.1^{+8.4}_{-8.3}$	19.6 ± 4.8	26.0 ± 8.2
Δ_0	-0.002	-0.51 ± 0.12	5.6 ± 1.6	$5.7^{+1.7}_{-1.6}$
$f_{5/2}$	0	0	0	$8.2^{+2.4}_{-2.5}$
Ω_{eff}	13.7	$17.6^{+8.5}_{-8.4}$	14.0 ± 4.0	$12.1^{+9.0}_{-8.8}$

$$\Omega_{\text{eff}} = 0.12 \pm 0.09 \equiv \Omega_{IB} - \Delta_0 - f_{5/2}$$

Phenomenological $K \to \pi\pi$ Fit

Cirigliano, Gisbert, Pich, Rodríguez-Sánchez, 2019

	LO-IC	LO-IB	NLO-IC	NLO-IB
Re g_8	4.99	5.00	3.60 ± 0.14	$3.58\,^{+0.15}_{-0.14}$
Re g_{27}	0.286	0.251	0.288 ± 0.014	$0.296\,^{+0.010}_{-0.003}$
$\chi_0 - \chi_2$	44.8°	48.0°	$(44.8 \pm 1.0)^\circ$	$(51.4 \pm 1.3)^\circ$

IC $\equiv$ $[m_u - m_d = \alpha = 0]$; IB $\equiv$ $[m_u - m_d \neq 0\,,\ \alpha \neq 0]$

$\pi\pi \to \pi\pi$: $\delta_0 - \delta_2 = (47.7 \pm 1.5)^\circ$ Colangelo–Gasser–Leutwyler '01

Modelling (some) non-factorizable $1/N_C$ corrections

Buras-Gérard, 1507.06326

$$B_6^{(1/2)} = 1 - \frac{3}{2}\left[\frac{F_\pi}{F_K - F_\pi}\right]\frac{(m_K^2 - m_\pi^2)}{(4\pi F_\pi)^2}\ln(1 + \frac{\Lambda^2}{\tilde{m}_6^2}) = 1 - 0.66\ \ln(1 + \frac{\Lambda^2}{\tilde{m}_6^2})$$

$$B_8^{(1/2)} = 1 + \frac{(m_K^2 - m_\pi^2)}{(4\pi F_\pi)^2}\ln(1 + \frac{\Lambda^2}{\tilde{m}_8^2}) = 1 + 0.08\ \ln(1 + \frac{\Lambda^2}{\tilde{m}_8^2})$$

$$B_8^{(3/2)} = 1 - 2\frac{(m_K^2 - m_\pi^2)}{(4\pi F_\pi)^2}\ln(1 + \frac{\Lambda^2}{\tilde{m}_8^2}) = 1 - 0.17\ \ln(1 + \frac{\Lambda^2}{\tilde{m}_8^2})$$

➡ $B_6^{(1/2)} \ \leq\ B_8^{(3/2)}\ <\ 1$

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➡ $B_6^{(1/2)} \leq B_8^{(3/2)} < 1$

- FSI $(1/N_C)$ not included ➡ $\delta_I = 0$
- Part of 1-loop χPT corrections (?)
- Difficult to account in a matching calculation

Modelling (some) non-factorizable $1/N_C$ corrections

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→ $B_6^{(1/2)} \;\leq\; B_8^{(3/2)} \;<\; 1$

Not true in QCD

- FSI $(1/N_C)$ not included → $\delta_I = 0$
- Part of 1-loop χPT corrections (?)
- Difficult to account in a matching calculation

BBG Model

Bardeen-Buras-Gerard

$$\mathcal{L}_{\rm eff} = \frac{f_\pi^2}{4} \left\{ \langle D_\mu U^\dagger D^\mu U \rangle + r \, \langle m \, (U + U^\dagger) \rangle - \frac{r}{\Lambda_\chi^2} \, \langle m \, (D^2 U + D^2 U^\dagger) \rangle \right\}$$

1. Equivalent to $\mathcal{O}(p^2)$ χPT + L_5 term $(L_i = 0\,,\ i \neq 5)$
   Most L_i are leading in N_C ➔ **$\mathcal{L}_{\rm eff}$ does not represent large-N_C QCD**

2. **Cut-off loop regularization:** $\mathbf{M \sim (0.8 - 0.9)}$ **GeV**

   $f_\pi^2(M^2) = F_\pi^2 + 2\, I_2(m_\pi^2) + I_2(m_K^2)$, $I_2(m_i^2) = \frac{1}{16\pi^2} \left[M^2 - m_i^2 \, \log\left(1 + \frac{M^2}{m_i^2}\right)\right]$

3. **Large-N_C factorization assumed to hold in the IR $(\mu = 0)$:** $\langle J \cdot J \rangle = \langle J \rangle \langle J \rangle$

4. **M identified with SD renormalization scale μ:** $C_i(\mu)$ **running**

   Meson evolution ⟷ **Quark evolution**

5. **Vector meson loops included through Hidden U(3) Gauge Symmetry**

   Could partially account for $L_{1,2,3,9,10}$

   L_8 still missing ➔ $\langle \bar{q} q \rangle$, $Q_{6,8}$ **not quite correct even at large-N_C**

6. $\pi\pi$ **re-scattering completely missing** ➔ $\delta_{0,2} = 0$, **FSI absent**

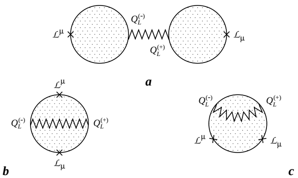
Dynamical understanding of the $\Delta I = 1/2$ rule

AP – E. de Rafael, PL B374 (1996) 186

$$\mathcal{L}_{\rm eff} = -\frac{G_F}{\sqrt{2}} F^4 \left[a\ \mathrm{Tr}(Q_L^{(-)} L_\mu)\,\mathrm{Tr}(Q_L^{(+)} L^\mu) + b\ \mathrm{Tr}(Q_L^{(-)} L_\mu Q_L^{(+)} L^\mu) + c\ \mathrm{Tr}(Q_L^{(-)} Q_L^{(+)} L_\mu L^\mu) \right]$$

$\mathcal{O}(N_C^2)$

$\mathcal{O}(N_C)$



$$Q_L^{(+)} = \begin{pmatrix} 0 & V_{\rm ud} & V_{\rm us} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad ; \quad Q_L^{(-)} = Q_L^{(+)\dagger}$$

$$g_8 = \frac{3}{5}(a+b) - b + c$$

$$g_{27} = \frac{3}{5}(a+b)$$

$$a = 1 + \mathcal{O}\left(\frac{1}{N_c^2}\right) \quad ; \quad c = \mathrm{Re}C_4 - 16\, L_5\, \mathrm{Re}C_6(\mu^2) \left[\frac{<\bar\psi\psi>}{f_\pi^3}\right]^2 + \mathcal{O}\left(\frac{1}{N_c^2}\right) \simeq 0.3 \pm 0.2$$

$$|g_{27}| \simeq 0.29 \;\longrightarrow\; b \simeq -0.52 + \mathcal{O}\left(\frac{1}{N_c^2}\right) \;\longrightarrow\; g_8 \simeq 1.1 + \mathcal{O}\left(\frac{1}{N_c^2}\right)$$

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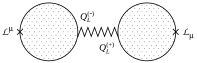

a

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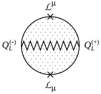
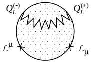
$\mathcal{O}(N_C)$



b



c

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$b < 0$ predicted through explicit calculations

AP–E. de Rafael, NP B358 (1991) 311
Bardeen-Buras-Gerard, Bijnens-Prades, Bertolini et al

Confirmed through inclusive QCD analysis

M. Jamin–AP, NP B425 (1994) 15

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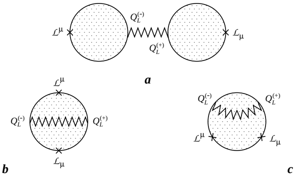

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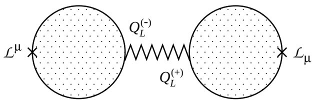
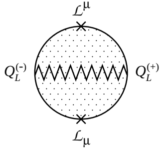
Confirmed through inclusive QCD analysis — M. Jamin–AP, NP B425 (1994) 15

Confirmed recently by lattice calculations — RBC-UKQCD, PRL 110 (2013) 15, 152001; PRD 91 (2015) 7, 074502

“A qualitative picture towards the understanding of the underlying physics begins to emerge”

AP – E. de Rafael, PL B374 (1996) 186





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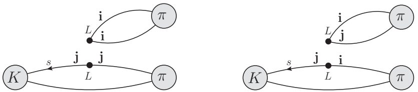
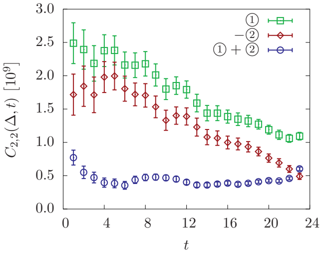
“Emerging understanding of the $\Delta I = 1/2$ Rule from Lattice QCD”

RBC-UKQCD, PRL 110 (2013) 15, 152001

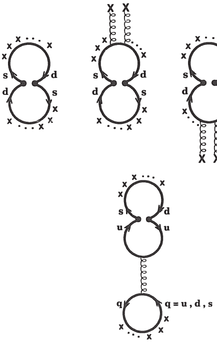


$$b \approx -0.7\, a$$



Effective Action Model: Bosonization in Gluonic Background

AP–E. de Rafael, NP B358 (1991) 311



$$\Delta \;=\; \frac{1}{N_C}\,\left[1-\frac{N_C}{2}\,\frac{\langle\frac{\alpha_s}{\pi}\,G^2\rangle}{16\pi^2 f_\pi^4}+\mathcal{O}(\alpha_s^2 N_C^2)\right]\;<\;0$$

$$g_{27}\;\approx\;\frac{3}{5}\,C_+(\mu^2)\;\left\{1+\Delta+\mathcal{O}(1/N_C^2)\right\}$$

$$g_8\;\approx\;\frac{1}{2}\,C_-(\mu^2)\;\left\{1-\Delta+\mathcal{O}(1/N_C^2)\right\}+\frac{1}{10}\,C_+(\mu^2)\;\left\{1+\Delta+\mathcal{O}(1/N_C^2)\right\}+c$$

$$c\;=\;C_4(\mu^2)-16\,C_6(\mu^2)\,L_5\,\left[\frac{\langle\bar\psi\psi\rangle}{f_\pi^3}\right]^2+\mathcal{O}(1/N_C^2)$$

Two-point Functions

AP–E. de Rafael, NP B358 (1991) 311, PL B374 (1996) 186
M. Jamin–AP, NP B425 (1994) 15

$$\Psi^{\Delta S=1,2}(q^2) \equiv i \int d^4x \, e^{iq\cdot x} \langle 0 | T \left(\mathcal{H}_{\rm eff}^{\Delta S=1,2}(x), \, \mathcal{H}_{\rm eff}^{\Delta S=1,2}(0)^\dagger \right) |0\rangle \;=\; \sum_{ij} C_i \, C_j^* \, \Psi_{ij}(q^2)$$

$$\frac{1}{\pi} \, \mathrm{Im}\Psi_{\pm\pm}(t) \;=\; \theta(t) \, \frac{2}{45} \, N_c^2 \left(1 \pm \frac{1}{N_c} \right) \frac{t^4}{(4\pi)^6} \, \alpha_s(t)^{-2a_\pm} \, C_\pm^2(M_W^2) \left[1 + \frac{3}{4} \, \frac{\alpha_s(t) N_c}{\pi} \, \mathcal{K}_\pm \right]$$

$$a_\pm \;=\; \pm \frac{9}{11 N_c} \, \frac{1 \mp 1/N_c}{1 - 6/11 N_c}$$

$$\mathcal{K}_+ \;=\; 1 - \frac{30587}{3630} \frac{1}{N_c} + \frac{164936}{19965} \frac{1}{N_c^2} - \frac{51591}{14641} \frac{1}{N_c^3} + \frac{440193}{322102} \frac{1}{N_c^4} + \cdots = -\frac{3649}{3645}$$

$$\mathcal{K}_- \;=\; 1 + \frac{30587}{3630} \frac{1}{N_c} + \frac{169706}{19965} \frac{1}{N_c^2} + \frac{70335}{14641} \frac{1}{N_c^3} + \frac{1810209}{322102} \frac{1}{N_c^4} + \cdots = +\frac{18278}{3645}$$