The Modern Physics of Compact Stars and Relativistic Gravity

Cosmological constant induced by a bulk scalar in braneworlds with compact dimensions

Yerevan, Armenia,
September 17-21, 2019

A. A. Saharian, H. G. Sargsyan

Yerevan State University,

Gourgen Sahakyan Chair of Theoretical Physics

http://www.ysu.am

Outline

- Motivation
- Bulk and boundary geometries
- The surface energy-momentum tensor
- VEV and the induced cosmological constant
- Conclusions

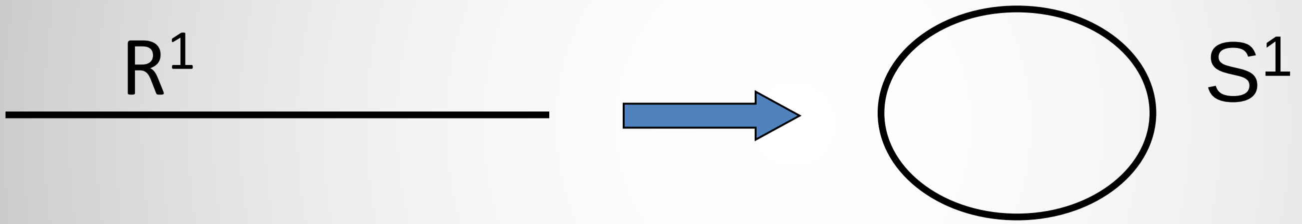
Models with extra dimensions

- Many of high energy theories of fundamental physics are formulated in higher-dimensional spacetimes
- Idea of extra dimensions has been extensively used in supergravity
- Extra dimensions are predicted by string theory, at present the most promising candidate for the consistent quantum gravity theory and for a unification of fundamental interactions
- Two types of models with extra dimensions
  - Kaluza-Klein type models: Extra dimensions are compact and they are accessible for all fields
  - Brane-world models: Standard model fields are localized on a hypersurface (brane). Gravity and possibly some other fields extend to all dimensions. Extra dimensions may be non-compact.

Boundary conditions in models with extra dimensions

- In Kaluza-Klein type models the extra dimensions are compactified

- Simplest example is the toroidal compactification



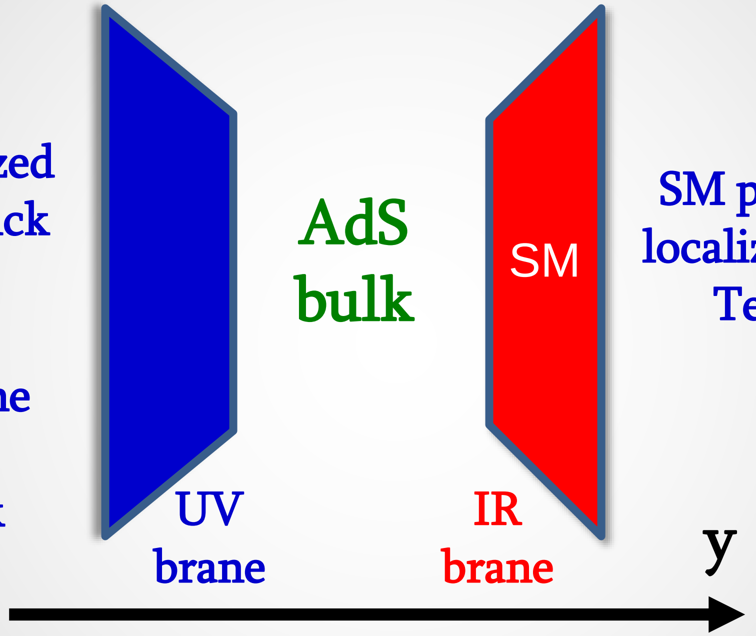
- Topologically inequivalent configurations for fields
  - Untwisted field → periodic boundary conditions $\varphi(x+L)=\varphi(x)$
  - Twisted field → antiperiodic boundary conditions $\varphi(x+L)=-\varphi(x)$
  - Generalized boundary conditions $\varphi(x+L)=e^{i\alpha}\varphi(x)$

Barneworld models: Randall-Sundrum 2-brane model

- Original Randall-Sundrum model (RS1) offers a solution to the hierarchy problem by postulating 5D AdS spacetime bounded by two (3+1)-dimensional branes

Gravity is localized near UV or Planck brane

Only graviton and some other non SM fields propagate in the bulk



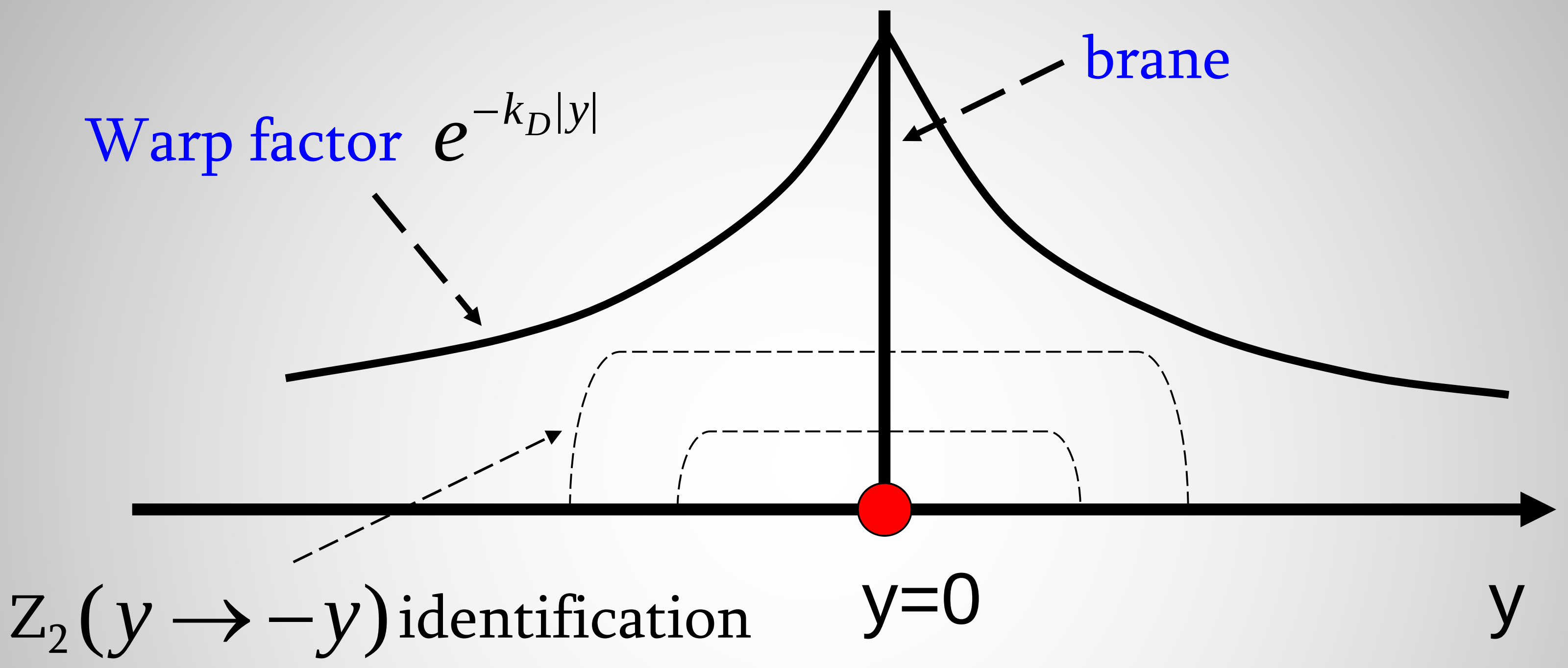
SM particles are localized on IR or TeV brane

$$ds^2 = e^{-2ky}\eta_{\mu\nu}dx^\mu dx^\nu - dy^2$$

- Hierarchy problem between the gravitational and electroweak scales is solved for

k·distance between branes = 40

Randall-Sundrum 1-brane model



Boundary condition on the brane $(1 + \beta n^\mu D_\mu)\varphi(x) = 0, \quad y = y_0$

Constant

Normal to the brane

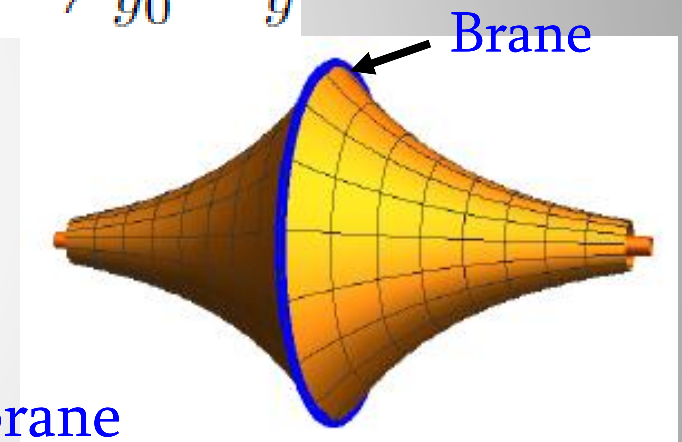
Generalized RS models with compact dimensions

- Background geometry: $ds^2 = e^{-2|y-y_0|/a}\eta_{ik}dx^i dx^k - dy^2$

  Topology $R^p \times T^q$

- Background geometry contains two patches $y > y_0$ of the AdS glued by the brane and related by the Z_2-symmetry identification $y - y_0 \longleftrightarrow y_0 - y$

- Spatial geometry in the case $D = 2$, embedded into a 3D Euclidean space



- For fields even under the reflection with respect to the brane (untwisted scalar field) the boundary condition is of the Robin type with $\beta = -1/(c + 2D\xi/a)$, with c being the brane mass term

- For fields odd with respect to the reflection (twisted fields) the boundary condition is reduced to the Dirichlet one

Fields

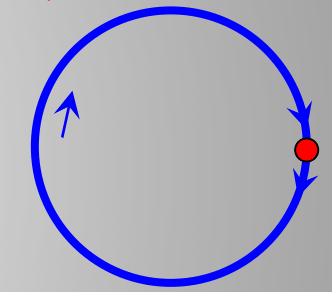
- Charged scalar field with general curvature coupling

$$\left(g^{\mu\nu}D_\mu D_\nu + m^2 + \xi R\right)\varphi(x) = 0,\quad D_\mu = \nabla_\mu + ieA_\mu$$

External classical gauge field

- In models with nontrivial topology one need also to specify the periodicity conditions obeyed by the field operator along compact dimension

$$\varphi(t, x^1, \ldots, x^{D-1} + L, y) = e^{2\pi i \alpha}\varphi(t, x^1, \ldots, x^{D-1}, y)$$



- Special cases:
  - Untwisted fields $\alpha = 0$
  - Twisted fields $\alpha = 1/2$

- Boundary condition on the brane $(1 + \beta^{(j)} n^i_{(j)} \nabla_i)\varphi(x) = 0,$ $n^i_{(j)}$ ← normal to the brane

$j = 1 \Rightarrow y = y_0 + 0,\qquad j = 2 \Rightarrow y = y_0 - 0$

- We assume that the gauge field is constant: $A_\mu = \mathrm{const}$

- Though the corresponding field strength vanishes, the nontrivial topology gives rise to Aharonov-Bohm-like effects

Bulk + Surface

- The **Gibbons-Hawking** surface term in GR, in manifolds with boundaries

$$S_{GR} = \frac{1}{2}\int d^{D+1}x\sqrt{|g|}R \longrightarrow S_{GH} = -\int d^{D}x\sqrt{|h|}\varepsilon K$$

- A complex scalar field on the background of a manifold with boundaries
  - The standard bulk action with coupling (to the background curvature) constant

$$S_b = \frac{1}{2}\int d^{D+1}x\sqrt{|g|}(\nabla_i\varphi\nabla^i\varphi^* - m^2\varphi\varphi^* - \xi R\varphi\varphi^*)$$

  - The surface action with the analog of Gibbons-Hawking term and surface mass term

$$S_s = -\int d^{D}x\sqrt{|h|}\varepsilon(\xi K\varphi\varphi^* + m_s\varphi\varphi^*)$$

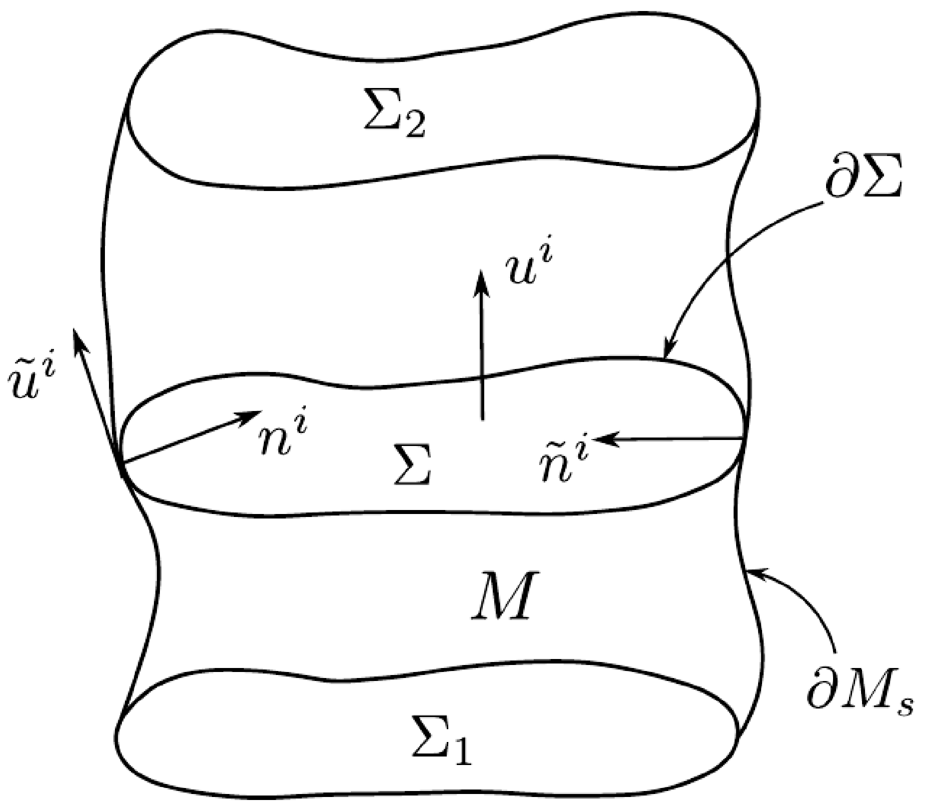
$$h_{ik} = g_{ik} - \varepsilon n_i n_k$$

$$K_{ik} = h_i^l h_k^m \nabla_l n_m$$

- The equations of motion

$$g^{lm}\nabla_l\nabla_m\varphi + m^2\varphi + \xi R\varphi = 0$$

$$2\left(\xi K + m_s\right)\varphi + n^l\nabla_l\varphi = 0$$



Surface energy-momentum tensor

- The energy-momentum tensor

$$T_{ik}^{(vol)} = \frac{1}{2}(\nabla_i\varphi\nabla_k\varphi^* + \nabla_k\varphi\nabla_i\varphi^*) + \left[\left(\xi - \frac{1}{4}\right) g_{ik}g^{lm}\nabla_l\nabla_m - \xi R_{ik} - \xi\nabla_i\nabla_k\right]\varphi\varphi^*$$

$$T_{ik}^{(surf)} = \delta(x;\partial M_s)\left[\xi\varphi\varphi^* K_{ik} - \left(\xi - \frac{1}{4}\right) h_{ik} n^l \nabla_l(\varphi\varphi^*)\right]$$

Extrinsic curvature tensor → K_{ik}; Induced metric → h_{ik}; Unit normal → n^l

- Vacuum expectation value of the surface EMT on the brane

$$\langle 0 | T_{\mu}^{(s)\nu} | 0 \rangle \equiv \langle T_{\mu}^{(s)\nu} \rangle = \mathrm{diag}(\varepsilon_j^{(s)}, \ldots, -p_j^{(s)}, \ldots), \quad \varepsilon_j^{(s)} = -p_j^{(s)}$$

- This corresponds to the generation of the cosmological constant on the branes by quantum effects

VEV of surface EMT

- VEV of the surface energy-momentum tensor is expressed in terms of the VEV for field squared $\langle T_{\mu}^{(s)\nu} \rangle = \delta(x; \partial M_s) \langle \tau_{\mu}^{(j)\nu} \rangle,$

$$\langle \tau_{\mu}^{(j)\nu} \rangle = -\delta_{\mu}^{\nu} \frac{n^{(j)}}{a} \left(\xi - \frac{2\xi - 1/2}{n^{(j)} \beta^{(j)}} a \right) \langle \varphi \varphi^{\dagger} \rangle_j, \quad \mu, \nu \neq D \qquad < \tau_{D}^{(j)D} >= 0$$

$$\varepsilon_j = \langle \tau_0^{(j)0} \rangle, \; p_j = -\varepsilon_j$$

- VEV for the field squared is divergent and renormalization is required

- We use the generalized zeta function technique

Cosmological constant on the brane

- Renormalized vacuum energy density on the brane fo $n^{(1)}\beta^{(1)} = n^{(2)}\beta^{(2)} = \beta$

$$\varepsilon = -\frac{1}{a^D}\left(\xi - \frac{2\xi - 1/2}{\beta} a\right)\left\{\frac{2(4\pi)^{(1-D)/2} z_0 \beta}{\Gamma((D-1)/2) La} \sum_{n=-\infty}^{\infty} \int_{k_{D-1}(n) z_0}^{\infty} du u \right.$$

$$\times [u^2 - z_0^2 k_{D-1}^2(n)]^{\frac{D-3}{2}} \left[\frac{K_\nu(u)}{\bar{K}_\nu^{(1)}(u)} + \frac{I_\nu(u)}{\bar{I}_\nu^{(2)}(u)} - \frac{a}{\beta} \sum_{l=1}^{[N/2]} \frac{2 v_{2l}^{(I,2)}}{u^{2l}}\right]$$

$$\left. + \frac{2\pi^{D-1/2}}{(L/z_0)^D} \sum_{l=1}^{[N/2]}{}' \frac{v_{2l}^{(I,2)} (L/z_0)^{2l}}{\Gamma(l)(2\pi)^{2l}} P\left(\frac{2l - D + 1}{2}, \tilde{\alpha}\right)\right\}.$$

$$\tilde{\alpha} = \alpha + eA_{D-1} L/(2\pi)$$

$$P(x,p) = \Gamma(x)\left[\zeta_H(2x,p) + \zeta_H(2x,1-p)\right],$$

Determines the magnetic flux enclosed by compact dimension

Hurwitz zeta function

Cosmological constant on the brane

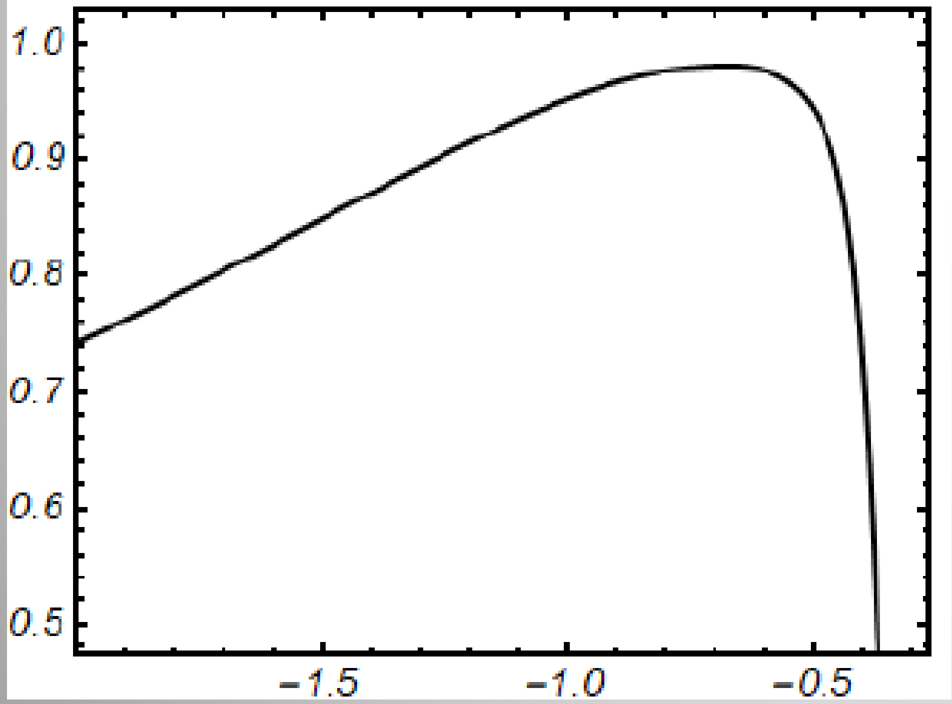
- Cosmological constant measured in units of the AdS curvature radius

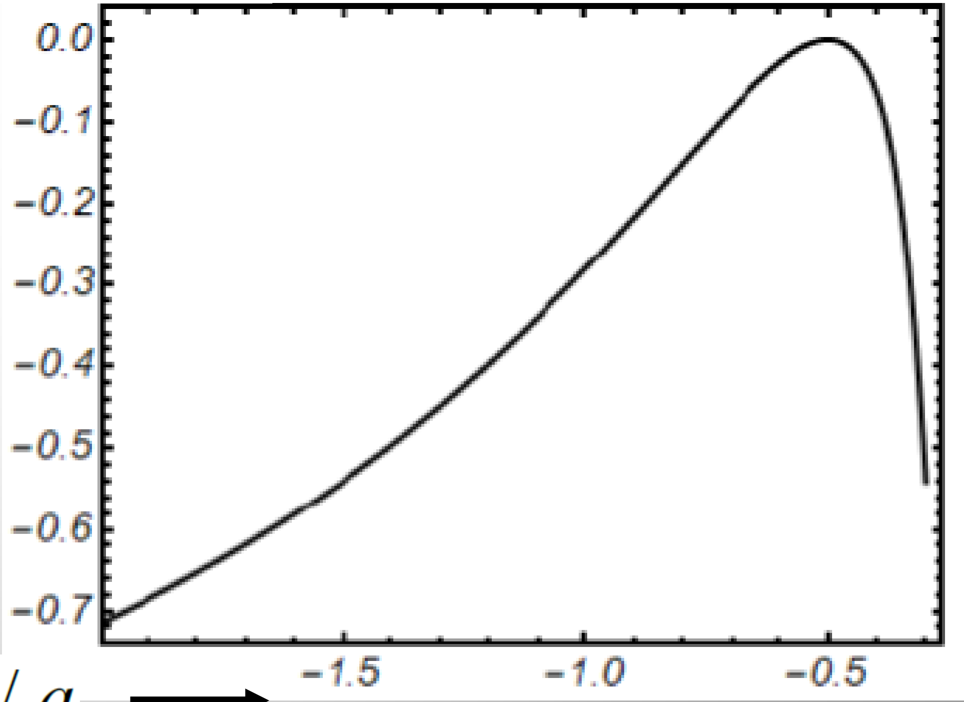
$a^D \varepsilon$ β / a $\tilde{\alpha}, L / z_0$ $z_0 = a e^{y_0 / a}$

- Depending on the values of the parameters the CC can be either positive or negative

Minimal coupling $z_0 / L = 1,\ \tilde{\alpha} = 0.45$ Conformal coupling

$a^D \varepsilon$





β / a →

The 2-brane model

- Separation into 1-brane and 2nd brane contributions

$$\zeta_{j\boldsymbol{k}_q}(s) = \zeta^{(J)}_{j\boldsymbol{k}_q}(s) - \frac{B_j}{\Gamma\left(-\frac{s}{2}\right)\Gamma\left(\alpha_s+1\right)\mu^{1+s}}\int_{k_q}^{\infty} \mathrm{d}\lambda\lambda\left(\lambda^2-k_q^2\right)^{\alpha_s}\Omega_{j\nu}(\lambda z_1,\lambda z_2)$$

- Cosmological constant induced on the visible brane by the hidden one

Conclusions

- Quantum fluctuations of bulk fields induce cosmological constant on the brane
- Induced cosmological constant is a periodic function of the magnetic flux enclosed by compact dimension with the period of the flux quantum
- Depending on the values of the parameter the CC on the brane can be either positive or negative
- If the length of the compact dimension is of the order of curvature radius for AdS, the induced energy density is of the order of dark energy driving the accelerated expansion of the Universe at recent epoch

Thank You!