Dynamics of dark energy universe embedded in magnetized fluid

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Objectives

To develop mathematical formulation for anisotropic dark energy models for studying dynamics of universe in General Relativity.

To investigate source pressure anisotropy and its contribution to late time accleration through dark energy models.

To construct Bianchi type models in GR and its modification by assuming valid conditions on the physical parameters.

To compare theoretical outcomes of models to recent observational data by calculating exact solutions and some physical parameters.

Topic Outlines

Part 1 (Stating and setting up the problem background)

- Introduction
- Research problem
- Problem Background

Part 2 (Investigating and solving Problem)

- Problems Investigated

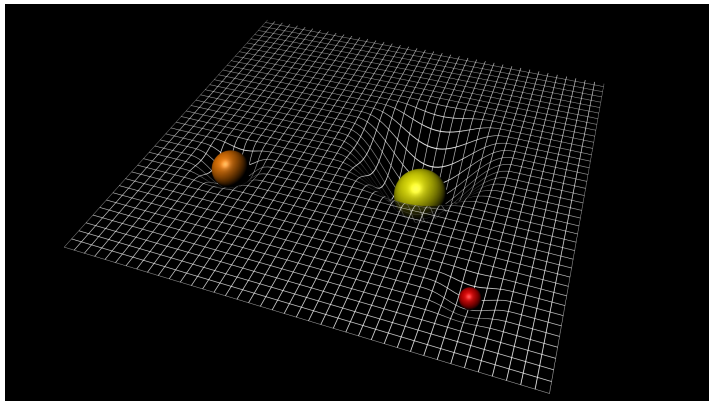
Part 3 (Summarizing major breakthroughs and further scope)

- General conclusion
- Future scope of research

Introduction

• **Theory of General Relativity**; A self contented theory, revealing many realistically famous predictions. (Recent discovery [1] of existence of gravitational waves)

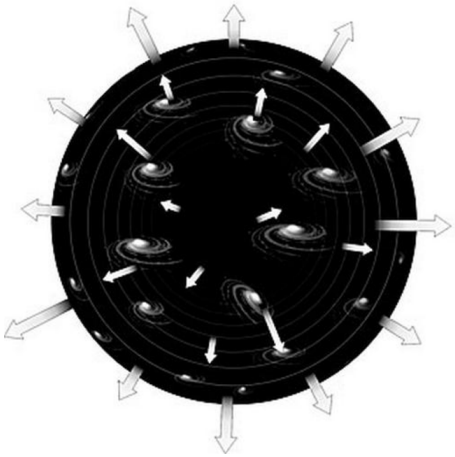
• **Einstein's field equation** ($G_{ij} = -\kappa T_{ij}$); Most significant feature of GR
(Geometry of space time $\backsim$ Source matter)



[1] P. Abbott et al., Phys. Rev. Lett. 116, 061102 (2016)

Introduction

• **Expanding universe**(Prediction and observation); From Einstein(1917), Friedmann(1922) to Hubble(1929) [$v = H_0 D$].



- **Accelerated expanding universe**; Supernova cosmology project[2], High red-shift supernova search team[3].
- **Dark energy**; The reason/source of accelerated expansion and an anti gravity energy that drives acceleration (predicted).

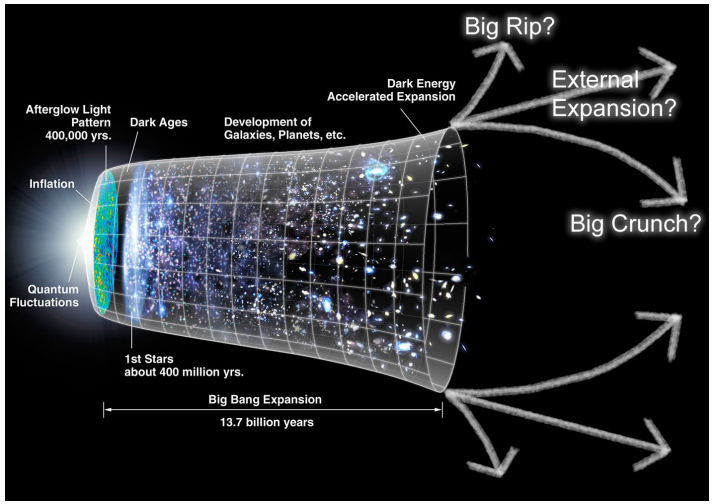
[2] Perlmutter, et al., Astrophys. J., 517 (1999) 565.
[3] G. Riess, et al., Astron. J., 116 (1998) 1009.

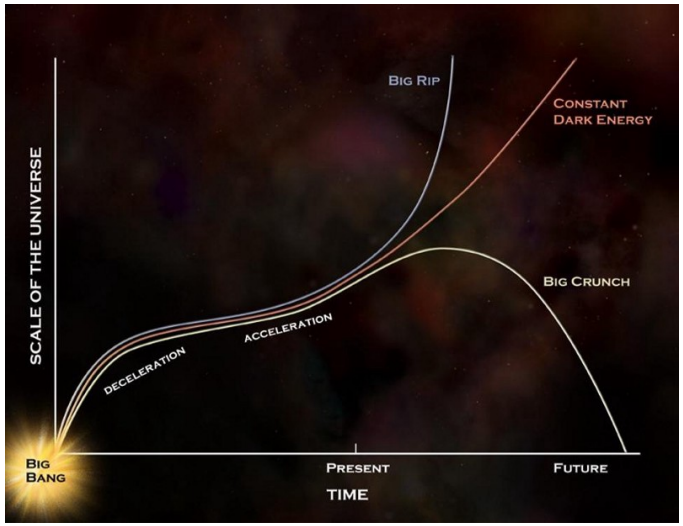
Research problem and background

- **Mathematical framework/model**; The only way to investigate nature of unknown DE.
- **Addressing anisotropic issue**; Since DE displays small scale anisotropic features [4], we need to incorporate anisotropy in the model to study their nature.
- **Why Bianchi space-time?**; Homogenous and anisotropic models with Generalization of open FRW universes [5].
- **Prediction of the fate of our universe**:

[4] S.W. Hawking, R.J. Taylor, Nature,**299**,1278 (1966)
[5] L.Bianchi, Memone di Mathematica **11**, 267,(1898)

Research problem and background





- **Requirement of two fluid source matter** (T_{ij}); Many models considering ordinary matter field failed to describe complete evolution as they are dominant only on past evolutionary epochs.
- Since DE plays the role in the late epoch including present time, combining both will describe the evolutionary history as well as nature of DE.

Model input: Space time and EMT of DE

- **Space time:** Bianchi V (Homogemeous and anisotropic)

$$ds^2 = dt^2 - a_1^2 dx^2 - e^{2\alpha x}\left(a_2^2 dy^2 + a_3^2 dz^2\right)$$

The directional scale factors are, $a_i(i = 1, 2, 3) = a_i(t)$,

- **Energy momentum tensor for dark energy is,**

$$T_{ij} = (\rho + p)u_i u_j + p g_{ij}$$

- In order to incorporate some amount of anisotropy to the model,

$$\begin{aligned} T^D_{\mu\nu} &= diag[-\rho_D, p_{D_x}, p_{D_y}, p_{D_z}] \\ &= diag[-1, \omega_D + \delta, \omega_D + \gamma, \omega_D + \eta]\rho_D, \end{aligned}$$

$\omega_D = \frac{p}{\rho_D}$: DE equation of state parameter(EoS).
δ, γ, η: Skewness parameters, deviations from ω_d in x, y, z axis.

Model input: EMT of electromagnetic fluid (EMF)

• **Energy momentum tensor of EMF** is E_{ij} and is considered to be distributed along latitudinal spatial direction.
• So, assuming infinite electrical conductivity, $f_{14} = f_{24} = f_{34} = 0$ and quantizing the field axis of the $f_{13} = f_{23} = 0$, the only non-vanishing field tensor is f_{12}.
• Assuming distribution of magnetic permeability along z-direction, with the help of Maxwell's equation, $f_{23} = -f_{23} = k$.
• Thus components of EMT , thus, can be expressed as, $E_{11} = \mathcal{M}_z a^2$, $E_{22} = -\mathcal{M}_z b^2 e^{2\alpha x}$, $E_{33} = -\mathcal{M}_z c^2 e^{2\alpha x}$ and $E_{44} = -\mathcal{M}_z$, where, $\mathcal{M}_z = \frac{k^2}{8\pi a_1^2 a_2^2 e^{2\alpha x}}$.
• Consider the EMF of DE T_{ij}^{de} and the required assumptions for physical parameters as usual.

The field equations

Then Einstein field equations ($G_{ij} = -T_{ij}$) for the metrics are, thus, found to be,

$$\frac{\ddot{a}_2}{a_2} + \frac{\ddot{a}_3}{a_3} + \frac{\dot{a}_2\dot{a}_3}{a_2 a_3} - \frac{\alpha^2}{a_1^2} = -(\omega_D + \delta)\rho_D - \mathcal{M}_z \tag{1}$$

$$\frac{\ddot{a}_1}{a_1} + \frac{\ddot{a}_3}{a_3} + \frac{\dot{a}_1\dot{a}_3}{a_1 a_3} - \frac{\alpha^2}{a_1^2} = -(\omega_D + \gamma)\rho_D + \mathcal{M}_z \tag{2}$$

$$\frac{\ddot{a}_1}{a_1} + \frac{\ddot{a}_2}{a_2} + \frac{\dot{a}_1\dot{a}_2}{a_1 a_2} - \frac{\alpha^2}{a_1^2} = -(\omega_D + \eta)\rho_D + \mathcal{M}_z \tag{3}$$

$$\frac{\dot{a}_1\dot{a}_2}{a_1 a_2} + \frac{\dot{a}_2\dot{a}_3}{a_2 a_3} + \frac{\dot{a}_3\dot{a}_1}{a_3 a_1} - \frac{3\alpha^2}{a_1^2} = \rho_D - \mathcal{M}_z \tag{4}$$

$$2\frac{\dot{a}_1}{a_1} - \frac{\dot{a}_2}{a_2} - \frac{\dot{a}_3}{a_3} = 0, \tag{5}$$

Few set ups and parametrizations

- Assumption 1: $a_2 = a_3^k$: Anisotropic proportion[a] in σ and θ
- Assumption 2: $H = \frac{\dot{R}}{R} = \frac{1}{3}\sum_{i=1}^{3} H_i$: The mean Hubble parameter.

$H_i = \frac{\dot{a}_i}{a_i}$: directional Hubble parameters along different spatial directions.

[a] S.D. Maharaj, & R. Naidoo, Astrophys Space Sci (1993) 208: 261

$V = R^3 = a_1 a_2 a_3$: The cosmic spatial volume

$\sigma^2 = \frac{1}{2}(\sum_{i=1}^{3} H_i^2 - \frac{\theta^2}{3})$:The shear scalar

R: A hybrid scale factor.

Why Hybrid Scale Factor

From the early stages of Big Bang, the effective energy densities of radiation and mass of the universe have been varying differently along with the cosmic inflation.

It leads to a radiation dominated era in the very early universe but a transition to a matter dominated era at a later time and, since about 5 billion years ago, a subsequent dark energy dominated era.

A cosmological evolved scale factor can be used to characterized the chronology of the universe, in terms of power law and exponential law or combiningly, hybrid law.

Role of Hybrid scale factor

The hybrid scale factor, $R = e^{at}t^{b}$,where a and b are constants, has 2 factors; one behaves like exponential expansion and the other behaves like power law expansion.

While the power law behavior dominates the cosmic dynamics in the early phase, the exponential factor dominates at late phase.

So, a cosmic transition from early deceleration to late time acceleration can be obtained by a hybrid scale factor.

Few set ups and parametrizations

$\mathcal{A} = \frac{1}{3}\sum\left(1 - \frac{H_i}{H}\right)^2 = \frac{2}{3}\left(\frac{k-1}{k+1}\right)^2$: The anisotropic parameter, $k \neq 1$

$q = \frac{d}{dt}\left(H^{-1}\right) - 1$: The deceleration parameter

Consequently, the scale factors are $a_1 = R,\ a_2 = R^{\frac{2k}{k+1}}$ and $a_3 = R^{\frac{2}{k+1}}$. Directional Hubble parametersare, $H_1 = H$, $H_2 = \left(\frac{2k}{k+1}\right)H$, $H_3 = \left(\frac{2}{k+1}\right)H$.

Hubble type field equation

Then Einstein field equations for the metrics are, thus, found to be,

$$2\dot{H} + 4\frac{(m^2+m+1)}{(m+1)^2}H^2 - \frac{\alpha^2}{a_1^2} = -(\omega_d+\delta)\rho_d - \mathcal{M}_z$$

$$\left(\frac{m+3}{m+1}\right)\dot{H} + \frac{(m^2+4m+7)}{(m+1)^2}H^2 - \frac{\alpha^2}{a_1^2} = -(\omega_d+\gamma)\rho_d + \mathcal{M}_z$$

$$\left(\frac{3m+1}{m+1}\right)\dot{H} + \frac{(8m^2+4m+1)}{(m+1)^2}H^2 - \frac{\alpha^2}{a_1^2} = -(\omega_d+\eta)\rho_d + \mathcal{M}_z$$

$$\frac{(2m^2+6m+4)}{(m+1)^2}H^2 - \frac{3\alpha^2}{a_1^2} = \rho_d - \mathcal{M}_z$$

$$2H - \left(\frac{2m}{m+1}\right)H - \left(\frac{m}{m+1}\right)H = 0,$$

- $\mathcal{M}_z = \mathcal{M} = \dfrac{k^2}{8\pi a_1^2 a_2^2 e^{2\alpha x}}.$

Basic formalism to derive model parameters

• The energy conservation equation for the anisotropic fluid, $\underline{T^{\mu\nu}_{;\nu} = 0}$, yields,

$$\dot{\rho_M}+3(p_M+\rho_M)H+\lambda H_1+\dot{\rho_D}+3\rho_D(\omega_D+1)H+\rho_D(\delta H_1+\gamma H_2+\eta H_3) = 0$$

• The non interacting mix fluid allows separation of above equation in three parts;

$$\dot{\rho_M} + 3H\left(\rho_M + p_M + \frac{\lambda}{3}\right) = 0 \quad (Matter \;\; parameters)$$

$$\dot{\rho_D} + 3H\rho_D(\omega_D + 1) \quad (DE \;\; Parameters)$$

$$\rho_D(\delta H_1 + \gamma H_2 + \eta H_3) = 0 \quad (Anisotropic \;\; parameters)$$

Dynamical model parameters

Solving the field equations, by mathematical formalism, with the help of energy conservation equation,

$$\rho_d = 2\phi_1(m)\frac{\dot{R}^2}{R^2} - 3\frac{\alpha^2}{R^2} + 2\mathcal{M}_z \tag{6}$$

$$\omega_d\rho_d = \left(\frac{\mathbb{A}}{2} - 1\right)\left[\frac{2\ddot{R}}{R} + \frac{\dot{R}^2}{R^2}\right] + \frac{2}{3}\mathcal{M}_z + \frac{\alpha^2}{R^2}, \tag{7}$$

$\phi_1(m) = \dfrac{(m^2+4m+1)}{(m+1)^2}$ and $\mathbb{A} = \frac{1}{3}\sum(\frac{\triangle H_i}{H})^2 = \frac{2}{3}\left(\dfrac{m-1}{m+1})^2\right)$, $m \neq 1$.

Skewness parameters

$$\delta = \frac{2}{3\rho_d}\left[\left(\frac{m-1}{m+1}\right)\epsilon(m)F(R) - \mathcal{M}(R,x)\right], \tag{8}$$

$$\gamma = -\frac{1}{3\rho_d}\left[\left(\frac{m+5}{m+1}\right)\epsilon(m)F(R) - \mathcal{M}(R,x)\right] \tag{9}$$

$$\eta = \frac{1}{3\rho_d}\left[\left(\frac{5m+1}{m+1}\right)\epsilon(m)F(R) + \mathcal{M}(R,x)\right] \tag{10}$$

Where $F(R) = \left(\frac{\ddot{R}}{R} + \frac{2\dot{R}^2}{R^2}\right)$ and $\mathcal{M}(R,x) = \frac{k^2 R^{-4}}{4\pi e^{4\alpha x}}$.

The functional $\epsilon(m) = \frac{1-m}{1+m}$, here, is a measure of deviation from isotropic behaviour.

Results and Analysis (EoS parameter)

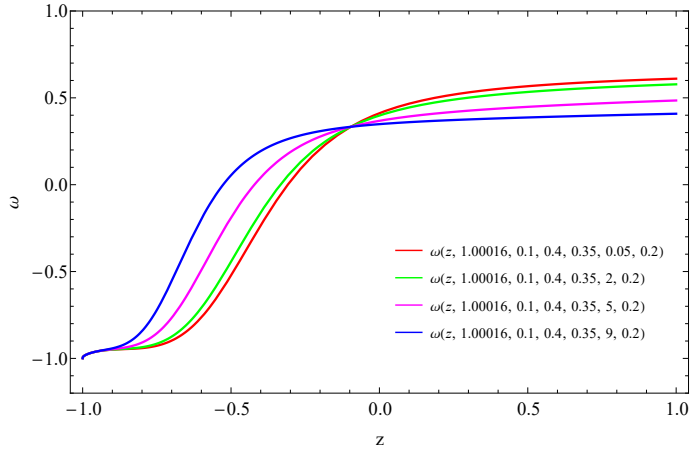


Figure 1: Evolution of EoS parameter when α varies

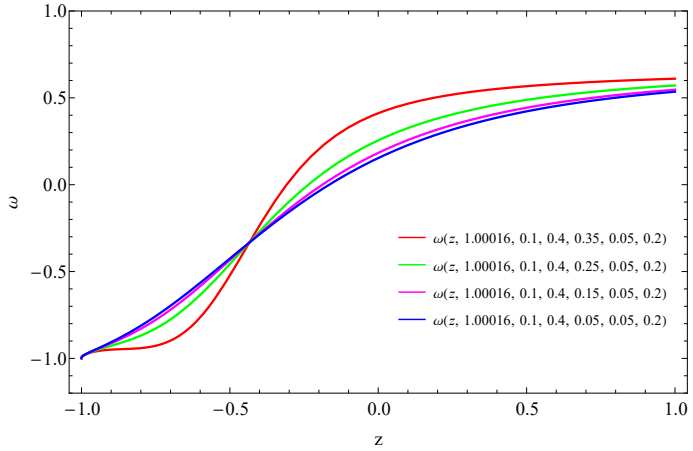


Figure 2: Evolution of EoS parameter when k varies

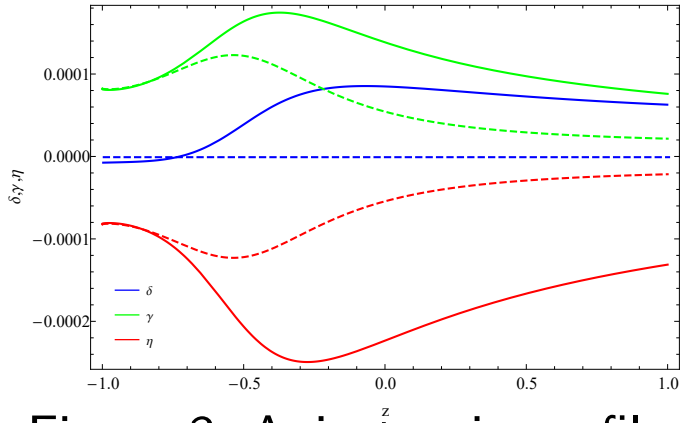


Figure 3: Anisotropic profile

Results and Analysis (Model diagnosis)

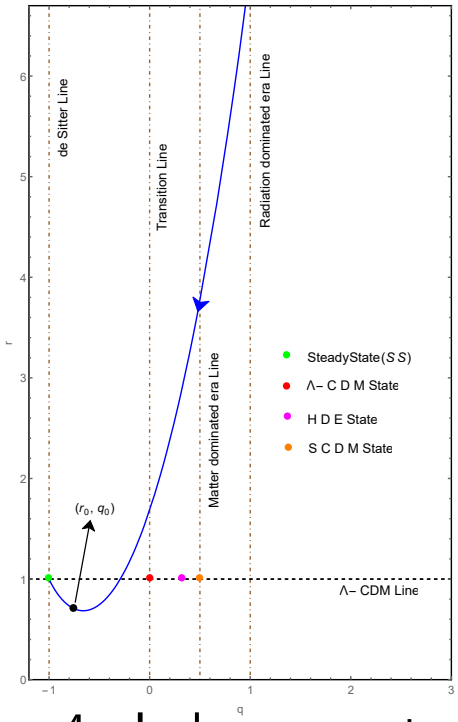


Figure 4: Jerk parameter vs q

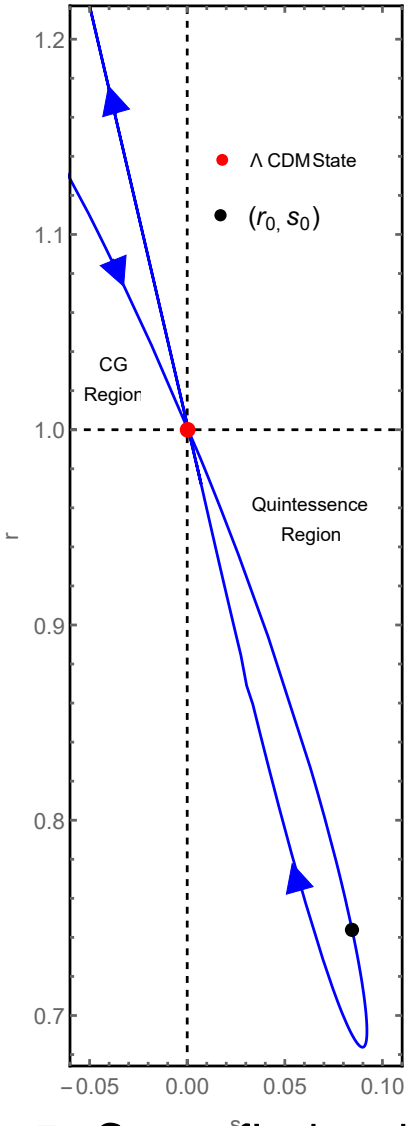


Figure 5: State finder diagnosis

Concluding remarks

- The DE model is developed in two fluid situations where EoS parameter of DE evolves within the predicted range.
- Skewness parameters shows that, the universe is mostly dominant by magnetic fluid than any other source of matter. But in the present epoch, the DE dominates the universe which may be attribute to the current accelerated expansion of the universe.
- There remain pressure anisotropies even at the late phase of cosmic evolution, though in the early phase pressure was assumed to be isotropic.
- Magnetic field has substantial effect on the dynamics of the universe. At early epoch, the impact is somewhat larger and can not be ignored at a later cosmic phase present epoch.
- State finder pairs imply that the derived models are physically realistic and behaves like ΛCDM universe at a non-perturbative level.