Modeling anisotropic magnetized white dwarfs with γ metric

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Outline

Motivation

Anisotropic magnetized EoS

Modeling magnetized compact stars with γ metric

Results

Summary

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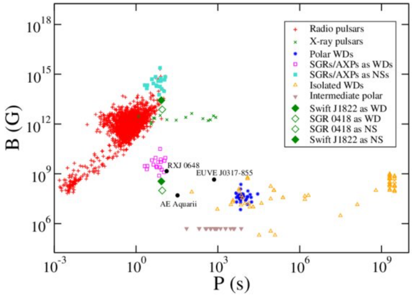
- Presence of magnetic fields in compact stars and its effects in the EoS and the structure.



arXiv:1307.5074 [astro-ph.SR]

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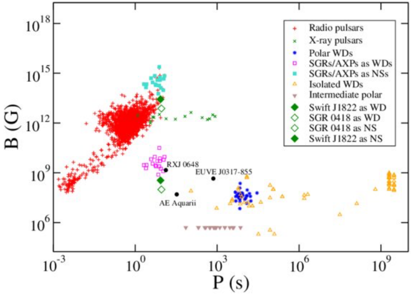
- Presence of magnetic fields in compact stars and its effects in the EoS and the structure.
- Anisotropic EoS featured by a fermion system in an external magnetic field $\mathbf{B} = B\hat{z}$.

Phys. Rev. Lett. 84, 5261 (2000).



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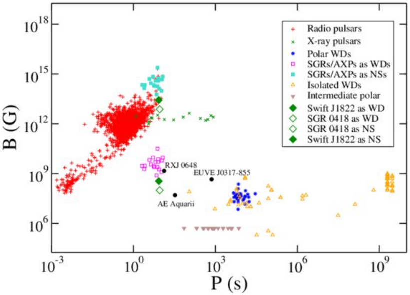

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- Modeling in the literature:
  - TOV avoiding (or neglecting) the anisotropy.
  - Numerical relativity schemes: Maxwell equation (usually not included in the EoS).

Anisotropic magnetized EoS

Effects of an external magnetic field $\mathbf{B} = B\hat{z}$ acting on a fermions system:

- Quantization of the particle's dispersion relation.
- Density of states:

$$2\int \frac{d^3\vec{p}}{(2\pi)^3} \to 2\sum_{l=0}^{\infty}(2-\delta_{l0})\int \frac{eB}{(2\pi)^2}dp_3$$

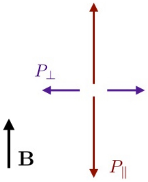
- Anisotropic stress-energy tensor:

$$T^{\mu}_{\nu} = \mathrm{diag}(-E, P_\perp, P_\perp, P_\parallel)$$

$$E = \Omega(B,\mu,T) + \mu N(B,\mu,T) + TS$$

$$P_\perp = -\Omega(B,\mu,T) - B\mathcal{M}(B,\mu,T)$$

$$P_\parallel = -\Omega(B,\mu,T)$$



Anisotropic EoS: E, $P_\perp$, $P_\parallel$.

Anisotropic magnetized EoS

$$\Omega(B,\mu,T) = -\frac{eB}{2\pi^2}\int_0^\infty dp_3 \sum_{l=0}^{\infty} g_l \left[\varepsilon_l + T\ln\left(1+e^{-\frac{\varepsilon_l-\mu}{T}}\right)\left(1+e^{-\frac{\varepsilon_l+\mu}{T}}\right)\right]$$

$$= \Omega^{\mathsf{vac}}(B) + \Omega^{\mathsf{st}}(B,\mu,T)$$

$$\varepsilon_l = \sqrt{p_3^2 + 2|eB|l + m^2}$$

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Vacuum term, after renormalization:

$$\Omega_w^{\mathsf{vac}}(B) = -\frac{m^4}{90(2\pi)^2}\left(\frac{B}{B_c}\right)^4, \quad B < B_c;$$

$$\Omega_s^{\mathsf{vac}}(B) = \frac{m^4}{24\pi^2}\left(\frac{B}{B_c}\right)^2 \ln\frac{B}{B_c}, \quad B > B_c.$$

$$B_c = m^2/e = 4.4\times 10^{13}\ \mathrm{G}$$

Anisotropic magnetized EoS

$$\Omega(B,\mu,T) = -\frac{eB}{2\pi^2}\int_0^\infty dp_3 \sum_{l=0}^{\infty} g_l \left[\varepsilon_l + T\ln\left(1+e^{-\frac{\varepsilon_l-\mu}{T}}\right)\left(1+e^{-\frac{\varepsilon_l+\mu}{T}}\right)\right]$$

$$= \Omega^{\mathsf{vac}}(B) + \Omega^{\mathsf{st}}(B,\mu,T)$$

$$\varepsilon_l = \sqrt{p_3^2 + 2|eB|l + m^2}$$

Statistical contribution ($T \to 0$):

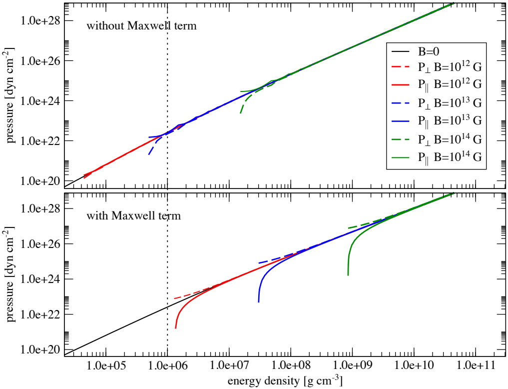
$$\Omega^{\mathsf{st}}(B,\mu,0) = \frac{m^2}{4\pi^2}\frac{B}{B_c}\sum_{l=0}^{l_{max}} g_l \left[\mu\, p_F - \varepsilon_l^2 \ln\left(\frac{\mu + p_F}{\varepsilon_l}\right)\right]$$

$$l_{max} = I[\frac{\mu^2 - m^2}{2eB}],\; p_F = \sqrt{\mu^2 - \varepsilon_l^2}$$

Anisotropic magnetized EoS in stellar equilibrium

$$E = \Omega + \mu N + m_N \frac{A}{Z} N + \frac{B^2}{8\pi}, \quad P_\parallel = -\Omega - \frac{B^2}{8\pi}, \quad P_\perp = -\Omega - B\mathcal{M} + \frac{B^2}{8\pi}$$

with Maxwell contribution.



γ metric (or Zipoy-Voorhees)

- Static, axisymmetric and asymptotically flat family of solutions to the Einstein equations in spherical coordinates,

$$ds^2 = -\Delta^\gamma dt^2 + \Delta^{\gamma^2-\gamma-1}\Sigma^{1-\gamma^2}dr^2 + r^2\Delta^{1-\gamma}\Sigma^{1-\gamma^2}d\theta^2 + r^2\sin^2\theta\Delta^{\gamma^2-\gamma}d\phi^2,$$

$$\Delta = \left(1 - \frac{2m}{r}\right),$$

$$\Sigma = \left(1 - \frac{2m}{r} + \frac{m^2}{r^2}\sin^2\theta\right)$$

- Parameters:

  m: related to the gravitational mass $M = \gamma m$.

  γ: related to the shape of the object, with the quadrupolar moment $Q = m^3\gamma(1-\gamma^2)/3$

$\gamma \to 0$: Minkowski ($M = Q = 0$)

$\gamma \to 1$: Schwarszchild ($Q = 0$)

γ metric: small deformations ($\gamma \simeq 1$)

$$ds^2 = -\left[1 - 2m(r)/r\right]^{\gamma} dt^2 + \left[1 - 2m(r)/r\right]^{-\gamma} dr^2 + r^2 \sin\theta d\phi^2 + r^2 d\theta^2$$

- With isotropic energy momentum tensor:

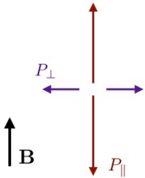
$$\frac{dP}{dr} = -\frac{(E+P)\left[\frac{r}{2} + 4\pi r^3 P - \frac{r}{2}\left(1 - \frac{2M}{r}\right)^{\gamma}\right]}{r^2\left(1 - \frac{2M}{r}\right)^{\gamma}}$$

radius R defined at $P(R) = 0$

IJMP CS vol.45,1760029 (2017)
1504.03006 [astro-ph.SR]
AIP Conference Proceedings, vol. 751, 03 (2005)
Phys. Lett. B, vol. 58, p. 357–360, 09 1975

Modeling magnetized compact stars with γ metric

- Small deformations ($\gamma \simeq 1$)
- From the pressure anisotropy in the magnetized EoS:
    - Spheroidal objects
    - Parametrization $z = \gamma r$
    - $P_\perp(r)$ and $P_\parallel(z(r))$
    - $\gamma = \frac{P_\parallel(r)}{P_\perp(r)} \approx \frac{P_{\parallel 0}}{P_{\perp 0}}$



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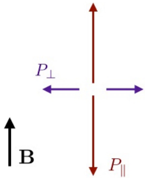
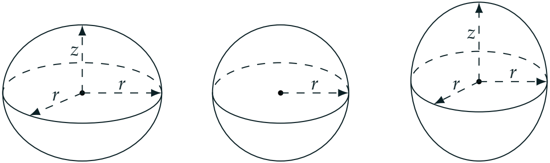


$\gamma < 1$, $P_\parallel < P_\perp$	$\gamma = 1$, $P_\parallel = P_\perp$	$\gamma > 1$, $P_\parallel > P_\perp$
oblate spheroid	sphere	prolate spheroid



Modeling magnetized compact stars with γ metric

$$\frac{dM}{dr} = 4\pi r^2 \frac{(E_\parallel + E_\perp)}{2}\gamma,$$

$$\frac{dP_\perp}{dr} = -\frac{(E_\perp + P_\perp)[\frac{r}{2} + 4\pi r^3 P_\perp - \frac{r}{2}(1-\frac{2M}{r})^\gamma]}{r^2(1-\frac{2M}{r})^\gamma},$$

$$\frac{dP_\parallel}{dz} = \frac{1}{\gamma}\frac{dP_\parallel}{dr}$$

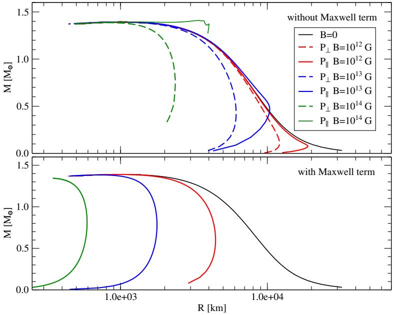
$$= -\frac{(E_\parallel + P_\parallel)[\frac{r}{2} + 4\pi r^3 P_\parallel - \frac{r}{2}(1-\frac{2M}{r})^\gamma]}{\gamma r^2(1-\frac{2M}{r})^\gamma},$$

$$E_\parallel = E(P_\parallel), E_\perp = E(P_\perp)$$

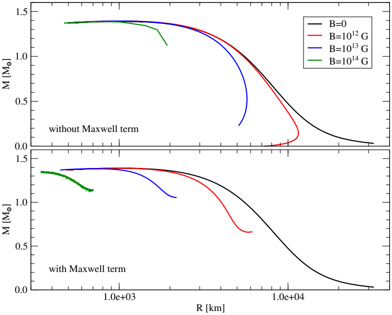
Reduces to TOV equations at $B = 0$: $\gamma = 1$, $P_\parallel = P_\perp$

Results: White dwarfs (WDs)

TOV:



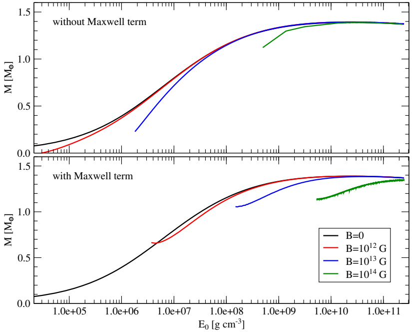
our model:

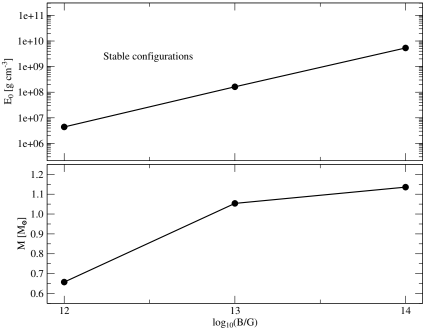


Results: WDs. Stability and super-Chandrasekhar masses

- Stability determined by small deformations and M vs E_0 minimum:

$$|E_\perp(r) - E_\parallel(r)|/E_0 \lesssim 10^{-3} \Rightarrow P_\parallel(r)/P_\perp(r) \approx P_{\parallel 0}/P_{\perp 0}$$

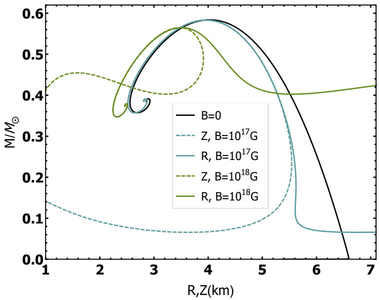




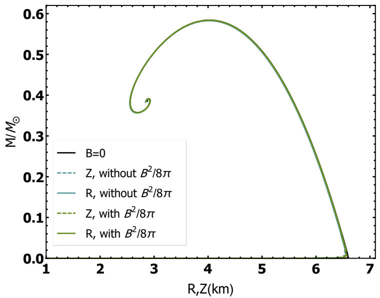
- $B > 10^{14}$ G: densities that yield stable objects are beyond WDs density range. *No super-Chandrasekhar masses.*

Results: BEC stars

our model:



our model (self-generated magnetic field):



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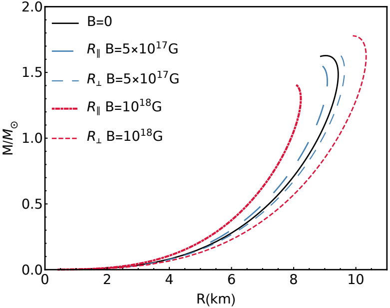
Results: Strange quark stars

TOV:



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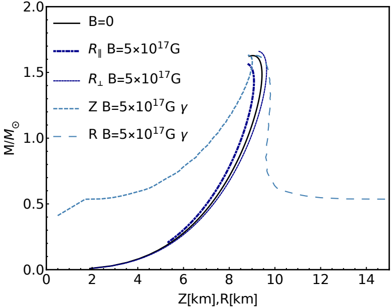

to appear in STARS/SMFNS 2019 Conference Proceedings

Summary

- Model that describes the structure of a deformed compact star, provided it is spheroidal.
- γ parameter relates the magnetic anisotropy on the EoS with the geometric deformation. Reasonable results for small deformations and TOV solutions when $\gamma = 1$.
- WDs:
    - Magnetic field effects are relevant at *low and intermediate* density regime with respect to B.
    - Prolate (oblate) deformation without (with) Maxwell term for stable compact stars with respect to the corresponding central densities solutions at $B = 0$. Maximum masses not affected.
- Model dependence of the observables mass and radii. Need better more realistic models.

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Thank you for your attention.