Vacuum fluxes from a plate in de Sitter spacetime with compact dimensions

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Outline

- Geometry of the problem and the Hadamard function
- VEV of the field squared
- VEV of the energy momentum tensor: energy flux
- Conclusions

Geometry of the problem

- We consider combined effects of topology and the presence of a boundary on the VEVs of the field squared and energy-momentum tensor for charged scalar field on the background of dS spacetime with an arbitrary number of toroidally compactified spatial dimensions
- dS spacetime is described by the line element: $ds^2 = dt^2 - e^{2t/\alpha} \sum_{l=1}^{D} (dz^l)^2,$

  Parameter α is related to the positive cosmological constant Λ by the formula $\alpha^2 = D(D-1)/(2\Lambda)$
- With conformal time $\tau = -\alpha e^{-t/\alpha}$ $-\infty < \tau \leq 0$

  the line element takes conformally flat form $ds^2 = (\alpha/\tau)^2 \eta_{\mu\nu} dx^\mu dx^\nu$
- Spatial Topology $R^p \times (S^1)^q,$

  Uncompact dimensions $\mathbf{x}_p = (x^1, ..., x^p)$ $-\infty < x^l < \infty,\ l = 1, \ldots, p$

  Compact dimensions $\mathbf{x}_q = (x^{p+1}, ..., x^D)$ L_l: $0 \leq x^l \leq L_l,\ l = p+1, \ldots, D.$

  $q = D - p$

  Length of the compact dimension
- Physical (comoving) length of compact dimension $\alpha L_l / \eta,\ \eta = |\tau|$

Geometry of the problem

- In addition, we assume planar boundary at $x^p = 0$ on which the field obeys the Robin boundary condition

$$(1 + \beta D_p)\varphi = 0, \quad x^p = 0$$

Special cases: Dirichlet
and Neumann conditions

- We investigate boundary-induced effects

Complex scalar field

- We consider a complex scalar field $\varphi(x)$ with a curvature coupling parameter ξ, in the presence of a classical abelian gauge field A_μ. The corresponding field equation has the form

$$\left(D_\mu D^\mu + m^2 + \xi R\right)\varphi(x) = 0$$

- In the most important special cases of minimally and conformally coupled scalars one has $\xi = 0$ and $\xi = (D-1)/(4D)$ respectively
  - $D_\mu = \nabla_\mu + ieA_\mu$ gauge-covariant derivative
  - e coupling between the scalar and gauge fields
  - $R = D(D+1)/\alpha^2$ scalar curvature

- One should specify the periodicity conditions along compact dimensions: $\varphi(t, \mathbf{x}_p, \mathbf{x}_q + L_l \mathbf{e}_l) = e^{i\alpha_l}\varphi(t, \mathbf{x}_p, \mathbf{x}_q)$

$\alpha_l = const$, $\mathbf{e}_l$ is the unit vector along the dimension $l = p+1, ..., D$

Gauge transformation

- We consider the simplest configuration of the gauge field with $A_\mu = \mathrm{const}$
  The gauge field can be excluded from the field equation by the gauge transformation
  $$A'_\mu = A_\mu + \partial_\mu \omega, \ \varphi'(x) = e^{-ie\omega}\varphi(x), \ \omega = -A_\mu x^\mu$$
- In the new gauge one has $A'_\mu = 0$ and $D'_\mu = \nabla_\mu$
- Quasi-periodicity condition after gauge transformation

$$\varphi'(t, \mathbf{x}_p, \mathbf{x}_q + L_l \mathbf{e}_l) = e^{i\tilde{\alpha}_l}\varphi'(t, \mathbf{x}_p, \mathbf{x}_q)$$

- The phases in the periodicity conditions and the value of the gauge field are related to each other through a gauge transformation.

$$\tilde{\alpha}_l = \alpha_l + eA_l L_l$$

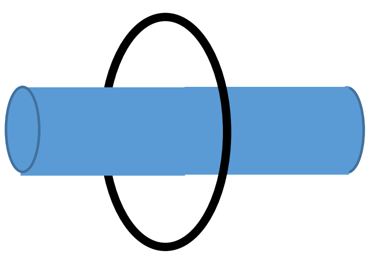
Effect of gauge field

- Although the field strength vanishes, a constant gauge field shifts the phases in the periodicity conditions along compact dimensions

- This leads to the Aharonov-Bohm-like effects on the expectation values

- The shift due to the gauge field may be written in the form,

$eA_l L_l = -e\mathbf{A}_l L_l = -2\pi\Phi_l / \Phi_0$, where $\Phi_0 = 2\pi / e$ is the flux quantum and Φ_l is the flux enclosed by the circle corresponding to the l -th compact dimension



Complete set of solutions and Hadamard function

- Mode-sum for the Hadamard function

$$G^{(1)}(x,x') = \sum_{\sigma}\sum_{s=\pm}\varphi_{\sigma}^{(s)}(x)\varphi_{\sigma}^{(s)*}(x')$$

Mode functions are specified by the set of quantum numbers

$$\sigma = (\mathbf{k}_p, \mathbf{n}_q) \quad \mathbf{n}_q = (n_{p+1}, \ldots, n_D)$$

Complete set of solutions to the field equation

- We assume that the scalar field is prepared in the Bunch-Davies vacuum

$$\varphi_{\sigma}^{(+)}(x) = C_{\sigma}^{(+)}\eta^{D/2}H_{\nu}^{(1)}(K\eta)\cos(k_p x^p + \alpha(k_p))e^{i\mathbf{k}_{p-1}\mathbf{x}_{p-1}+i\mathbf{k}_q\mathbf{x}_q}$$

$$\nu = \left[D^2/4 - D(D+1)\xi - m^2\alpha^2\right]^{1/2} \qquad \varphi_{\sigma}^{(-)}(x) = \varphi_{\sigma}^{(+)*}(x)$$

$H_{\nu}^{(1)}(z)$ Hankel function

Momentum along uncompactified dimensions

$$-\infty < k_l < +\infty, \; l = 1, \ldots, p, \qquad 0 \le k_p < +\infty$$

Along compactified dimensions

$$k_l = (2\pi n_l + \tilde{\alpha}_l)/L_l, \quad n_l = 0, \pm 1, \pm 2, \ldots, \qquad l = p+1, \ldots, D$$

Complete set of solutions and Hadamard function

- The Hadamard function is split into two parts

$$G^{(1)}(x,x') = G_0^{(1)}(x,x') + G_b^{(1)}(x,x')$$

without boundary

boundary-induced part

$$G_b^{(1)}(x,x') = \frac{(\eta\eta')^{D/2}}{2^p\pi^p V_q \alpha^{D-1}} \int d\mathbf{k}_{p-1} e^{i\mathbf{k}_{p-1}\Delta\mathbf{x}_{p-1}} \sum_{\mathbf{n}_q} e^{i\mathbf{k}_q\Delta\mathbf{x}_q} \int_k^\infty du e^{-u(x^p+x^{p\prime})} \frac{\beta u+1}{\beta u-1}$$
$$\times \{K_\nu(\eta y)[I_{-\nu}(\eta' y) + I_\nu(\eta' y)] + [I_{-\nu}(\eta y) + I_\nu(\eta y)]K_\nu(\eta' y)\}_{y=\sqrt{u^2-k^2}}$$

VEV of the field squared

- Given the Hadamard function, the vacuum expectation value of the field squared is evaluated as the coincidence limit

$$\langle \varphi\varphi^\dagger \rangle = \frac{1}{2} \lim_{x' \to x} G^{(1)}(x, x')$$

- $$\langle \varphi\varphi^+ \rangle = \langle \varphi\varphi^+ \rangle_0 + \langle \varphi\varphi^+ \rangle_b$$

without boundary

boundary-induced part

$$\langle \varphi\varphi^+ \rangle_b = \frac{2^{1-p}\pi^{-(p+1)/2}\eta^D}{\Gamma((p-1)/2)V_q\alpha^{D-1}} \sum_{\mathbf{n}_q} \int_0^\infty dx\, x^p f(\sqrt{x^2 + k_{\mathbf{n}_q}^2}, x^p) h_\nu(\eta x)$$

$$h_\nu(u) = \int_0^1 dz\, z(1 - z^2)^{(p-3)/2} g_\nu(uz)$$

$$k_{\mathbf{n}_q}^2 = \sum_{l=p+1}^{D} (2\pi n_l + \tilde{\alpha}_l)^2 / L_l^2$$

$$g_\nu(x) = K_\nu(x)[I_{-\nu}(x) + I_\nu(x)],$$

$$f(u, x^p) = \frac{e^{-2ux^p}}{u} \frac{\beta u + 1}{\beta u - 1}$$

VEV of the field squared

- For a conformally coupled massless scalar field

$$\langle \varphi\varphi^{+}\rangle_b = \frac{(\eta/\alpha)^{D-1}}{(4\pi)^{p/2}\Gamma(p/2)V_q}\sum_{\mathbf{n}_q}\int_{k_{\mathbf{n}_q}}^{\infty} dx\,(x^2-k_{\mathbf{n}_q}^2)^{p/2-1}e^{-2xx^p}\frac{\beta x+1}{\beta x-1}$$

Conformal factor

Conformally related to the result in Minkwoski bulk

- For Dirichlet and Neumann boundary conditions

$$\langle \varphi\varphi^{+}\rangle_b = \mp\frac{2\eta^D(2x^p)^{2-p}}{(2\pi)^{p/2+1}V_q\alpha^{D-1}}\int_0^{\infty} dy\,y g_\nu(\eta y)\sum_{\mathbf{n}_q} f_{p/2-1}(2x^p\sqrt{y^2+k_{\mathbf{n}_q}^2})$$

VEV of the energy-momentum tensor

- Vacuum expectation value of the energy-momentum tensor

$$\langle T_{ik}\rangle_{p,q} = \frac{1}{2}\lim_{x'\to x}\partial_i\partial'_k G_{p,q}(x,x') + \frac{1}{2}\left[\left(\xi-\frac{1}{4}\right)g_{ik}\nabla_l\nabla^l - \xi\nabla_i\nabla_k - D\xi g_{ik}/\alpha^2\right]G_{p,q}(x,x)$$

- $\langle T_{ik}\rangle = \langle T_{ik}\rangle_0 + \langle T_{ik}\rangle_b$

without boundary

boundary-induced part

$$\langle T_i^k\rangle_b = \frac{A_p}{V_q\alpha^{D+1}}\sum_{\mathbf{n}_q}\int_0^\infty dx\, x^{p-D} f(\sqrt{x^2+k_{\mathbf{n}_q}^2}, x^p) \times\left[\hat{F}_i^k(\eta)h_\nu^{(0)}(\eta x) + \eta^2 x^2 F_i^k h_\nu^{(1)}(\eta x)\right],$$

- The only nonzero off-diagonal component $\langle T_0^p\rangle$
- Boundary-induced part obeys the covariant continuity equation

$$\nabla_k\langle T_i^k\rangle_b = 0$$

VEV of the energy-momentum tensor

- Off-diagonal component $\langle T_0^p \rangle_b$ appears (energy flux)

$$\langle T_0^p \rangle_b = \frac{\pi^{-(p+1)/2}(\eta/\alpha)^{D+1}}{2^{p-2}\Gamma((p-1)/2)V_q} \sum_{\mathbf{n}_q} \int_0^\infty dy\, y F_\nu(\eta y)$$
$$\times \int_0^\infty dw\, w^{p-2} \frac{\beta u + 1}{\beta u - 1} e^{-2x^p u} \Big|_{u=\sqrt{y^2+w^2+k_{\mathbf{n}_q}^2}},$$

where $F_\nu(x) = \left[\left(\xi - \frac{1}{4}\right) x\partial_x + (D+1)\xi - \frac{D}{4}\right] g_\nu(x)$

- In special cases of Dirichlet and Neumann boundary conditions

$$\langle T_0^p \rangle = \mp \frac{4(\eta/\alpha)^{D+1}}{(2\pi)^{p/2+1} V_q (2x^p)^{p-1}} \sum_{\mathbf{n}_q} \int_0^\infty dy\, y f_{p/2}(2x^p \sqrt{y^2 + \mathbf{k}_q^2}) F_\nu(\eta y)$$

- For Dirichlet and Neumann boundary conditions the energy flux has opposite signs

Asymptotic behavior

- For $x^p/\eta \ll 1$ (x^p/η is the proper distance from the boundary in units of the curvature radius 1/α)

  the influence of the gravitational field is weak

- For $x^p/\eta \gg 1$
  - $\nu^2 > 0$, in addition $x^p \gg L_l$

$$\langle\varphi\varphi^+\rangle_b \approx \mp\frac{2^{2\nu-p}\Gamma(\nu)\Gamma(p/2-\nu)\eta^{D-2\nu}}{(2\pi)^{p/2+1}V_q\alpha^{D-1}(x^p)^{3p/2-1-2\nu}}$$

→ For $\tilde{\alpha}_l = 0,\ l = p+1,\dots,D$

Power-law decay

$$\langle\varphi\varphi^+\rangle_b \approx \mp\frac{2^{2\nu-p}\Gamma(\nu)\eta^{D-2\nu}k_{(0)}^{(p-1)/2-\nu}e^{-2x^p k_{(0)}}}{(2\pi)^{(p+1)/2}V_q\alpha^{D-1}(x^p)^{p-\nu-1/2}}$$

→ If, at least one $\tilde{\alpha}_l \neq 0$

Exponential decay

  - $\nu^2 < 0$, in addition $x^p \gg L_l$

$$\langle\varphi\varphi^+\rangle_b \approx \mp\frac{\pi^{-p/2-1}A_p\eta^D}{2^{p+1}V_q\alpha^{D-1}(x^p)^p}\cos\left[2|\nu|\ln\left(2x^p/\eta\right)+\phi_1\right]$$

↑ For $\tilde{\alpha}_l = 0,\ l = p+1,\dots,D$

$$\langle\varphi\varphi^+\rangle_b \approx \mp\frac{\pi^{-p/2}\eta^D k_{(0)}^{(p-1)/2}e^{-2x^p k_{(0)}}}{2^{p+1}V_q\alpha^{D-1}(x^p)^{(p+1)/2}}\frac{\cos\left[|\nu|\ln\left(4x^p/\eta^2k_0\right)+\phi_2\right]}{\sqrt{|\nu|\sinh(\pi|\nu|)}}$$

↗ If, at least one $\tilde{\alpha}_l \neq 0$

Asymptotic behavior

- At early stages of the expansion ($t \to -\infty$)

$$\langle T_0^p \rangle \approx \mp \frac{2D\,(\xi - \xi_D)\,(\eta/\alpha)^D}{(2\pi)^{(p+1)/2}\,\alpha V_q (2x^p)^p} \sum_{\mathbf{n}_q} f_{(p+1)/2}(2x^p k_{\mathbf{n}_q})$$

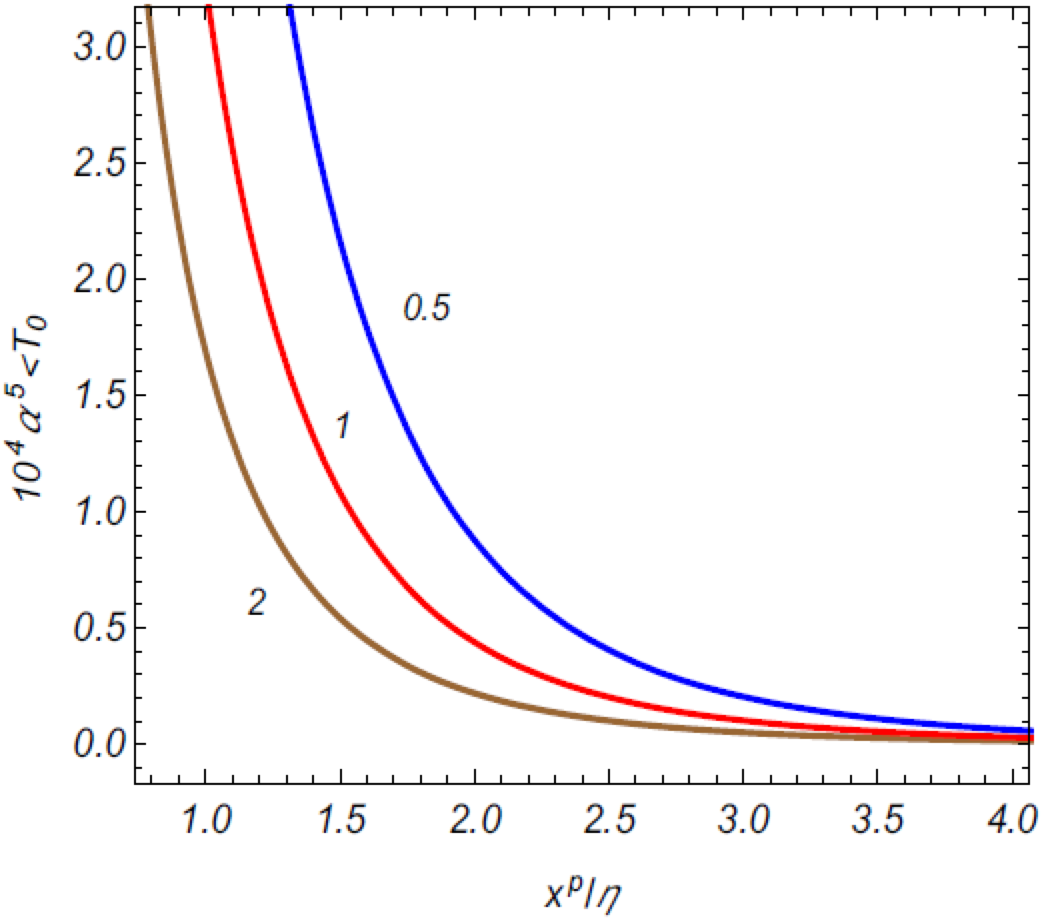
- At late stages of the expansion ($t \to +\infty$)
  - $\nu^2 > 0$ Decay is monotonically

$$\langle T_0^p \rangle \approx \mp 2^{p/2+\nu+2}\Gamma(\nu)(\eta/\alpha)^{D+1-2\nu} \frac{2\nu\,(1/4 - \xi) + (D+1)\xi - D/4}{(4\pi)^{p/2+1}\,\alpha^{2\nu} V_q (2x^p)^{p+1-2\nu}} \sum_{\mathbf{n}_q} g_{p/2+1-\nu}(2x^p k_{\mathbf{n}_q})$$

  - $\nu^2 < 0$ Decay is oscillatory

$$\langle T_0^p \rangle \approx -\frac{(\eta/\alpha)^{D+1}}{2^{p-1}\pi^{(p+1)/2} V_q} \sum_{\mathbf{n}_q} A(x^p, k_{\mathbf{n}_q}) \sin[2|\nu| \ln(\eta/2) - \phi(x^p, k_{\mathbf{n}_q})]$$

Energy flux

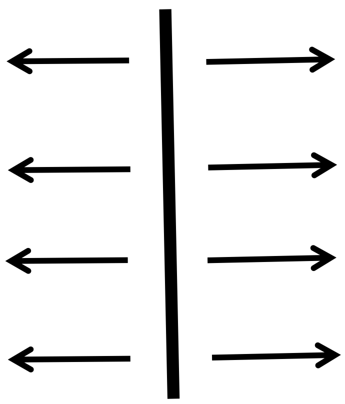


$D = 4$

Model with a single compact dimension

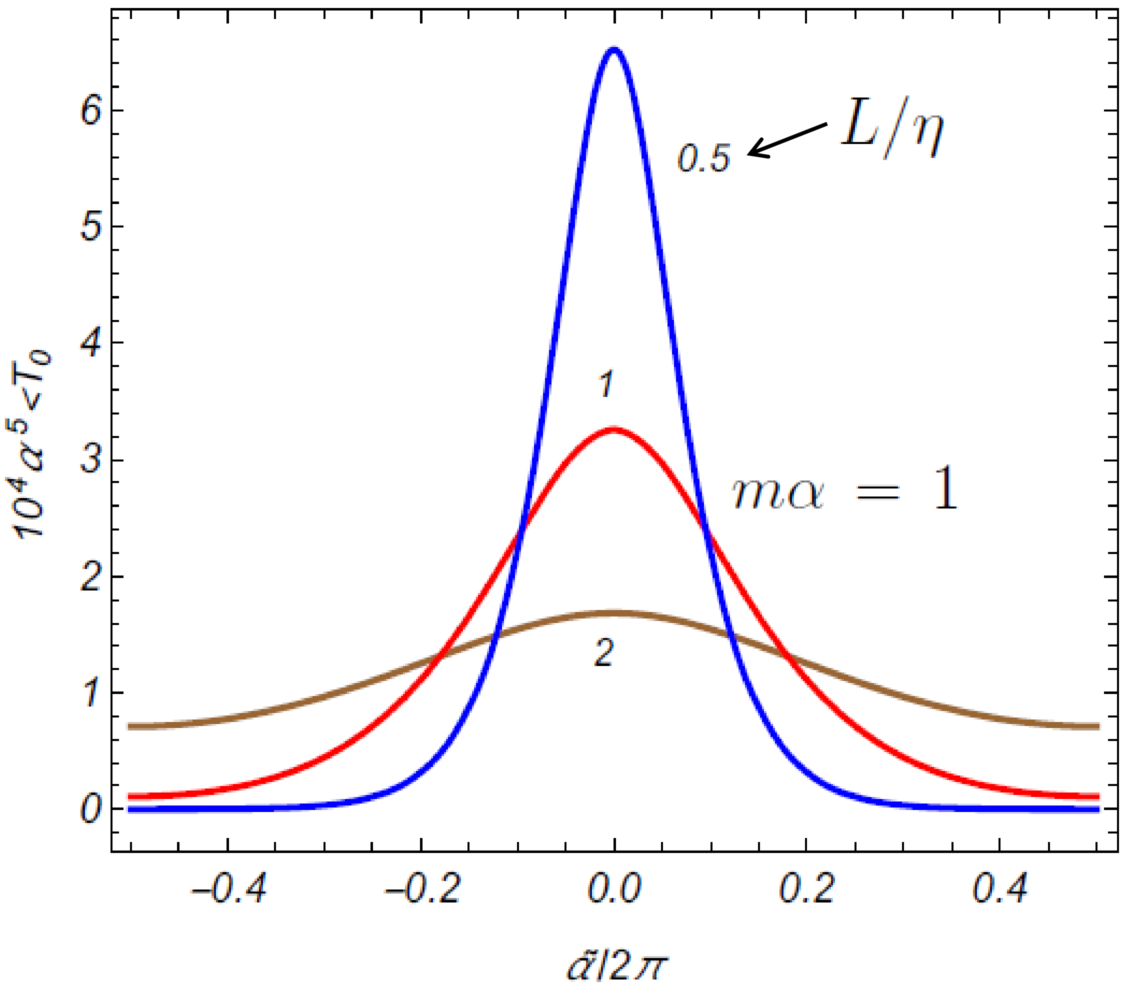
Dirichlet boundary condition

$m\alpha = 1, \ \tilde{\alpha} = \pi/4$



Energy flux is directed from the boundary (to the boundary for Neumann b.c.)

Energy flux



$D = 4$

Model with a single compact dimension

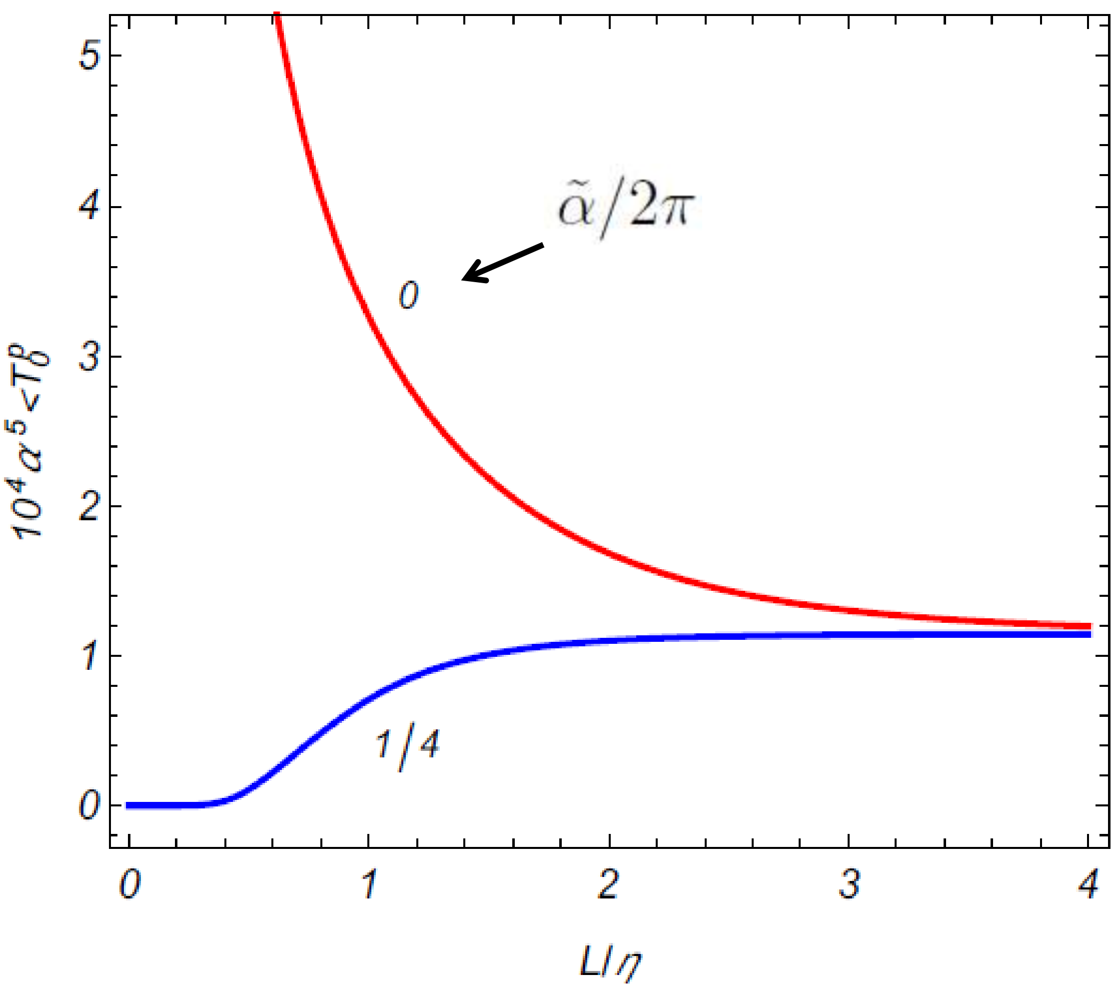
Dirichlet boundary condition

$x^p/\eta = 1 \quad m\alpha = 1$

Energy flux

$D = 4$

Model with a single compact dimension



Conclusion

- The presence of a planar boundary give rise to the energy flux in the vacuum state along the direction normal to the plate

- Vacuum expectation values are decomposed into the boundary-free and boundary-induced parts

- The flux is an even periodic function of magnetic fluxes enclosed by compact dimensions with the period equal to flux quantum

- Near the boundary the inuence of the gravitational field on boundary-induced quantum effects is weak

THANK YOU