General relativistic Mass twin stars and their astrophysical aspects

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- A brief introduction to the neutron star equation of state and its location within the QCD phase diagram.
- Multi polytrope Equation of state
- The compact star mass twins hypothesis.
- Effects of the Mixed phase (pasta phases)
- Rotation of compact Stars
- Maximum mass constraints.
Superdense objects – what is inside?
Compact Star Sequences
(M-R $\Leftrightarrow$ EoS)

- TOV Equations
- Equation of State (EoS)

\[ \frac{dp}{dr} = \frac{(\varepsilon + p/c^2)G(m + 4\pi r^3 p/c^2)}{r^2(1 - 2Gm/rc^2)} \]

\[ \frac{dm}{dr} = 4\pi r^2 \varepsilon \]

\[ p(\varepsilon) \]

arXiv: 1305.3510
Massive Neutron Stars

PSR J0740+6620: $M = 2.17^{+0.11, -0.10}\, M_{\odot}$, at 68.3% credibility
Support a CEP in QCD phase diagram with Astrophysics?

Crossover at finite $T$ (Lattice QCD) + First order at zero $T$ (Astrophysics)

$\Rightarrow$ Critical endpoint exists!

A. Ohnishi @ SQM-2015 in Dubna
Compact Star Mass Twins and the AHP scheme

• First order PT can lead to a stable branch of hybrid stars with quark matter cores which, depending on the size of the “latent heat” (jump in energy density), can even be disconnected from the hadronic one by an unstable branch → “third family of CS”.

• Measuring two disconnected populations of compact stars in the M-R diagram would represent the detection of a first order phase transition in compact star matter and thus the indirect proof for the existence of a critical endpoint (CEP) in the QCD phase diagram!

Piecewise polytrope EoS


\[ P_i(n) = \kappa_i n^{\Gamma_i} \]

\( i = 1 : n_1 \leq n \leq n_{12} \)

\( i = 2 : n_{12} \leq n \leq n_{23} \)

\( i = 3 : n \geq n_{23} \)

Here, 1st order PT in region 2:

\[ \Gamma_2 = 0 \text{ and } P_2 = \kappa_2 = P_{\text{crit}} \]

\[ P(n) = n^2 \frac{d(\varepsilon(n)/n)}{dn}, \]

\[ \varepsilon(n)/n = \int dn \frac{P(n)}{n^2} = \int dn \kappa n^{\Gamma - 2} = \frac{\kappa n^{\Gamma - 1}}{\Gamma - 1} + C, \]

\[ \mu(n) = \frac{P(n) + \varepsilon(n)}{n} = \frac{\kappa \Gamma}{\Gamma - 1} n^{\Gamma - 1} + m_0, \]

Seidov criterion for instability:

\[ \frac{\Delta \varepsilon}{\varepsilon_{\text{crit}}} \geq \frac{1}{2} + \frac{3}{\varepsilon_{\text{crit}}} \]

\[ n(\mu) = \left[ \frac{\mu - m_0}{\kappa \Gamma} \right]^{1/(\Gamma - 1)} \]

\[ P(\mu) = \kappa \left[ \frac{\mu - m_0}{\kappa \Gamma} \right]^{\Gamma/(\Gamma - 1)} \]

Maxwell construction:

\[ P_1(\mu_{\text{crit}}) = P_3(\mu_{\text{crit}}) = P_{\text{crit}} \]

\[ \mu_{\text{crit}} = \mu_1(n_{12}) = \mu_3(n_{23}) \]
Compact Star Twins

Alvarez-Castillo, Blaschke (2017)
High mass twins from multi-polytrope equations of state
Massive Neutron stars: Is there a concrete upper limit for the mass?
Upper limit on the Maximum Mass of static compact stars?

Universal relation for maximum mass increase upon rigid rotation

\[ \frac{M_{\text{crit}}}{M_{\text{TOV}}} = 1 + a_2 \left( \frac{j}{j_{\text{Kep}}} \right)^2 + a_4 \left( \frac{j}{j_{\text{Kep}}} \right)^4 \]

\[ M_{\text{max}} := M_{\text{crit}}(j = j_{\text{Kep}}) = (1 + a_2 + a_4) M_{\text{TOV}} \]

\[ \simeq (1.203 \pm 0.022) M_{\text{TOV}} \]

“universal” increase of maximum mass by 20% due to rigid rotation at maximum (critical) angular momentum

Breu & Rezzolla et al., MNRAS (2016)
ACB4 and ACB5 M-R: observations and constraints

Mixed phase effects
(Pasta phases)

High mass twin stars
(ACB4)

Low mass twin stars
(ACB5)
High mass twin stars (ACB4)

Low mass twin stars (ACB5)
Mass-Radius relations

ACB4

ACB5
Effect of Rotation and mixed phase

ACB4
Maxwell construction

ACB5
Critical value of Mixed-phase parameter

Full GR
Perturbative Approach ~Ω
Static = TOV
Effect of Rotation and mixed phase

ACB4

Maxwell construction

ACB5

Critical value of Mixed-phase parameter
Universal relation for rotating compact stars

\[ \alpha = \frac{M_{\text{rot}}}{M_{\text{TOV}}} \]

- ACB4, \( \Delta_p = 4\% \)
- ACB5, \( \Delta_p = 2\% \)

\( M_{\text{max}} > 2.07 \, M_{\odot} \)

\( M_{\text{max}} > 2.17 \, M_{\odot} \)
Conclusions

1) The existence of mass twins invariably signals the existence of a strong phase transition in compact star matter at supersaturation densities. “Strong transition”: energy density changes by about the value of the critical energy density in a sufficiently small region of pressures around the critical one of the Maxwell construction. This can also support the existence of a critical endpoint (CEP) of first order phase transitions in the QCD phase diagram.

2) The mixed phase construction mimics the pasta phase in accordance with a full pasta calculation. The result is a “broadening” of the phase transition over a certain pressure region, similar to the Gibbs construction in matter with more than one conserved charge and global charge conservation [Glendenning (1992)]. The construction makes the approach more realistic and Has advantages for numerical treatment of hybrid stars in general relativity.

3) The conjecture of an upper limit on the maximum mass of nonrotating compact stars derived from GW170817 is revisited. We find a criterion for the minimal central energy density in the maximum mass configuration that would correspond to the core of GW170817. The equation of state at high densities must be effectively soft, either as a relatively soft hadronic one or a hybrid one with a strong phase transition. The NICER radius measurement could be decisive.