Hybrid Star Properties within the Nambu-Jona-Lasinio Model for Quark Matter and Relativistic Mean Field Model for Hadronic Matter

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Heavy Ion Colliders, Large Hadron Collider $\Longrightarrow$ Hot dense QCD matter properties

Neutron Stars as cosmic laboratories $\Longrightarrow$ Cold dense QCD matter properties

$\Downarrow$

Tolman, Oppenheimer and Volkoff equations:

$$\frac{dP(r)}{dr} = -\frac{G[\varepsilon(r)+P(r)][m(r)+4\pi r^3 P(r)]}{r^2[1-2Gm(r)/r]}$$

$$\frac{dm(r)}{dr} = 4\pi r^2 \varepsilon(r)$$

Equation of State ?

$$\varepsilon(P)$$

$$P_c \Longrightarrow m(r), \quad P(r), \quad \varepsilon(r)$$

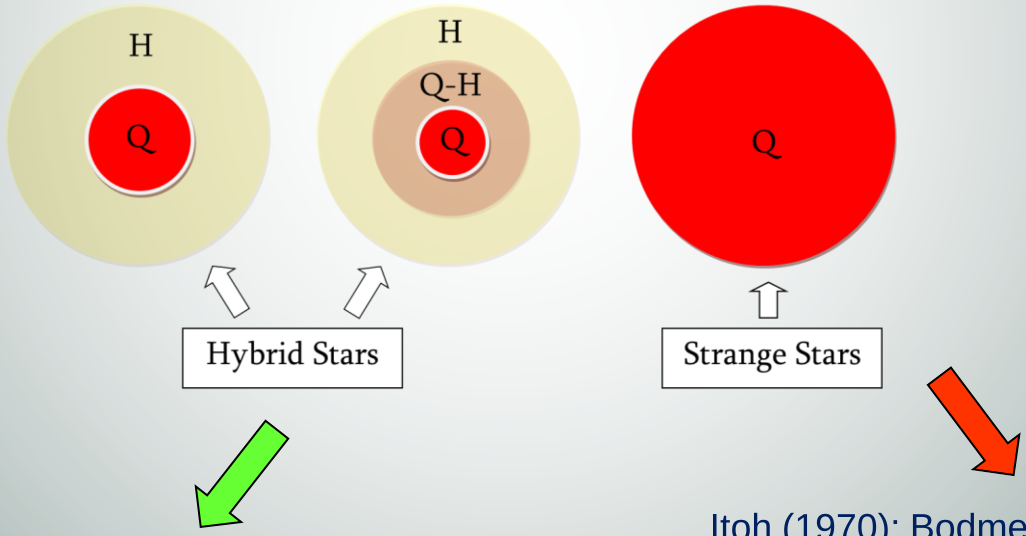
$$P(R) = 0 \Longrightarrow R, \quad m(R) = M$$

Discoveries of massive neutron stars

P. Demorest et al., *Nature* 467, 1081 (2010).
PSR J1614–2230 ⟹ $M = 1.97 \pm 0.04\ M_{\odot}$

J. Antoniadis, et al., *Science* 340, 1233232 (2013)
PSR J0348+0432 ⟹ $M = 2.01 \pm 0.04\ M_{\odot}$

H.T.Cromartie et.al., *Nature Astronomy* (16 sept. 2019)
MSP J0740+6620 ⟹ $M = 2.14\ ^{+\,0.10}_{-\,0.09}\ M_{\odot}$



Itoh (1970); Bodmer (1971); Witten (1984)

$\varepsilon_{uds} < 930.4\,MeV$

Quark phase: Local three-flavor NJL model

Hadronic phase: RMF model with $\sigma\omega\rho\delta$ meson fields

Quark Matter EOS: Local Nambu-Jona-Lasinio Model

Y.Nambu, G.Jona-Lasinio, Phys. Rev. **122**, 345, 1961

Y.Nambu, G.Jona-Lasinio, Phys. Rev. **124**, 246, 1961

$$\mathcal{L}_{NJL} = \bar{\psi}\left(i\gamma^{\mu}\partial_{\mu} - \hat{m}_0\right)\psi + G\sum_{a=0}^{8}\left[\left(\bar{\psi}\lambda_a\psi\right)^2 + \left(\bar{\psi}i\gamma_5\lambda_a\psi\right)^2\right] -$$

$$-K\left\{\det{}_f\left(\bar{\psi}(1+\gamma_5)\psi\right) + \det{}_f\left(\bar{\psi}(1-\gamma_5)\psi\right)\right\} \quad ,$$

ψ_f^c Quark spinor fields of flavor f=u, d, s and color c=r, g, b

$\hat{m}_0 = diag(m_{0u}, m_{0d}, m_{0s})$

$\lambda_a \; (a = 1, 2, \ldots 8)$ Gell-Mann SU(3) matrices $\qquad \lambda_0 = \sqrt{\frac{2}{3}}\,\hat{I}$

G four-quark scalar channel interaction constant

K Kobayashi-Maskawa-’t Hooft six-quark interaction constant

Quark chiral condensates: $\sigma_u = \langle \bar{\psi}_u \psi_u \rangle, \quad \sigma_d = \langle \bar{\psi}_d \psi_d \rangle, \quad \sigma_s = \langle \bar{\psi}_s \psi_s \rangle$

Gap equations of quark dynamic mass:

$$m_u = m_{0u} - 4G\sigma_u + 2K\sigma_d \sigma_s \,,$$
$$m_d = m_{0d} - 4G\sigma_d + 2K\sigma_s \sigma_u \,,$$
$$m_s = m_{0s} - 4G\sigma_s + 2K\sigma_u \sigma_d \,.$$

$$\sigma_i = -\frac{3}{\pi^2} \int_{p_F(n_i)}^{\Lambda} \frac{m_i}{\sqrt{k^2 + m_i^2}} k^2 dk \qquad p_F(n_i) = (\pi^2 n_i)^{1/3} \qquad (i = u, d, s)$$

Electrical neutrality condition: $\frac{2}{3} n_u - \frac{1}{3} n_d - \frac{1}{3} n_s - n_e = 0$

$\beta -$ equilibrium condition:

$$\begin{cases} \mu_d(n_d, m_d) = \mu_u(n_u, m_u) + \mu_e(n_e), \\ \mu_s(n_s, m_s) = \mu_d(n_d, m_d). \end{cases}$$

$$\varepsilon_{udse}(n_B)=\frac{3}{\pi^2}\sum_{i=u,d,s}\int_0^{\left(\pi^2 n_i(n_B)\right)^{1/3}}\sqrt{k^2+m_i(n_B)^2}\,k^2dk+2G\left[\sigma_u(n_B)^2+\sigma_d(n_B)^2+\sigma_s(n_B)^2\right]-$$

$$-4K\sigma_u(n_B)\,\sigma_d(n_B)\,\sigma_s(n_B)+\frac{1}{\pi^2}\int_0^{\left(3\pi^2 n_e(n_B)\right)^{1/3}}\sqrt{k^2+m_e^2}\,k^2dk+$$

$$+\frac{3}{\pi^2}\sum_{i=u,d,s}\int_0^{\Lambda}\left(\sqrt{k^2+m_i(0)^2}-\sqrt{k^2+m_i(n_B)^2}\right)k^2dk-$$

$$-2G\left[\sigma_u(0)^2+\sigma_d(0)^2+\sigma_s(0)^2\right]+4K\sigma_u(0)\,\sigma_d(0)\,\sigma_s(0)\ ,$$

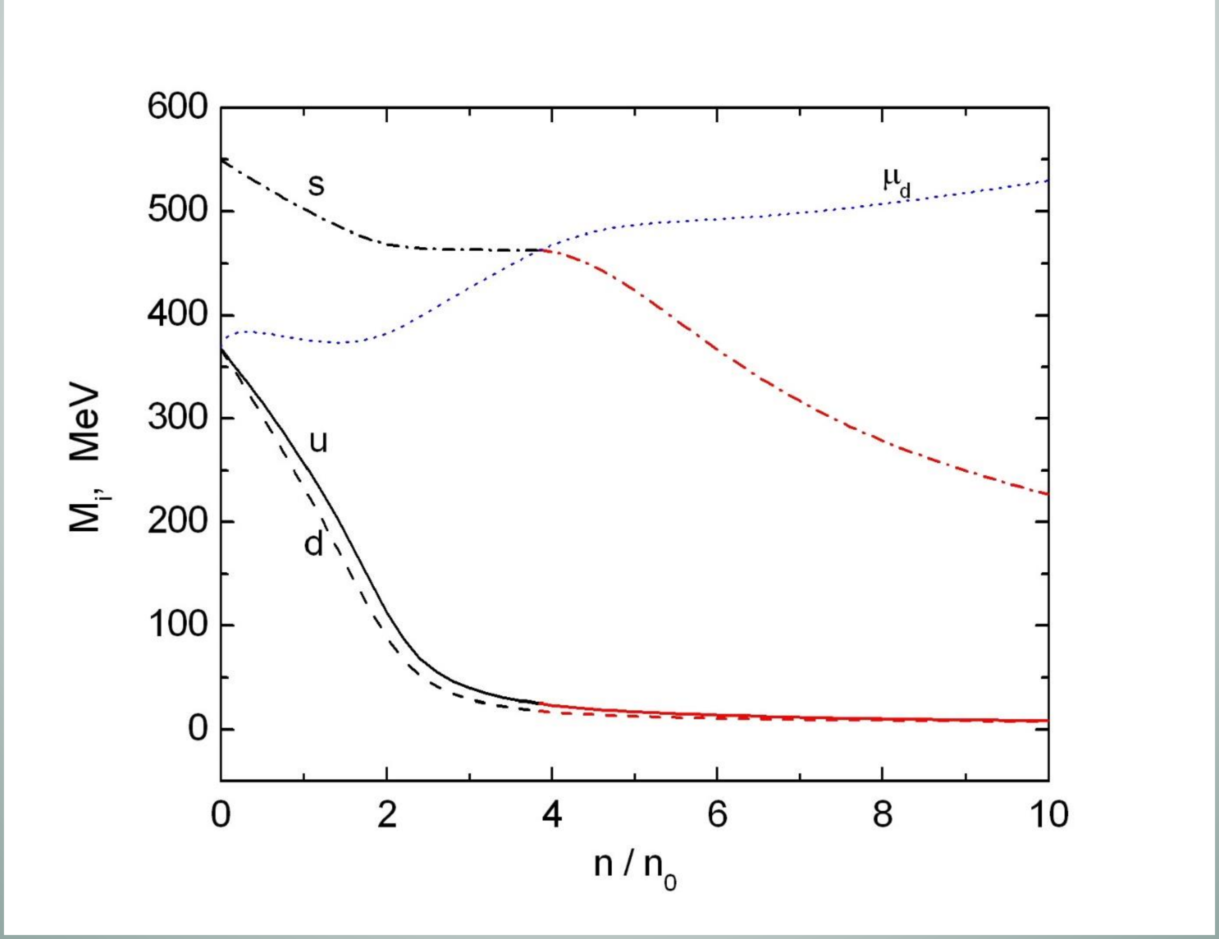
$$P_{udse}(n_B)=\sum_{i=u,d,s}n_i(n_B)\sqrt{\left(\pi^2 n_i(n_B)\right)^{2/3}+m_i(n_B)^2}+n_e(n_B)\sqrt{\left(3\pi^2 n_e(n_B)\right)^{2/3}+m_e^{\,2}}-\varepsilon_{udse}(n_B)$$

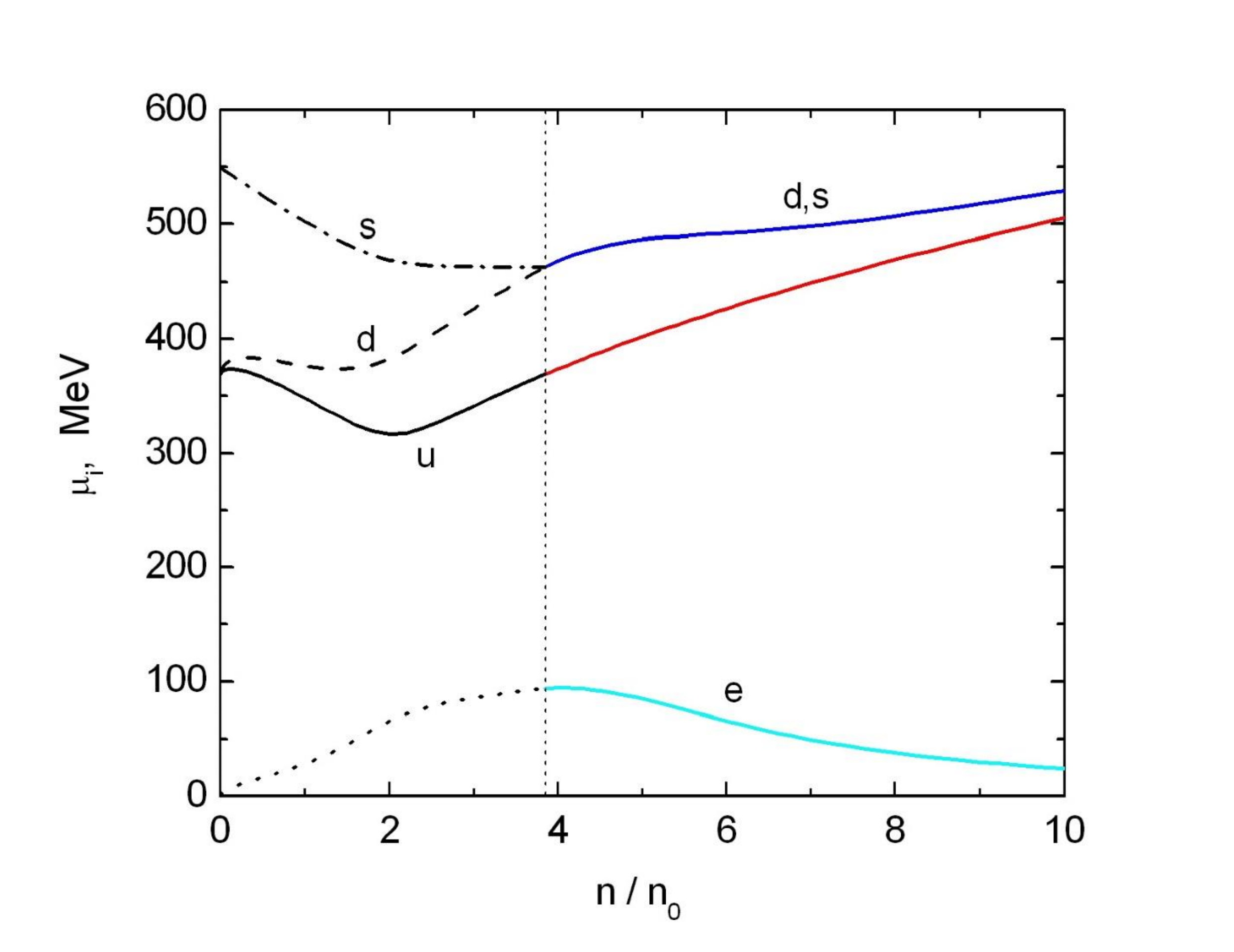
Three-flavor NJL model Parameters

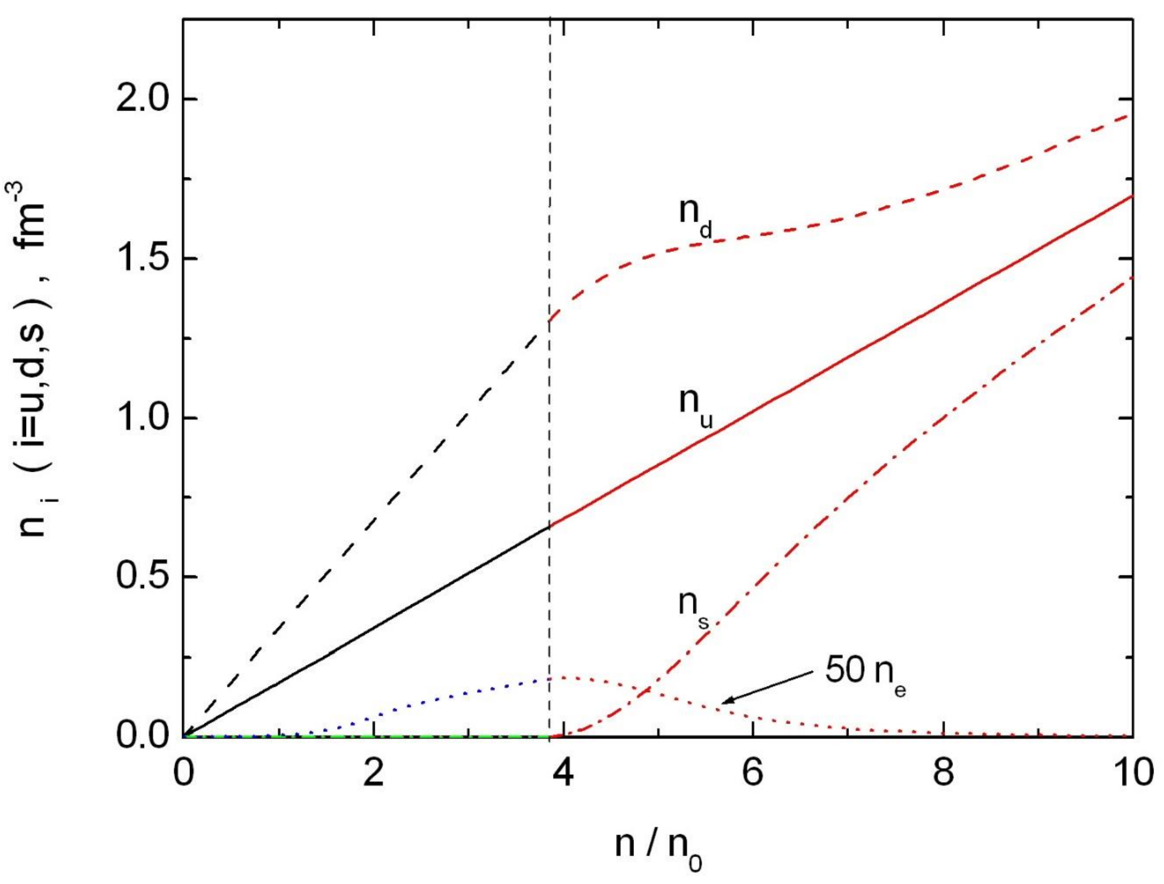
P.Rehberg, S.P.Klevansky, J.Hufner, Phys. Rev. C **53**, 410, 1996.

$$m_{0u} = m_{0d} = 5,5 MeV, \;\; m_{0s} = 140,7 MeV, \;\; \Lambda = 602,3 MeV,$$

$$G = 1.835/\Lambda^2 \qquad K = 12,36/\Lambda^5.$$




600
500
400
300
200
100
0
0
2
4
6
8
10
μ_i, MeV
n / n_0
s
d
u
d,s
e



2.0
1.5
1.0
0.5
0.0
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4
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8
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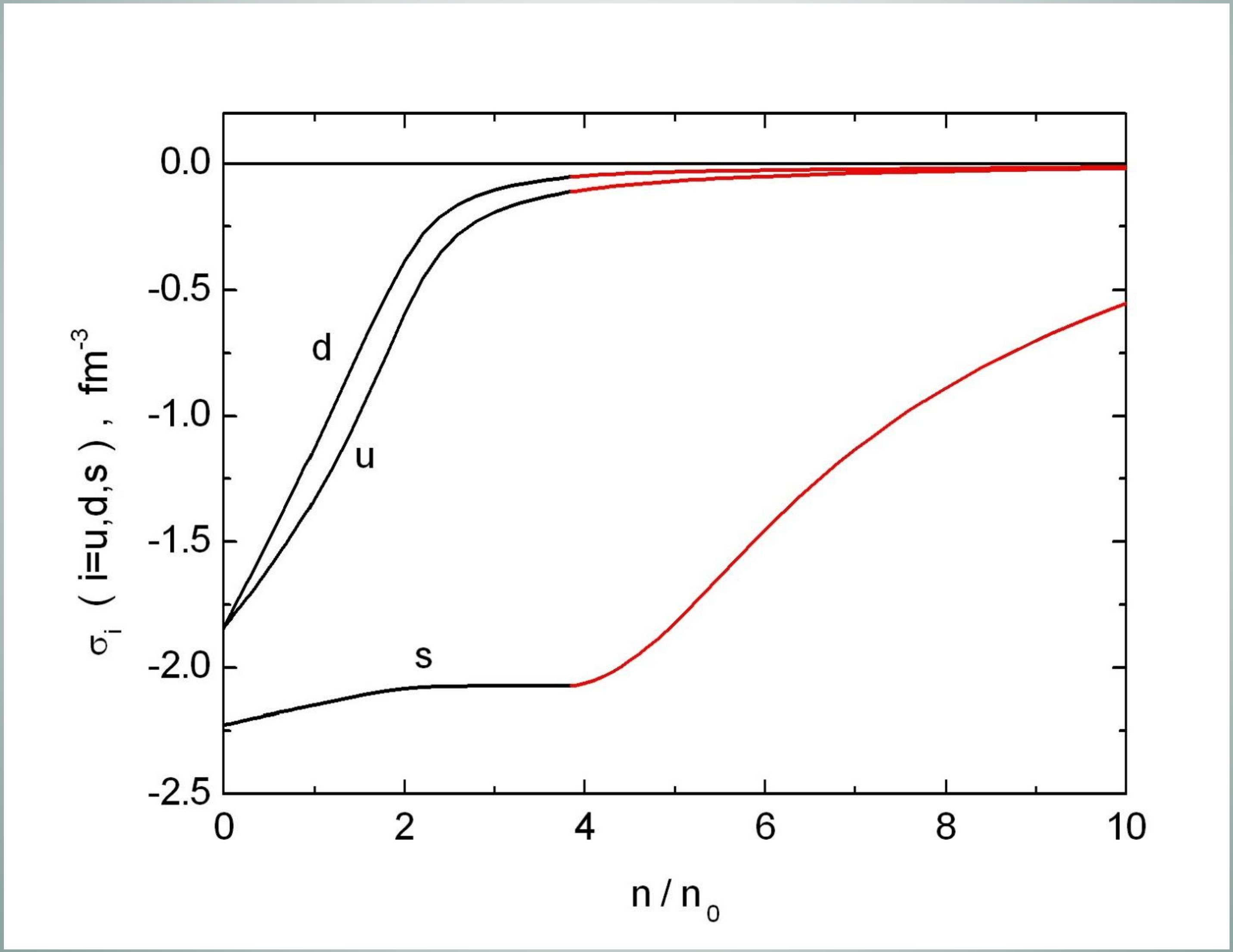
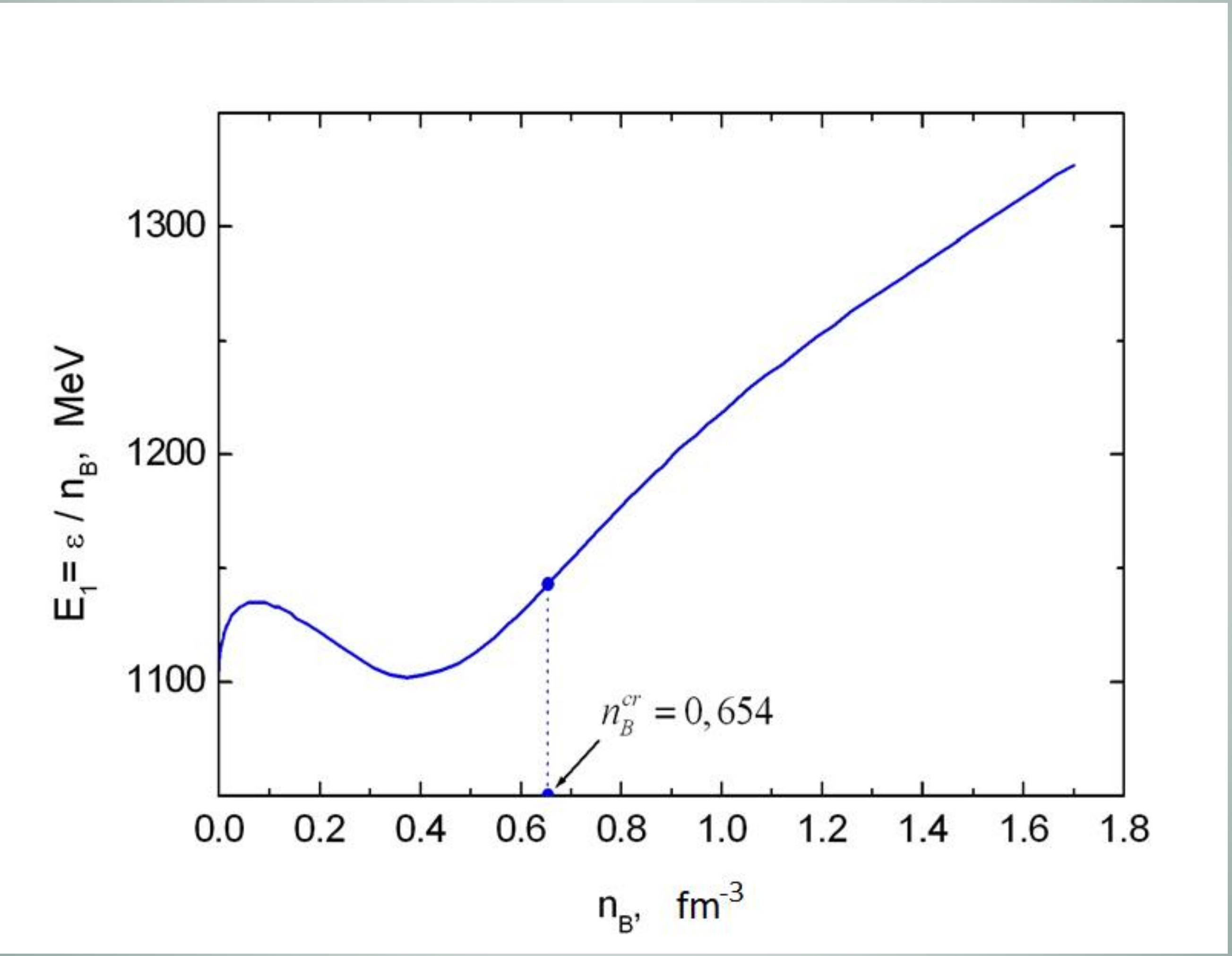
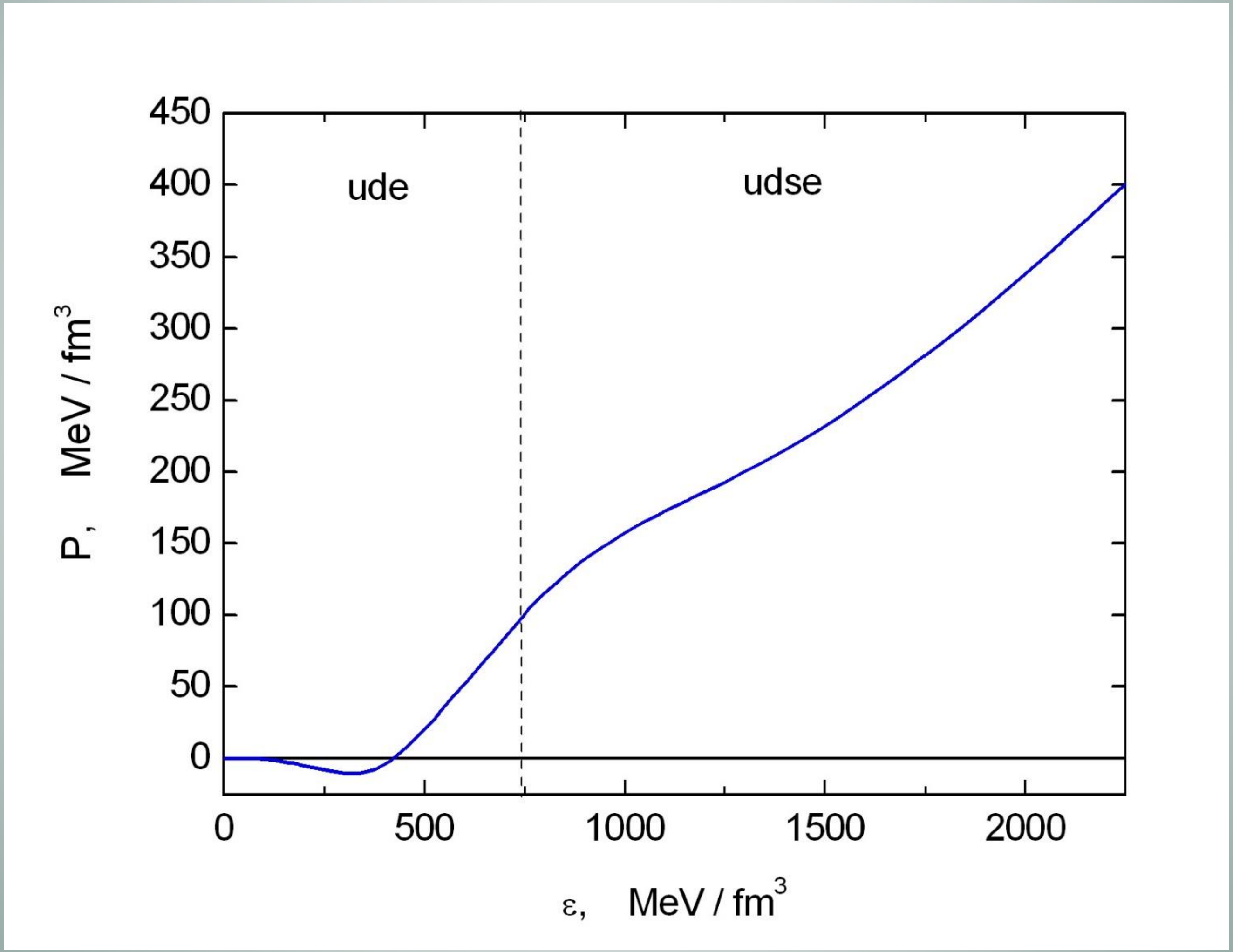
n / n0
n i (i=u,d,s) , fm-3
nd
nu
ns
50 ne


0.0
-0.5
-1.0
-1.5
-2.0
-2.5
0
2
4
6
8
10
d
u
s
σ_i (i=u,d,s) , fm^{-3}
n / n_0



1300
1200
1100
$E_1 = \varepsilon / n_B$, MeV
$n_B^{cr} = 0,654$
0.0 0.2 0.4 0.6 0.8 1.0 1.2 1.4 1.6 1.8
n_B, fm^{-3}

Deconfined Quark Matter EOS: Three-flavor Local NJL Model



Hadronic Matter EOS: RMF$\sigma\omega\rho\delta$ Model

$\sigma\omega\,\rho$

J.D.Walecka, Ann.Phys. 87, 4951, 1974
B.D.Serot, J.D.Walecka, Int.J.Mod.Phys. E6, 515, 1997.

	Scalar	Vector
Isoscalar	σ	ω
Isovector	δ	ρ

Neutron stars without quark deconfinement:

$\sigma\omega\,\rho\,\delta$ B.Liu, H.Guo, M.Di Toro, V.Greco, arXiv Nucl-th/0409014 v2, 2005

Neutron stars with quark deconfinement:

$\sigma\omega\,\rho\,\delta$

G.B.Alaverdyan, Astrophysics 52, 132, 2009.
G.B.Alaverdyan, Gravitation and Cosmology 15, 5, 2009.
G.B.Alaverdyan, Research in Astronomy and Astrophysics, 10, 1255, 2010.

Lagrangian density of many-particle system of p,n.σ,ω,ρ,δ

$$\mathcal{L} = \bar{\psi}_N \left[\gamma^\mu \left(i\partial_\mu - g_\omega \omega_\mu(x) - \frac{1}{2} g_\rho \vec{\tau}_N \cdot \vec{\rho}_\mu(x) \right) - \left(m_N - g_\sigma \sigma(x) - g_\delta \vec{\tau}_N \cdot \vec{\delta}(x) \right) \right] \psi_N +$$

$$+ \frac{1}{2}\left(\partial_\mu \sigma(x) \partial^\mu \sigma(x) - m_\sigma^2 \sigma(x)^2 \right) - U(\sigma(x)) + \frac{1}{2} m_\omega^2 \omega^\mu(x) \omega_\mu(x) - \frac{1}{4} \Omega_{\mu\nu}(x) \Omega^{\mu\nu}(x) +$$

$$+ \frac{1}{2}\left(\partial_\mu \vec{\delta}(x) \partial^\mu \vec{\delta}(x) - m_\delta^2 \vec{\delta}(x)^2 \right) + \frac{1}{2} m_\rho^2 \vec{\rho}^\mu(x) \vec{\rho}_\mu(x) - \frac{1}{4} R_{\mu\nu}(x) R^{\mu\nu}(x),$$

$$x = x_\mu = (t, x, y, z) \qquad \sigma(x),\ \omega_\mu(x),\ \vec{\delta}(x),\ \vec{\rho}_\mu(x) \qquad \psi_N = \begin{pmatrix} \psi_p \\ \psi_n \end{pmatrix}$$

$$U(\sigma) = \frac{b}{3} m_N (g_\sigma \sigma)^3 + \frac{c}{4} (g_\sigma \sigma)^4,$$

$$\Omega_{\mu\nu}(x) = \partial_\mu \omega_\nu(x) - \partial_\nu \omega_\mu(x),$$

$$\Re_{\mu\nu}(x) = \partial_\mu \rho_\nu(x) - \partial_\nu \rho_\mu(x).$$

Relativistic mean-field approach

$$\frac{\partial \mathcal{L}}{\partial \phi(x)} - \partial_\mu \frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi(x))} = 0$$

$$e_p(k) = \sqrt{k^2 + m_p^{*\,2}} + g_\omega\, \bar{\omega}_0 + \frac{1}{2} g_\rho\, \bar{\rho}_0^{(3)},$$

$$e_n(k) = \sqrt{k^2 + m_n^{*\,2}} + g_\omega\, \bar{\omega}_0 - \frac{1}{2} g_\rho\, \bar{\rho}_0^{(3)},$$

$$m_p^* = m_N - g_\sigma\, \bar{\sigma} - g_\delta\, \bar{\delta}^{(3)},$$

$$m_n^* = m_N - g_\sigma\, \bar{\sigma} + g_\delta\, \bar{\delta}^{(3)}.$$

$$m_\sigma^{\;2}\, \bar{\sigma} = g_\sigma \left(n_{sp} + n_{sn} - \frac{dU(\bar{\sigma})}{d\bar{\sigma}} \right),$$

$$m_\omega^{\;2}\, \bar{\omega} = g_\omega \left(n_p + n_n \right),$$

$$m_\delta^{\;2}\, \bar{\delta}^{(3)} = g_\delta \left(n_{sp} - n_{sn} \right),$$

$$m_\rho^{\;2}\, \bar{\rho}_0^{(3)} = \frac{1}{2} g_\rho \left(n_p - n_n \right),$$

$$n_p = \frac{k_{Fp}^{\;3}}{3\pi^2}, \qquad n_n = \frac{k_{Fn}^{\;3}}{3\pi^2},$$

$$n_{sp} = \frac{1}{\pi^2} \int_0^{k_{Fp}} \frac{m_p^*}{\sqrt{k^2 + m_p^{*\,2}}}\, k^2 dk,$$

$$n_{sn} = \frac{1}{\pi^2} \int_0^{k_{Fn}} \frac{m_n^*}{\sqrt{k^2 + m_n^{*\,2}}}\, k^2 dk.$$

$$\mu_p = e_p(k_{Fp}) = \sqrt{k_{Fp}^{\;2} + m_p^{*\,2}} + g_\omega\, \bar{\omega}_0 + \frac{1}{2} g_\rho\, \bar{\rho}_0^{(3)},$$

$$\mu_n = e_n(k_{Fn}) = \sqrt{k_{Fn}^{\;2} + m_n^{*\,2}} + g_\omega\, \bar{\omega}_0 - \frac{1}{2} g_\rho\, \bar{\rho}_0^{(3)}.$$

Parametric EOS for hadronic matter

$$g_\sigma \bar{\sigma} \equiv \sigma, \quad g_\omega \bar{\omega}_0 \equiv \omega, \quad g_\delta \bar{\delta}^{(3)} \equiv \delta, \qquad g_\rho \bar{\rho}^{(3)} \equiv \rho,$$

$$\left(\frac{g_\sigma}{m_\sigma}\right)^2 \equiv a_\sigma, \quad \left(\frac{g_\omega}{m_\omega}\right)^2 \equiv a_\omega, \quad \left(\frac{g_\delta}{m_\delta}\right)^2 \equiv a_\delta, \quad \left(\frac{g_\rho}{m_\rho}\right)^2 \equiv a_\rho$$

$\alpha = \dfrac{n_n - n_p}{n}$, the asymmetry parameter

$$P(n,\alpha) = \frac{1}{\pi^2} \int_0^{k_F(n)(1-\alpha)^{1/3}} \left(\sqrt{k_F(n)^2 (1-\alpha)^{2/3} + (m_N - \sigma - \delta)^2} - \sqrt{k^2 + (m_N - \sigma - \delta)^2} \right) k^2 dk +$$
$$+ \frac{1}{\pi^2} \int_0^{k_F(n)(1+\alpha)^{1/3}} \left(\sqrt{k_F(n)^2 (1+\alpha)^{2/3} + (m_N - \sigma + \delta)^2} - \sqrt{k^2 + (m_N - \sigma + \delta)^2} \right) k^2 dk -$$
$$- \tilde{U}(\sigma) + \frac{1}{2} \left(-\frac{\sigma^2}{a_\sigma} + \frac{\omega^2}{a_\omega} - \frac{\delta^2}{a_\delta} + \frac{\rho^2}{a_\rho} \right).$$

$$\varepsilon(n,\alpha) = \frac{1}{\pi^2} \int_0^{k_F(n)(1-\alpha)^{1/3}} \sqrt{k^2 + (m_N - \sigma - \delta)^2}\, k^2 dk +$$
$$+ \frac{1}{\pi^2} \int_0^{k_F(n)(1+\alpha)^{1/3}} \sqrt{k^2 + (m_N - \sigma + \delta)^2}\, k^2 dk + \tilde{U}(\sigma) + \frac{1}{2} \left(\frac{\sigma^2}{a_\sigma} + \frac{\omega^2}{a_\omega} + \frac{\delta^2}{a_\delta} + \frac{\rho^2}{a_\rho} \right),$$

Parameters of RMF theory

$$a_\sigma\,,\ a_\omega\,,\ a_\delta\,,\ a_\rho\,,\ b,\ c$$

Symmetric nuclear matter $(\alpha = 0)$ Saturation density $(n = n_0)$

$$m_N^* = \gamma\, m_N\,, \qquad \sigma_0 = (1-\gamma)\, m_N$$

$$\left.\frac{d\varepsilon(n,\alpha)}{dn}\right|_{\substack{n=n_0\\ \alpha=0}} = \frac{\varepsilon(n_0,0)}{n_0} = m_N + f_0\,, \qquad f_0 = \frac{B}{A},$$ Binding energy per baryon

$$a_\omega = \frac{1}{n_0}\left(m_N + f_0 - \sqrt{k_F(n_0)^2 + (m_N - \sigma_0)^2}\right)$$

$$\omega_0 = a_\omega n_0 = m_N + f_0 - \sqrt{k_F(n_0)^2 + (m_N - \sigma_0)^2}$$

$$\frac{\sigma_0}{a_\sigma} = \frac{2}{\pi^2}\int_0^{k_F(n_0)} \frac{(m_N - \sigma_0)}{\sqrt{k^2 + (m_N - \sigma_0)^2}}\, k^2 dk \, - b m_N \sigma_0^{\,2} - c\sigma_0^{\,3}$$

Parameters of RMF theory

$$\varepsilon_0 = n_0(m_N + f_0) = \frac{2}{\pi^2}\int_0^{k_F(n_0)} \sqrt{k^2 + (m_N - \sigma_0)^2}\, k^2 dk + \frac{b}{3} m_N \sigma_0^3 + \frac{c}{4}\sigma_0^4 + \frac{1}{2}\left(\frac{\sigma_0^2}{a_\sigma} + n_0^2 a_\omega\right)$$

$$K = 9 n_0^2 \frac{d^2}{dn^2}\left(\frac{\varepsilon(n,\alpha)}{n}\right)\Bigg|_{\substack{n=n_0\\ \alpha=0}}$$ compressibility module

$$\frac{\varepsilon_{sym}}{n} = E_{sym}(n)\alpha^2 \quad \Longrightarrow \quad E_{sym}(n) = \frac{1}{2n}\frac{d^2\varepsilon(n,\alpha)}{d\alpha^2}\Bigg|_{\alpha=0}$$

Symmetry energy

$m_N = 938{,}93\ MeV$

$n_0 = 0{,}153\ fm^{-3}$

$\gamma = \frac{m_N^*}{m_N} = 0{,}78$

$K = 300\ MeV$

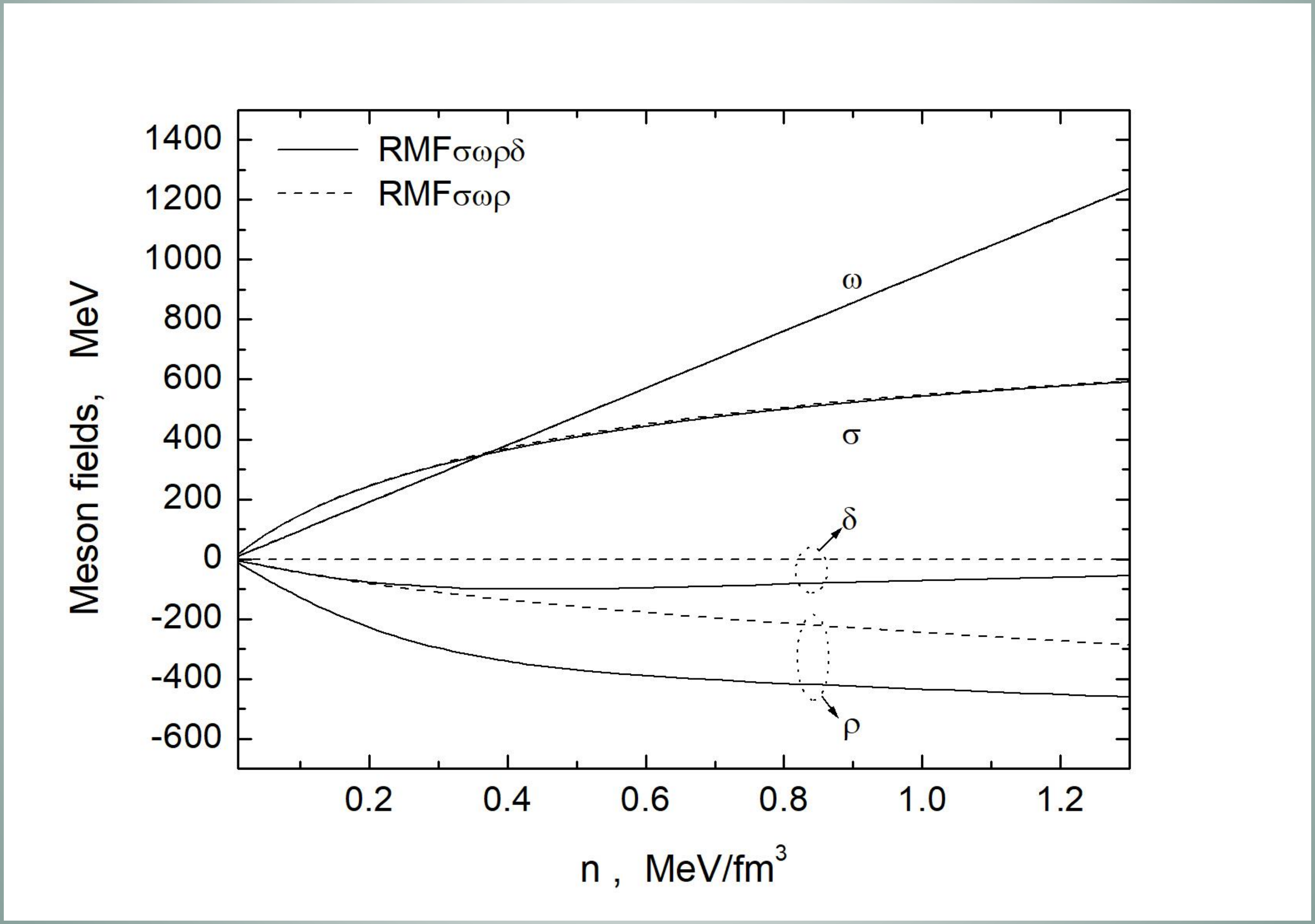
$f_0 = -16{,}3\ MeV$

$E_{sym}^{(0)} = 32{,}5\ MeV$

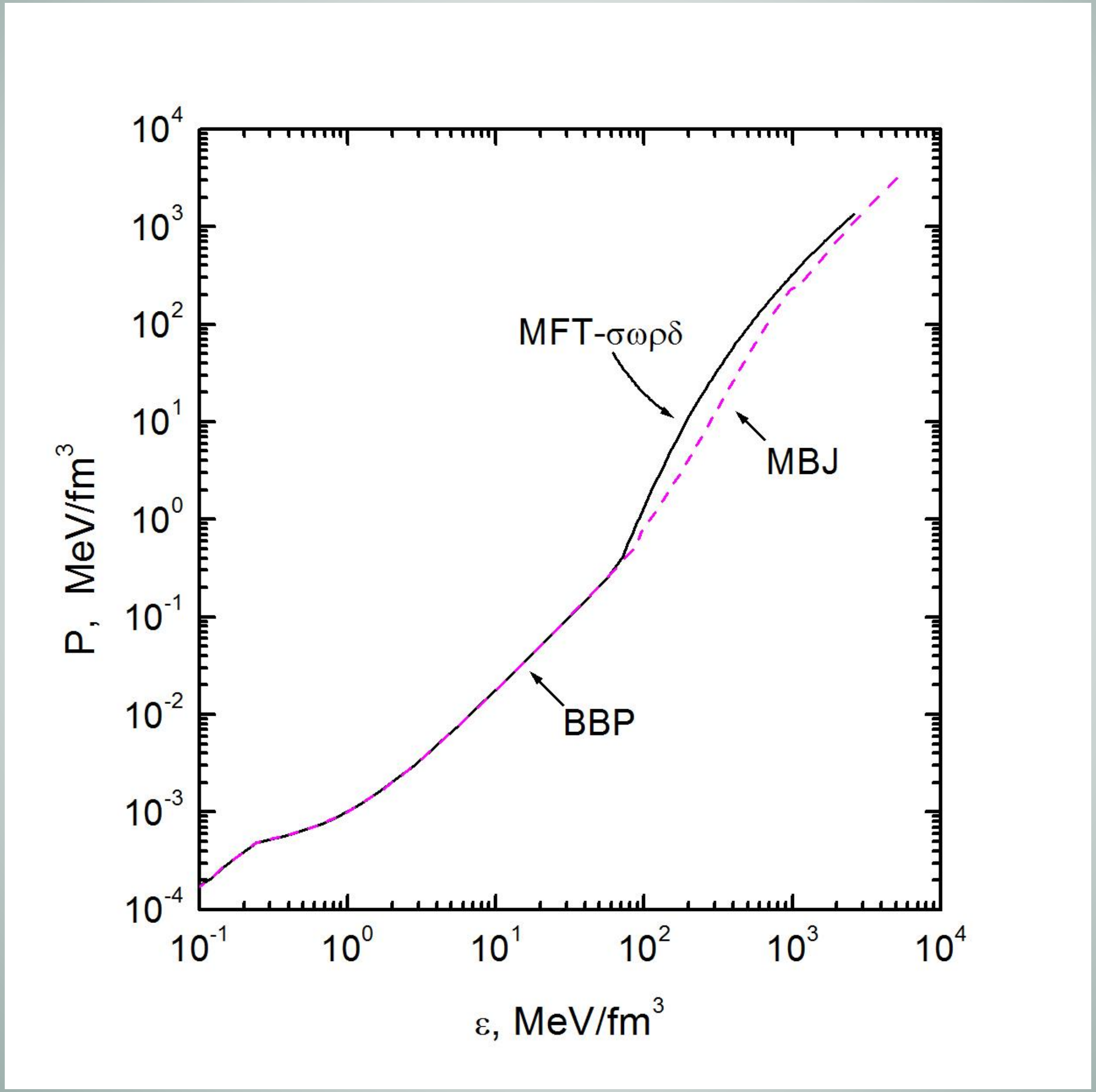
$\Longrightarrow$

Parameters	σωρ	σωρδ
a_σ , fm^2	9.154	9.154
a_ω , fm^2	4.828	4.828
a_δ , fm^2	0	2.5
a_ρ, fm^2	4.794	13.621
b , fm^{-1}	$1.654\ 10^{-2}$	$1.654\ 10^{-2}$
c	$1.319\ 10^{-2}$	$1.319\ 10^{-2}$

Meson mean-fields: $\sigma\omega\rho\delta$



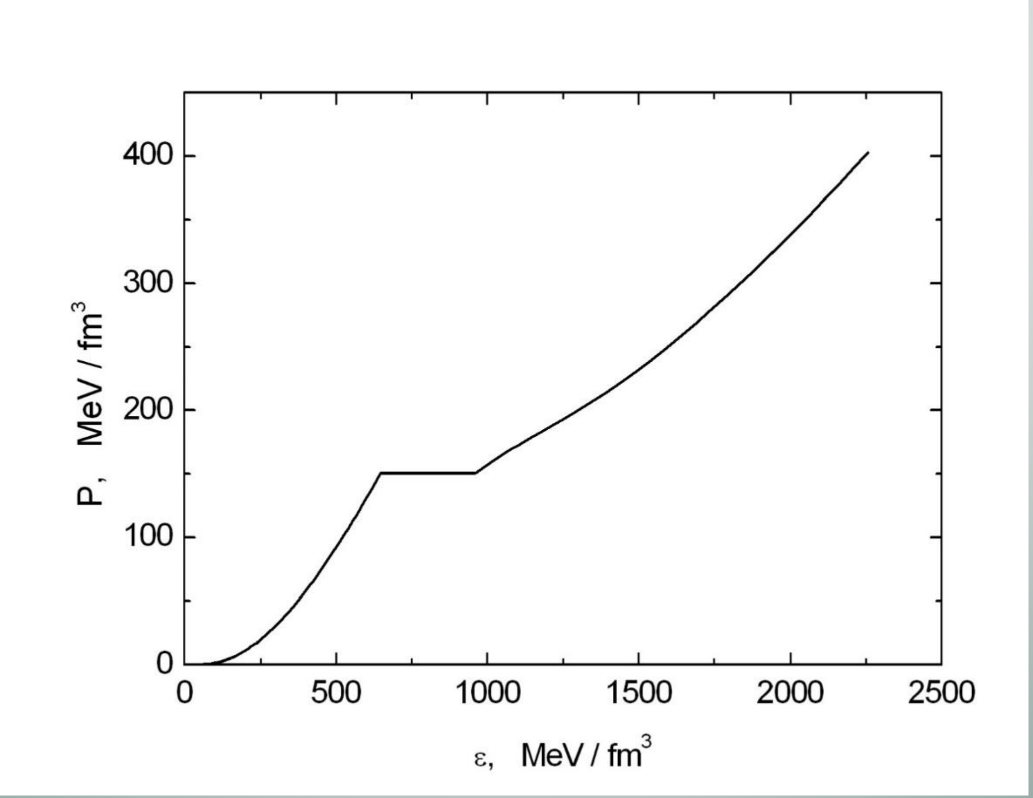
EOS of neutron star matter in hadronic phase

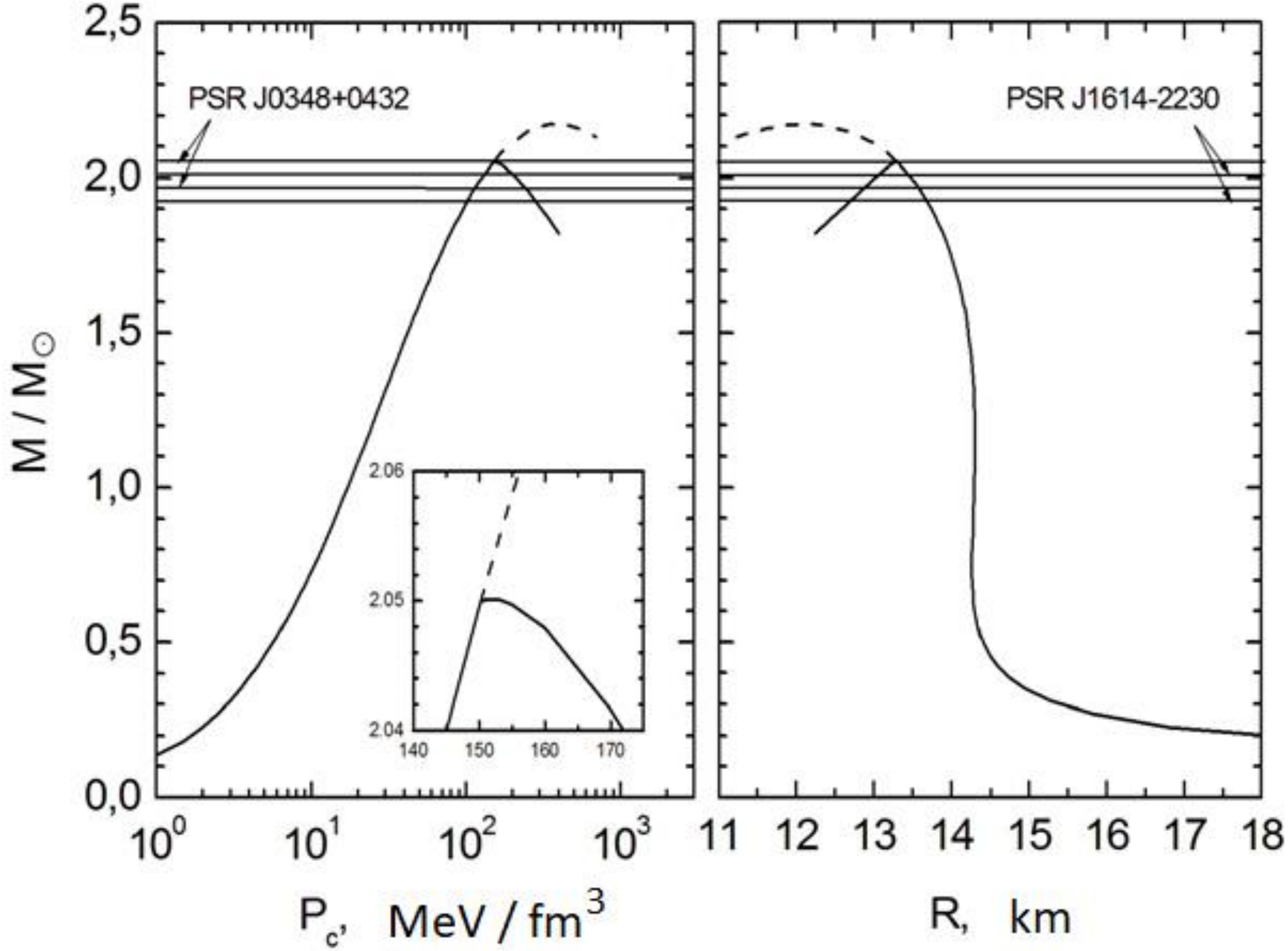


EOS of Neutron star matter with firs-order hadron-quark phase transition

Maxwell construction parameters

	P_0, MeV/fm³	n_B, fm⁻³	ε, MeV/fm³
Hadronic matter	150,5	$n_B^H = 0,584$	$\varepsilon^H = 647,1$
Quark matter	150,5	$n_B^Q = 0,814$	$\varepsilon^Q = 960,9$





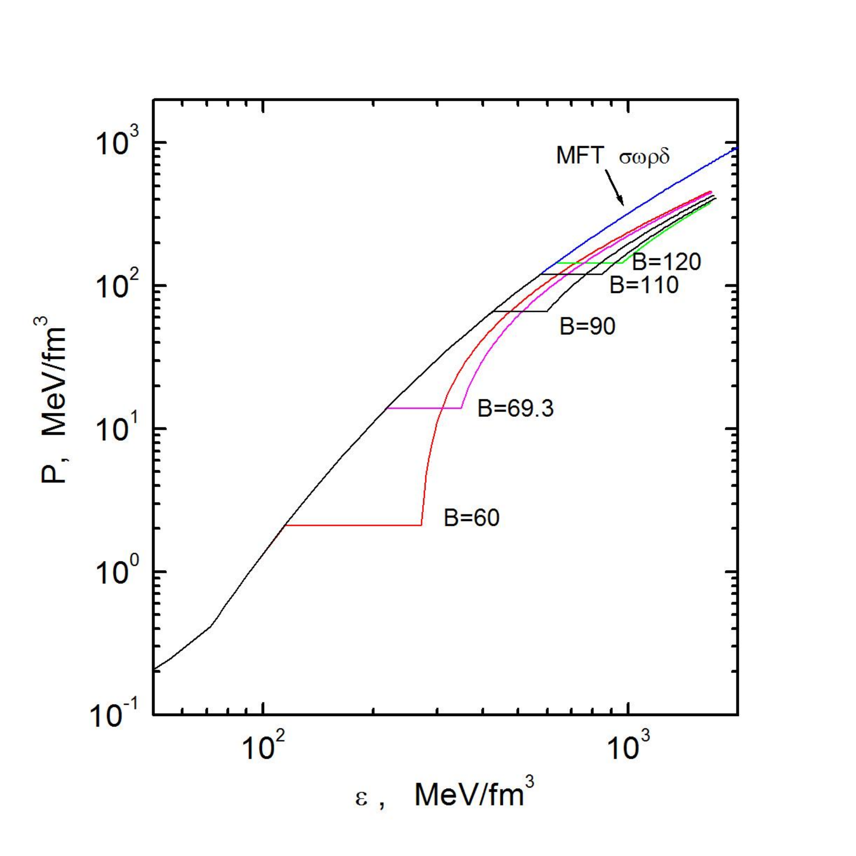
2,5
2,0
1,5
1,0
0,5
0,0
M / M$_\odot$
PSR J0348+0432
PSR J1614-2230
10^0
10^1
10^2
10^3
11 12 13 14 15 16 17 18
P_c, MeV / fm^3
R, km
2.06
2.05
2.04
140 150 160 170

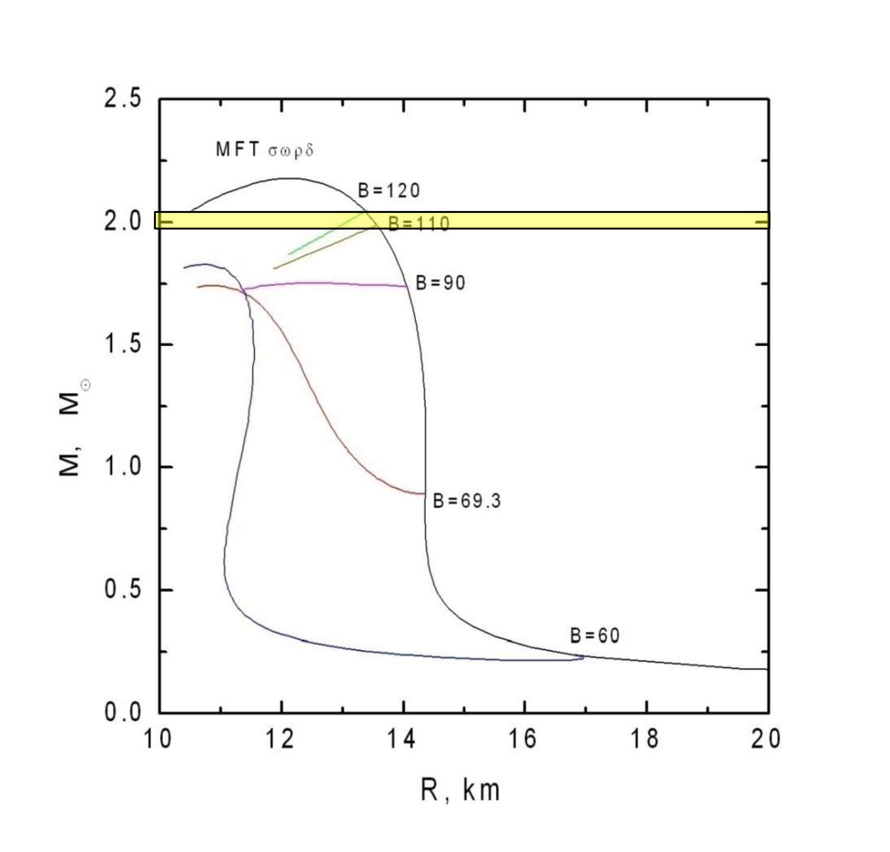
Parameters of some of the characteristic configurationsions

A - Star with mass 1.44 $M_\odot$ (PSR B1913+16)
B - Critical configuration with central pressure P_0
C - Hybrid star with maximal mass

Configurations	P_c MeV/fm³	ρ_c 10^{15} g/cm³	R km	R_{core} km	M $M_\odot$	M_{core} $10^{-4} M_\odot$	I 10^{45} g sm²	Z_s	Z_c
A	38	0.586	14.25	0	1.44	0	2	0.194	0.444
B	150.5	1.71	13.313	0	2.0506	0	2.860	0.354	0.996
C	152	1.73	13.309	0.64	2.0509	9.56	2.858	0.355	0.998

Neutron stars with quark core: Model RMF + MIT





Summary

- The integral parameters of neutron stars are investigated by taking into account the hadron-quark phase transition, as a result of which in the central part of the star is formed quark matter core. To describe the quark matter, the local three-flavor Nambu-Jona-Lasinio (NJL) model is used. The thermodynamic characteristics of the hadronic matter are calculated within the framework of the extended version of the relativistic mean field (RMF) model, in which the contribution of the scalar-isovector δ -meson effective field also taken into account.

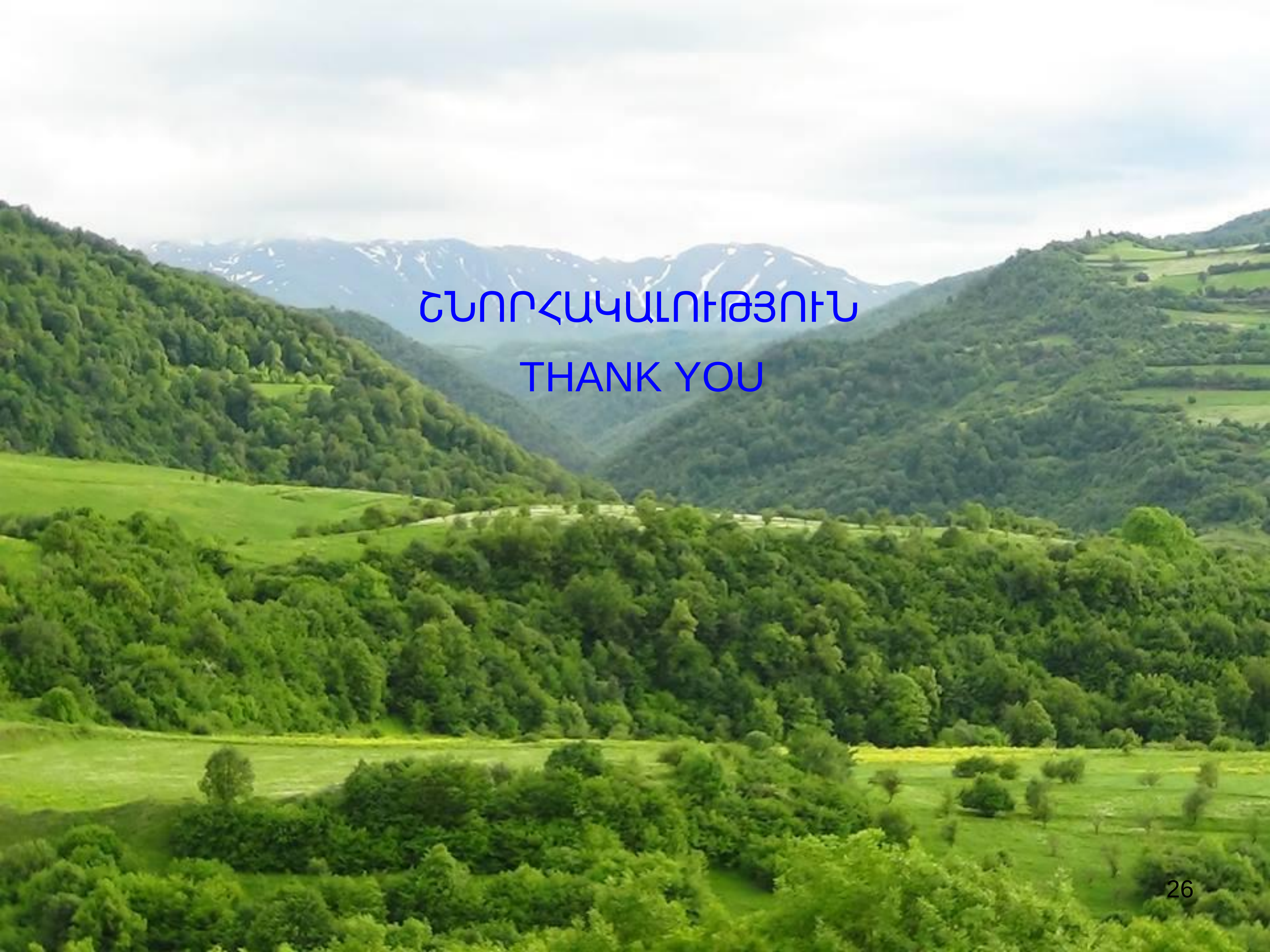
- It has been shown that Witten's hypothesis that the true ground state of matter at zero pressure is a β-equilibrium three-flavor quark matter does not hold in this case. As opposed to the MIT quark bag model, the version of the NJL model examined here, does not allow the existence of such exotic stellar objects as strange stars.

- The parameters of the phase transition have been calculated assuming that the phase transition proceeds in accordance with a Maxwell construction and it has been shown that in the variant of the model considered here, the phase transition between hadron and quark matter takes place at fairly high concentrations $n_B^H = 0.584\, fm^{-3}$, $n_B^Q = 0.814\, fm^{-3}$.

- Integral characteristics of compact stars, such as the gravitational mass M, radius R, quark core mass M_{core} and radius R_{core}, moment of inertia I, and gravitational red shifts Z_c for photons from the star's center and Z_s from its surface, have been calculated for different values of the central pressure. It has been shown that, for the EOS examined here, stable hybrid stars correspond to a fairly narrow range of central densities $\rho_c \in (1.71 \div 1.73]10^{15}$g/cm^3.

- The maximum gravitational mass of a stable hybrid star, $M_{max} = 2.05 M_{\odot}$, is attained for a central density of $\rho_m = 1.73]10^{15}$g/cm^3. Our result for the maximum mass of a hybrid star satisfies the criterion $M_{max} > 2\, M_{\odot}$, in agreement with observations that indicate the existence of pulsars with masses on the order of twice the sun's.

- The configuration with a maximum mass in our case has a quark core with a mass on the order of $M_{core} \approx 10^{-3}\, M_{\odot}$ and a radius on the order of $R_{core} \approx 0.6\, km$.



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THANK YOU