Electromagnetic vacuum densities around a cosmic string in de Sitter spacetime

Aram Saharian, Vardan Manukyan, Nvard Saharyan

Gourgen Sahakian Chair of Theoretical Physics, Yerevan State University, Yerevan, Armenia

Shirak State University, Gyumri, Armenia

The Modern Physics of Compact Stars and Relativistic Gravity,
17 September – 21 September 2019, Yerevan, Armenia

Outline

- Bulk geometry
- Electromagnetic field modes
- VEVs of the field squared
- VEVs of the energy-momentum tensor
- Energy flux
- Conclusion

Cosmic strings

- Cosmic strings are among the most popular topological defects formed by the symmetry breaking phase transitions in the early universe within the framework of the Kibble mechanism
- They are sources of a number of interesting physical effects that include the generation of gravitational waves, high-energy cosmic rays, and gamma ray bursts
- Among the other signatures are the gravitational lensing and the creation of small non-Gaussianities in the cosmic microwave background
- Cosmic strings produced in phase transitions before or during early stages of inflation are diluted by the expansion
- Formation of defects near or at the end of inflation can be triggered by several mechanisms
- Depending on the underlying microscopic model, there exist several kinds of cosmic strings: They can be either nontrivial field configurations or more fundamental objects in superstring theories

Problem

- We investigate combined effects of curved background and nontrivial topology on the properties of the electromagnetic vacuum
- Background geometry is given by de Sitter (dS) spacetime
- Nontrivial topology due to the presence of a straight cosmic string
- We assume that the electromagnetic field is in the Bunch-Davies vacuum state

Bulk geometry

We consider (D + 1) - dimensional locally dS background geometry described in cylindrical spatial coordinates $(r, \phi, \mathbf{z})$, $\mathbf{z} = (z^3, ..., z^D)$, by the interval

$$ds^2 = (\alpha/\tau)^2 [d\tau^2 - dr^2 - r^2 d\phi^2 - (d\mathbf{z})^2],$$

with the conformal time coordinate τ, $-\infty < \tau < 0$.

conformal time defined as $\tau = -\alpha e^{-t/\alpha}$

For the remaining coordinates we assume that

$$0 \leqslant r < \infty,\ 0 \leqslant \phi \leqslant \phi_0,\ -\infty < z^l < +\infty,\ l = 3, \ldots, D.$$

The cosmological constant Λ is expressed in terms of the parameter α by the relation $\Lambda = D(D-1)/(2\alpha^2)$.

Field Mode Functions

We have a single mode of the TE type (σ = 1) and D − 2 modes of the TM type (σ = 2, . . . ,D − 1).

For the polarization σ = 1 the cylindrical electromagnetic modes corresponding to the Bunch-Davies vacuum are presented as

$$A_{(\beta)\mu}(x) = c_\beta \eta^{D/2-1} H^{(1)}_{D/2-1}(\omega\eta) \left(0, \frac{iqm}{r}, -r\partial_r, 0, \ldots, 0\right) J_{q|m|}(\gamma r) e^{iqm\phi + i\mathbf{k}\cdot\mathbf{z}},$$

and for the polarizations σ = 2, . . . , D − 1 we have

$$A_{(\beta)\mu}(x) = c_\beta \omega \eta^{D/2-1} H^{(1)}_{D/2-1}(\omega\eta) \left(0, \epsilon_{\sigma l} + i\frac{\mathbf{k}\cdot\epsilon_\sigma}{\omega^2}\partial_l\right) J_{q|m|}(\gamma r) e^{iqm\phi + i\mathbf{k}\cdot\mathbf{z}},$$

where $\eta = |\tau|$, $q = 2\pi/\phi_0$, $H^{(1)}_{D/2-1}(x)$ and $J_m(x)$ are the Hankel and Bessel functions $\mathbf{k} = (k_3, \ldots, k_D)$ $k = |\mathbf{k}|$, $\omega^2 = \gamma^2 + k^2$,

Field Mode Functions

For the components of the polarization vector we have $\epsilon_{\sigma 1} = \epsilon_{\sigma 2} = 0,\ \sigma = 2, \ldots, D-1,$ and the relations

$$\sum_{l,n=3}^{D} \left(\omega^2 \delta_{nl} - k_l k_n\right) \epsilon_{\sigma l} \epsilon_{\sigma' n} \ = \ \gamma^2 \delta_{\sigma\sigma'},$$

$$\omega^2 \sum_{\sigma=2}^{D-1} \epsilon_{\sigma n} \epsilon_{\sigma l} - k_n k_l \ = \ \gamma^2 \delta_{nl},$$

The normalization coefficient C_β is determined from the standard orthonormalization condition for the vector potential:

$$|c_\beta|^2 = \frac{q}{4(2\pi\alpha)^{D-3}\gamma}$$

for all the polarizations $\sigma = 1, \ldots, D-1$.

Electromagnetic Field Squared

$$\langle E^2\rangle = -g^{00}g^{lm}\langle F_{0l}(x)F_{0m}(x)\rangle \qquad \langle B^2\rangle = \frac{1}{2}g^{lm}g^{np}\langle F_{ln}F_{mp}\rangle = \frac{1}{2}g^{lm}g^{np}\sum_\beta F_{(\beta)ln}F^*_{(\beta)mp}$$

$$\langle E^2\rangle = \langle E^2\rangle_{\mathrm{dS}} + \frac{8\alpha^{-D-1}}{(2\pi)^{D/2}}\left[\sum_{l=1}^{[q/2]} g_E(r/\eta, s_l) - \frac{q}{\pi}\sin(q\pi)\int_0^\infty dy \frac{g_E(r/\eta,\cosh y)}{\cosh(2qy)-\cos(q\pi)}\right]$$

$$\langle B^2\rangle = \langle B^2\rangle_{\mathrm{dS}} + \frac{8\alpha^{-D-1}}{(2\pi)^{D/2}}\left[\sum_{l=1}^{[q/2]} g_M(r/\eta, s_l) - \frac{q}{\pi}\sin(q\pi)\int_0^\infty dy \frac{g_M(r/\eta,\cosh y)}{\cosh(2qy)-\cos(q\pi)}\right]$$

renormalized VEVs in the absence of the cosmic string

parts are induced by the cosmic string (topological parts): $\langle E^2\rangle_t$ and $\langle B^2\rangle_t$

where $s_l = \sin(\pi l/q)$

$$g_E(x,y) = \int_0^\infty du\, u^{D/2}K_{D/2-2}(u)e^{u-2x^2y^2u}\left[2ux^2y^2\left(2y^2-D+1\right)+(D-1)\left(D/2-2y^2\right)\right]$$

$$\begin{aligned} g_M(x,y) \;&=\; \int_0^\infty du\, u^{D/2}K_{D/2-1}(u)e^{u-2x^2y^2u}\{(D-1)D/2 \\ &\quad -4(D-2)y^2+2x^2y^2u\left[2(D-2)y^2-D+1\right]\}\,. \end{aligned}$$

Electromagnetic Field Squared

For odd values of **D** $g_E(x,y)$ and $g_M(x,y)$ are expressed in terms of elementary functions. As a result, the corresponding VEVs are also presented in terms of elementary functions. In particular, for **D = 3** and **D = 5** one has

$$\langle B^2\rangle_{\mathrm{t}} = \langle E^2\rangle_{\mathrm{t}} = -\frac{\left(q^2-1\right)\left(q^2+11\right)}{180\pi(\alpha r/\eta)^4},\ D=3,$$

$$\langle E^2\rangle_{\mathrm{t}} = -\frac{\left(q^2-1\right)\left(q^4+22q^2+211\right)}{1890\pi^2\left(\alpha r/\eta\right)^6},\ D=5.$$

$$\langle B^2\rangle_{\mathrm{t}} = \frac{\left(3+r^2/\eta^2\right)c_4(q)-c_6(q)}{2\pi^2\left(\alpha r/\eta\right)^6},\ D=5.$$

where

$$c_4(q) = \frac{q^2-1}{90}\left(q^2+11\right),\ c_6(q) = \frac{q^2-1}{1890}(2q^4+23q^2+191).$$

Asymptotic Behavior of The Electric and Magnetic Field Squared

At large distances from the string $(r/\eta \gg 1)$

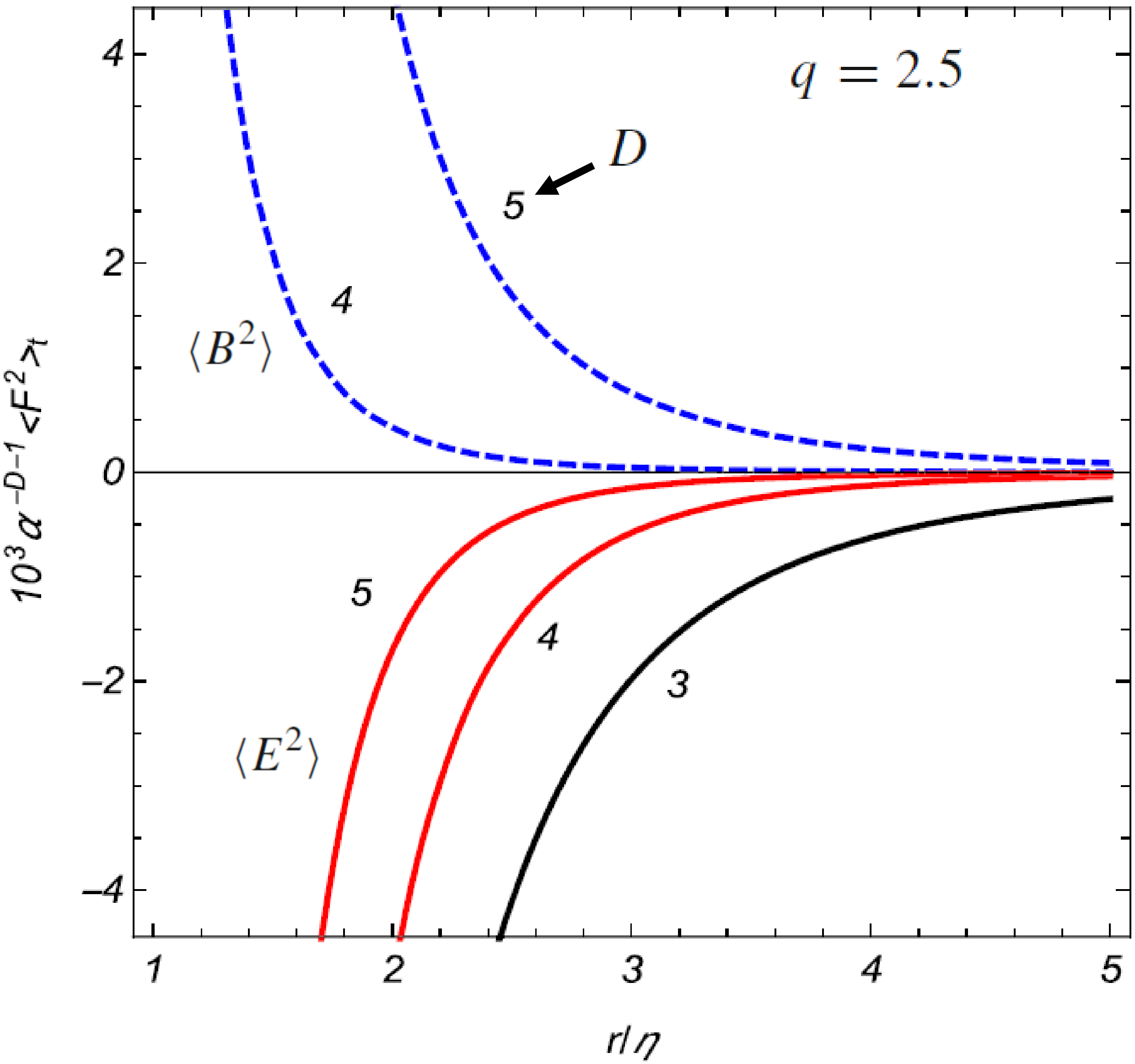
The topological part in the VEV of the **electric field** squared decays as

$(\eta/r)^4$ for D=3, $\ln(r/\eta)(\eta/r)^6$ for D=4, $(\eta/r)^6$ for D>4.

The topological part in the VEV of the **magnetic field** squared decays as

$(\eta/r)^4$ for D=3 and D>4, $(\eta/r)^6$ for D=4.

VEVs of Electric and Magnetic Field Squared



Classicalization of quantum fluctuations

- Among the most interesting features of the inflation is the transition from quantum-to-classical behavior of quantum fluctuations during the quasi-exponential expansion of the universe
- An important example of this type of effects is the classicalization of the vacuum fluctuations of the inflaton field which underlies the most popular models of generation of large-scale structure in the universe
- A similar effect of classicalization should take place for the electromagnetic fluctuations
- Quantum-to-classical transition of super-Hubble magnetic modes during inflation has been considered as a possible mechanism for the generation of galactic and galaxy cluster magnetic fields

Large scale magnetic fields around strings

- Presence of a cosmic string induces shifts in the VEVs of the squared electric and magnetic fields
- As a consequence of the quantum-to-classical transition of the corresponding fluctuations during the dS expansion, after inflation these shifts will be imprinted as classical stochastic fluctuations of the electric and magnetic fields surrounding the cosmic string
- In the post inflationary radiation dominated era the conductivity is high and the currents in the cosmic plasma eliminate the electric fields whereas the magnetic counterparts are frozen
- As a consequence, the cosmic strings will be surrounded by large-scale magnetic fields
- These fields would be among the distinctive features of the cosmic strings produced during the inflation and also of the corresponding inflationary models

Energy-momentum tensor

$$\langle T_\mu^\nu \rangle = \frac{1}{16\pi}\delta_\mu^\nu g^{\rho l} g^{\sigma m} \sum_\beta F_{(\beta)\rho\sigma} F^*_{(\beta)lm} - \frac{1}{4\pi} g^{\nu k} g^{\rho\sigma} \sum_\beta F_{(\beta)\mu\rho} F^*_{(\beta)k\sigma}$$

Diagonal components of the VEV

$$\langle T_i^i \rangle = \frac{q 2^{2-D} \eta^{D+2}}{\pi^{D/2+1}\Gamma(D/2-1)\alpha^{D+1}} \sum_{m=0}^{\infty}{}' \int_0^\infty dk k^{D-3} \int_0^\infty d\gamma \gamma$$
$$\times \sum_{l=0,1} t_l^{(i)}[k,\gamma,J_{qm}(\gamma r)] L_{D/2-1-l}(\sqrt{\gamma^2+k^2}\,\eta),$$

where

$$t_l^{(i)}(k,\gamma,J_{qm}(\gamma r)) = (a_l^{(i)}k^2 + b_l^{(i)}\gamma^2) Z_{qm}^{(i)}[J_{qm}(\gamma r)] + ((D-3)c_l^{(i)}k^2 + d_l^{(i)}\gamma^2) J_{qm}^2(\gamma r),$$

$$L_\nu(x) = K_\nu(xe^{-i\pi/2}) K_\nu(xe^{i\pi/2}),$$

$K_\nu(x)$ is a Macdonald function,

$$Z_{qm}^{(i)}[f(y)] = f'^2(y) + q^2 m^2 f^2(y)/y^2, \ i = 0, 3, \ldots, D,$$
$$Z_{qm}^{(i)}[f(y)] = f'^2(y) - q^2 m^2 f^2(y)/y^2, \ i = 1, 2.$$

Numerical coefficients

$$a_l^{(0)} = \begin{pmatrix} 2 & 2 \end{pmatrix},\ b_l^{(0)} = \begin{pmatrix} D-2 & 1 \end{pmatrix},\ c_l^{(0)} = \begin{pmatrix} 1 & 1 \end{pmatrix},\ d_l^{(0)} = \begin{pmatrix} 1 & D-2 \end{pmatrix},$$

$$a_l^{(1)} = \begin{pmatrix} 0 & 0 \end{pmatrix},\ b_l^{(1)} = \begin{pmatrix} 2-D & -1 \end{pmatrix},\ c_l^{(1)} = \begin{pmatrix} 1 & -1 \end{pmatrix},\ d_l^{(1)} = \begin{pmatrix} -1 & 2-D \end{pmatrix},$$

$$a_l^{(2)} = \begin{pmatrix} 0 & 0 \end{pmatrix},\ b_l^{(2)} = \begin{pmatrix} D-2 & 1 \end{pmatrix},\ c_l^{(2)} = \begin{pmatrix} 1 & -1 \end{pmatrix},\ d_l^{(2)} = \begin{pmatrix} -1 & 2-D \end{pmatrix},$$

$$a_l^{(p)} = \frac{2}{D-2}\begin{pmatrix} D-4 & 2-D \end{pmatrix},\ b_l^{(p)} = \begin{pmatrix} D-4 & -1 \end{pmatrix},$$

$$c_l^{(p)} = \frac{1}{D-2}\begin{pmatrix} D-6 & 4-D \end{pmatrix},\ d_l^{(p)} = \begin{pmatrix} 1 & 4-D \end{pmatrix}.$$

$$b_l^{(0)} = b_l^{(2)} = -b_l^{(1)},\ d_l^{(0)} = -d_l^{(1)} = -d_l^{(2)},$$

$$c_l^{(2)} = c_l^{(1)},\ a_l^{(2)} = a_l^{(1)} = 0,$$

Energy-momentum tensor

We will regularize the VEV by introducing the cutoff function $e^{-b\omega^2}$ with $b > 0$:

$$\langle T_i^i \rangle_{reg} = \frac{q 2^{2-D} \eta^{D+2}}{\pi^{D/2+1} \Gamma(D/2-1) \alpha^{D+1}} \sum_{m=0}^{\infty}{}' \int_0^\infty dk k^{D-3} \int_0^\infty d\gamma \gamma \boxed{e^{-b\omega^2}} \times \sum_{l=0,1} t_l^{(i)}[k, \gamma, J_{qm}(\gamma r)] L_{D/2-1-l}(\sqrt{\gamma^2 + k^2}\, \eta)$$

$$\langle T_i^i \rangle_{reg} = \frac{(\eta/r)^{D+2} q}{2^{D/2} \pi^{D/2+1} \alpha^{D+1}} \sum_{m=0}^{\infty}{}' \int_0^\infty \frac{dx}{x} w^{D/2+1} e^{2\eta^2/x} \sum_{l=0,1} K_{D/2-1-l}(2\eta^2/x) W_l^{(i)}(e^{-w} I_{qm}(w)),$$

where
$$W_l^{(i)} = b_l^{(i)} w \partial_w^2 + (a_l^{(i)}(D/2-1) + b_l^{(i)}(1+w) + d_l^{(i)} w)\partial_w + (D/2-1)(a_l^{(i)} + (D-3)c_l^{(i)}) + b_l^{(i)} + d_l^{(i)}, \quad \text{for } i = 0, 3, \ldots, D,$$

$$W_l^{(i)} = (d_l^{(i)} - b_l^{(i)}) w \partial_w + (D-3)(D/2-1)c_l^{(i)} + d_l^{(i)}, \quad \text{for } i = 1, 2,$$

$$w = \frac{r^2}{2(b + x/4)}.$$

Energy-momentum tensor

Next we use the formula

$$\sum_{m=0}^{\infty}{}' I_{qm}(w) = \frac{1}{q}\sum_{j=0}^{[q/2]} e^{w\cos(2j\pi/q)} - \frac{\sin(q\pi)}{2\pi}\int_0^\infty dy \frac{e^{-w\cosh y}}{\cosh(qy)-\cos(q\pi)} .$$

Regularized VEV in dS spacetime in the absence of cosmic string:

$q = 1 \quad j = 0$

$$\langle T_i^i \rangle_{reg}^{dS} = \frac{(\eta/r)^{D+2}}{(2\pi)^{D/2+1}\alpha^{D+1}}\int_0^\infty \frac{dx}{x} w^{D/2+1} e^{2\eta^2/x} \sum_{l=0,1} K_{D/2-1-l}(2\eta^2/x)\, W_{[dS]l}^{(i)},$$

where

$$W_{[dS]l}^{(i)} = (D/2-1)(a_l^{(i)} + (D-3)c_l^{(i)}) + b_l^{(i)} + d_l^{(i)},$$

for $i = 0, 3, \ldots, D$, and

$$W_{[dS]l}^{(i)} = (D-3)(D/2-1)c_l^{(i)} + d_l^{(i)},$$

for $i = 1, 2$.

Energy-momentum tensor

The **topological part** of the VEV

$$\langle T_i^i \rangle_t = \lim_{b \to 0} [\langle T_i^i \rangle_{reg} - \langle T_i^i \rangle_{reg}^{dS}]$$

$$\langle T_i^i \rangle_t = \frac{2\alpha^{-D-1}}{(2\pi)^{D/2+1}} \left[\sum_{j=1}^{[q/2]} t^{(i)}\left(\frac{r}{\eta}, s_j\right) - \frac{q}{\pi} \sin(q\pi) \int_0^\infty dz \frac{t^{(i)}(r/\eta, \cosh z)}{\cosh(2qz) - \cos(q\pi)} \right],$$

where

$$s_j = \sin(j\pi/q),$$

$$t^{(i)}(x,y) = \int_0^\infty du u^{D/2} e^{u-2ux^2y^2} \sum_{l=0,1} K_{D/2-1-l}(u) t_l^{(i)}(x,y,u),$$

$$t_l^{(i)}(x,y,u) = 4b_l^{(i)} ux^2y^4 - 2(b_l^{(i)} + d_l^{(i)})ux^2y^2 - (a_l^{(i)}(D-2) + 2b_l^{(i)})y^2 + (D/2-1)(a_l^{(i)} + (D-3)c_l^{(i)}) + b_l^{(i)} + d_l^{(i)}, \quad \text{for } i = 0, 3, \ldots, D,$$

$$t_l^{(i)}(x,y,u) = 2(b_l^{(i)} - d_l^{(i)})ux^2y^2 + (D/2-1)(D-3)c_l^{(i)} + d_l^{(i)}, \quad \text{for } i = 1, 2.$$

Energy-momentum tensor

For odd values of **D** $t^{(i)}(x,y)$ are expressed in terms of elementary functions. As a result, the corresponding VEVs are also presented in terms of elementary functions. In particular, for **D = 3** and **D = 5** one has

$$\langle T_i^i \rangle_t = -\frac{A^{(i)} c_4(q)}{8\pi^2 (\alpha r/\eta)^4}, \quad \text{for D=3}$$

$$\langle T_i^i \rangle_t = \frac{B^{(i)} c_4(q) + (r/\eta)^2 C^{(i)} c_4(q) + D^{(i)} c_6(q)}{16\pi^3 (\alpha r/\eta)^6}, \quad \text{for D=5.}$$

where

$$c_4(q) = \frac{(q^2-1)}{90}(q^2+11), \; c_6(q) = \frac{(q^2-1)}{1890}(2q^4+23q^2+191).$$

$$A^{(0)} = A^{(1)} = A^{(3)} = 1, \; A^{(2)} = -3, \; B^{(0)} = B^{(i)} = 2, \; B^{(1)} = B^{(2)} = 0,$$

$$C^{(0)} = -C^{(i)} = -C^{(1)} = 1, \; C^{(2)} = 5, \; D^{(0)} = D^{(i)} = D^{(1)} = -2, \; D^{(2)} = 10,$$

$$i = 3, 4, 5.$$

Asymptotic Behavior of The Energy-momentum tensor

At large distances from the string $(r/\eta \gg 1)$ the topological part in the VEV of the $\langle T_0^0 \rangle$ decays as

$\dfrac{\ln(r/\eta)}{(r/\eta)^6}$ for D=4, and $(\eta/r)^4$ for D>4.

$$\langle T_1^1 \rangle_t = \langle T_i^i \rangle_t = \frac{(D-3)(D-6)-2}{32(r/\eta)^4 \pi^{D/2+1} \alpha^{D+1}} c_4(q) \Gamma(D/2-1)$$

$$\langle T_2^2 \rangle_t = \frac{\Gamma(D/2-1)}{\pi^{D/2+1} \alpha^{D+1}} \frac{(D-1)D}{32(r/\eta)^4} c_4(q)$$

for $D \geqslant 4$.

In the case D = 3, the asymptotic coincides with the exact result.

Energy flux

$$\langle T_0^1 \rangle = -\frac{1}{4\pi} g^{\rho\rho} g^{11} \sum_\beta F_{(\beta)0\rho} F^*_{(\beta)1\rho}$$

$$\langle T_0^1 \rangle = \frac{i(D-3)}{\pi^3} \frac{\eta^{D+2}}{\alpha^4} \frac{q}{(2\pi\alpha)^{D-3}} \frac{\pi^{D/2-1}}{\Gamma(D/2-1)} \sum_{m=0}^{\infty}{}' \int_0^\infty dk k^{D-3} \int_0^\infty d\gamma \gamma \omega$$
$$\times K_{D/2-2}(\omega\eta e^{-i\pi/2}) K_{D/2-1}(\omega\eta e^{i\pi/2}) J_{qm}(\gamma r) \partial_r J_{qm}(\gamma r).$$

The off-diagonal component vanishes:

- **In the case D=3**
- **In absence of cosmic string**

Energy flux

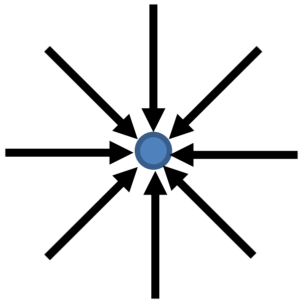
$$\langle T_0^1\rangle = 4(D-3)\frac{r}{\eta}\frac{\alpha^{-D-1}}{(2\pi)^{D/2+1}}\int_0^\infty du\, u^{D/2+1}e^u[K_{D/2-2}(u)-K_{D/2-1}(u)]$$

$$\times\left\{\sum_{j=1}^{[q/2]}\sin^2(j\pi/q)e^{-2(r/\eta)^2 u\sin^2(j\pi/q)}-\frac{q\sin(q\pi)}{\pi}\int_0^\infty dz\frac{\cosh^2(z)e^{-2(r/\eta)^2 u\cosh^2 z}}{\cosh(2qz)-\cos(q\pi)}\right\}$$

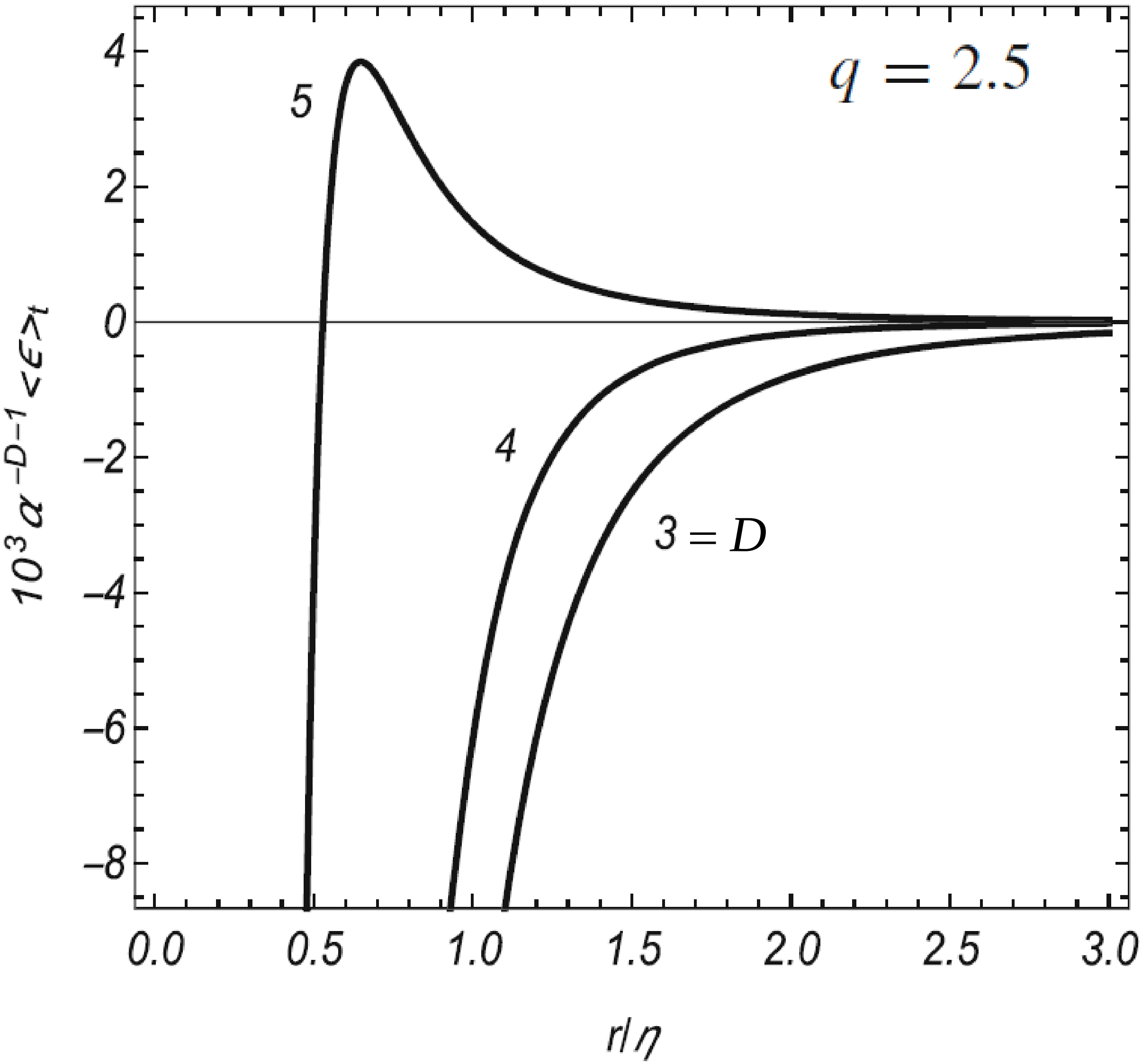
For odd values of **D** $\langle T_0^1\rangle$ is expressed in terms of elementary functions. In particular, for **D = 5** one has

$$\langle T_0^1\rangle_{D=5} = -\frac{c_4(q)}{\alpha^6(2\pi)^3(r/\eta)^5},$$

$$c_4(q) = \frac{q^2-1}{90}(q^2+11)$$



Topological part in the VEV of the energy density



Conclusions

- Topological parts in the VEVs depend on the time and the radial coordinate through the ratio r/η, which represents the proper distance from the string measured in units of the dS curvature radius
- Near the string, the dominant contribution to the VEVs comes from the fluctuations with short wavelengths and the VEVs coincide with those for the string in Minkowski bulk with the distance from the string replaced by the proper distance ar/η
- Influence of the gravitational field on the topological contributions in the VEVs is crucial at proper distances larger than the curvature radius of the background geometry
- Modifications of the electromagnetic field vacuum fluctuations during the dS expansion phase will be imprinted in large-scale stochastic perturbations of the electromagnetic fields surrounding the cosmic string in the post-inflationary radiation dominated era