The fourth family of compact stars

A Sedrakian

Introduction
Dense QCD
Constructing EoS
Sequential phase transitions
Low-mass twins and GW

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Outline

1. Introduction to compact star equilibria
2. Dense QCD and the construction of EoS
3. New equilibria via sequential phase transitions within QCD
4. Low-mass twins and GW
5. Remarks on cooling
6. Conclusions
Equilibria of compact objects

- **White dwarfs** - first family, $M \leq 1.5M_\odot$, [S. Chandrasekhar, L. Landau (1930-32)]
- **Neutron Stars** - second family, $M \leq 2M_\odot$, [Oppenhimer-Volkoff (1939)]
- **Hybrid Stars** - third family, $M \leq 2M_\odot$, [Gerlach (1968), Glendenning-Kettner (2000)]

S. Shapiro, S. Teukolsky, “Black holes, White dwarfs and Neutron Stars”
Einstein’s field equations:

\[ G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = -8\pi T_{\mu\nu}, \]

Energy-momentum tensor:

\[ T_{\mu\nu} = -P(r) g_{\mu\nu} + [P(r) + \epsilon(r)] u_\mu u_\nu \]

TOV equations:

\[ \frac{dP(r)}{dr} = -\frac{G\epsilon(r)M(r)}{c^2 r^2} \left(1 + \frac{P(r)}{\epsilon(r)}\right) \left(1 + \frac{4\pi r^3 P(r)}{M(r)c^2}\right) \left(1 - \frac{2GM(r)}{c^2 r}\right)^{-1}. \]

\[ M(r) = 4\pi \int_0^r r^2 \epsilon(r) dr. \]
The Lagrangian of QCD is written for $\psi_q = (\psi_{qR}, \psi_{qG}, \psi_{qB})^T$ as

$$\mathcal{L}_{QCD} = \overline{\psi}_q ^i (i \gamma^\mu ) (D_\mu )_{ij} \psi_q ^j - m_q \overline{\psi}_q ^i \psi_q ^i - \frac{1}{4} F^a_{\mu \nu} F^{a \mu \nu},$$

where $(D_\mu )_{ij} = \delta_{ij} \partial_\mu - ig_s t^a t^a_\mu$ , and $F^{\mu \nu} = \partial^\mu A^\nu - \partial^\nu A^\mu - 2q (A^\mu \times A^\nu)$

covariant derivative

gluons (Yang–Mills)

gluonic field (Yang–Mills) field tensor

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SOME METHODS

LQCD
$\chi$PT
pQCD
NJL-like

colour superconductors $\langle \psi C^a \gamma_5 \psi \rangle$

CFL

Picture courtesy: M. Mannarelli
Color-superconductivity within the NJL model

\[ \mathcal{L}_{NJL} = \bar{\psi}(i\gamma^\mu \partial_\mu - \hat{m})\psi + G_V(\bar{\psi}i\gamma^\mu \psi)^2 + G_S \sum_{a=0}^{8} \left[ (\bar{\psi}\lambda_a\psi)^2 + (\bar{\psi}i\gamma_5\lambda_a\psi)^2 \right] + \left[ \bar{\psi}\alpha i\gamma_5 \epsilon^{\alpha\beta\gamma}\epsilon_{abc}(\psi_C)^b_i \right] \left[ (\bar{\psi}_C)^r_\rho i\gamma_5 \epsilon^{\rho\sigma\gamma}\epsilon_{rsc}\psi^s_\sigma \right] - K \{ \det_f[\bar{\psi}(1 + \gamma_5)\psi] + \det_f[\bar{\psi}(1 - \gamma_5)\psi] \}, \]

- quarks: \( \psi^a_\alpha \), color \( a = r, g, b \), flavor \( (\alpha = u, d, s) \); mass matrix: \( \hat{m} = \text{diag}_f(m_u, m_d, m_s) \);
- other notations: \( \lambda_a, a = 1, ..., 8 \), \( \psi_C = C\bar{\psi}^T \) and \( \bar{\psi}_C = \psi^T C \), \( C = i\gamma^2\gamma^0 \).

Parameters of the model:
- \( G_S \) the scalar coupling and cut-off \( \Lambda \) are fixed from vacuum physics
- \( G_D \) is the di-quark coupling \( \simeq 0.75G_S \) (via Fierz) but free to change
- \( G_V \) and \( \rho_{tr} \) are treated as free parameters
Dense QCD

**QCD interactions pairing interactions and gaps**

\[
\Delta \propto \langle 0 | \psi^a_{\alpha \sigma} \psi^b_{\beta \tau} | 0 \rangle
\]

- Symmetric in space wave function (isotropic interaction)
- Antisymmetry in colors \(a, b\) for attraction
- Antisymmetry in spins \(\sigma, \tau\) (Cooper pairs as spin-0 objects)
- Antisymmetry in flavors \(\alpha, \beta\)

### 2SC phase:

Low densities, large \(m_s\) (strange quark decoupled)

\[
\Delta(2SCs) \propto \Delta \epsilon^{ab3} \epsilon_{\alpha \beta} \quad \delta \mu \ll \Delta,
\]

### Crystalline or gapless phases:

Intermediate densities, large \(m_s\) (strange quark decoupled)

\[
\Delta(\text{cryst.}) \propto \epsilon_{\alpha \beta} \Delta_0 e^{i \vec{Q} \cdot \vec{r}} \quad \delta \mu \geq \Delta,
\]

### CFL phase:

High densities nearly massless \(u, d, s\) quarks

\[
\Delta(CFL) \propto \langle 0 | \psi^a_{\alpha L} \psi^b_{\beta L} | 0 \rangle = -\langle 0 | \psi^a_{\alpha R} \psi^b_{\beta R} | 0 \rangle = \Delta \epsilon^{abC} \Delta \epsilon_{\alpha \beta C}.
\]
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Constructing EoS

EOS including (hyper)nuclear, 2SC and CFL phases of matter

Choose Maxwell (large surface tension) or Glendenning (low surface tension) constructions. Matching condition for Maxwell is simply

\[ P_N(\mu_B) = P_Q(\mu_B), \]

i.e., with low-density nuclear and high-density quark phases
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Synthetic equations of state with constant speed of sound

- Instead of full NJL-model EoS with 2SC-CFL transition use synthetic EoS
- Realistic DD-ME2 EoS below the deconfinement (Colucci-Sedrakian EoS)
- Parametrize synthetic EoS via Constant Speed of Sound (CSS) parameterization (Alford-Han-Prakash 2013), also Haensel-Zdunik (2012).
Relativistic DFT theory

\[ \mathcal{L} = \sum_B \bar{\psi}_B \left[ \gamma^\mu \left( i \partial_\mu - g_{BB} \omega_\mu - \frac{1}{2} g_{BB} \tau \cdot \rho_\mu \right) - (m_B - g_{BB} \sigma) \right] \psi_B \]

- B-\text{sum is over the baryonic octet } B \equiv p, n, \Lambda, \Sigma^{\pm,0}, \Xi^{-,0}
- Meson fields include \( \sigma \)-meson, \( \rho_\mu \)-meson and \( \omega_\mu \)-meson
- Leptons include electrons, muons and neutrinos for } T \neq 0
Fixing the couplings: nucleonic sector

\[ g_{iN}(\rho_B) = g_{iN}(\rho_0)h_i(x), \quad i = \sigma, \omega, \quad h_i(x) = a_i \frac{1 + b_i(x + d_i)^2}{1 + c_i(x + d_i)^2} \]
\[ g_{\rho N}(\rho_B) = g_{\rho N}(\rho_0) \exp[-a_\rho(x - 1)]. \]


<table>
<thead>
<tr>
<th></th>
<th>( \sigma )</th>
<th>( \omega )</th>
<th>( \rho )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( m_i ) [MeV]</td>
<td>550.1238</td>
<td>783.0000</td>
<td>763.0000</td>
</tr>
<tr>
<td>( g_{Ni}(\rho_0) )</td>
<td>10.5396</td>
<td>13.0189</td>
<td>3.6836</td>
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<tr>
<td>( a_i )</td>
<td>1.3881</td>
<td>1.3892</td>
<td>0.5647</td>
</tr>
<tr>
<td>( b_i )</td>
<td>1.0943</td>
<td>0.9240</td>
<td>—</td>
</tr>
<tr>
<td>( c_i )</td>
<td>1.7057</td>
<td>1.4620</td>
<td>—</td>
</tr>
<tr>
<td>( d_i )</td>
<td>0.4421</td>
<td>0.4775</td>
<td>—</td>
</tr>
</tbody>
</table>

Total number of parameters 8: boundary conditions on \( h(x) \) at \( x = 1 \).
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**Constructing EoS**

- saturation density
  \[ \rho_0 = 0.152 \text{ fm}^{-3} \]
- binding energy per nucleon
  \[ E/A = -16.14 \text{ MeV} , \]
- incompressibility
  \[ K_0 = 250.90 \text{ MeV} , \]
- symmetry energy
  \[ J = 32.30 \text{ MeV} , \]
- symmetry energy slope
  \[ L = 51.24 \text{ MeV} , \]
- symmetry incompressibility
  \[ K_{\text{sym}} = -87.19 \text{ MeV} \]

\[ K_0 = k_F^2 \frac{\partial E/A}{\partial k^2} \bigg|_{k=k_F} = 9 \rho_0^2 \frac{\partial^2 E/A}{\partial \rho^2} \bigg|_{\rho=\rho_0}, \quad S(\rho) = \frac{1}{2} \frac{\partial^2 \epsilon/\rho}{\partial \delta^2} \bigg|_{\delta=0}. \]

\[ S(\rho) = J + L \left( \frac{\rho - \rho_0}{3\rho_0} \right) + \frac{1}{2} K_{\text{sym}} \left( \frac{\rho - \rho_0}{3\rho_0} \right)^2. \]
Zero temperature equations of state of hypernuclear matter for fixed $x_{\sigma \Lambda} = 0.6164$ and a range of values $0.15 \leq x_{\sigma \Sigma} \leq 0.65$. These values generate the shaded area, which is bound from below by the softest EoS (dashed red line) corresponding to $x_{\sigma \Sigma} = 0.65$ and from above by the hardest EoS (solid line) corresponding to $x_{\sigma \Sigma} = 0.15$. 

EoS
CSS parameterization

\[ \varepsilon(p) = \begin{cases} 
\varepsilon_{NM}(p) & p < p_{trans} \\
\varepsilon_{NM}(p_{trans}) + \Delta \varepsilon + c_{QM}^{-2} (p - p_{trans}) & p > p_{trans}
\end{cases} \]

Phase diagram in \( M-R \) space

Phase diagram for hybrid star branches in the mass-radius relation of compact stars. The left panel shows schematically the possible topological forms of the mass-radius relation in each region of the diagram.

\[
\frac{\Delta \varepsilon_{\text{crit}}}{\varepsilon_{\text{trans}}} = \frac{1}{2} + \frac{3}{2} \frac{p_{\text{trans}}}{\varepsilon_{\text{trans}}}. 
\]

**EoS with sequential phase transitions**

![Graph showing pressure and energy density with phase transitions labeled as nuclear, 2SC, Δε_1, Δε_2SC, and CFL.]

**Parameters of the models:**

\[(\epsilon_1, P_1) \quad \Delta\epsilon_1, \quad \Delta\epsilon_{2SC} \quad (\epsilon_2, P_2) \quad \Delta\epsilon_2\]

Note that there are five independent parameters.
The EOS is analytically given

\[
P(\varepsilon) = \begin{cases} 
  P_1, & \varepsilon_1 < \varepsilon < \varepsilon_1 + \Delta\varepsilon_1 \\
  P_1 + s_1 [\varepsilon - (\varepsilon_1 + \Delta\varepsilon_1)], & \varepsilon_1 + \Delta\varepsilon_1 < \varepsilon < \varepsilon_2 \\
  P_2, & \varepsilon_2 < \varepsilon < \varepsilon_2 + \Delta\varepsilon_2 \\
  P_2 + s_2 [\varepsilon - (\varepsilon_2 + \Delta\varepsilon_2)], & \varepsilon > \varepsilon_2 + \Delta\varepsilon_2.
\end{cases}
\]

Need to specify:

- the two speeds of sounds: \( s_1 \) and \( s_2 \)
- the point of transition from NM to QM \( \varepsilon_1, P_1 \)
- the magnitude of the first jump \( \Delta\varepsilon_1 \)
- the size of the 2SC phase, i.e., the second transition point \( \varepsilon_2, P_2 \)
- the size of the second jump \( \Delta\varepsilon_2 \)
Varying parameters of EoS with sequential phase transition
Sequential phase transitions

.... and resulting topologies of sequences

![Graph showing sequential phase transitions](image)
The stellar mass as a function of the star’s central pressure for four different values of $\Delta \epsilon_2$. The other parameters of the EOS are fixed at $P_1 = 1.7 \times 10^{35}$ dyn cm$^{-2}$, $s_1 = 0.7$, $\Delta \epsilon_{2SC}/\epsilon_1 = 0.27$, $\Delta \epsilon_1/\epsilon_1 = 0.6$, and $s_2 = 1$. The vertical dotted lines mark the two phase transitions at $P_1$ and $P_2$. Stable branches are solid lines, unstable branches are dashed lines. We see the emergence of separate 2SC and CFL hybrid branches along with the occurrence of triplets.
Sequential phase transitions

... and resulting topologies of mass-radius relations
The \( M-R \) relations for the parameter values defined above. We have fixed the properties of the nuclear \( \rightarrow \) 2SC transition and the speed of sound in 2SC and CFL matter. For the 2SC \( \rightarrow \) CFL transition we have fixed the critical pressure and we vary the energy-density discontinuity \( \Delta \varepsilon_2 \). The separate 2SC and CFL hybrid branches are clearly visible, along with the occurrence of triplets.
Profiles of triplets stars (same mass)

The profiles (here the log of pressure as a function of the internal radius) of the three members of a triplet with masses $M = 1.975 \, M_\odot$. Here “N” means the nuclear phase. The parameter values are as above, with $\Delta \varepsilon_2/\Delta \varepsilon_1 = 0.23$. 
## Stability conditions for our models

<table>
<thead>
<tr>
<th>$\Delta \varepsilon_2 / \Delta \varepsilon_1$</th>
<th>$\Delta \varepsilon_1 / \varepsilon_1$</th>
<th>0.4</th>
<th>0.5</th>
<th>0.6</th>
<th>0.7</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>$s, s$</td>
<td>$s, s$</td>
<td>$us, s$</td>
<td>$u, us$</td>
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<td>0.2</td>
<td>$s, s$</td>
<td>$s, s$</td>
<td>$us, us$</td>
<td>$u, us$</td>
<td>N-2SC</td>
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<tr>
<td>0.3</td>
<td>$s, s$</td>
<td>$s, s$</td>
<td>$us, us$</td>
<td>$u, us$</td>
<td>N-2SC;N-CFL</td>
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<tr>
<td>0.4</td>
<td>$s, s$</td>
<td>$s, us$</td>
<td>$us, u$</td>
<td>$u, u$</td>
<td>2SC-CFL</td>
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<td></td>
</tr>
<tr>
<td>0.5</td>
<td>$s, s$</td>
<td>$s, us$</td>
<td>$us, u$</td>
<td>$u, u$</td>
<td>2SC-CFL</td>
</tr>
</tbody>
</table>

In each entry stable/unstable branches are referred by $s/u$, the 2SC and CFL phases are separated by comma, and the pressure increases from left to right. The presence of twin hybrid configurations or triplet configurations is marked by the underbraces with information about the involved phases (“N” means nuclear).
Lower mass triplets

- Low-mass triplets via early transition NM → QM
- Still 2-solar mass members possible but only with the NM-2SC-CFL composition
GW170817: First gravitational waves from a neutron star merger
(Ligo-Virgo-Collaboration)

The associated EM events observed by over 70 observatories:

- +2 sec gamma ray burst is detected
- +10 h 52 min bright source in optical
- +11 h 36 min infrared emission; +15 h ultraviolet
- +9 days X-rays; +16 days radio
TABLE I. Source properties for GW170817: we give ranges encompassing the 90% credible intervals for different assumptions of the waveform model to bound systematic uncertainty. The mass values are quoted in the frame of the source, accounting for uncertainty in the source redshift.

|                          | Low-spin priors (|\chi| \leq 0.05)               | High-spin priors (|\chi| \leq 0.89)               |
|-------------------------|----------------|----------------|
| Primary mass \( m_1 \)  | 1.36–1.60 \( M_\odot \) | 1.36–2.26 \( M_\odot \) |
| Secondary mass \( m_2 \) | 1.17–1.36 \( M_\odot \) | 0.86–1.36 \( M_\odot \) |
| Chirp mass \( \mathcal{M} \) | 1.188_{-0.002}^{+0.004} \( M_\odot \) | 1.188_{-0.002}^{+0.004} \( M_\odot \) |
| Mass ratio \( m_2/m_1 \) | 0.7–1.0 | 0.4–1.0 |
| Total mass \( m_{\text{tot}} \) | 2.74_{-0.01}^{+0.04} \( M_\odot \) | 2.82_{-0.09}^{+0.47} \( M_\odot \) |
| Radiated energy \( E_{\text{rad}} \) | > 0.025\( M_\odot \) \( c^2 \) | > 0.025\( M_\odot \) \( c^2 \) |
| Luminosity distance \( D_L \) | \( 40^{+8}_{-14} \) Mpc | \( 40^{+8}_{-14} \) Mpc |
| Viewing angle \( \Theta \) | \( \leq 55^\circ \) | \( \leq 56^\circ \) |
| Using NGC 4993 location | \( \leq 28^\circ \) | \( \leq 28^\circ \) |
| Combined dimensionless tidal deformability \( \tilde{\Lambda} \) | \( \leq 800 \) | \( \leq 700 \) |
| Dimensionless tidal deformability \( \Lambda(1.4M_\odot) \) | \( \leq 800 \) | \( \leq 1400 \) |
New nuclear physics laboratories

- extreme high temperatures $\sim 100 \text{ MeV}$
- supra-nuclear densities $\sim 5 \times n_s$
- high and differential rotation rates

pictures courtesy: J. Pappenfort
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(a) Mass-radius relation for hybrid stars with a single QCD phase translation, with different hadronic envelopes. (b) Mass-deformability relation for stars featuring nucleonic envelopes. The inset shows the results for the case $M^H_{\text{max}}/M_\odot = 1.20$. 
(a) Mass-radius relation for hybrid stars with a single QCD phase translation, with different hadronic envelopes. (b) Mass-deformability relation for stars featuring nucleonic envelopes. The inset shows the results for the case $M_{\text{max}}^H/M_\odot = 1.20$. 
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Low-mass twins and GW

a) Tidal deformabilities of compact objects in the binary with chirp mass $\mathcal{M} = 1.186M_\odot$
(b) Prediction by an EoS with maximal hadronic mass $M_{\text{max}}^H = 1.365M_\odot$. The inset shows the mass-radius relation around the phase transition region. The circles $M_2$ are two possible companions for circle $M_1$, generating two points in the $\Lambda_1$-$\Lambda_2$ curves while one point is located below the diagonal line.
The case of double phase transition a) Tidal deformabilities of compact objects in the binary with chirp mass $\mathcal{M} = 1.186M_\odot$ (b) Prediction by an EoS with maximal hadronic mass $M^H_{\text{max}} = 1.365M_\odot$. The inset shows the mass-radius relation around the phase transition region. The circles $M_2$ are two possible companions for circle $M_1$, generating two points in the $\Lambda_1$-$\Lambda_2$ curves while one point is located below the diagonal line.
Near future experimental advances:
- NICER (X-ray studies of neutron stars)
- LIGO-VIRGO (Gravitational waves from BNS and pulsars)
- SKA (radio timing of pulsars)

Theory questions:
- Dense QCD phases: static and dynamic properties
- Astrophysical properties of compact stars with quark phases
- Triplets and twins
- Gravity wave and QCD

Thank you for your attention!