Bulk viscosity of baryonic matter with trapped neutrinos

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Outline

- Introduction & motivation
- Urca processes and bulk viscosity
- Numerical results
- Conclusions
Introduction & motivation

The structure of a neutron star

- Quark-hybrid star
- Hyperon star
- Neutron star with pion condensate
- Nucleon star
- Hydrogen/He atmosphere
- Strange quark matter (u,d,s quarks)
- Strange star
- 2SC
- CFL
- n superfluid
- n superconducting Protons
- N+e
N+e+n
n,p,e, μ
Λ, Σ, Δ, Ω
μ
π−

$\begin{align*}
\text{Fe} & \quad 10^6 \text{ g/cm}^3 \\
& \quad 10^{11} \text{ g/cm}^3 \\
& \quad 10^{14} \text{ g/cm}^3 \\
\end{align*}$

$R \sim 10 \text{ km}$
Compact-star binaries

- Compact stars are natural laboratories which allow us to study the properties of nuclear matter under extreme physical conditions (strong gravity, strong magnetic fields, etc.).
- The recent detection of gravitational and electromagnetic waves originating from black hole or neutron star mergers motivates studies of compact binary systems.
- Various physical processes in the compact binary systems can be modelled in the framework of general-relativistic hydrodynamics simulations.
- The bulk viscosity might affect the hydrodynamic evolution of neutron star mergers by damping the density oscillations which can be detected from gravitational signals \(^1\).

- Our aim is to study the bulk viscosity in dense baryonic matter for temperatures relevant to neutron star mergers and supernovas \(T \geq 5\) MeV.
- At these temperatures neutrinos are trapped in matter, and the bulk viscosity arises from weak interaction (neutron decay and electron capture) processes.

\(^1\) M. G. Alford, et al., On the importance of viscous dissipation and heat conduction in binary neutron-star mergers, 2017.
Introduction & motivation

Literature on bulk viscosity

- M. Alford and S. Harris, *Damping of density oscillations in neutrino-transparent nuclear matter*, 1907.03795.
Urca processes and bulk viscosity

Urca process rates

- We consider a simple composition of baryonic matter consisting of neutrons, protons, electrons and neutrinos. The simplest weak-interaction processes are the following (direct) Urca processes

\[
\begin{align*}
n & \rightleftharpoons p + e^- + \bar{\nu}_e \quad (\text{neutron decay process}) \\
p + e^- & \rightleftharpoons n + \nu_e \quad (\text{electron capture process})
\end{align*}
\]

- In $\beta$-equilibrium the chemical potentials of particles obey the relation $\mu_n + \mu_\nu = \mu_p + \mu_e$. Out of $\beta$-equilibrium in general implies an imbalance

\[
\mu_\Delta \equiv \mu_n + \mu_\nu - \mu_p - \mu_e \neq 0.
\]

- The $\beta$-equilibration rate for the neutron decay $n \rightarrow p + e^- + \bar{\nu}_e$ is given by

\[
\Gamma_{1p}(\mu_\Delta) = \int d\Omega \sum_{s_i} |\mathcal{M}_{\text{Urca}}|^2 f(p') \bar{f}(k') \bar{f}(k) \bar{f}(p) (2\pi)^4 \delta^{(4)}(p + k + k' - p'),
\]

with $\bar{f}(p) = 1 - f(p)$. Similar expressions can be written also for $\Gamma_{1n}$, $\Gamma_{2p}$ and $\Gamma_{2n}$.

- The squared matrix element of Urca processes is

\[
\sum_{s_i} |\mathcal{M}_{\text{Urca}}|^2 = 32G^2 (k \cdot p')(p \cdot k') \simeq 32G^2 p_0 p_0' k_0 k_0',
\]
Consider now small-amplitude density oscillations in baryonic matter with frequency $\omega$

$$n_B(t) = n_{B0} + \delta n_B(t), \quad n_L(t) = n_{L0} + \delta n_L(t), \quad \delta n_B(t), \delta n_L(t) \sim e^{i\omega t}. $$

The baryon and lepton number conservation $\partial n_i / \partial t + \text{div} (n_i v) = 0$ implies

$$\delta n_i(t) = -\frac{\theta}{i\omega} n_{i0}, \quad i = \{B, L\}, \quad \theta = \text{div} v. $$

The oscillations cause perturbations in particle densities $n_j(t) = n_{j0} + \delta n_j(t)$, due to which the chemical equilibrium of matter is disturbed leading to a small shift $\mu_\Delta = \delta \mu_n + \delta \mu_\nu - \delta \mu_p - \delta \mu_e$, which can be written as

$$\mu_\Delta = (A_{nn} - A_{pn})\delta n_n + A_{\nu\nu}\delta n_\nu - (A_{pp} - A_{np})\delta n_p - A_{ee}\delta n_e, \quad A_{ij} = \left( \frac{\partial \mu_i}{\partial n_j} \right)_0. $$

If the weak processes are turned off, then a perturbation conserves all particle numbers

$$\frac{\partial}{\partial t} \delta n_j(t) + \theta n_{j0} = 0, \quad \delta n_j(t) = -\frac{\theta}{i\omega} n_{j0}. $$
Out of equilibrium the chemical equilibration rate to linear order in $\mu_\Delta$ is given by

$$\Gamma_p - \Gamma_n = \lambda \mu_\Delta, \quad \lambda > 0.$$ 

The rate equations which take into account the loss and gain of particles read as

$$\frac{\partial}{\partial t} \delta n_n(t) = -\theta n_0 - \lambda \mu_\Delta(t), \quad \frac{\partial}{\partial t} \delta n_p(t) = -\theta n_0 + \lambda \mu_\Delta(t).$$

Solving these equations we can compute the pressure out of equilibrium

$$p = p(n_j) = p(n_j + \delta n_j) = p_0 + \delta p = p_{eq} + \delta p',$$

where the non-equilibrium part of the pressure - the bulk viscous pressure, is given by

$$\Pi \equiv \delta p' = \sum_j \left(\frac{\partial p}{\partial n_j}\right)_0 \delta n'_j = \sum_{ij} n_{i0} A_{ij} \delta n'_j.$$

The bulk viscosity is then identified from $\Pi = -\zeta \theta$

$$\zeta = \frac{C^2}{A} \frac{\lambda A}{\omega^2 + \lambda^2 A^2}$$

with susceptibilities $A = -\frac{1}{n_B} \left(\frac{\partial \mu_\Delta}{\partial x_p}\right)_{n_B}$ and $C = n_B \left(\frac{\partial \mu_\Delta}{\partial n_B}\right)_{x_p}$.
In chemical equilibrium the conditions of the detailed balance are satisfied

\[ \Gamma_{1p} = \Gamma_{1n} \equiv \Gamma_1, \quad \Gamma_{2p} = \Gamma_{2n} \equiv \Gamma_2. \]

In the low-temperature limit (degenerate matter) we have the following results

\[ \Gamma \equiv \Gamma_1 + \Gamma_2 = \frac{m^* \tilde{G}^2}{12\pi^3} T^3 p_{Fe} p_{F\nu} (p_{Fe} + p_{F\nu} - |p_{Fn} - p_{Fp}|), \quad \lambda = \frac{\Gamma}{T}. \]

The “beta-disequilibrium–proton-fraction” susceptibility is given by

\[ A = \frac{\pi^2}{m^*} \left( \frac{1}{p_{Fn}} + \frac{1}{p_{Fn}} \right) + \frac{\pi^2}{p_{Fe}^2} + \frac{2\pi^2}{p_{F\nu}^2} + \left( \frac{g_\rho}{m_\rho} \right)^2. \]

The “beta-disequilibrium–baryon-density” susceptibility reads

\[ C = \frac{p_{Fn}^2 - p_{Fp}^2}{3m^*} + \frac{p_{F\nu} - p_{Fe}}{3} + \frac{n_n - n_p}{2} \left( \frac{g_\rho}{m_\rho} \right)^2 + n_B \frac{p_{Fn}^2 - p_{Fp}^2}{2m^*} \left( \frac{g_\sigma}{m_\sigma} \right)^2. \]
Beta-equilibrated nuclear matter

We use the density functional theory approach to the nuclear matter, which is based on phenomenological baryon-meson Lagrangians of the type proposed by Walecka and others. The Lagrangian density of matter is written as $\mathcal{L} = \mathcal{L}_N + \mathcal{L}_L$, where

$$
\mathcal{L}_N = \sum_N \bar{\psi}_N \left[ \gamma^\mu \left( i \partial_\mu - g_{\omega B} \omega_\mu - \frac{1}{2} g_{\rho N} \tau \cdot \rho_\mu \right) - (m_N - g_{\sigma N} \sigma) \right] \psi_N 
+ \frac{1}{2} \partial^\mu \sigma \partial_\mu \sigma - \frac{1}{2} m^2_\sigma \sigma^2 - \frac{1}{4} \omega^{\mu \nu} \omega_{\mu \nu} + \frac{1}{2} m^2_\omega \omega^{\mu \nu} \omega_\mu \omega_\nu - \frac{1}{4} \rho^{\mu \nu} \rho_{\mu \nu} + \frac{1}{2} m^2_\rho \rho^\mu \cdot \rho_\mu,
$$

The leptonic contribution is given by

$$
\mathcal{L}_L = \sum_\lambda \bar{\psi}_\lambda (i \gamma^\mu \partial_\mu - m_\lambda) \psi_\lambda.
$$

The pressure of baryonic matter is given by

$$
P_N = -\frac{m^2_\sigma}{2} \sigma^2 + \frac{m^2_\omega}{2} \omega^2 + \frac{m^2_\rho}{2} \rho^{03} + \frac{g_\lambda}{3 \pi^2} \sum_\lambda \int_0^\infty \frac{k^4 \, dk}{(k^2 + m^2_\lambda)^{1/2}} \left[ f(E^\lambda_k - \mu_\lambda) + f(E^\lambda_k + \mu_\lambda) \right] 
+ \frac{1}{3} \sum_N \frac{2 J_N + 1}{2 \pi^2} \int_0^\infty \frac{k^4 \, dk}{(k^2 + m^*_N)^{1/2}} \left[ f(E^N_k - \mu^*_N) + f(E^N_k + \mu^*_N) \right],
$$

where $m^*_N = m_N - g_{\sigma N} \sigma$ and $\mu^*_N = \mu_N - g_{\omega N} \omega_0 - g_{\rho N} \rho^{03} I_3$ are the nucleon effective mass and effective chemical potentials, respectively.
Numerical results

Particle fractions in equilibrium

The particle fractions are found from $\beta$-equilibrium conditions $\mu_n + \mu_\nu = \mu_p + \mu_e$ and $\mu_\mu = \mu_e$, the charge neutrality condition $n_p = n_e + n_\mu$, the baryon number conservation $n_B = n_n + n_p$, and the lepton number conservation $n_l + n_{\nu_l} = n_L = Y_L n_B$.

We consider two cases: (i) $Y_L = 0.1$ for both flavors, typical for neutron star mergers; (ii) $Y_{Le} = 0.4$ and $Y_{L\mu} = 0$ typical for matter in supernovae and proto-neutron stars.

The particle fractions are not sensitive to the temperature for the given value of $Y_L$.

In the low-density and high-temperature regime the net neutrino density becomes negative, indicating that there are more anti-neutrinos than neutrinos in that regime.

Merger matter has much smaller electron neutrino fraction than supernova matter.
The neutron decay rate $\Gamma_1$ is exponentially suppressed at low temperatures because of damping of anti-neutrino population in the degenerate matter.

The electron capture rate $\Gamma_2$ has a finite low-temperature limit which is $\propto T^3$.

In the regime of interest $\Gamma_1 \ll \Gamma_2$, therefore the electron capture process dominates in the $\beta$-equilibration and the bulk viscosity.
The beta equilibrium relaxation rate $\lambda A$ determines at which frequency the bulk viscosity reaches its resonant maximum.

The relaxation rate is slowest in the neutrino-transparent case, and increases with the lepton fraction in the neutrino-trapped case.

In neutrino-trapped matter $\lambda A \gg \omega$ for oscillation frequencies typical to neutron star mergers and supernovas $\Rightarrow$ the bulk viscosity takes the form $\zeta \approx C^2/(\lambda A^2)$.

The neutrino-transparent matter instead features a relaxation rate which is comparable to the oscillation frequencies at typical temperatures $2 \div 7$ MeV.
The susceptibility prefactor $C^2/A$

The susceptibility $A$ does not depend strongly on the density and temperature and has roughly the same order of magnitude $A \sim 10^{-3}$ MeV$^{-2}$.

The susceptibility $C$ increases with density and at sufficiently high temperatures $T \gtrsim 30$ MeV $C$ crosses zero at critical values of the density, close to saturation density.
The density dependence of the bulk viscosity follows that of the susceptibility \( C^2/A \).

The temperature dependence of \( \zeta \) arises mainly from the temperature dependence of the beta relaxation rate \( \lambda A \propto T^2 \).

Bulk viscosity decreases as \( \zeta \propto T^{-2} \) in the neutrino-trapped regime.

This scaling breaks down at high temperatures \( T \geq 30 \text{ MeV} \) where the bulk viscosity has sharp minimums \( \zeta \to 0 \) when the matter becomes scale-invariant.
The bulk viscosity in neutrino-transparent matter is frequency-dependent.
It attains its maximum value at temperature $T \simeq 2 \div 7$ MeV, where $\lambda A = \omega$.
This is the temperature range which is relevant for neutron-star mergers.
We interpolate the numerical results for the bulk viscosity between the two regimes in the interval $5 \leq T \leq 10$ MeV.

The relaxation rate is slower for neutrino-transparent matter, so the resonant peak of the bulk viscosity occurs within its regime of validity.

The bulk viscosity in the neutrino transparent regime is larger, and drops by orders of magnitude as the matter enters the neutrino-trapped regime.

Likely bulk viscosity will have its greatest impact on neutron star mergers in regions that are neutrino transparent rather than neutrino trapped.
THANK YOU FOR ATTENTION!