Rare kaon decays in general and recent results on $K \to \pi \ell^+ \ell^-$

$(K, \pi) = (K^\pm, \pi^\pm), (K_S, \pi^0)$

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OUTLINE

I. Remarks on rare kaon decays

II. Toward predicting the amplitude for $K^\pm \to \pi^\pm \ell^+ \ell^-$

III. Summary - Conclusions

based on:

G. D'Ambrosio, D. Greynat and M. K., JHEP 1902, 049 (2019)
G. D'Ambrosio, D. Greynat and M. K., Phys. Lett. B 797, 134891 (2019)
V. Bernard, S. Descotes-Genon, M.K. and B. Moussallam, work in progress

I. Remarks on rare kaon decays

Rare kaon decays proceed through FCNC, are suppressed in the SM

$\longrightarrow$ interesting window into new physics $\longrightarrow$ talks by D. Marzocca, A. Buras, G. Valencia

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Two broad classes:

- SD dominated rare decays $\Longrightarrow K \to \pi\nu\bar{\nu}$, $K \to \pi\pi\nu\bar{\nu}$

- LD dominated rare decays $\Longrightarrow K \to \gamma^{(*)}\gamma^{(*)}, K \to \pi\gamma^{(*)}, K \to \pi\gamma\gamma^{(*)}, K \to \pi\pi\gamma^{(*)}$...

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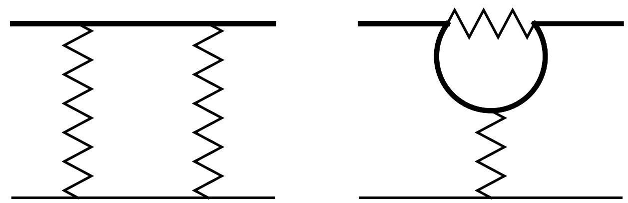

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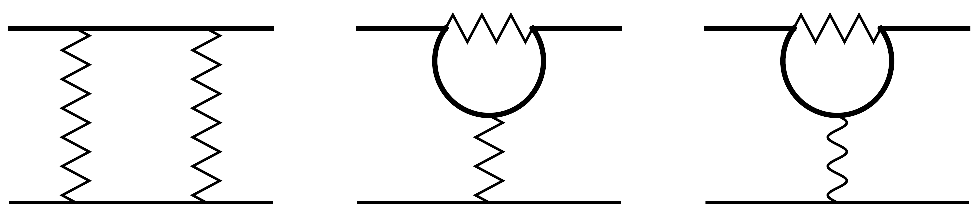

For reviews, see V. Cirigliano et al, Rev. Mod. Phys. 84, 399 (2012)
L. Littenberg, G. Valencia in PDG

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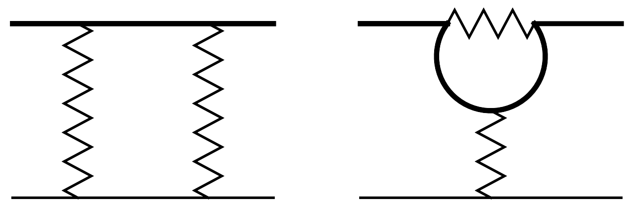

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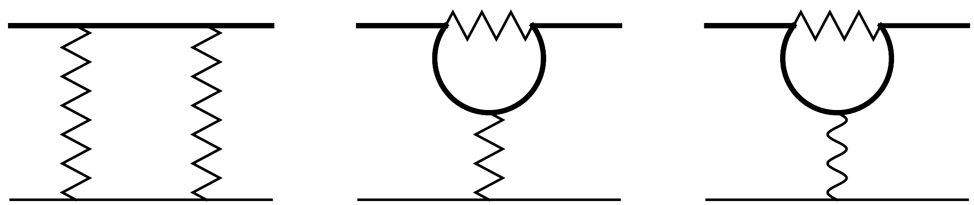

LD part difficult to assess

Several tools have been explored in order to evaluate the corresponding matrix elements

Taking $K \to \pi \ell^+ \ell^-$ as a case study

- Chiral perturbation theory $[4m_\ell^2 \leq s \leq (M_K - M_\pi)^2]$
  - one loop
    G. Ecker et al., Nucl Phys B 291, 692 (1987)
    B. Ananthanarayan and I. S. Imsong, J. Phys. G 39, 095002 (2012)
  - beyond one loop
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$\longrightarrow$ limitation: unknown low-energy constants

- Chiral perturbation theory and large-N_c
  S. Friot et al., Phys Lett B 595, 301 (2004)
  E. Coluccio Leskov et al, Phys Rev D 93, 094031 (2016)

- Lattice QCD $\longrightarrow$ talk by F. Ó hÓgáin
  G. Isidori et al., Phys Lett B 633, 75 (2006)
  N. H. Christ et al, Phys Rev D 92, 094512 (2015); D 94, 114516 (2016)

- $\cdots$ see review
  J. Portolés, J Phys Conf Series 800, 012030 (2017)

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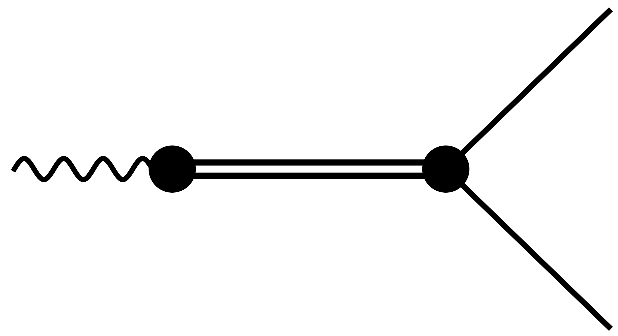
Why is it so difficult to implement simple resonance saturation that works so well in the strong sector?

$$\langle\pi|j_\mu(0)|\pi\rangle_{\rm QCD} \quad \longrightarrow \quad F_V^\pi(t)$$



$$F_V^\pi(t) = \frac{f_V\, g_V}{M_V^2 - t} \sim \frac{f_V\, g_V}{M_V^2}\left[1 + \frac{t}{M_V^2} + \cdots\right]$$

$$f_V \longrightarrow \Gamma_{\rho\to e^+e^-} \qquad\qquad g_V \longrightarrow \Gamma_{\rho\to\pi\pi}$$

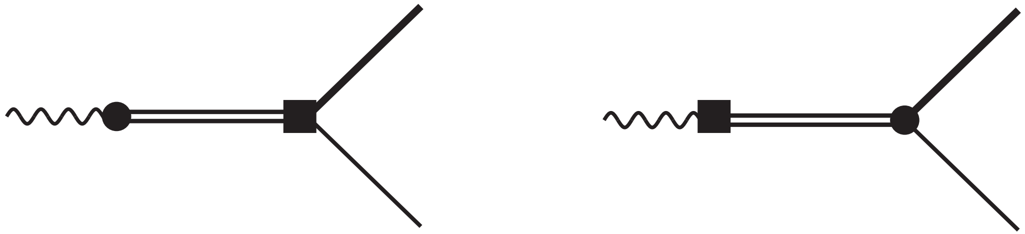
$$F_V^\pi(0) = 1 \quad \longrightarrow \quad f_V\, g_V = M_V^2$$

Systematize: effective lagrangian with resonances

G. Ecker, J. Gasser, A. Pich and E. de Rafael, Nucl. Phys. B 321, 311 (1989)

G. Ecker, J. Gasser, H. Leutwyler, A. Pich and E. de Rafael, Phys. Lett. B 223, 425 (1989)

$$\langle \pi | j_\mu(0) | K \rangle_{\mathcal{O}(G_F)} \qquad \longrightarrow \qquad W_{K\pi}(t)$$



$$W^{\mathrm{VMD}}_{K\pi}(t) = \frac{f_V \widetilde{g}_V}{M_V^2 - t} + \frac{g_V \widetilde{f}_V}{M_V^2 - t} \sim \frac{f_V \widetilde{g}_V + g_V \widetilde{f}_V}{M_V^2} \left[1 + \frac{t}{M_V^2} + \cdots \right]$$

$$\widetilde{f}_V \longrightarrow \Gamma_{K^* \to e^+ e^-} \;? \qquad \widetilde{g}_V \longrightarrow \Gamma_{\rho \to K\pi} \;?$$

$W_{K\pi}(0)$ not fixed by theory! $\quad [W'_{K\pi}(0)/W_{K\pi}(0) \sim 1/M_V^2]$

Effective lagrangians with resonances contain arbitrary parameters $\longrightarrow$ substantial model dependence

G. Ecker, A. Pich and E. de Rafael, Phys. Lett. B 237, 481 (1990)

G. Ecker, J. Kambor and D. Wyler, Nucl. Phys. B 394, 101 (1993)

G. D'Ambrosio and J. Portolés, Nucl. Phys. B 533, 494 (1998)

II. The case of the CPC $K \to \pi \ell^+ \ell^-$ decays

Focus on the CP conserving decays

$$K^{\pm} \to \pi^{\pm}\gamma^{*} \to \pi^{\pm}\ell^{+}\ell^{-} \qquad\qquad K_S \to \pi^{0}\gamma^{*} \to \pi^{0}\ell^{+}\ell^{-}$$

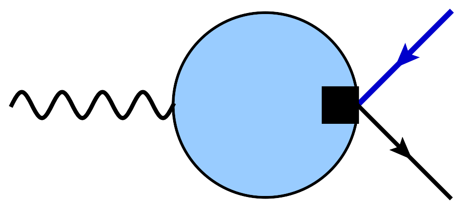
Good reasons to do so:

- already quite well studied experimentally (esp. $K^{\pm}$)
- more data will become available in the future (NA62, LHCb)
- analogues, in the kaon sector, of $b \to s\ell^{+}\ell^{-}$ transitions
- any LFUV effect invoked in order to explain the anomalies seen at LHCb

might also manifest itself here A. Crivellin et al., Phys. Rev. D 93, 074038 (2016)

General structure of the amplitude [to first order in G_F and in α]

$$\begin{aligned}
\mathcal{A}^{K\to\pi\ell^+\ell^-}(s) &= e^2 \times \bar{\mathrm{u}}(p_-)\gamma_\sigma \mathrm{v}(p_+) \times \frac{(-1)}{s}\left[\eta^{\sigma\rho} - (1-\xi)\,\frac{(k-p)^\sigma (k-p)^\rho}{s}\right] \times \langle \pi(p)|j_\rho(0)|K(k)\rangle_{\mathrm{SM}} \\
&= -e^2 \times \bar{\mathrm{u}}(p_-)\gamma_\rho \mathrm{v}(p_+) \times \frac{1}{s} \times \left[s(k+p)_\rho - (M_K^2 - M_\pi^2)(k-p)_\rho\right] \times \frac{W_{K\pi}(z)}{16\pi^2 M_K^2} \\
&= -e^2 \times \bar{\mathrm{u}}(p_-)\gamma^\rho \mathrm{v}(p_+)(k+p)_\rho \times \frac{W_{K\pi}(z)}{16\pi^2 M_K^2} \qquad [z \equiv s/M_K^2]
\end{aligned}$$



Differential decay rate

$$\frac{d\Gamma}{dz} = \frac{\alpha^2 M_K}{3(4\pi)^5}\,\lambda^{3/2}(1, z, M_\pi^2/M_K^2)\sqrt{1 - 4\frac{m_\ell^2}{zM_K^2}}\left(1 + 2\frac{m_\ell^2}{zM_K^2}\right)|W_{K\pi}(z)|^2$$

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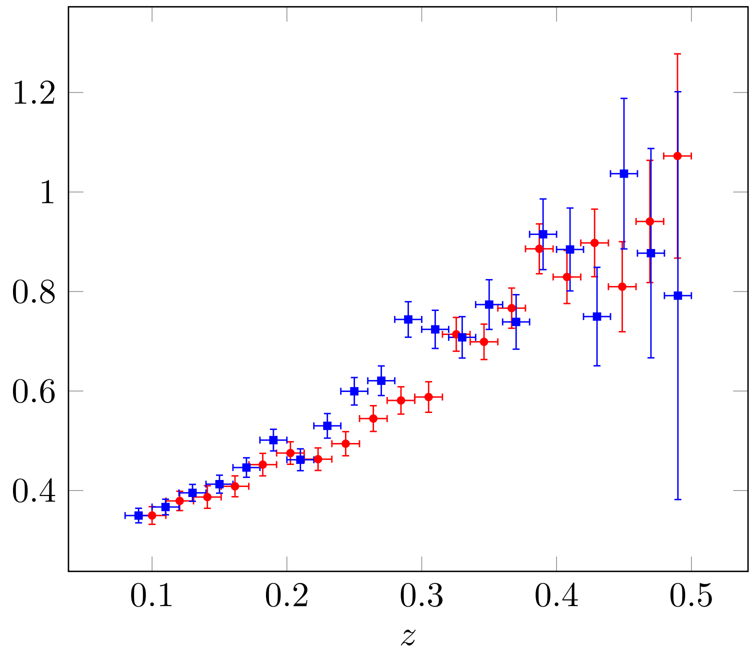
NA48/2 (21 bins, $K^\pm \to \pi^\pm e^+ e^-$)

E865 (20 bins, $K^+ \to \pi^+ e^+ e^-$)



$|W_{K\pi}(z)|^2$

Experimental situation

exp.	ref.	mode	number of events
BNL*	[1]	$K^+ \to \pi^+ e^+ e^-$	~ 500
BNL-E865*	[2]	$K^+ \to \pi^+ e^+ e^-$	$10\,300$
NA48/2*	[3]	$K^\pm \to \pi^\pm e^+ e^-$	$7\,263$
BNL-E787	[4]	$K^+ \to \pi^+ \mu^+ \mu^-$	~ 200
BNL-E865	[5]	$K^+ \to \pi^+ \mu^+ \mu^-$	~ 400
FNAL-E871	[6]	$K^\pm \to \pi^\pm \mu^+ \mu^-$	~ 100
NA48/2*	[7]	$K^\pm \to \pi^\pm \mu^+ \mu^-$	3120
NA48/1	[8]	$K_S \to \pi^0 e^+ e^-$	7
NA48/1	[9]	$K_S \to \pi^0 \mu^+ \mu^-$	6

*: decay distribution

[1] C. Alliegro *et al.*, Phys. Rev. Lett. 68, 278 (1992)

[2] R. Appel *et al.* [E865 Collaboration], Phys. Rev. Lett. 83, 4482 (1999)

[3] J. R. Batley *et al.* [NA48/2 Collaboration], Phys. Lett. B **677**, 246 (2009)

[4] S. Adler *et al.* [E787 Collaboration], Phys. Rev. Lett. **79**, 4756 (1997)

[5] H. Ma *et al.* [E865 Collaboration], Phys. Rev. Lett. **84**, 2580 (2000)

[6] H. K. Park *et al.* [HyperCP Collaboration], Phys. Rev. Lett. **88**, 111801 (2002)

[7] J. R. Batley *et al.* [NA48/2 Collaboration], Phys. Lett. B **697**, 107 (2011)

[8] J. R. Batley *et al.* [NA48/1 Collaboration], Phys. Lett. B **576**, 43 (2003)

[9] J. R. Batley *et al.* [NA48/1 Collaboration], Phys. Lett. B **599**, 197 (2004)

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$$= -e^2 \times \bar{\mathrm{u}}(p_-)\gamma_\rho \mathrm{v}(p_+) \times \frac{1}{s} \times \left[s(k+p)_\rho - (M_K^2 - M_\pi^2)(k-p)_\rho\right] \times \frac{W_{K\pi}(z)}{16\pi^2 M_K^2}$$

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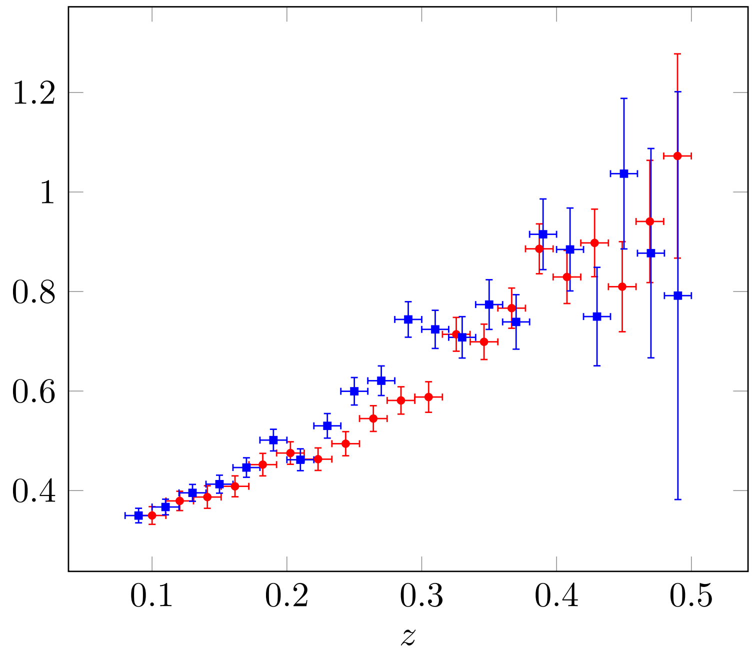

$|W_{K\pi}(z)|^2$

Main features

- slope, $W'_{K\pi}(0) = G_F M_K^2 b_+$
- cusp-like structure at $z \sim 0.3$ ($s = 4M_\pi^2$), fixed by unitarity and described by $K\pi \to \pi^+\pi^-$ (α_+, β_+)
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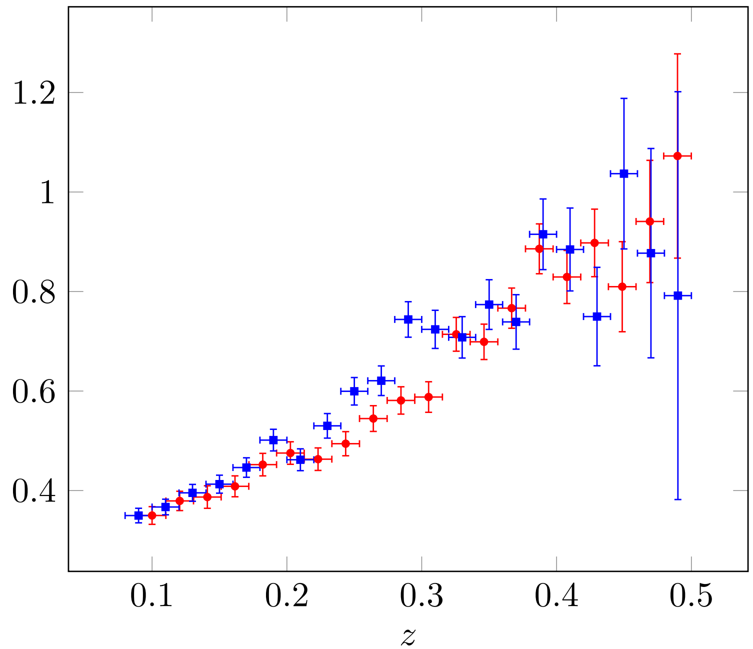

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- value at $z = 0$, $W_{K\pi}(0) = G_F M_K^2 a_+$ $\quad a_+ = -0.593(9)_{\mathrm{stat}}(1)_{\alpha_+}(6)_{\beta+}$

$$\frac{a_+}{b_+}\frac{M_K^2}{M_V^2} \sim 0.4 \longrightarrow \text{strong deviation from VMD}$$

G. D'Ambrosio et al., JHEP 9808, 004 (1998)

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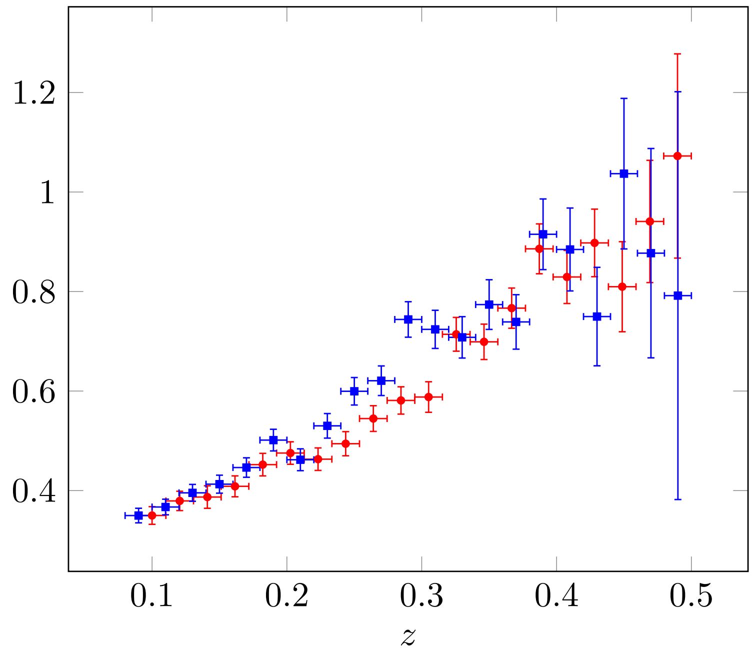

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$$R_{K^\pm} \equiv \frac{\mathrm{Br}[K^\pm \to \pi^\pm \mu^+ \mu^-]}{\mathrm{Br}[K^\pm \to \pi^\pm e^+ e^-]} = \begin{cases} 0.313(71) & \text{[PDG average]} \\ 0.309(43) & \text{[NA48/2 alone]} \end{cases}$$

What does the SM predict?

General structure of the $W_{K\pi}(z)$ form factors

$$W_{K\pi}(z) = W_{K\pi}^{\mathrm{LD}}(z;\nu) + W_{K\pi}^{\mathrm{SD}}(z;\nu)$$

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$$\mathcal{L}_{\rm non-lept}^{\Delta S=1}(x) = -\frac{G_{\rm F}}{\sqrt{2}} V_{us}^* V_{ud} \sum_{I=1}^{6} C_I(\nu) Q_I(x;\nu)$$

$\longrightarrow$ pure three-flavour QCD problem

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$$\nu\frac{d}{d\nu} j_\rho = 0 \qquad \nu\frac{d}{d\nu}\mathcal{L}_{\rm non-lept}^{\Delta S=1} = 0$$

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$\longrightarrow$ pure three-flavour QCD problem

$$\nu\frac{d}{d\nu} j_\rho = 0 \qquad \nu\frac{d}{d\nu}\mathcal{L}^{\Delta S=1}_{\rm non-lept} = 0 \qquad \text{but } \nu\frac{d}{d\nu} T\{j_\rho(0)\mathcal{L}^{\Delta S=1}_{\rm non-lept}(x)\} \neq 0$$

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- Long-distance part (first order in G_F and in α)

$$\mathcal{A}_{\rm LD} = e^2 \times \bar{\rm u}(p_{\ell^-})\gamma_\sigma {\rm v}(p_{\ell^+}) \times \frac{(-1)}{s}\left[\eta^{\sigma\rho} - (1-\xi)\,\frac{(k-p)^\sigma(k-p)^\rho}{s}\right] \times \langle\pi(p)|j_\rho(0)|K(k)\rangle_{\rm SM}$$

$$\begin{aligned}\langle\pi(p)|j_\rho(0)|K(k)\rangle_{\rm SM} &= i\int d^4x\,\langle\pi(p)|T\{j_\rho(0)\mathcal{L}_{\rm non-lept}^{\Delta S=1}(x)\}|K(k)\rangle_{\rm QCD} \\ &= \left[s(k+p)_\rho - (M_K^2 - M_\pi^2)(k-p)_\rho\right] \times \frac{W_{K\pi}^{\rm LD}(s;\nu)}{16\pi^2 M_K^2}\end{aligned}$$

$$\mathcal{L}_{\rm non-lept}^{\Delta S=1}(x) = -\frac{G_{\rm F}}{\sqrt{2}} V_{us}^* V_{ud} \sum_{I=1}^{6} C_I(\nu) Q_I(x;\nu)$$

$\longrightarrow$ pure three-flavour QCD problem

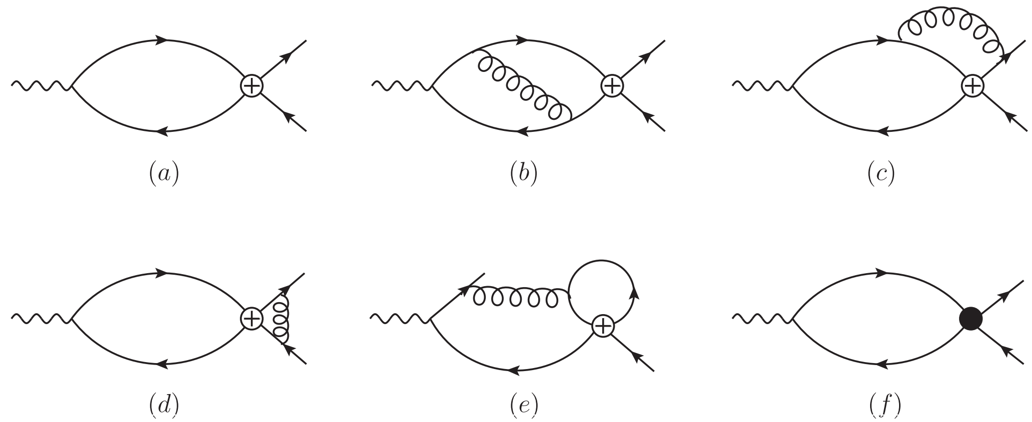
$$\nu\frac{d}{d\nu}j_\rho = 0 \qquad \nu\frac{d}{d\nu}\mathcal{L}_{\rm non-lept}^{\Delta S=1} = 0 \qquad \text{but } \nu\frac{d}{d\nu}T\{j_\rho(0)\mathcal{L}_{\rm non-lept}^{\Delta S=1}(x)\} \neq 0$$

$$W_{K\pi}^{\rm LD}(z;\nu) = W_{K\pi}^{\pi\pi}(z) + W_{K\pi}^{K\pi}(z) + W_{K\pi}^{K\bar{K}}(z) + \cdots + W_{K\pi}^{\rm pQCD}(z;\nu)$$

General structure of the $W_{K\pi}(z)$ form factors

$$W_{K\pi}(z) = W_{K\pi}^{\mathrm{LD}}(z;\nu) + W_{K\pi}^{\mathrm{SD}}(z;\nu)$$

$$W_{K\pi}^{\mathrm{pQCD}}(z;\nu) \longrightarrow$$



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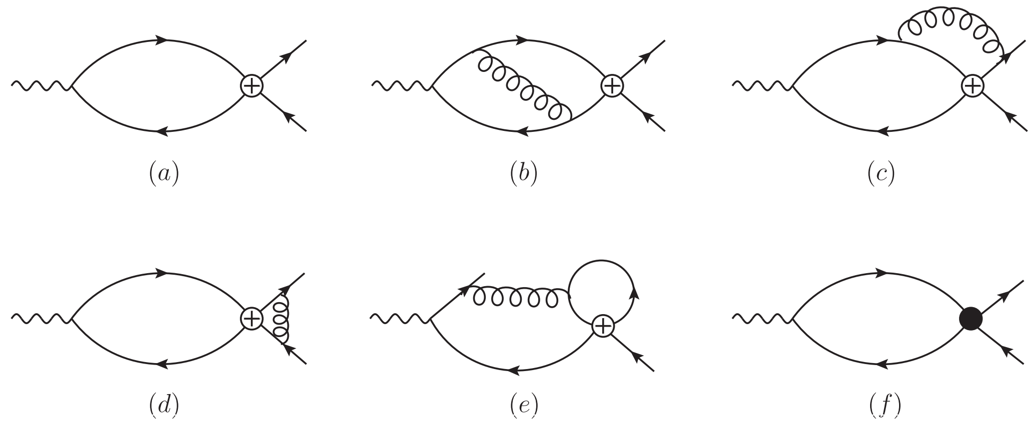

- Short-distance part

$$\mathcal{L}_{\rm lept}^{\Delta S=1}(x) = -\frac{G_{\rm F}}{\sqrt{2}} V_{us}^* V_{ud} \left[C_{7V}(\nu) Q_{7V}(x) + C_{7A} Q_{7A}(x)\right]$$

$$Q_{7V} = (\bar{s}^i d_i)_{V-A}(\bar{\ell}\ell)_V,\ Q_{7A} = (\bar{s}^i d_i)_{V-A}(\bar{\ell}\ell)_A$$

E. Witten, Nucl. Phys. B 122, 109 (1977)

F. J. Gilman, M. B. Wise, Phys Rev D 21, 3150 (1980)

C. Dib et al., Phys Lett B 218, 487 (1989); Phys Rev D 39, 2639 (1989)

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A. J. Buras et al., Nucl Phys B 423, 349 (1994)

General structure of the $W_{K\pi}(z)$ form factors

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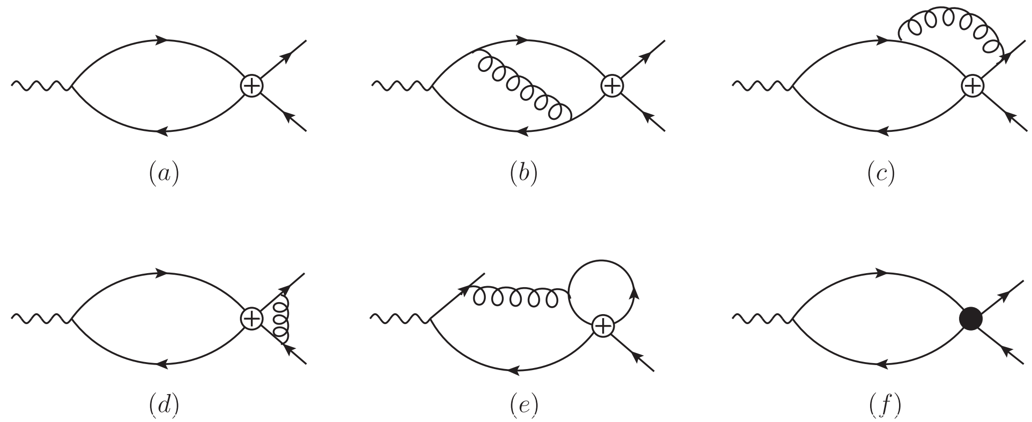

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A. J. Buras et al., Nucl Phys B 423, 349 (1994)

$$\frac{W_{K\pi}^{\rm SD}(z;\nu)}{16\pi^2 M_K^2} = -\left(-\frac{G_{\rm F}}{\sqrt{2}} V_{us} V_{ud} \right) \times \frac{C_{7V}(\nu)}{4\pi\alpha} \times C_{K\pi} f_+(s)$$

$$\langle \pi(p) | \, (\bar{s}\,\gamma_\rho d)(0) | K(k) \rangle = C_{K\pi} \left[(k+p)_\rho f_+^{K\pi}(s) + (k-p)_\rho f_-^{K\pi}(s) \right]$$

$$\nu \frac{dC_{7V}(\nu)}{d\nu} = \frac{\alpha}{\alpha_s(\nu)} \sum_{J=1}^{6} \gamma_{J,7}(\alpha_s) C_J(\nu)$$

Outline of a phenomenological construction of $W_{K\pi}(z)$

$$W_{K\pi}(z) = W_{K\pi}^{\pi\pi}(z) + W_{K\pi}^{K\pi}(z) + W_{K\pi}^{K\bar{K}}(z) + \cdots + W_{K\pi}^{\mathrm{pQCD}}(z;\nu) + W_{K\pi}^{\mathrm{SD}}(z;\nu)$$

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$$\downarrow$$

$$W_{K\pi}(z) = W_{K\pi}^{\pi\pi}(z) + W_{K\pi}^{K\pi}(z) + W_{K\pi}^{K\bar{K}}(z) + W_{K\pi}^{\mathrm{res}}(z;\nu) + W_{K\pi}^{\mathrm{SD}}(z;\nu)$$

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- $W_{K\pi}^{\mathrm{res}}(z;\nu)$

- should reproduce the leading short-distance behaviour of $W_{K\pi}^{\mathrm{LD}}(z)$ for large euclidian z ($q^2 = s < 0$)

$$\lim_{q\to\infty} i \int d^4x\, e^{iq\cdot x} T\{j^\mu(x)\mathcal{L}_{\Delta S=1}(0)\}_{\overline{\mathrm{MS}}} =$$

$$= \left(-\frac{G_{\mathrm{F}}}{\sqrt{2}} V_{us}^* V_{ud}\right) [q^\mu q^\rho - q^2 \eta^{\mu\rho}] \times \bar{s}\gamma_\rho(1-\gamma_5)d \times \frac{1}{4\pi} \sum_{I=1}^{6} C_I(\nu)\, \xi_I(\alpha_s\, ; \nu^2/q^2) + \mathcal{O}(q)$$

$$\xi_I(\alpha_s\, ; \nu^2/q^2) = \sum_{p\geq 0} \sum_{r=0}^{p+1} \xi_{pr}^I \alpha_s^p(\nu) \ln^r(-\nu^2/q^2)$$

Outline of a phenomenological construction of $W_{K\pi}(z)$

$$W_{K\pi}(z) = W^{\pi\pi}_{K\pi}(z) + W^{K\pi}_{K\pi}(z) + W^{K\bar{K}}_{K\pi}(z) + \cdots + W^{\rm pQCD}_{K\pi}(z;\nu) + W^{\rm SD}_{K\pi}(z;\nu)$$

$$\downarrow$$

$$W_{K\pi}(z) = W^{\pi\pi}_{K\pi}(z) + W^{K\pi}_{K\pi}(z) + W^{K\bar{K}}_{K\pi}(z) + W^{\rm res}_{K\pi}(z;\nu) + W^{\rm SD}_{K\pi}(z;\nu)$$

- $W^{\rm res}_{K\pi}(z;\nu)$

- should reproduce the leading short-distance behaviour of $W^{\rm LD}_{K\pi}(z)$ for large euclidian z ($q^2 = s < 0$)

$$\lim_{q\to\infty} i\int d^4x\, e^{iq\cdot x} T\{j^\mu(x)\mathcal{L}_{\Delta S=1}(0)\}_{\overline{\rm MS}} =$$

$$= \left(-\frac{G_{\rm F}}{\sqrt{2}} V^*_{us}V_{ud}\right)[q^\mu q^\rho - q^2\eta^{\mu\rho}] \times \bar{s}\gamma_\rho(1-\gamma_5)d \times \frac{1}{4\pi}\sum_{I=1}^{6} C_I(\nu)\,\xi_I(\alpha_s\,;\nu^2/q^2) + \mathcal{O}(q)$$

$$\xi_I(\alpha_s\,;\nu^2/q^2) = \sum_{p\geq 0}\sum_{r=0}^{p+1}\xi^I_{pr}\alpha^p_s(\nu)\ln^r(-\nu^2/q^2)$$

One loop:

$$\xi^I_{01} = \frac{1}{4\pi}\,\frac{8}{9}\times(N_c\,,\,1\,,\,-1\,,\,-N_c\,,\,0\,,\,0)$$

$$\xi^I_{00} = \frac{\xi^I_{01}}{3}\times\begin{cases} 2 & \text{NDR} \\ 5 & \text{HV}\end{cases}$$

Outline of a phenomenological construction of $W_{K\pi}(z)$

$$W_{K\pi}(z) = W_{K\pi}^{\pi\pi}(z) + W_{K\pi}^{K\pi}(z) + W_{K\pi}^{K\bar{K}}(z) + \cdots + W_{K\pi}^{\mathrm{pQCD}}(z;\nu) + W_{K\pi}^{\mathrm{SD}}(z;\nu)$$

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$$\xi_I(\alpha_s\,;\nu^2/q^2) = \sum_{p\geq 0}\sum_{r=0}^{p+1} \xi_{pr}^I \alpha_s^p(\nu)\ln^r(-\nu^2/q^2)$$

One loop:

the relations

$$\frac{\gamma_{I,7}^{(0)}}{4\pi} + 2\xi_{01}^I = 0$$

entail

$$\nu\,\frac{dW_{K\pi}(z)}{d\nu} = 0 + \mathcal{O}(\alpha_s)$$

Outline of a phenomenological construction of $W_{K\pi}(z)$

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$$= \left(-\frac{G_{\mathrm{F}}}{\sqrt{2}} V_{us}^* V_{ud}\right) [q^\mu q^\rho - q^2 \eta^{\mu\rho}] \times \bar{s}\gamma_\rho(1-\gamma_5)d \times \frac{1}{4\pi} \sum_{I=1}^{6} C_I(\nu)\, \xi_I(\alpha_s\,; \nu^2/q^2) + \mathcal{O}(q)$$

$$\xi_I(\alpha_s\,;\nu^2/q^2) = \sum_{p\geq 0} \sum_{r=0}^{p+1} \xi_{pr}^I \alpha_s^p(\nu) \ln^r(-\nu^2/q^2)$$

Two loops:

turn the argument around!

$$\text{order } \mathcal{O}(\alpha_s):\ \ \xi_{11}^I = -\frac{1}{2}\frac{\gamma_{I,7}^{(1)}}{(4\pi)^2} - \frac{1}{2}\sum_{J=1}^{6}\frac{\gamma_{IJ}^{(0)}}{4\pi}\xi_{00}^J \qquad \xi_{12}^I = -\frac{1}{4}\sum_{J=1}^{6}\frac{\gamma_{IJ}^{(0)}}{4\pi}\xi_{01}^J$$

$$\nu\,\frac{dW_{K\pi}(z)}{d\nu} = 0 + \mathcal{O}(\alpha_s^2)$$

Outline of a phenomenological construction of $W_{K\pi}(z)$

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$$\downarrow$$

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$$\xi_I(\alpha_s\,;\nu^2/q^2) = \sum_{p\geq 0}\sum_{r=0}^{p+1} \xi_{pr}^I \alpha_s^p(\nu)\ln^r(-\nu^2/q^2)$$

Two loops:

turn the argument around!

$$\xi_{12}^I = \frac{1}{(4\pi)^2}\,\frac{4}{27}\left(N_c - \frac{1}{N_c}\right) \times (0\,,\,-8\,,\,+11\,,\,N_f\,,\,0\,,\,N_f)$$

$$\xi_{11}^I = \frac{1}{(4\pi)^2}\,\frac{8}{3}\left(N_c - \frac{1}{N_c}\right) \times \begin{cases} \left(\frac{N_c}{2}\,,\,-\frac{19}{18}\,,\,\frac{17}{9}\,,\,\frac{7}{6} - \frac{N_c}{2}\,,\,0\,,\,\frac{7}{6}\right) & \mathrm{NDR} \\ \left(\frac{N_c}{2}\,,\,-\frac{5}{18}\,,\,\frac{13}{9}\,,\,\frac{7}{6} - \frac{N_c}{2}\,,\,0\,,\,\frac{7}{6}\right) & \mathrm{HV} \end{cases}$$

G. Altarelli et al., Nucl. Phys. B 187, 461 (1981)

A. J. Buras and P. H. Weisz, Nucl. Phys. B 333, 66 (1990)

A. J. Buras et al., Nucl. Phys. B 370, 69 (1992)

A. J. Buras et al., Nucl. Phys. B 400, 37 (1993)

M. Ciuchini et al., Nucl. Phys. B 415, 403 (1994)

$$\xi_{10}^I = ?$$

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$$\lim_{q\to\infty} i\int d^4x\, e^{iq\cdot x} T\{j^\mu(x)\mathcal{L}_{\Delta S=1}(0)\} =$$

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$$\xi_I(\alpha_s\,;\nu^2/q^2) = \sum_{p\geq 0}\sum_{r=0}^{p+1} \xi_{pr}^I \alpha_s^p(\nu)\ln^r(-\nu^2/q^2)$$

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A. J. Buras et al., Nucl. Phys. B 400, 37 (1993)
M. Ciuchini et al., Nucl. Phys. B 415, 403 (1994)

$$\xi_{10}^I = ? \qquad \longrightarrow \qquad -\xi_{11}^I \leq \xi_{10}^I \leq -\xi_{11}^I$$

Outline of a phenomenological construction of $W_{K\pi}(z)$

$$W_{K\pi}(z) = W_{K\pi}^{\pi\pi}(z) + W_{K\pi}^{K\pi}(z) + W_{K\pi}^{K\bar{K}}(z) + \cdots + W_{K\pi}^{\mathrm{pQCD}}(z;\nu) + W_{K\pi}^{\mathrm{SD}}(z;\nu)$$

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- $W_{K\pi}^{\mathrm{res}}(z;\nu)$

- should reproduce the leading short-distance behaviour of $W_{K\pi}^{\mathrm{LD}}(z)$ for large euclidian z ($q^2 = s < 0$)

- described by an infinite sum of equally-spaced (in mass^2) zero-width resonances (Regge-type spectrum)

M. A. Shifman, hep-ph/0009131

M. Golterman and S. Peris, JHEP 0101, 028 (2001)

M. Golterman, S. Peris, B. Phily and E. De Rafael, JHEP 0201, 024 (2002)

S. Friot, D. Greynat and E. De Rafael, Phys. Lett. B 628, 73 (2005)

E. de Rafael, Pramana 78, 927 (2012)

D. Greynat, E. de Rafael and G. Vulvert, JHEP 1403, 107 (2014)

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E. de Rafael, Pramana 78, 927 (2012)
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$$W_{K\pi}^{\mathrm{res}}(z;D) = -16\pi^2 M_K^2 \left(-\frac{G_{\mathrm{F}}}{\sqrt{2}} V_{us} V_{ud}\right) \frac{C_{K\pi} f_+^{K\pi}(zM_K^2)}{4\pi} \times \mathcal{W}_{K\pi}^{\mathrm{res}}(z;D)$$

$$\mathcal{W}_{K\pi}^{\mathrm{res}}(z;D) = \int dx \, \frac{\rho_{K\pi}^{\mathrm{res}}(x;D)}{x - zM_K^2 - i0}$$

$$\rho_{K\pi}^{\mathrm{res}}(s;D) = \sum_{n\geq 1} M^2 \mu_n(D)\, \delta(s - nM^2) \qquad [M \sim 1\ \mathrm{GeV}]$$

G. D'Ambrosio, D. Greynat and M. Knecht, JHEP 1902, 049 (2019)
G. D'Ambrosio, D. Greynat and M. Knecht, Phys. Lett. B 797, 134891 (2019)

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- $W_{K\pi}^{\mathrm{res}}(z;\nu)$

- should reproduce the leading short-distance behaviour of $W_{K\pi}^{\mathrm{LD}}(z)$ for large euclidian z ($q^2 = s < 0$)

- described by an infinite sum of equally-spaced (in mass^2) zero-width resonances (Regge-type spectrum)

$$\mathcal{W}_{K\pi}^{\mathrm{res}}(z;\nu) = \sum_{I=1}^{6} C_I(\nu)\Bigg\{\xi_{00}^I + \alpha_s(\nu)\xi_{10}^I + \ln\frac{\nu^2}{M^2}\left[\xi_{01}^I + \alpha_s(\nu)\left(\xi_{11}^I + \xi_{12}^I\ln\frac{\nu^2}{M^2}\right)\right]$$
$$-\psi(1+w)\left[\xi_{01}^I + \alpha_s(\nu)\left(\xi_{11}^I + 2\xi_{12}^I\ln\frac{\nu^2}{M^2}\right)\right] + \alpha_s(\nu)\,\xi_{12}^I\left(\widetilde{\psi}(w) - \frac{\pi^2}{3} - 2\gamma_1\right)\Bigg\}$$

$$w \equiv -zM_K^2/M^2 = -s/M^2$$

$$\psi(1+w) = -\gamma_E + M^2 w\int\frac{dx}{x}\,\frac{1}{x+M^2w}\sum_{n\geq 1}M^2\delta(x-nM^2) = -\gamma_E + \sum_{n\geq 1}\frac{w}{n(n+w)} \underset{w\to+\infty}{\sim} \ln w + \cdots$$

$$\widetilde{\psi}(w) = 2M^2 w\int\frac{dx}{x}\,\frac{1}{x+M^2w}\sum_{n\geq 1}M^2(\ln n)\delta(x-nM^2) = 2w\sum_{n\geq 1}\frac{\ln n}{n(n+w)} \underset{w\to+\infty}{\sim} \ln^2 w + 2\left(\frac{\pi^2}{6}+\gamma_1\right) + \cdots$$

G. D'Ambrosio, D. Greynat and M. Knecht, JHEP 1902, 049 (2019)

G. D'Ambrosio, D. Greynat and M. Knecht, Phys. Lett. B (2019), to appear

Outline of a phenomenological construction of $W_{K\pi}(z)$

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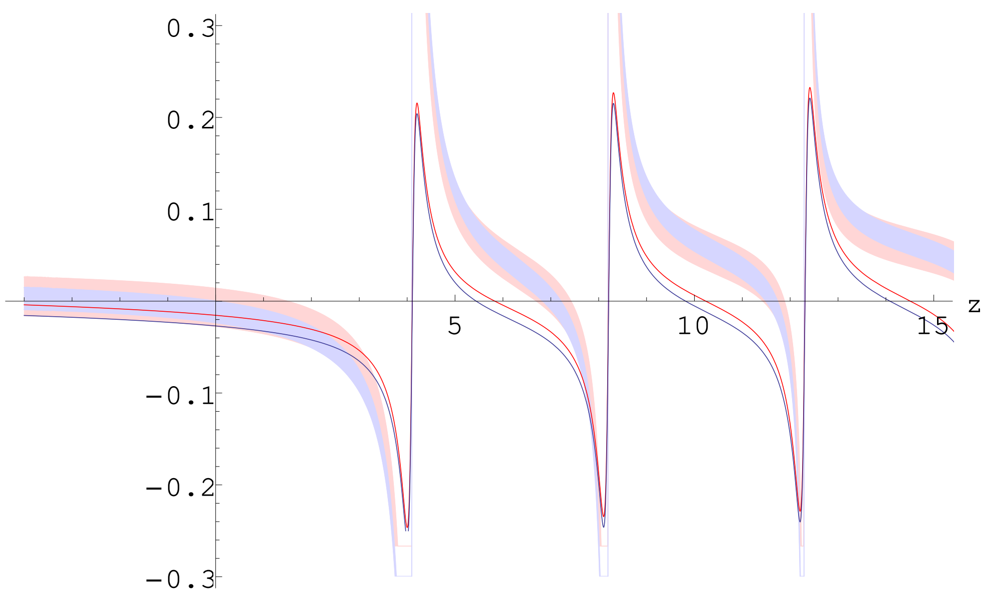
- $W_{K\pi}^{\rm res}(z;\nu)$

- should reproduce the leading short-distance behaviour of $W_{K\pi}^{\rm LD}(z)$ for large euclidian z ($q^2 = s < 0$)

- described by an infinite sum of equally-spaced (in mass^2) zero-width resonances (Regge-type spectrum)

$$\mathcal{W}_{K\pi}^{\rm res}(z;\nu) = \sum_{I=1}^{6} C_I(\nu)\Bigg\{\xi_{00}^I + \alpha_s(\nu)\xi_{10}^I + \ln\frac{\nu^2}{M^2}\left[\xi_{01}^I + \alpha_s(\nu)\left(\xi_{11}^I + \xi_{12}^I \ln\frac{\nu^2}{M^2}\right)\right]$$
$$-\psi(1+w)\left[\xi_{01}^I + \alpha_s(\nu)\left(\xi_{11}^I + 2\xi_{12}^I\ln\frac{\nu^2}{M^2}\right)\right] + \alpha_s(\nu)\,\xi_{12}^I\left(\widetilde{\psi}(w) - \frac{\pi^2}{3} - 2\gamma_1\right)\Bigg\}$$



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- describe the contribution from $\pi\pi$ intermediate state by an unsubtracted DR

$$W_{+}^{\pi\pi}(z) = \frac{1}{\pi}\int_0^\infty dx\, \frac{\mathrm{Abs}\, W_{+}^{\pi\pi}(x/M_K^2)}{x - zM_K^2 - i0}$$

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$$\frac{\mathrm{Abs}\, W_{+}^{\pi\pi}(s/M_K^2)}{16\pi^2 M_K^2} = \theta(s-4M_\pi^2) \times \frac{s-4M_\pi^2}{s}\lambda_{K\pi}^{-1/2}(s) F_V^{\pi *}(s) \times f_1^{\pi^+\pi^-\to K^+\pi^-}(s)$$

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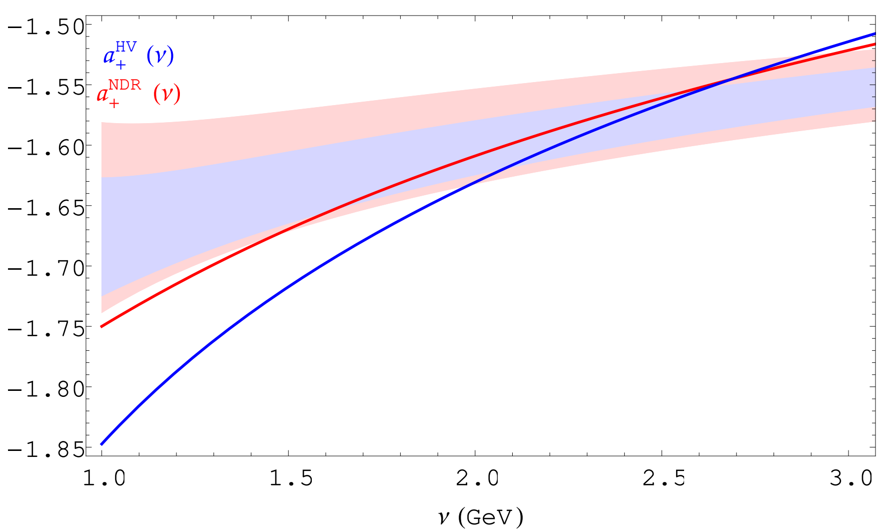

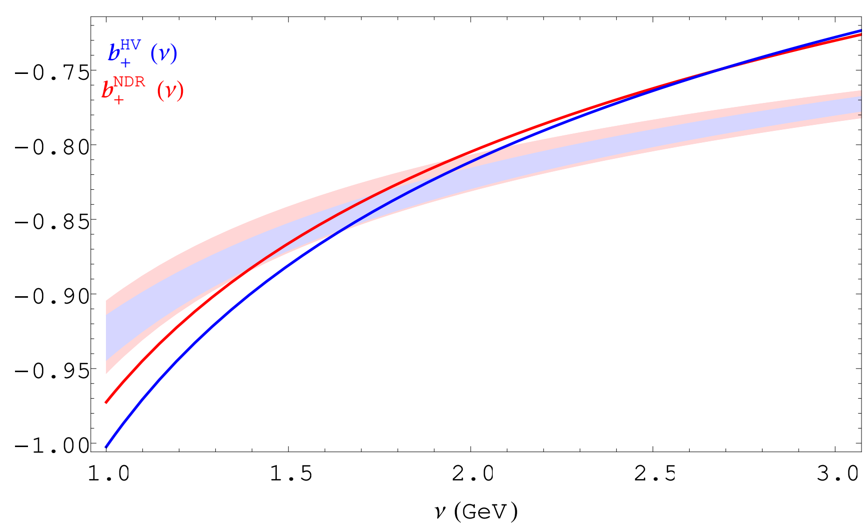

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- future plans: add also $K\pi$ (ISI)...

- ... and $K\bar{K}$ (coupled-channel KT analysis, cf M. Albaladejo and B. Moussallam, Eur. Phys. J. C 77, 508 (2017))

III. Summary - Conclusions

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  - simple treatment of $W_{+}^{\pi\pi}(z)$ leads to values of a_+ and b_+ in the right ballpark
  - deviation from VMD behaviour requires to also include $W_{K\pi}^{K\pi}(z)$ and $W_{K\pi}^{K\bar{K}}(z)$
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  - tools to address these issues are available

Thanks for your attention!