Scalar Field and Quintessence in Gauge Symmetry Group $SU(2) \otimes U(1)$

A. S. Gevorkyan[1,2]

[1] Institute for Informatics and Automation Problems, NAS of Armenia, E-mail: g_ashot@sci.am
[2] Institute of Chemical Physics, NAS of Armenia

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Content

Justification of the research problem

The problem of quintessence, as the basis of everything, is the most discussed idea of all times, from the ancient to the present times. History of this problem very long from Aristotle and Plato to Ratra and Peebles (1988) and Steinhardt et al. (1999).

In cosmology, quintessence is a hypothetical form of dark energy or scalar field, postulated as an explanation of observation, namely, the expansion of the universe with acceleration.

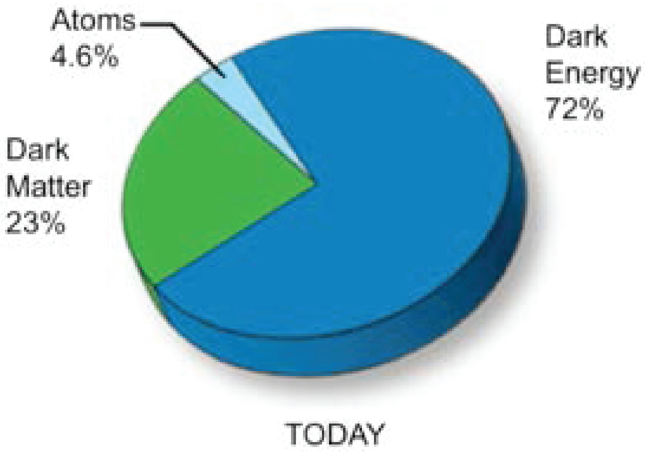
Since quintessence – dark energy or, more precisely, a scalar field, determines main properties of observed matter in universe its accurate quantitative investigation is an important problem of modern theoretical and mathematical physics.

It is well known that perturbation theory for QFT is destroyed at low energies, and field operators can have nonzero values of vacuum expectation, called condensates.

In this regard, the mathematical problem consists in rigorous proof of the existence of a scalar field.

A. S. Gevorkyan, Quantum Vacuum: The Structure of Empty Space –Time and Quintessence with Gauge Symmetry Group $SU(2) \otimes U(1)$, *Particles 2019*, **2**, *281–308*; doi:10.3390/particles2020019

Energy-mass distribution in the universe today



Standard model for free vacuum fields

The Yang-Mills theory is a special example of gauge field theory with a non-Abelian gauge symmetry group, whose Lagrangian for the case of free vacuum fields (VF) has the following form:

$$\mathcal{L}_{gf} = -\frac{1}{2}\mathrm{Tr}(\mathcal{F}^2) = -\frac{1}{4}\mathcal{F}_a^{\mu\nu}\mathcal{F}^a_{\mu\nu}, \tag{1}$$

where $\mathcal{F}$ is the 2-form of the Yang-Mills field strength, which is represented by the tensor potential A^a_μ as follows:

$$\mathcal{F}^a_{\mu\nu} = \partial_\mu \mathcal{A}^a_\nu - \partial_\nu \mathcal{A}^a_\mu + \mathfrak{g}\mathfrak{f}^{abc}\mathcal{A}^b_\mu \mathcal{A}^c_\nu, \qquad \mu,\nu = 0,1,2,3. \tag{2}$$

Note that $\partial_\mu = (ic_0^{-1}\partial_t,\, \partial_x,\, \partial_y,\, \partial_z) = (ic_0^{-1}\partial_t, \boldsymbol{\nabla})$ denotes the covariant derivative in the four-dimensional Minkowski space-time, which in Galilean coordinates is reduced to the usual partial derivative. In addition, $\mathfrak{f}^{abc} = \mathfrak{f}_{abc}$ are called structural constants of the group (Lie algebra), $\mathfrak{g}$ is the self-action constant and for the group $SU(N)$, the number of isospins generators varies $a, b, c = [1, N^2 - 1]$.

Yang-Mills equations and its extension

From the Lagrangian (1) one can derive the equations of motion for the classical free Yang-Mills fields:

$$\partial^{\mu}\mathcal{F}^{a}_{\mu\nu} + \mathfrak{g}\mathfrak{f}^{abc}\mathcal{A}^{\mu b}\mathcal{F}^{c}_{\mu\nu} = 0, \tag{3}$$

where the second term characterizing self-action plays a key role in the representation. In the case of a small coupling constants $\mathfrak{g} < 1$, the perturbation theory is applicable for solving these equations. However, as shown by numerous studies, in this case, massless vector bosons with spin 1 are not formed. For the case of $\mathfrak{g} > 1$, there is no idea how to solve these equations?

Recall that this is one of the millennium problem by classification of the Clay Mathematics Institute.

Our proposal is as follows: $\Rrightarrow$ since in QV at small scales of space-time, continuous random multi-scale fluctuations are observed, the system (3) should be considered as a system of SDE for complex probabilistic processes $\mathcal{F}^{a}_{\mu\nu}$.

Equation of vector fields

We consider the case, when space-time is described by the Lorentz metric $\chi_{\mu\nu} = \mathrm{diag}(+\ -\ -\ -)$, the self-action constant $\mathfrak{g} = 0$ and, the fields satisfy the symmetry group $SU(2) \otimes U(1)$. In this case, obviously, there are three isospins $a = 1, 2, 3.$ The latter means that we consider the unified electroweak interaction within the Abelian gauge group, but using stochastic field equations.

We determine the covariant antisymmetric tensor of the quantum vacuum fields (QVF) in the form:

$$\mathcal{F}^{a}_{\mu\nu} = \begin{pmatrix} 0 & \psi_x^{\pm} & \psi_y^{\pm} & \psi_z^{\pm} \\ \psi_x^{\pm} & 0 & \pm\psi_z^{\pm} & \mp\psi_y^{\pm} \\ \psi_y^{\pm} & \mp\psi_z^{\pm} & 0 & \mp\psi_x^{\pm} \\ \psi_z^{\pm} & \pm\psi_y^{\pm} & \mp\psi_x^{\pm} & 0 \end{pmatrix}, \qquad (4)$$

where $\psi_\sigma^{\pm}$ denotes the component of the wave function. Substituting (4) into (3), we can obtain the following vector equation, which we will call the Langevin-Weyl equation:

$$\Downarrow$$

$$\dot{\psi}^{\pm}\big(\mathbf{r},t;\mathbf{f}(t)\big) \mp c\big(\mathbf{S}\cdot\boldsymbol{\nabla}\big)\psi^{\pm}\big(\mathbf{r},t;\mathbf{f}(t)\big)=0, \quad \dot{\chi}=\partial_t\chi, \tag{5}$$

where $\mathbf{S}=(S_x,S_y,S_z)$ denotes the set of matrix:

$$S_x=\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix},\ S_y=\begin{pmatrix} 0 & 0 & i \\ 0 & 0 & 0 \\ -i & 0 & 0 \end{pmatrix},\ S_z=\begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}.$$

In addition, $\mathbf{f}(t)$ is a random function characterizing QV fluctuations, c is VF propagation speed, which can be different from the speed of light c_0, the symbol $\psi^{\pm}\big(\mathbf{r},t;\mathbf{f}(t)\big)$ denotes a complex probabilistic process, which can be represented as a three-component vector in the Hilbert space:

$$\psi^{\pm}(\mathbf{r},t;\mathbf{f}(t))=\left[\begin{array}{c} \psi^{\pm}_x(\mathbf{r},t;\mathbf{f}(t)) \\ \psi^{\pm}_y(\mathbf{r},t;\mathbf{f}(t)) \\ \psi^{\pm}_z(\mathbf{r},t;\mathbf{f}(t)) \end{array}\right]. \tag{6}$$

In the case when $\mathbf{f}\equiv 0$ the equation (5) passes to the well-known Weyl-type equation for light, taking into account the spin of light.

Quantization of stochastic vacuum fields

Multiscale random-fluctuations of VF can be described by relaxation times $\{\tau\} = (\tau_0, \tau_1, ..)$ and fluctuations powers $\{\varepsilon\} = (\varepsilon_0, \varepsilon_1, ...)$.

Theorem. If QVF obeys the Langevin–Weyl SDE (5), then for the symmetry group $SU(2) \otimes U(1)$ on the main relaxation scale $(\tau_0, \varepsilon_0^a)$, in the limit of statistical equilibrium, a massless Bose particle with spin 1 is formed as 2D topological structure in 3D space.

Obviously, in a localized quantum state, the four-dimensional interval of the propagated signal s should be zero:

$$s^2 = c^2 t^2 - r^2 = 0, \qquad r^2 = x^2 + y^2 + z^2. \tag{7}$$

Using the equations (7), it is easy to calculate the following derivatives:

$$c_{,t} = -\frac{c^2}{r}, \quad c_{,x} = \frac{cx}{r^2}, \quad c_{,y} = \frac{cy}{r^2}, \quad c_{,z} = \frac{cz}{r^2}, \tag{8}$$

where $c_{,\sigma} = \partial_\sigma c$.

Second-order equations for quantum vacuum fields

Taking into account (5), (6) and (8), we obtain the following second-order PDF for QVF:

$$\begin{aligned}
\Box\psi_x^+ &= \frac{c_{,y}}{c}\left(\partial_x\psi_y^+ - \partial_y\psi_x^+\right) - \frac{c_{,z}}{c}\left(\partial_z\psi_x^+ - \partial_x\psi_z^+\right) - \frac{c_{,t}}{c^3}\dot{\psi}_x^+,\\
\Box\psi_y^+ &= \frac{c_{,z}}{c}\left(\partial_y\psi_z^+ - \partial_z\psi_y^+\right) - \frac{c_{,x}}{c}\left(\partial_x\psi_y^+ - \partial_y\psi_x^+\right) - \frac{c_{,t}}{c^3}\dot{\psi}_y,\\
\Box\psi_z^+ &= \frac{c_{,x}}{c}\left(\partial_z\psi_x^+ - \partial_x\psi_z^+\right) - \frac{c_{,y}}{c}\left(\partial_y\psi_z^+ - \partial_z\psi_y^+\right) - \frac{c_{,t}}{c^3}\dot{\psi}_z.
\end{aligned} \tag{9}$$

The following additional conditions are imposed on the components of the derived fields:

$$\begin{aligned}
\left(c_{,z} - c_{,y}\right)\dot{\psi}_x &= c_{,z}\dot{\psi}_y^+ - c_{,y}\dot{\psi}_z^+,\\
\left(c_{,x} - c_{,z}\right)\dot{\psi}_y &= c_{,x}\dot{\psi}_z^+ - c_{,z}\dot{\psi}_x^+,\\
\left(c_{,y} - c_{,x}\right)\dot{\psi}_z &= c_{,y}\dot{\psi}_x^+ - c_{,x}\dot{\psi}_y^+.
\end{aligned} \tag{10}$$

Note that this allows us to write the system of equations in a canonical form, which looks quite natural.

The canonical form of fields equations

Using the conditions (10), the system of equations (9) can be easily leaded to the canonical form:

$$\Big\{\Box + \big[i(c_{,z} - c_{,y}) + c_{,t}c^{-1}\big]c^{-2}\partial_t\Big\}\psi_x^+ = 0,$$
$$\Big\{\Box + \big[i(c_{,x} - c_{,z}) + c_{,t}c^{-1}\big]c^{-2}\partial_t\Big\}\psi_y^+ = 0,$$
$$\Big\{\Box + \big[i(c_{,y} - c_{,x}) + c_{,t}c^{-1}\big]c^{-2}\partial_t\Big\}\psi_z^+ = 0. \qquad (11)$$

For further investigations, it is convenient to represent the wave function component in the form:

$$\psi_\sigma^+\big(\mathbf{r}, t; f_\sigma(t)\big) = \exp\Big\{\int_{-\infty}^{t}\zeta_\sigma(t')dt'\Big\}\phi_\sigma^+(\mathbf{r}), \qquad (12)$$

where $\zeta_\sigma(t)$ denotes the random function, and $f_\sigma(t)$ is the corresponding projection of a random vector force $\mathbf{f}(t)$.

$$\Downarrow$$

Substituting (12) into (11) and taking into account (8), we get the following system of differential equations:

$$\Big\{\triangle - \Big[\Big(\frac{\xi_x(t)}{c}\Big)^2 + \frac{r - i(z-y)}{cr^2}\zeta_x(t)\Big]\Big\}\phi_x^+(\mathbf{r}) = 0,$$
$$\Big\{\triangle - \Big[\Big(\frac{\xi_y(t)}{c}\Big)^2 + \frac{r - i(x-z)}{cr^2}\zeta_y(t)\Big]\Big\}\phi_y^+(\mathbf{r}) = 0,$$
$$\Big\{\triangle - \Big[\Big(\frac{\xi_z(t)}{c}\Big)^2 + \frac{r - i(y-x)}{cr^2}\zeta_z(t)\Big]\Big\}\phi_z^+(\mathbf{r}) = 0, \qquad (13)$$

where the following designations are made:

$$\xi_\sigma^2(t) = \dot{\zeta}_\sigma(t) + \zeta_\sigma^2(t), \qquad \dot{\zeta}_\sigma = \partial_t\zeta_\sigma, \quad \sigma = x, y, z.$$

Averaging the equations (13) on the main relaxation time scale τ_0, we obtain the following system of stationary differential equations:

Equations of vector fields after relaxation

$$\left\{\triangle - \left[\left(\frac{\omega_x}{c}\right)^2 + \frac{r - i(z-y)}{cr^2}\varrho(\omega_x)\right]\right\}\phi_x^+(\mathbf{r}) = 0,$$
$$\left\{\triangle - \left[\left(\frac{\omega_y}{c}\right)^2 + \frac{r - i(x-z)}{cr^2}\varrho(\omega_y)\right]\right\}\phi_y^+(\mathbf{r}) = 0,$$
$$\left\{\triangle - \left[\left(\frac{\omega_z}{c}\right)^2 + \frac{r - i(y-x)}{cr^2}\varrho(\omega_z)\right]\right\}\phi_z^+(\mathbf{r}) = 0, \qquad (14)$$

where ω_σ and $\varrho(\omega_\sigma)$ are regular parameters of the problem, which are defined as follows:

$$\omega_\sigma^2 = \langle \xi_\sigma^2(t) \rangle = \langle \dot{\zeta}_\sigma(t) + \zeta_\sigma^2(t) \rangle, \qquad \varrho(\omega_\sigma) = \langle \zeta_\sigma(t) \rangle. \qquad (15)$$

In the (15) the bracket $\langle \ldots \rangle$ denotes the averaging by τ_0.

Now the main question to which we must answer the following: is it possible the emergence of statistical equilibrium in the system under consideration, which can lead to the stable distribution of the parameter $\varrho(\omega_\sigma)$?

Distribution density of excitations' mod

Using the first relation in (15), we can get the following Langevin type equation:

$$\dot{\zeta}_\sigma = -(\zeta_\sigma^2 - \omega_\sigma^2) + U_\sigma(t), \qquad U_\sigma(t) = U_{0\sigma} + f_\sigma(t), \tag{16}$$

where $U_{0\sigma} = \langle U_\sigma(t) \rangle < 0$ is an unknown constant. As for function $f_\sigma(t)$, it denotes a random force that satisfies the white noise conditions:

$$\langle f_\sigma(t) \rangle = 0, \qquad \langle f_\sigma(t) f_\sigma(t') \rangle = \varepsilon_{0\sigma}^a \delta(t - t'). \tag{17}$$

Note that the set of constants $\boldsymbol{\varepsilon}_0^a = (\varepsilon_{0x}^a, \varepsilon_{0y}^a, \varepsilon_{0z}^a)$ denote fluctuations powers of isospin a along different axes. It is natural to assume that each isospin consists from terms $\boldsymbol{\varepsilon}_0^a = (\varepsilon_{0x}^a, \varepsilon_{0y}^a, \varepsilon_{0z}^a)$, in addition, $\boldsymbol{\varepsilon}_0^a \neq \boldsymbol{\varepsilon}_0^b$. Note that in $SU(2) \times U(1)$ gauge group there are three W bosons of weak isospin from $SU(2)$, namely $(W_1, W_2$ and W_3, and the B boson of weak hypercharge from $U(1)$, respectively, all of which are massless. These bosons obviously determined by a set of constants $\boldsymbol{\varepsilon}_0 = (\varepsilon_0^1, \varepsilon_0^2, \varepsilon_0^3)$.

$$\Downarrow$$

Using SDE (16) and relations (17), as well as assuming that $U_{0\sigma} = -2\,\omega_\sigma^2$, one can obtain the equation for the distribution:

$$\frac{\partial \mathcal{P}^0}{\partial t} = \left\{ \frac{\partial}{\partial \zeta}\left(\zeta^2 + \omega^2\right) + \frac{\varepsilon_0}{2}\frac{\partial^2}{\partial \zeta^2} \right\} \mathcal{P}^0. \qquad (18)$$

Recall that in (18) and below, to simplify writing, we will omit both the isospin index a and the coordinate index σ. Solving the equation (18):

$$\mathcal{P}^0(\bar{\zeta};\bar{\omega}) = 2\varepsilon_0^{-1}\mathcal{J}(\bar{\omega})e^{-2\Phi(\bar{\zeta})}\int_{-\infty}^{\bar{\zeta}} e^{2\Phi(\bar{\zeta}')}d\bar{\zeta}', \qquad (19)$$

where $\bar{\zeta} = \zeta/\varepsilon_0^{1/3}$ and $\Phi(\bar{\zeta}) = (\bar{\zeta}^3 + 3\bar{\omega}^2\bar{\zeta})/3$, in addition, from the condition for normalizing the distribution (19) to unity:

$$\mathcal{J}^{-1}(\bar{\omega}) = \sqrt{\pi}\left(\frac{2}{\varepsilon_0}\right)^{1/3}\int_0^{\infty} \exp\Big[-\frac{x^3}{6} - 2\,\bar{\omega}^2 x\Big]\frac{dx}{\sqrt{x}},$$

where $\bar{\omega} = \omega/\varepsilon_0^{1/3}$ is the dimensionless frequency.

The wave function of a massless particle with spin 1

Since the equations in the system (14) are independent, we can investigate them separately. For definiteness, consider the first equation of the system (14), which describes x component of QVF. Representing the wave function in the form:

$$\phi_x^+(\mathbf{r}) = \phi_x^{+(r)}(\mathbf{r}) + i\phi_x^{+(i)}(\mathbf{r}), \tag{20}$$

from the first equation of (14), we can get two equations:

$$\begin{aligned} &\left\{\triangle - \left[\left(\frac{\omega}{c}\right)^2 + \frac{\lambda}{r}\right]\right\}\phi_x^{+(r)}(\mathbf{r}) - \lambda\frac{z-y}{r^2}\phi_x^{+(i)}(\mathbf{r}) = 0, \\ &\left\{\triangle - \left[\left(\frac{\omega}{c}\right)^2 + \frac{\lambda}{r}\right]\right\}\phi_x^{+(i)}(\mathbf{r}) + \lambda\frac{z-y}{r^2}\phi_x^{+(r)}(\mathbf{r}) = 0, \end{aligned} \tag{21}$$

where the parameter:

$$\lambda = -\varrho(\bar{\omega})/c < 0. \tag{22}$$

$$\Downarrow$$

It is easy to show that the equations (21) are invariant with respect to permutations:

$$\phi_x^{+(r)}(\mathbf{r}) \mapsto \phi_x^{+(i)}(\mathbf{r}), \qquad \phi_x^{+(i)}(\mathbf{r}) \mapsto -\phi_x^{+(r)}(\mathbf{r}).$$

Using this symmetry from (21) we can get two independent equations of the form:

$$\left\{\triangle + \left[-\left(\frac{\omega}{c}\right)^2 + |\lambda|\frac{r-(y-z)}{r^2}\right]\right\}\phi_x^{+(r)}(\mathbf{r}) = 0,$$
$$\left\{\triangle + \left[-\left(\frac{\omega}{c}\right)^2 + |\lambda|\frac{r+(y-z)}{r^2}\right]\right\}\phi_x^{+(i)}(\mathbf{r}) = 0. \tag{23}$$

Recall that the real term of the wave function $\phi_x^{+(r)}(\mathbf{r})$ describes the electrical field, while the imaginary term $\phi_x^{+(i)}(\mathbf{r})$ corresponds to the magnetic field. Now the main question is that: do these equations have quantized solutions that can be interpreted as the solution of localized particles.

Quantization conditions

Consider the solution $\phi_x^{+(r)}(\mathbf{r})$ on the plane:

$$r - y + z = \mu r, \tag{24}$$

where parameter μ varies within $\mu \in [1-\sqrt{2}, 1+\sqrt{2}]$. Given (24), the first equation in (23) can be written as:

$$\left\{\triangle + \left[-\left(\frac{\omega}{c}\right)^2 + \frac{|\lambda|\mu}{r}\right]\right\}\phi_x^{+(r)}(\mathbf{r}) = 0. \tag{25}$$

Representing the wave function in the form:

$$\phi_x^{+(r)}(\mathbf{r}) = \Lambda(r)Y(\theta,\varphi), \qquad r = |\mathbf{r}|, \tag{26}$$

where $Y(\theta,\varphi)$ denotes spherical function, for the function $\Lambda(r)$ with the given parameter $\mu > 0$ we get the following equation for a hydrogen-like atom:

$$\frac{d^2\Lambda}{d\rho^2} + \frac{2}{\rho}\frac{d\Lambda}{d\rho} + \left[-\beta^2 + \frac{2}{\rho} - \frac{l(l+1)}{\rho^2}\right]\Lambda = 0, \quad \rho = r/a_p, \tag{27}$$

where $a_p = 2/(|\lambda|\mu_0)$ denotes the particle size and $l = 0, 1, \ldots$.

$$\Downarrow$$

Note that the equation (27) has discrete solutions:

$$\Lambda_{nl}(r) = \frac{(b)^{3/2}(br)^l e^{-br/2}}{\sqrt{2n(n-l-1)!(n+l)!}} L_{n-l-1}^{2l+1}(br), \quad b = \frac{2}{na_p}, \quad (28)$$

($L_{n-l-1}^{2l+1}(br)$ are associated Laguerre polynomials) if the following condition holds:

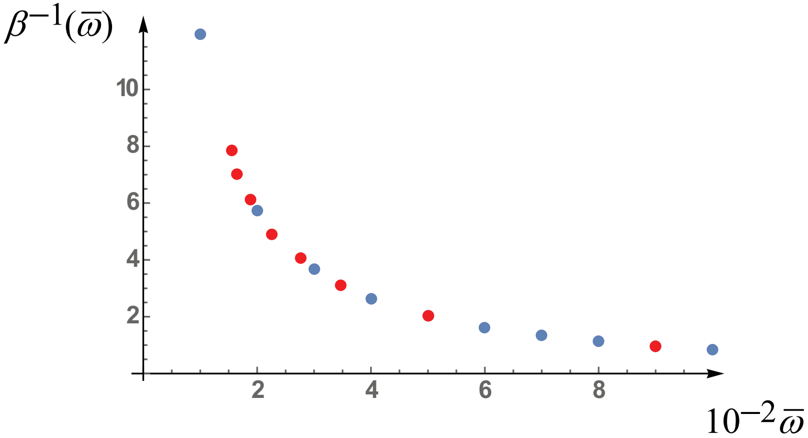
$$n_r + l + 1 = n + l = \beta^{-1}, \qquad n_r = 0, 1, 2..., \qquad (29)$$

where n_r is the radial quantum number, n is the principal quantum number and l- quantum number of angular momentum. In other words, the quantization condition is the integer value of the term β^{-1}, which implies satisfying the following conditions:

$$\left[\beta^{-1}\right] = \left[\frac{\breve{\varrho}(\bar{\omega})}{\bar{\omega}}\right] = n, \qquad \left\{\beta^{-1}\right\} = \left\{\frac{\breve{\varrho}(\bar{\omega})}{\bar{\omega}}\right\} = 0, \qquad (30)$$

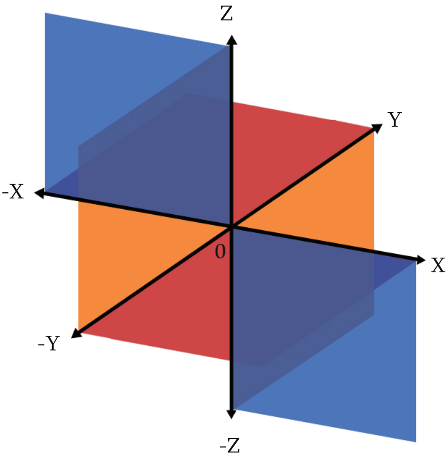
where $\breve{\varrho}(\bar{\omega}) = \varepsilon_0^{1/3}\varrho(\bar{\omega})$ - dimensionless function, $[...]$ and $\{...\}$ brackets denote the integer and fractional parts of the function, respectively.

A set of values $\bar{\omega}$ for which the system is quantized



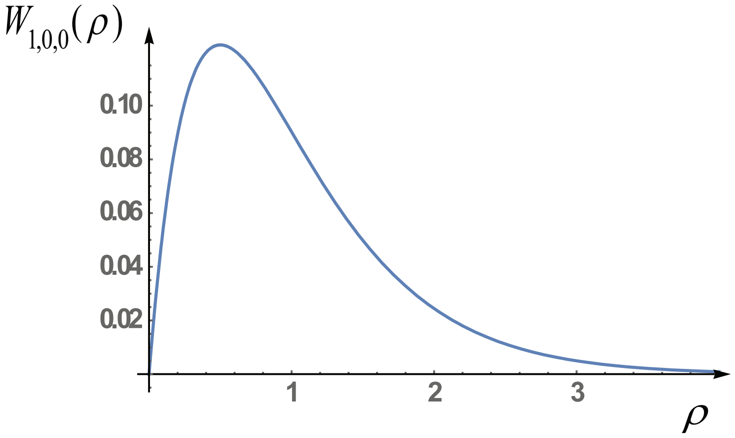
Obrázek: The dependence of a quantity $\beta^{-1}(\bar{\omega})$ on the dimensionless frequency $\bar{\omega}$. It is easy to see that the red dots satisfy the quantization conditions (29) - (30), while the blue dots do not satisfy these conditions.

The manifold on which hion (bose particle) is localized



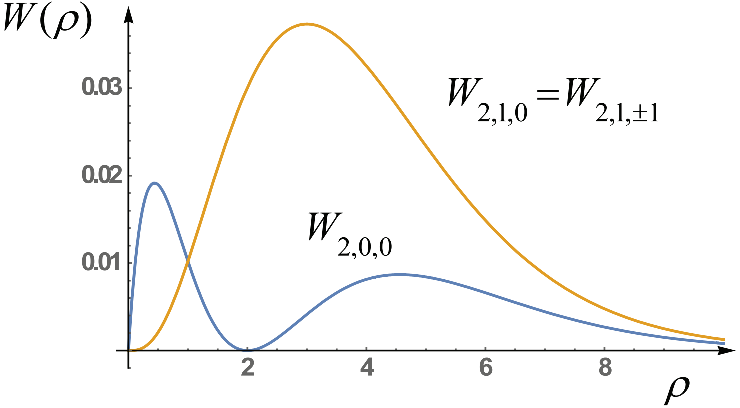
Obrázek: Boson of a vector field (hion) with projection of spin $+1$ is a $2D$ structure consisting of six components localized on the manifold consisting of planes $\phi_x^+[(-Y,Z)\cup(Y,-Z)]$, $\phi_y^+[(-X,Z)\cup(X,-Z)]$ and $\phi_z^+[(-X,Y)\cup(X,-Y)]$, respectively.

Ground state of hion radial wave function



Obrázek: The probability distribution of hion in the ground state depending on the radius. The distance $\rho_0 = 1/2$, or more precisely $\varrho_0 = a_p/2$, at which the maximum value of the amplitude of the hion probability is reached.

The first few excited radial wave functions of hion



Obrázek: *The probability distributions of the first four excited states of the hion depending on radius. The orange curve in the graph shows the probability distributions in various three excited states.*

Conclusion

1. Main result of this study is justification of new paradigm and extension Standard Model.
2. A new approach allows to rigorous proof of possibility formation massless bose particles (hions) with spin 1 for the gauge group symmetry $SU(2) \otimes U(1)$.
3. It is shown that, when two hions with opposite spins are entangled up, bosons with 0 spins can be formed. Further, these 0-spin particles by way of condensation form a scalar field- dark energy-quintessence.
4. These researches opens up new possibilities for implementation real space-time engineering.

Details are available. A. S. Gevorkyan, Quantum Vacuum: The Structure of Empty Space–Time and Quintessence with Gauge Symmetry Group $SU(2) \otimes U(1)$, *Particles 2019*, **2**, *281–308*; doi:10.3390/particles2020019

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