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Electrodynamics of axion-active system: polarization and stratification of plasma in an axionic dyon magnetosphere

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Introduction

- ① 1) We extend classical Pannekoek-Rosseland in the framework of the Einstein - Maxwell - Fermi - Dirac - axion theory.
- ② 2) We consider an "axion dyon".
- ③ 3) Questions about the physical sense of such model:
profile of the resulting electric field in the polarized plasma
polarization of electron-positron plasma
problem of axionic modification of the Lorentz force

Rosseland-Pannekoek effect

The effect of electric polarization of the equilibrium multicomponent plasma in the external gravitational field has been known since the 1920s due to the classical works of Pannekoek [1] and Rosseland [2]. This effect was described first in the context of electric polarization of the nonrelativistic electron-ion isothermal plasma in the external Newtonian gravitational field. **This effect is absent in electron-positron plasma!**

- ❶ 1) A. Pannekoek, Ionization in stellar atmospheres, Bull. Astron. Instit. Neth. 1, 107118 (1922).
- ❷ 2) S. Rosseland, Electrical state of a star, Mon. Not. R. Astron. Soc. 84, 720 (1924).

Axion dyon

Axion - hypothetical pseudo-Goldstone boson which introduced by the Peccei and Quinn in 1977 [3] to resolve the strong CP problem in quantum chromodynamics. Axion produce extended electrodynamics by interacting with photons.

Axion dyon - hypothetical (Wilchek, 1987 [4]) object which consist of magnetic monopole in the axion environment. In such an object, the pseudoscalar (axion) field produces the radial electric field in the presence of the radial magnetic field.

- ① 3) R. D. Peccei and H. R. Quinn, CP Conservation in the Presence of Instantons, Phys. Rev. Lett. 38, 1440 (1977).
- ② 4) F. Wilczek, Two Applications of Axion Electrodynamics, Phys. Rev. Lett. 58, 1799 (1987).

Hierarchical model

- ① First level - "frozen" space-time metrics.
- ② Second level - multicomponent plasma, pseudoscalar (axion) field, electromagnetic field.

General metric(1)

The first level of hierarchical model.

Reissner-Nordström type solution

$$ds^2 = c^2 N(r) dt^2 - \frac{1}{N(r)} dr^2 - r^2 (d\theta^2 + \sin^2 \theta d\varphi^2)$$

$$N(r) = 1 - \frac{r_g}{r} + \frac{r_\mu^2}{r^2}$$

$r_g = \frac{2GM}{c^2}$ - the Schwarzschild radius, $r_\mu^2 = \frac{G\mu^2}{c^4}$.
 μ - the magnetic charge of the monopole.

General metric(2)

$$N(r_{\pm}) = 0 \Rightarrow r_{\pm} = \frac{1}{2}r_g \left[1 \pm \sqrt{1 - \frac{4r_{\mu}^2}{r_g^2}} \right].$$

Killing vector

$$\xi^i = \mathcal{B}\delta_0^i, \xi(r) \equiv \sqrt{g_{mn}\xi^m\xi^n} = \mathcal{B}\sqrt{N(r)}.$$

We assume that the radius of the object, r_0 , exceeds the radius of outer horizon r_+ ; we consider the plasma and electromagnetic field in the region $r > r_0 > r_+$.

Relativistic axionically active plasma in the equilibrium state: Kinetic description(1)

The second level of hierarchical model.

Covariant relativistic kinetic equations for the multi-component plasma

$$\frac{p^i}{m_a c} \hat{\nabla}_i f_a + \frac{\partial}{\partial p^i} (\mathcal{F}_a^i f_a) = \sum_b I_{ab}$$

The Cartan derivative

$$\hat{\nabla}_i = \nabla_i - \Gamma_{ik}^j p^k \frac{\partial}{\partial p^j}$$

Relativistic axionically active plasma in the equilibrium state: Kinetic description(2)

Extended Lorentz force

$$\mathcal{F}_a^i \equiv \frac{e_a}{m_a c^2} [F^i_k + \nu \phi F^{*i}_k] p^k$$

e_a - the electric charge of particles of the sort a .

F_{mn} - the Maxwell tensor, $F^{*ik} \equiv \frac{1}{2} \epsilon^{ikmn} F_{mn}$ - its dual tensor.

ϕ - the dimensionless pseudoscalar (axion) field.

- ① $\nu = 0$ - standard Lorentz force.
- ② $\nu = 1$ - extended Lorentz force.

Gyroscopic type force

$$\frac{\partial}{\partial p^i} \mathcal{F}_a^i = 0 \rightarrow \frac{\partial}{\partial p^i} (\mathcal{F}_a^i f_a) = \mathcal{F}_a^i \left(\frac{\partial f_a}{\partial p^i} \right)$$

Relativistic axionically active plasma in the equilibrium state: Kinetic description(3)

Collision integral

$$I_{ab} = \int \int \int dP' dQ dQ' W(p, q | p', q') \delta(p + q - p' - q') \times \\ \times [n_a(p) n_b(q) (1 - n_a(p')) (1 - n_b(q')) - n_a(p') n_b(q') (1 - n_a(p)) (1 - n_b(q))]$$

The invariant measure of integration in the momentum space

$$dP \equiv d^4 p \sqrt{-g} \delta[p_k p^k - m_a^2 c^2] \mathcal{H}(p_k V^k)$$

In equilibrium state $I_{ab} = 0$

Relativistic axionically active plasma in the equilibrium state: Kinetic description(4)

We obtain that n_a is the Fermi-Dirac function:

$$n_a(x, p) = \frac{1}{e^{\mathcal{U}+1}}$$
$$\mathcal{U} \equiv -\mathcal{M}_a(x) + \xi_k(x) p^k$$

Equation for set of functions $\mathcal{M}_a(x)$

$$\frac{1}{2} p^i p^k (\nabla_i \xi_k + \nabla_k \xi_i) + p^k \left[-\frac{\partial}{\partial x^k} \mathcal{M}_a + \frac{e_a}{c} \xi^i (F_{ik} + \nu \phi F_{ik}^*) \right] = 0$$

Equations of axion electrodynamics(1)

Axion functional

$$S_0 = \int d^4x \sqrt{-g} \left\{ \frac{R}{2\kappa} + \frac{1}{4} F^{mn} F_{mn} + \frac{1}{4} \phi F_{mn}^* F^{mn} + \frac{1}{2} \Psi_0^2 \left[-\nabla_k \phi \nabla^k \phi + \mathcal{V} \right] + L_{(m)} \right\}$$

$$g_{A\gamma\gamma} = \frac{1}{\Psi_0} \quad (g_{A\gamma\gamma} < 1.47 \cdot 10^{-10} \text{GeV}^{-1})$$

$$\nabla_k [F^{ik} + \phi F^{*ik}] = -\frac{4\pi}{c} I^i, \quad I^i = \frac{c}{4\pi} \frac{\delta}{\delta A_i} L_{(m)}$$

$I^i = \sum_a e_a c \mathcal{N}_a^i$ - sum of the first momenta of the plasma distribution functions.

Equations of axion plasma electrodynamics(2)

Nonlinear equations of axion electrodynamics in the axionically active plasma

$$\nabla_k F^{ik} = -F^{*ik} \nabla_k \phi - 4\pi \sum_a e_a \int dP f_a^{(\text{eq})} p^i$$

Key electrostatic equation for the Boltzmann plasma

$$\frac{1}{r^2} \frac{d}{dr} \left[r^2 \frac{d\Theta}{dr} + \mu \phi \right] = -\frac{32e\pi^2}{c^3 h^3 N^2(r)} \left\{ \left[c \sqrt{N(r_*)} \sqrt{m_e^2 c^2 + \mathcal{P}_F^2(r_*)} - e\Theta(r) - e\nu\mu \int_{r_*}^r \frac{dz}{z^2} \phi(z) \right]^2 - m_e^2 c^4 N(r) \right\}^{\frac{3}{2}}$$

$$\Theta(r) \equiv A_0(r) - A_0(r^*), \quad \Theta(r_*) = 0$$

We assume, that $\frac{dA_0(r)}{dr}(r \rightarrow \infty) \propto \frac{1}{r^{2+\varepsilon}} \rightarrow 0$

Key equation for the pseudoscalar (axion) field

$$\nabla^k \nabla_k \phi + \phi \frac{\partial \mathcal{V}}{\partial \phi^2} = -\frac{1}{4\Psi_0^2} F_{mn}^* F^{mn}$$

This equation can be written as

$$\frac{d}{dr} \left[r^2 N \frac{d\phi}{dr} + \frac{\mu}{\Psi_0^2} A_0 \right] = r^2 \phi \frac{\partial \mathcal{V}}{\partial \phi^2}$$

Our ansatz is that the potential of the axion field is of the Higgs type.

Specific periodic potential

$$\mathcal{V}_{(P)} = \mathcal{V}_0 \left[1 - \cos \left(\frac{2\pi\phi}{\Phi(\xi)} \right) \right]$$

$$\mathcal{V} = \frac{1}{2} \gamma \left[\phi^2 - \Phi(N(r))^2 \right]^2,$$

Two special versions of the set of key equations for the Boltzmann plasma(1)

Solution of the key equation for the pseudoscalar (axion) field

$$Nr^2 \frac{d\Phi}{dr} + \frac{\mu}{\Psi_0^2} \Theta = K, \quad K = \left[Nr^2 \frac{d\Phi}{dr} \right]_{|r=r^*}$$

Two special versions of the set of key equations for the Boltzmann plasma

1) The case $\nu = 0$: There is no axionic modification of the Lorentz force.

$$\frac{1}{r^2} \frac{d}{dr} \left(r^2 \frac{d\Theta}{dr} \right) - \frac{\mu^2}{r^4 N(r) \Psi_0^2} \Theta = - \frac{\mu}{r^4 N(r)} K - \sum_a \frac{4\pi e_a N_a(r^*)}{\sqrt{N(r)}} \left[\frac{\lambda_a(r^*) K_2(\lambda_a(r))}{\lambda_a(r) K_2(\lambda_a(r^*))} \right] \exp \left[- \frac{e_a \Theta}{k_B T_0} \right]$$

Two special versions of the set of key equations for the Boltzmann plasma(2)

$$\frac{d\Phi}{dr} = \frac{1}{r^2 N(r)} \left[K - \frac{\mu}{\Psi_0^2} \Theta(r) \right]$$

Solution of the 1st type equation

$$\Phi(r) = \Phi(r_*) + \frac{K}{(r_+ - r_-)} \ln \left| \frac{(r - r_+)(r_* - r_-)}{(r - r_-)(r_* - r_+)} \right| - \\ - \frac{\mu}{\Psi_0^2} \int_{r_*}^r \frac{dz \Theta(z)}{(z - r_+)(z - r_-)}$$

Coulombian asymptote case

$$\Phi(\infty) - \Phi(r_*) = \frac{K}{(r_+ - r_-)} \ln \left| \frac{(r_* - r_-)}{(r_* - r_+)} \right|$$

Two special versions of the set of key equations for the Boltzmann plasma(3)

The case $\nu = 1$: There is intrinsic symmetry in the axionic modification of the Lorentz force.

Super-potential

$$\Psi(r) \equiv \Theta(r) + \mu \int_{r^*}^r \frac{dz \Phi(z)}{z^2}$$

Master equation

$$\frac{1}{r^2} \frac{d}{dr} \left(r^2 \frac{d\Psi}{dr} \right) = - \sum_a \frac{4\pi e_a \mathcal{N}_a(r^*) \lambda_a(r^*) K_2(\lambda_a(r))}{\sqrt{N(r)} \lambda_a(r) K_2(\lambda_a(r^*))} \exp \left[-\frac{e_a \Psi(r)}{k_B T_0} \right]$$

Two special versions of the set of key equations for the Boltzmann plasma(4)

Solution for electric potential

$$\Theta(r) = \frac{\Psi_0^2}{\mu} \left[r_*^2 N(r_*) \frac{d\tilde{\Phi}}{dr} \Big|_{r=r_*} - r^2 N(r) \frac{d\tilde{\Phi}}{dr} \right]$$

$$\tilde{\Phi} = \Phi - \frac{\mathcal{M}}{\mu}, \quad \mathcal{M} \equiv \lim_{r \rightarrow \infty} \left[\mu \Phi(r) + r^2 \frac{d\Theta}{dr} \right]$$

Key equation for the axion field, and its fundamental solutions(1)

Key equation for the axion field

$$r^2 \frac{d}{dr} \left(r^2 N \frac{d\tilde{\Phi}}{dr} \right) = \frac{\mu^2}{\Psi_0^2} \tilde{\Phi}$$

$$x = \frac{r}{r_+}, \quad r_+ = \frac{1}{2} r_g \left[1 + \sqrt{1 - \frac{4r_g^2 \mu}{r_g^2}} \right]$$
$$a \equiv \frac{r_g}{r_+} - 1 = \frac{r_-}{r_+} < 1, \quad r_A \equiv \frac{\mu}{\Psi_0}$$

Rewritten form

$$\frac{d^2 \tilde{\Phi}}{dx^2} + \frac{d\tilde{\Phi}}{dx} \left[\frac{1}{x-1} + \frac{1}{x-a} \right] - \left(\frac{r_A^2}{r_+^2} \right) \frac{\tilde{\Phi}}{x^2(x-1)(x-a)} = 0$$

Key equation for the axion field, and its fundamental solutions(2)

Solution of this equation.

$$\tilde{\Phi}(x) = C_1 Y_{(1)}(x) + C_2 Y_{(2)}(x)$$

$$C_1 = \frac{\tilde{\Phi}(x_*)Y'_{(2)}(x_*) - \tilde{\Phi}'(x_*)Y_{(2)}(x_*)}{W(x_*)}, C_2 = \frac{\tilde{\Phi}'(x_*)Y_{(1)}(x_*) - \tilde{\Phi}(x_*)Y'_{(1)}(x_*)}{W(x_*)}$$

Key equation for the axion field, and its fundamental solutions(3)

Fuchs-type equation

$$Y''(x) + Y'(x) \left[\frac{\gamma}{x} + \frac{\delta}{x-1} + \frac{\epsilon_1}{x-a} + \frac{\epsilon_2}{x-b} \right] + Y \frac{(\alpha\beta x^2 + p_1 x + p_2)}{x(x-1)(x-a)(x-b)} = 0$$

Set of parameters

$$b=\alpha=p_1=0, \quad \epsilon_2=-\gamma, \quad \delta=\epsilon_1=1, \quad p_2 = -\frac{r_A^2}{r_+^2}$$

Key equation for the axion field, and its fundamental solutions (4)

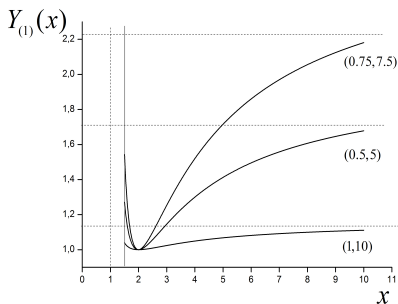


Figure: Plot of the Fuchs-type function $Y_{(1)}(x, a, p_2)$, the first fundamental solution to the Fuchs-Type equation, which corresponds to the standard boundary conditions $Y_{(1)}(x_*)=1$, $Y'_{(1)}(x_*)=0$. For illustration, we depict three curves corresponding to three sets of the parameters (a, p_2) (these quantities are indicated on the plot near the corresponding curves). We put $x_*=2$. The vertical line $x=1$ relates to the outer horizon; the value $x=x_0=1.5$ relates to the radius of the dyon. The function is monotonic for $x > x_*=2$. The profiles of the function $Y_{(1)}(x, a, p_2)$ have horizontal asymptotes at $x \rightarrow \infty$; these asymptotic values are indicated on the plot by the corresponding horizontal lines.

Key equation for the axion field, and its fundamental solutions (5)

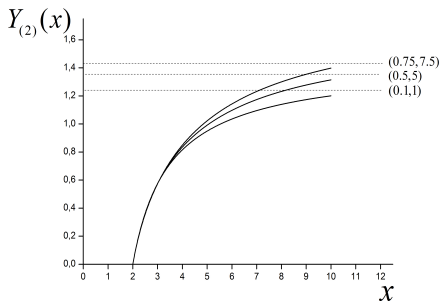


Figure: Plot of the Fuchs-type function $Y_{(2)}(x, a, p_2)$, the second fundamental solution to the Fuchs-type equation, which corresponds to the standard boundary conditions $Y_{(2)}(x_*)=0$, $Y'_{(1)}(x_*)=1$. For illustration, we depict three curves corresponding to three sets of the parameters (a, p_2) in the outer domain $x \geq x_*=2 > x_0$. The function is monotonic; the profiles of the function $Y_{(2)}(x, a, p_2)$ have horizontal asymptotes at $x \rightarrow \infty$, indicated by the corresponding horizontal lines.

Relativistic Boltzmann electron - ion plasma(1)

Plasma with the standard Lorentz force $\nu = 0$

Master equation

$$\frac{1}{r^2} \frac{d}{dr} \left(r^2 \frac{d\Theta}{dr} \right) - \left[\frac{1}{\mathcal{R}_*^2} + \frac{\mu^2}{r^4 N(r) \Psi_0^2} \right] \Theta = \mathcal{I}_* - \frac{\mu K}{r^4 N(r)} .$$

Screening radius

$$\frac{1}{\mathcal{R}_*^2} \equiv \sum_a \frac{4\pi e_a^2 \mathcal{N}_a(r^*)}{k_B T_0 \sqrt{N(r)}} \left[\frac{\lambda_a(r^*) K_2(\lambda_a(r))}{\lambda_a(r) K_2(\lambda_a(r^*))} \right] .$$

Relativistic Boltzmann electron - ion plasma(2)

Recovering the standard Pannekoek-Rosseland effect

Classical ($\mu = 0$) nonrelativistic ($\lambda_a = \frac{m_a c^2}{k_B T_0} \gg 1$) plasma.

The asymptotic decomposition of the McDonald function

$$K_2(\lambda) \rightarrow \sqrt{\frac{\pi}{2\lambda}} e^{-\lambda} \left[1 + \frac{15}{8\lambda} + \dots \right] .$$

Electric potential

$$\Theta(z) = 4\pi \sum_a \frac{e_a \mathcal{N}_a(R_0) \lambda_D^2}{(1 - \lambda_D^2 \alpha_a^2)} \left[e^{-\alpha_a z} - e^{-\frac{z}{\lambda_D}} \right] ,$$

where $\alpha_a \equiv \frac{m_a g}{k_B T_0}$.

Relativistic Boltzmann electron - ion plasma(3)

Relativistic case ($\lambda_a = \frac{m_a c^2}{k_B T_0} \ll 1$)

The decomposition of the McDonald function

$$K_2(z) \rightarrow \frac{2}{z^2} - \frac{1}{2} + \dots$$

The corresponding key equation for the electric potential

$$\frac{1}{r^2} \frac{d}{dr} \left[r^2 \frac{d\Theta}{dr} \right] - \left[\frac{1}{\lambda_D^2 N^2(r)} + \frac{\mu^2}{r^4 \Psi_0^2 N(r)} \right] \Theta = \left[\frac{N(r) - N(r_*)}{N^2(r)} \right] \mathcal{Q}_* - \frac{\mu K}{r^4 N(r)}.$$

Relativistic Boltzmann electron - ion plasma(4)

Far zone $\frac{r^2}{\sqrt{N(r)}} \gg r_A \lambda_D$

The basic equation for the electric potential

$$\frac{d^2\Theta}{dx^2} + \left(\frac{2}{x}\right) \frac{d\Theta}{dx} - \left(\frac{r_+^2}{\lambda_D^2}\right) \Theta = \mathcal{J}(x), \mathcal{J}(x) \equiv \left[\frac{N(x) - N(x_*)}{N^2(x)} \right] \mathcal{Q}_* r_+^2.$$

Solution for axion field

$$\Phi(x) = \Phi(x_*) + \frac{K}{r_+(1-a)} \ln \left| \frac{(x-1)(x_*-a)}{(x-a)(x_*-1)} \right| + \frac{r_A^2 \lambda_D^2 r_g \mathcal{Q}_*}{\mu r_+ r_* a (1-a)} \ln \left[\left(\frac{x-a}{x_*-a} \right)^{\gamma_1} \left(\frac{x-1}{x_*-1} \right)^{\gamma_2} \left(\frac{x}{x_*} \right)^{\gamma_3} \right].$$

Relativistic Boltzmann electron - ion plasma(5)

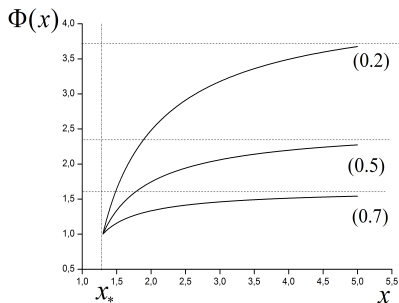


Figure: Plot of the function $\Phi(x, a)$ given by the formula (??). The values of the guiding parameter a are indicated in parentheses near the corresponding curve. All curves have the same initial value $\Phi(x_*)=1$; asymptotic values $\Phi(\infty)$ depend on the parameter a and are marked by the corresponding horizontal lines. The coefficients in front of logarithmic functions in (??) are chosen equal to 1.5 and 2, respectively, for the sake of simplicity.

Relativistic Boltzmann electron - ion plasma(6)

Near zone $\frac{r^2}{\sqrt{N(r)}} \ll r_A \lambda_D$

Equation for electric potential

$$\frac{1}{r^2} \frac{d}{dr} \left(r^2 \frac{d\tilde{\Theta}}{dr} \right) - \frac{r_A^2 \tilde{\Theta}}{r^4 N(r)} = \left[\frac{N(r) - N(r_*)}{N^2(r)} \right] \mathcal{Q}_*, \quad \tilde{\Theta} \equiv \Theta - \frac{\mu K}{r_A^2}.$$

Solution

$$\tilde{\Theta} = C_1 Y_{(1)} + C_2 Y_{(2)} + Y^*,$$

$$Y^*(x) = \frac{1}{x_*^2 W(x_*)} \int_{x_*}^x z^2 dz \tilde{\mathcal{J}}(z) \times [Y_{(1)}(z) Y_{(2)}(x) - Y_{(1)}(x) Y_{(2)}(z)] .$$

Relativistic Boltzmann electron - ion plasma(7)

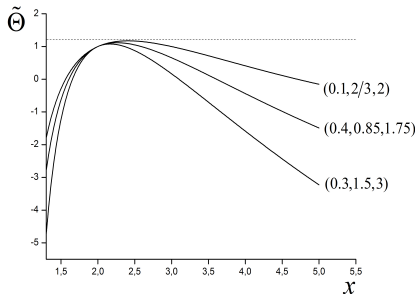


Figure: Plot of the function $\tilde{\Theta}$, which illustrates the profile of the reduced electric potential in the near zone. The curves are distinguished by three parameters indicated in parentheses near the corresponding curve; the first value relates to the parameter a , the second value describes the guiding parameter $\frac{r_A^2}{r_+^2}$; the third value relates to the parameter Q_* . The general feature of these curves is the presence of maximum of the electric potential at $x = x_{\max}$ in the near zone; at $x < x_{\max}$ the electric field $E(x) = \frac{1}{r_+} \frac{d\tilde{\Theta}}{dx}$ is positive, at $x > x_{\max}$ it is negative. Since the electric field changes sign, we deal with the example of plasma stratification.

Relativistic Boltzmann electron-positron plasma(1)

$$m_p = m_e = m, e_p = -e_e = e, \mathcal{N}_p = \mathcal{N}_e = \mathcal{N}$$

Electron-positron plasma with symmetric generalized Lorentz force,
 $\nu = 1$

Electric potential

$$\Theta(r) = -\mu \int_{r_*}^r \frac{dr \Phi(r)}{r^2} = -\frac{\Psi_0^2}{\mu} \left[r^2 N(r) \frac{d\Phi}{dr} - r_*^2 N(r_*) \frac{d\Phi}{dr} \Big|_{r=r_*} \right].$$

Equation for axion field

$$r^2 \frac{d}{dr} \left[r^2 N(r) \frac{d\Phi}{dr} \right] = \frac{\mu^2 \Phi}{\Psi_0^2}.$$

Relativistic Boltzmann electron-positron plasma(2)

Electron-positron plasma with $\nu = 0$

Electric potential

$$\Theta(x) = \frac{\mu K \lambda_D}{x r_+} \left\{ C^{**} \sinh \left[\frac{r_+}{\lambda_D} (x - x_*) \right] - \int_{x_*}^x \frac{dz}{z(z-1)(z-a)} \sinh \left[\frac{r_+}{\lambda_D} (x - z) \right] \right\},$$

$$C^{**} = \int_{x_*}^{\infty} \frac{dz}{z(z-1)(z-a)} \exp \left[\frac{r_+}{\lambda_D} (x_* - z) \right].$$

Conclusions

- 1 In case of relativistic Boltzmann electron - ion plasma, the plasma polarization and electric field near the axionic dyon can be formed as follows: first, the simple plasma polarization where the axionically induced electric field is directed along the Pannekoek-Rosseland field; second, the plasma stratification where the Pannekoek- Rosseland electric field is directed contrarily to the axionically induced electric field; third, the profile related to the compensation of two mentioned electric field.
- 2 When we deal with relativistic Boltzmann electron-positron plasma, the Pannekoek-Rosseland effect is absent because of the equivalence of the masses of the electrons and positrons. Nevertheless, the axionically induced electric field again polarizes the plasma, and plasma backreaction and screening form the final sophisticated profile of the coupled electric and axionic fields.

Thanks for you attention!