KAON 2019
theory summary

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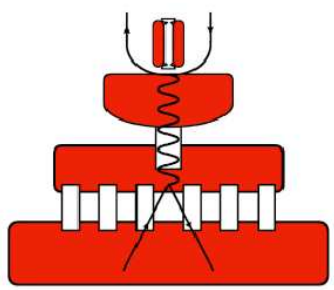

- 13 theory talks out of 48 presentations

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KAON2016: $\sim 40\%$

KAON2013: $\sim 40\%$

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an issue to improve upon for KAON2022!

- 13 theory talks out of 48 presentations

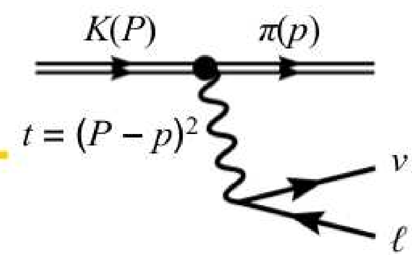
- but broad spectrum of topics covered
    - CKM unitarity
    - $K \to \pi\nu\bar{\nu}$
    - other rare kaon decays
    - contributions from BSM physics, connection with B anomalies
    - HNLs
    - $K \to \pi\pi$, ϵ'/ϵ

$$V_{us}$$

CKM unitarity

V_{us} from K_{l3}

2.1 V_{us} from K_{l3} : Master Formula



- Master formula for $K \to \pi l \nu_l$: K = {K^+,K^0}, l={e,μ}

$$\boxed{\Gamma\left(K \to \pi l \nu[\gamma]\right) = Br(K_{l3})/\tau = C_K^2 \frac{G_F^2 m_K^5}{192\pi^3} S_{EW}^K \left|V_{us}\right|^2 \left|f_+^{K^0\pi^-}(0)\right|^2 I_{Kl}\left(1 + 2\Delta_{\mathrm{EM}}^{Kl} + 2\Delta_{\mathrm{SU(2)}}^{K\pi}\right)}$$

Experimental inputs:

$\Gamma(K_{l3})$ Rates with well-determined treatment of radiative decays
- Branching ratios
- Kaon lifetimes

$I_{Kl}(\lambda_{Kl})$ Integral of form factor over phase space: λs parametrize evolution in $t=q^2$

Inputs from theory:

S_{EW}^K Universal short distance EW corrections

$f_+^{K^0\pi^-}(0)$ Hadronic matrix element (form factor) at zero momentum transfer (t=0)

$\Delta_{\mathrm{EM}}^{Kl}$ Form-factor correction for long-distance EM effects

$\Delta_{\mathrm{SU(2)}}^{K\pi}$ Form-factor correction for SU(2) breaking

- IB corrections (e.m. and $m_d \neq m_u$) under good theoretical control
- phase-space integral available from experiment

$$|V_{us}|f_+(0) = 0.21652(41)$$

- $f_+(0)$ without LEC at one loop, but not at two loops $\longrightarrow$ lattice QCD

$f_+(0)$ from lattice

2019 averages for $f_+(0)$

N_f = 2+1+1

$$f_+(0) = \mathbf{0.9698(17)}$$

FNAL/MILC18 preliminary replaces FNAL/MILC13E in FLAG average

ETM16	$0.9709(44)(9)(11)_{\rm ext}$
FNAL/MILC18	0.9696(15)(11)

N_f = 2+1

$$f_+(0) = \mathbf{0.9677(27)}$$

FLAG average, Nov 2016 update

JLQCD17 not included because only 1 lattice spacing used

FNAL/MILC12I	0.9667(23)(33)
RBC/UKQCD15A	0.9685(34)(14)

V_{us} from V_{us}/V_{ud}

3.1 Master formula for V_{us}/V_{ud} from $K_{\ell 2}/\pi_{\ell 2}$ decays

- From K_{l2}/π_{l2}:

$$\frac{|V_{us}|}{|V_{ud}|}\frac{f_K}{f_\pi} = \left(\frac{\Gamma_{K_{\mu 2(\gamma)}} m_{\pi^\pm}}{\Gamma_{\pi_{\mu 2(\gamma)}} m_{K^\pm}}\right)^{1/2} \frac{1-m_\mu^2/m_{\pi^\pm}^2}{1-m_\mu^2/m_{K^\pm}^2}\left(1-\frac{1}{2}\delta_{EM}-\frac{1}{2}\delta_{SU(2)}\right)$$

Inputs from theory:

Cirigliano, Neufeld '11

$\delta_{EM} = -0.0069(17)$
Long-distance EM corrections

$\delta_{SU(2)} = -0.0043(5)(11)$
Strong isospin breaking
$f_K/f_\pi \to f_{K\pm}/f_{\pi\pm}$

Lattice: f_K/f_π
Cancellation of lattice-scale uncertainties from ratio
NB: Most lattice results already corrected for $SU(2)$-breaking: $f_{K\pm}/f_{\pi\pm}$

Inputs from experiment:

Updated $K^\pm$ BR fit:
$BR(K^\pm_{\mu 2(\gamma)}) = 0.6358(11)$
$\tau_{K\pm} = 12.384(15)$ ns

PDG:
$BR(\pi^\pm_{\mu 2(\gamma)}) = 0.9999$
$\tau_{\pi\pm} = 26.033(5)$ ns

$$\boxed{|V_{us}/V_{ud}| \times f_{K\pm}/f_{\pi\pm} = 0.27599(37)}$$

No $SU(2)$-breaking correction

radiative corrections in V_{ud}

New Radiative Corrections for $0^+ \to 0^+$

$$|V_{ud}|^2 = \frac{2984.432(3)\,\mathrm{s}}{\mathcal{F}t(1+\Delta_R^V)}$$

- Conventional calculation:

$$\Delta_R^V = 0.02361(38)$$

Marciano & Sirlin'06

- Dispersion Relations:

$$\Delta_R^V = 0.02467(22)$$

Seng, Gorchtein, Patel & Ramsey-Musolf'18

$$\boxed{|V_{ud}| = \mathbf{0.97418(10)_{Ft}(18)_{\Delta_R^V}}} \Rightarrow \boxed{|V_{ud}| = \mathbf{0.97370(10)_{Ft}(11)_{\Delta_R^V}}}$$

~1.8σ smaller

⇒ New Analysis $\boxed{|V_{ud}| = \mathbf{0.97389(19)}}$ *Czarnecki, Marciano, Sirlin'19*

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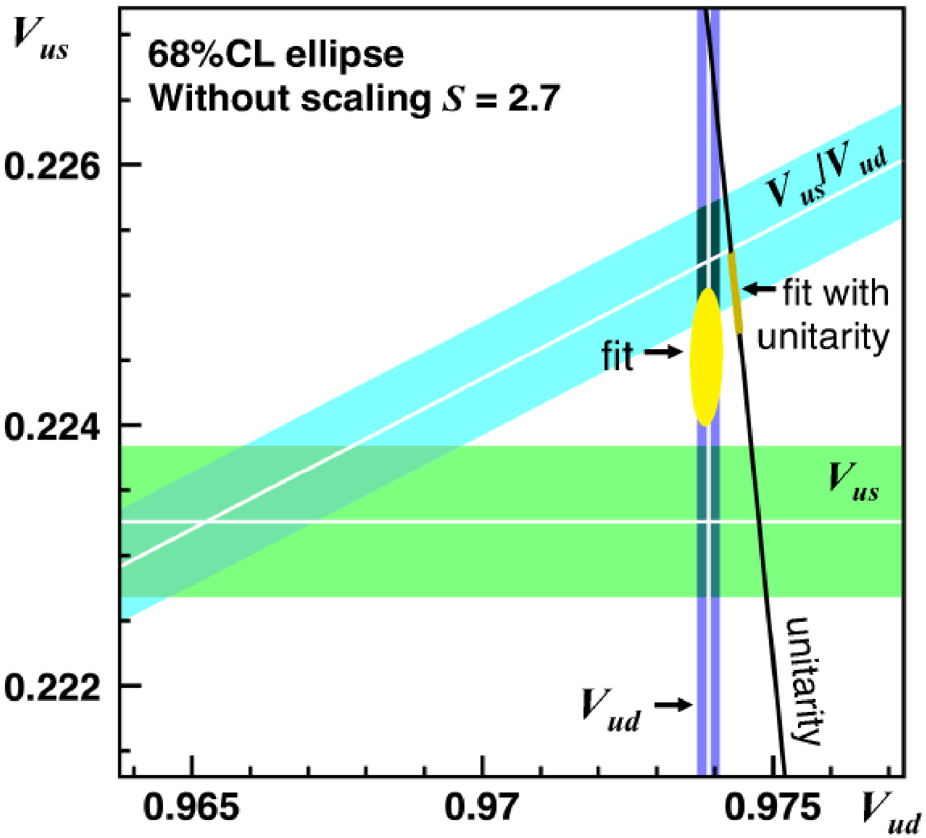
Better understanding of inputs necessary

CKM unitarity?

$$|V_{ud}|^2 + |V_{us}|^2 + \underbrace{|V_{ub}|^2}_{\sim 2\cdot 10^{-5}} = 1 + \Delta_{\mathrm{CKM}}$$

4.5 V_{us} and CKM unitarity: All data, New Vud

N_f **= 2+1+1:** Fit to results for $|V_{ud}|$, $|V_{us}|$, $|V_{us}|/|V_{ud}|$
$f_+(0)$ **= 0.9698(17),** f_K/f_π **= 1.1967(18)**



$|V_{ud}|$ **= 0.97389(19)**
$|V_{us}|$ = 0.2233(6)
$|V_{us}|/|V_{ud}|$ = 0.2313(5)

Fit results, no constraint

V_{ud} **= 0.97386(19)**
V_{us} **= 0.22452(35)**
χ^2/ndf = 7.5/1 (0.6%)
Δ_{CKM} **= −0.00119(41)**
-2.9σ

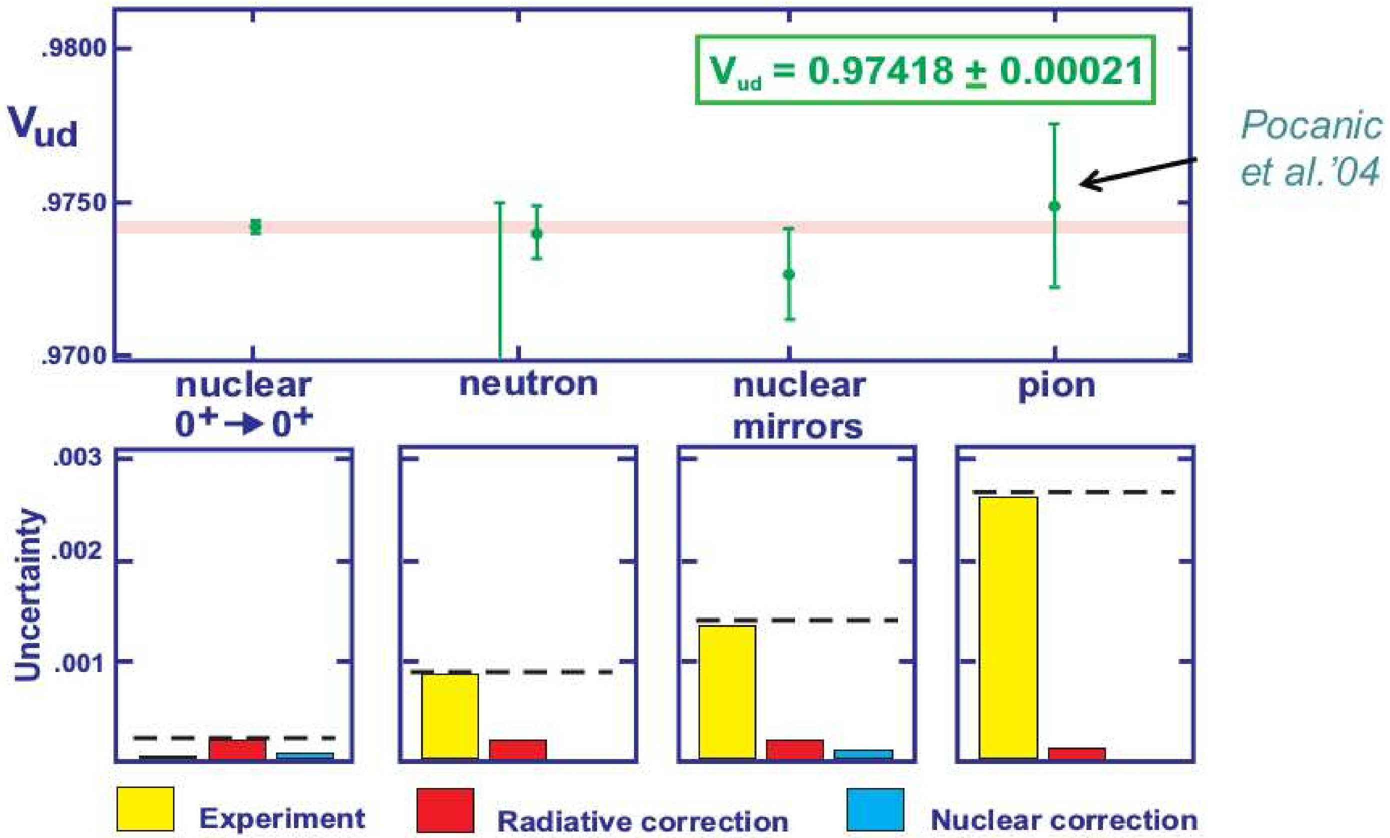
With scale factor S = 2.7

V_{ud} **= 0.97368(53)**
V_{us} **= 0.2245(10)**

Other sources for V_{ud}?

Extraction of V_{ud} : summary

Hardy@Amherst'19



π_β as an alternative?

- vector from factor only, like $0^+ \to 0^+$ transitions but unlike neutron β decay
- no structure-dependent correction, unlike $0^+ \to 0^+$ transitions
- cleanest way to measure $|V_{ud}|$
- but...

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$$R_{\pi_\beta}^{\rm PIBETA} = [1.036 \pm 0.004_{\rm stat} \pm 0.004_{syst} \pm 0.003_{\pi_{e2}}] \cdot 10^{-8}$$

D. Pocanić et al (PIBETA Coll.), Phys. Rev. Lett. 93, 181803 (2004)

$$\sim 10^6 \text{ stopped } \pi^+/\text{sec} \qquad 6.4 \cdot 10^4 \; \pi_\beta \text{ events} \qquad 6.8 \cdot 10^8 \; \pi_{e2} \text{ events}$$

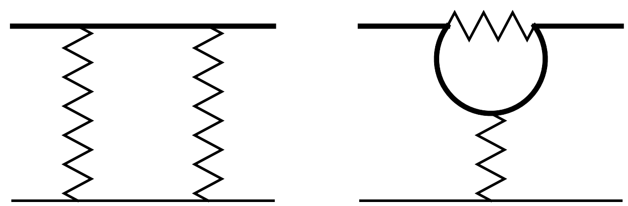
$$V_{ud}^{\rm PIBETA} = 0.9749(26) \times \left[\frac{BR(\pi^+ \to e^+ \nu_e(\gamma))}{1.2352 \times 10^{-4}} \right]^{\frac{1}{2}}$$

using

$$f_+^{\pi_\beta}(0) = 1.0046(5)$$

V. Cirigliano, M. K., H. Neufeld, H. Pichl, Eur. Phys. J. C 27, 255 (2003)

Rare kaon decays (SD) $K \to \pi \nu \bar{\nu}$



Expressions for $K \to \pi\bar{\nu}\nu$

$$\mathcal{B}(K^+ \to \pi^+ \nu\bar{\nu}) = \kappa_+ (1 + \Delta_{\rm EM}) \cdot \left[\left(\frac{{\rm Im}\lambda_t}{\lambda^5} X(x_t) \right)^2 + \left(\frac{{\rm Re}\lambda_c}{\lambda} P_c(X) + \frac{{\rm Re}\lambda_t}{\lambda^5} X(x_t) \right)^2 \right]$$

$$\mathcal{B}(K_L \to \pi^0 \nu\bar{\nu}) = \kappa_L \cdot \left(\frac{{\rm Im}\lambda_t}{\lambda^5} X(x_t) \right)^2$$

- ${\rm Im}\lambda_t = \eta A^2 \lambda^5$, ${\rm Re}\lambda_t = \frac{\lambda^2-2}{2} A^2 \lambda^2 (1 - \bar{\rho})$, ${\rm Re}\lambda_c = \lambda \frac{\lambda^2-2}{2}$
- $\kappa_+, \kappa_L, \Delta_{EM}$ strong and em iso-spin breaking [0705.2025]
- $\mathrm{P_c} = \mathrm{P_c^{pert.}} + \delta\, \mathrm{P_{c,u}} = 0.372(15) + 0.04(2) \leftarrow$ (NNLO + EW) [ph/0603079] [0805.4119] + χ PT & Lattice [ph/0503107] [1806.11520]

Very well predicted in the SM

Expressions for $K \to \pi\bar{\nu}\nu$

$$\mathcal{B}(K^+ \to \pi^+\nu\bar{\nu}) = \kappa_+(1+\Delta_{\rm EM})\cdot\left[\left(\frac{{\rm Im}\lambda_t}{\lambda^5}X(x_t)\right)^2 + \left(\frac{{\rm Re}\lambda_c}{\lambda}P_c(X) + \frac{{\rm Re}\lambda_t}{\lambda^5}X(x_t)\right)^2\right]$$

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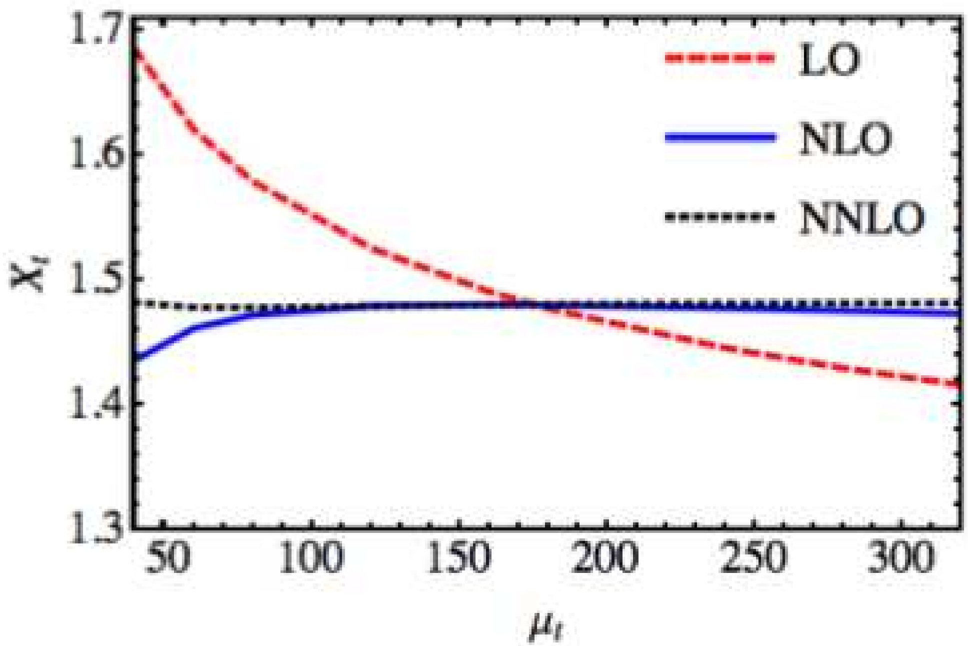
NNLO corrections to $X(x_t)$ worked out

Possible Scale Dependence @ NNLO

- NNLO finite result with correct μ_t dependence
- Numerics not checked: Toy numerics
  - fix $X_t^{NNLO}(\mu_t = 170\text{ GeV}) = X_t^{NLO}(\mu_t = 170\text{ GeV})$
- Absolute size of NNLO corrections blinded



only scale-dependence at NNLO shown!

New physics in $K \to \pi\nu\bar{\nu}$?

Several scenarios explored

- general lagrangian with additional spin 0, 1/2 and 1 fields
    - relations between couplings fixed by tree-level unitarity
    - checked against results from various special cases

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$K \to \pi\nu\bar{\nu}$ additional neutrino

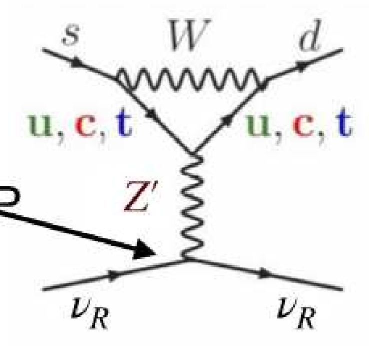
- New light sterile neutrino

$$\mathcal{H} = \frac{G_F}{\sqrt{2}} \frac{\alpha}{\pi s_W^2} V_{ts}^{\star} V_{td} \bar{s}\gamma_\mu d \left((X(x_t) + X_N) \sum_{\ell} \bar{\nu}_\ell \gamma^\mu P_L \nu_\ell + \tilde{X}_N \bar{\nu}_R \gamma^\mu P_R \nu_R \right)$$

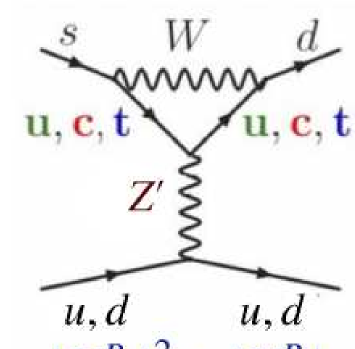
new RH ν

- <u>No interference</u> with SM, can only add to SM rates

sterile:
couples only to NP



NOT directly related
to ϵ, ϵ'

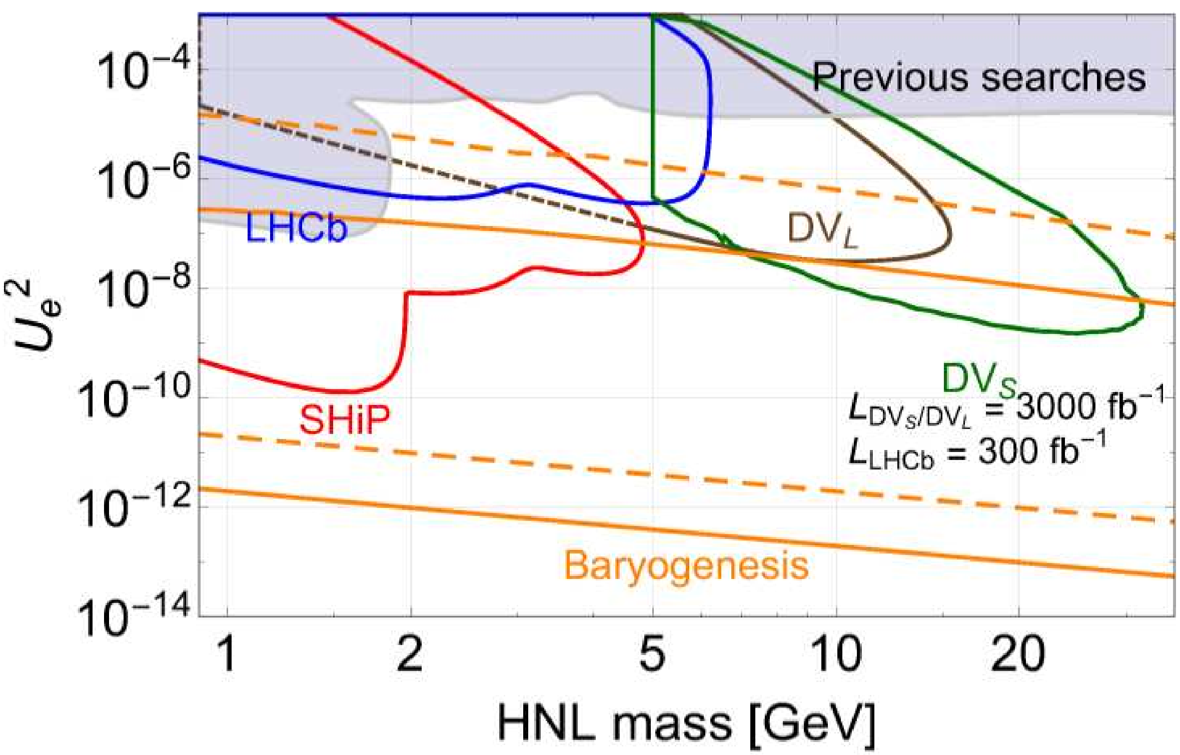


$|V_{bd}^R|^2 + |V_{tu}^R|$

new parameters
can be tiny

High intensity experiments and LHC: Synergy

1902.04535



HL LHC

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- add all four-quark operators not generated by SM
  - matrix elements evaluated in dual QCD approach
  - test various scenarios

Induced Z-mediated FCNCs

LH FCNCs

Enhancement of ε'/ε implies
suppression of $K_L \to \pi^0 \nu\bar{\nu}$
$\mathrm{Br}\left(K^+ \to \pi^+ \nu\bar{\nu}\right) \leq 2\ \mathrm{Br}\left(K^+ \to \pi^+ \nu\bar{\nu}\right)_{SM}$

($K_L \to \mu\bar{\mu}$ bound)

RH FCNCs

Suppression of $K_L \to \pi^0 \nu\bar{\nu}$
$\mathrm{Br}\left(K^+ \to \pi^+ \nu\bar{\nu}\right) \leq 1.5\ \mathrm{Br}\left(K^+ \to \pi^+ \nu\bar{\nu}\right)_{SM}$

(ε_K bound SMEFT)

LH+RH FCNCs

$K_L \to \pi^0 \nu\bar{\nu}$ and $K^+ \to \pi^+ \nu\bar{\nu}$
can be both enhanced if necessary
(no definite prediction)

Literature:

AJB (1601.00005), Bobeth, AJB, Celis, Jung (1703.04753)
Endo, Kitahara, Mishima, Yamamoto (1612.08839)

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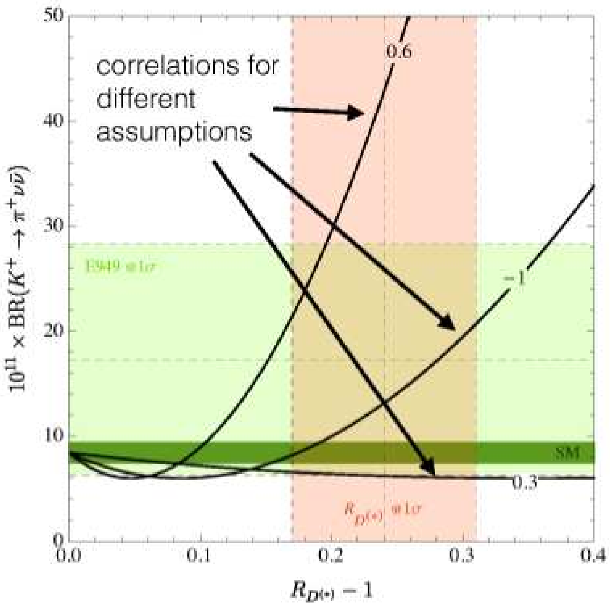
- implications of B anomalies for kaon physics
  - requires some symmetry structure in the flavour sector: $U(2)^5$, $\longrightarrow$ larger NP effects in 3rd generation leptons, correlation between $R(D^{(*)})$ and $K \to \pi\nu\bar{\nu}$

Kaon Physics and R(D(*))

> The flavor symmetry predicts larger NP effects in 3rd gen. leptons
> In Kaon physics the only chance is with tau-neutrino in $K \to \pi\nu\nu$
> The main correlation is with $R(D^{(*)})$
For the connection with R(K) see [Fajfer et al. 1802.00786]

Connection in the SMEFT, assuming U(2)^5 structure

[Bordone, Buttazzo, Isidori, Monnard 1705.10729]



While the precise correlation depends on the details of the model, it is clear that a future measurements by **NA62**, **KOTO**, and **KLEVER** will cover most of the parameter space.

For a complete analysis it is necessary to take into account the bounds from $B \to K^{(*)} \nu\nu$, $\Delta F=2$, LFV, LEP data, and direct searches.

Need a full UV model which can address the anomalies.

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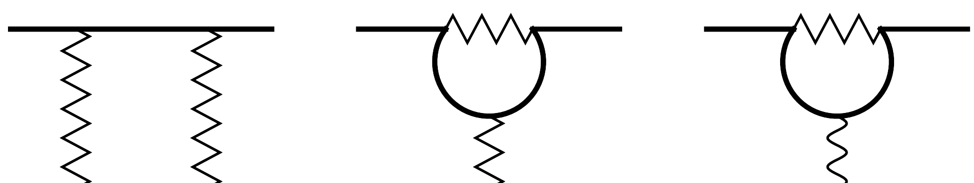
    - more general flavour structure needed to also make contatc with $R(K^{(*)})$ (ROFV)

Rare kaon decays (LD)



Issue: determination of LECs (predictivity)

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use some decay modes to fix LECs, and predict the remaining modes

15 modes available, depending on only 5 LECs $N_{14}, \ldots N_{18}$ (at one loop)

- Measured modes:

$$
\begin{array}{llllll}
K^\pm \to \pi^\pm \gamma^* & [10^{-7}]_{3\%}\,; & K_S \to \pi^0 \gamma^* & [10^{-9}]_{50\%}\,; & K_L \to \pi^0 \gamma^* & [< 10^{-10}] \\
K^\pm \to \pi^\pm \pi^0 \gamma & [10^{-6}]_{7\%}\,; & K_S \to \pi^+ \pi^- \gamma & [10^{-3}]_{3\%}\,; & K_L \to \pi^+ \pi^- \gamma & [10^{-5}]_{4\%} \\
K^\pm \to \pi^\pm \gamma\gamma & [10^{-6}]_{6\%}\,; & K_S \to \pi^0 \gamma\gamma & [10^{-8}]_{37\%}\,; & K_L \to \pi^0 \gamma\gamma & [10^{-6}]_{3\%} \\
K^\pm \to \pi^\pm \pi^0 \gamma^* & [10^{-6}]_{3\%}\,; & K_S \to \pi^+ \pi^- \gamma^* & [10^{-5}]_{3\%}\,; & K_L \to \pi^+ \pi^- \gamma^* & [10^{-7}]_{6\%}
\end{array}
$$

- Near-future upgrades:

$$
\begin{array}{ll}
K^\pm \to \pi^\pm \pi^0 \gamma^* & \mathrm{NA62\,(?)} \\
K_S \to \pi^+ \pi^- \gamma^* & \mathrm{LHCb}
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\end{array}
$$

Potential difficulties

- one-loop may not be enough to describe data

- IB component dominates, LECs $\in$ SD part

Alternatives?

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phenomenological construction of form factor, e.g. in $K \to \pi\ell^+\ell^-$

$$W_{K\pi}(z) = W_{K\pi}^{\pi\pi}(z) + W_{K\pi}^{K\pi}(z) + W_{K\pi}^{K\bar{K}}(z) + W_{K\pi}^{\mathrm{res}}(z;\nu) + W_{K\pi}^{\mathrm{SD}}(z;\nu)$$

Alternatives?

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$$W_{K\pi}(z) = W^{\pi\pi}_{K\pi}(z) + W^{K\pi}_{K\pi}(z) + W^{K\bar{K}}_{K\pi}(z) + W^{\mathrm{res}}_{K\pi}(z;\nu) + W^{\mathrm{SD}}_{K\pi}(z;\nu)$$

- essential to have correct matching to short-distance behaviour
- extension to other processes?

Alternatives?

lattice QCD

RBC-UKQCD kaon program

- ΔM_K: 153 configs., 1/a=2.38 GeV, 64^3 x 128, all masses physical (Bigeng Wang).
- $(\varepsilon_K)_{LD}$: exploratory calculation done (Ziyuan Bai)
- $(K^+ \to \pi^+ \nu \overline{\nu})_{LD}$: 37 configs., 1/a=2.38 GeV 64^3 x 128, all masses physical. (Xu Feng)
- $K^+ \to \pi^+ l^+ l^-$: Fionn Ó hÓgáin's talk on Thurs.
- $(K_L \to \mu^+ \mu^-)_{\gamma\gamma}$: under study, $\pi \to e^+ e^-$ done (Y. Zhao)
- E&M corrections:
  - $\pi \to \mu \nu$
  - $K \to \mu/e\, \nu$
  - $K \to \mu/e\, \nu\, \gamma$

- QED_L or
- new IVR method without power-law corrections, (Feng & Jin: arXiv:1812)

Summary

- Exploratory calculations show successful extraction of matrix elements
- Physical point calculations of non-eye diagrams look promising
- Intercept of $V(z) = a + bz$ will be known soon
- Possible that single π state is not the only exponential contribution
- Further result to follow as more kinematics are run

work in progress, looking forward to see results

$$K \to \pi\pi, \, \epsilon'/\epsilon$$

$K \to \pi\pi$, ϵ'/ϵ

$$10^4 \cdot \mathrm{Re}(\epsilon'/\epsilon) = \begin{cases} 19.2\,(2.1) & \text{KTeV'11} \\ 14.7(2.2) & \text{NA48'02} \\ 7.4(5.2)(2.9) & \text{E731'93} \\ 23.0(6.3) & \text{NA31'93} \end{cases}$$

$$10^4 \cdot \mathrm{Re}(\epsilon'/\epsilon) = 16.8(2.2) \quad \text{PDG average}$$

- Short-distance aspects under control

- Long-distance issues are crucial and quite involved

    - Lattice QCD: ab initio approach (QCD and only QCD), several difficulties need to be overcome

        - discretization effects

        - finite volume effects

        - physical quark masses

        - disconnected diagrams

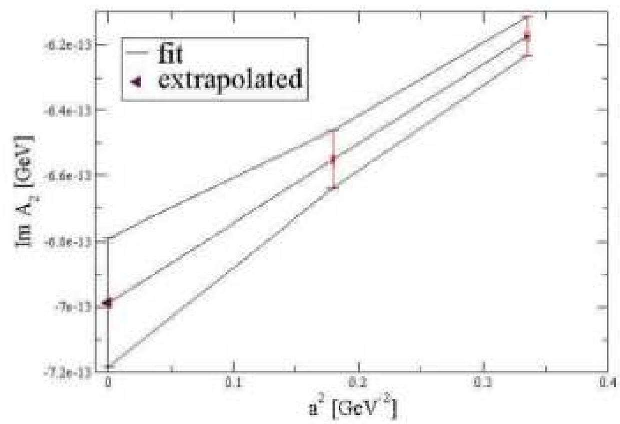
        - exp. growing contributions from states with $E < E_K(k)$

        - two-pion states with vacuum quantum numbers

$\Delta I = 3/2$ – Continuum Results

(M. Lightman, E. Goode T. Janowski)

- Use two large ensembles to remove a^2 error (m_π=135 MeV, L=5.4 fm)
  - $48^3 \times 96$, $1/a$=1.73 GeV
  - $64^3 \times 128$, $1/a$=2.28 GeV



- Continuum results:
  - $\mathrm{Re}(A_2) = 1.50(0.04_{\mathrm{stat}})\,(0.14)_{\mathrm{syst}} \times 10^{-8}$ GeV
  - $\mathrm{Im}(A_2) = -6.99(0.20)_{\mathrm{stat}}\,(0.84)_{\mathrm{syst}} \times 10^{-13}$ GeV
- Experiment: $\mathrm{Re}(A_2) = 1.479(4)\ 10^{-8}$ GeV
- $E_{\pi\pi} \rightarrow \delta_2 = -11.6(2.5)(1.2)^{\circ}$
- [Phys.Rev. **D91**, 074502 (2015)]

2015 Results

[Phys. Rev. Lett. 115 (2015) 212001]

- $E_{\pi\pi}$(499 MeV) determines δ_0 :
  - $I = 0$ $\pi\pi$ phase shift: $\delta_0 = 23.8(4.9)(2.2)^o$
  - Dispersion theory result: $\delta_0 = 34^o$ [G. Colangelo, *et al.*]
- Re(ε'/ε) = $(1.38 \pm 5.15_{stat} \pm 4.59_{sys}) \times 10^{-4}$
  - Expt.: $(16.6 \pm 2.3) \times 10^{-4}$
  - 2.1 σ difference
- Unanswered questions:
  - Is this 2.1 σ difference real? → Reduce errors
  - Why is δ_0 so different from the dispersive result? → Introduce more $\pi\pi$ operators to distinguish excited states

Final results available very soon!

- Short-distance aspects under control

- Long-distance issues are crucial and quite involved

    - phenomenological approaches: QCD as far as possible

        - PP approach

        - BBG approach

Common starting point

$$\langle \pi\pi | \mathcal{L}^{|\Delta S|=1}_{\mathrm{non-lept}}(x) | K \rangle \qquad \mathcal{L}^{|\Delta S|=1}_{\mathrm{non-lept}}(x) = -\frac{G_{\mathrm{F}}}{\sqrt{2}} V^*_{us} V_{ud} \sum_I C_I(\nu) Q_I(x;\nu) + \mathrm{h.\,c.}$$

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different points of view when it comes to the evaluation of the matrix elements

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$$\downarrow$$

- symmetry structure
- low-energy expansion

$$\downarrow$$

$$\mathcal{L}_{\rm ChPT}$$

- ν-dependence not an issue anymore
- FSI interactions follow from chiral loops $\longrightarrow$ large corrections, unambiguous at one loop
- LECs capture the information from short distances
- LECs not all known $\longrightarrow$ large N_c, small corrections but large uncertainties

Anatomy of ε'/ε calculation

$$\boxed{\mathrm{Re}\left(\frac{\varepsilon'}{\varepsilon}\right) = -\frac{\omega_+}{\sqrt{2}|\varepsilon|}\left\{\frac{\mathrm{Im}\,A_0^{(0)}}{\mathrm{Re}\,A_0^{(0)}}\,(1-\Omega_{\mathrm{eff}}) - \frac{\mathrm{Im}\,A_2^{\mathrm{emp}}}{\mathrm{Re}\,A_2^{(0)}}\right\}}$$

$$\mathcal{A}_n^{(X)} = a_n^{(X)}\left[1 + \Delta_L\mathcal{A}_n^{(X)} + \Delta_C\,\mathcal{A}_n^{(X)}\right]$$

Cirigliano-Gisbert-Pich-Rodríguez 2019

1. $\mathcal{O}(p^4)$ χPT **Loops:** **Large correction** (NLO in $1/N_C$) FSI

   $$\Delta_L\,\mathcal{A}_{1/2}^{(8)} = 0.27 + 0.47\,i \qquad ; \qquad \Delta_L\,\mathcal{A}_{3/2}^{(g)} = -0.50 - 0.21\,i$$

2. $\mathcal{O}(p^4)$ **LECs fixed at** $N_C \to \infty$: **Small correction**

   $$\Delta_C\,[\mathcal{A}_{1/2}^{(8)}]^- = 0.11 \pm 0.05 \qquad ; \qquad \Delta_C\,[\mathcal{A}_{3/2}^{(g)}]^- = -0.19 \pm 0.19$$

3. **Isospin Breaking** $O\left[(m_u - m_d)\,p^2, e^2 p^2\right]$: **Sizeable correction**

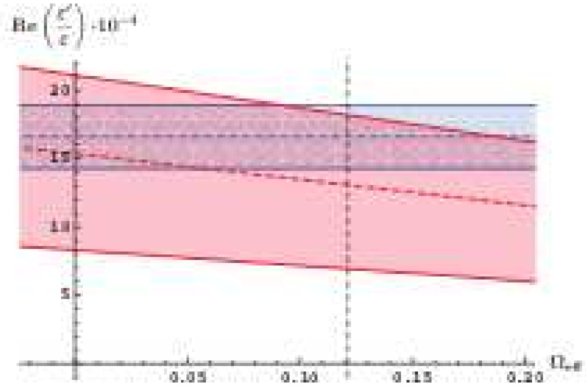
   $$\Omega_{\mathrm{eff}} = 0.12 \pm 0.09$$

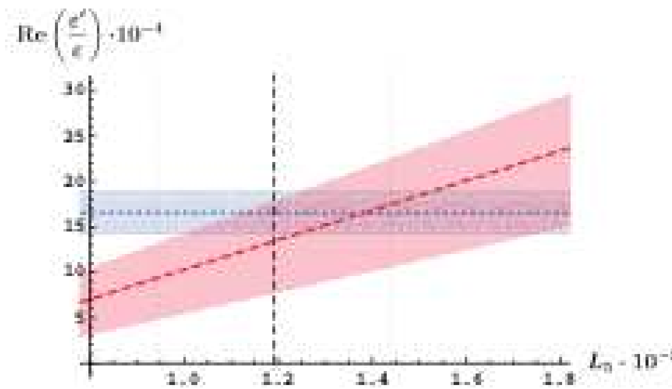
4. $\mathrm{Re}(g_8)$, $\mathrm{Re}(g_{27})$, $\chi_0 - \chi_2$ **fitted to data**

SM Prediction of ε'/ε

$$\boxed{\mathrm{Re}\,(\varepsilon'/\varepsilon)_{\rm SM} \;=\; \left(13\,{}^{+\,6}_{-\,7}\right)\cdot 10^{-4}}$$

Cirigliano, Gisbert, Pich, Rodríguez-Sánchez





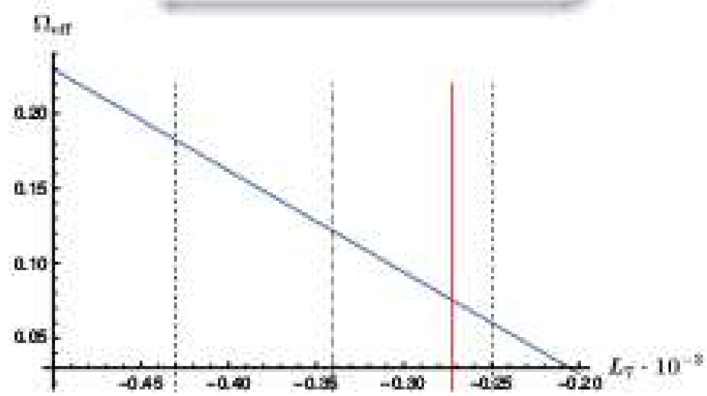
$$\mathrm{Re}\,(\varepsilon'/\varepsilon)_{\rm SM} \;=\; \left(13.1 \pm 0.4_{m_s}\,{}^{+\,2.2}_{-\,4.0}{}_{\mu}\,{}^{+\,3.0}_{-\,3.2}{}_{\nu_\chi} \pm 1.2_{\gamma_S} \pm 4.3_{L_{5,8}} \pm 1.1_{L_7} \pm 0.2_{K_i} \pm 0.3_{\chi_i}\right)\cdot 10^{-4}$$

Large uncertainty but no anomalies!

Improvements needed on $\Omega_{\rm eff}$, C_I's at NNLO, FSI at NNLO, LECs,...

Results

$$\Omega_{eff} = (12.1^{+9.0}_{-8.8}) \cdot 10^{-2}$$



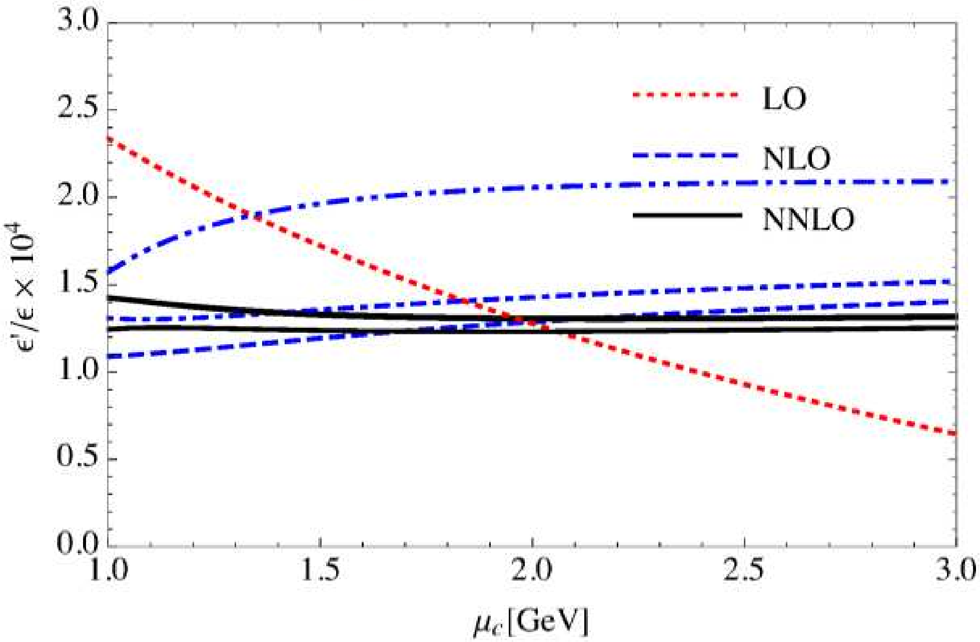
Those analyses that include in A_0 the EM penguin contributions should subtract them from Δ_0 :

$$\hat{\Omega}_{eff} \equiv \Omega_{IB} - \Delta_0|_{\alpha=0} - f_{5/2} = (18.3^{+9.6}_{-9.5}) \cdot 10^{-2}$$

ε'/ε & μ_c

[Cerdà-Sevilla, Gorbahn, Jäger, Kokulu]

Residual μ_c scale dependence



- **Uncertainty is significantly reduced by going to NNLO**

- **There are still improvements: e.g. better as implementation & better incorporation of sub-leading corrections.**

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$$\downarrow$$

$$\langle\pi\pi|Q_I(x;\nu)|K\rangle \qquad \text{different philosophy}$$

- ν-dependence remains an issue (as well as scheme-dependence)
- inclusion of meson evolution mandatory
- pion loops considered with a cut-off Λ
- $1/N_c$ corrections described by keeping only Λ-dependent contributions from loops
- identification of Λ with ν

AJB Claims 2016 - 2019

ε'/ε anomaly could turn out to be the largest anomaly in flavour physics !

$$(\varepsilon'/\varepsilon)_{exp} = (16.6 \pm 2.3) \cdot 10^{-4}$$

$$(\varepsilon'/\varepsilon)_{SM} < (6.0 \pm 2.4) \cdot 10^{-4}$$ Dual QCD

AJB Expectation
(May 2018)
1805.11096

$$(\varepsilon'/\varepsilon)_{SM} = (5 \pm 2) \cdot 10^{-4}$$

1812.06102 (Christmas Story)

Possible NP has to enhance ε'/ε (important message for model builders)

Will new RBC-UKQCD results support this claim?

Conclusion

Despite an (anomalously!) low number of theory presentations, a broad range of topics has been covered

- CKM unitarity
- $K \to \pi\nu\bar{\nu}$
- other rare kaon decays
- contributions from BSM physics, connection with B anomalies
- HNLs
- $K \to \pi\pi$, ϵ'/ϵ

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A rich variety of scenarios are being explored in order to explain any significant deviation from SM that would show up in the flavour sector

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results eagerly awaited (not only by Andrej)

Thanks for your attention!

thanks to the speakers for providing slides and additional useful insights