First-order phase transition from hypernuclear matter to deconfined quark matter obeying new constraints from compact star observations

Mahboubeh Shahrbaft
Collaborators: David Blaschke, Ana Gabriela Grunfeld, Hamid Reza Moshfegh

The Modern Physics of Compact Stars and Relativistic Gravity
September 18, 2019, Yerevan, Armenia
The number of nucleons is supposed to be infinite.

Coulomb interaction is disregarded because of the strong interaction between nucleons.

The density of nuclear matter is supposed to be finite

\[ \rho = \lim_{N,V \to \infty} \frac{N}{V} \]

\[
\begin{array}{|c|c|}
\hline
\rho_0 (fm^{-3}) & 0.1748 \\
E_0/A (MeV) & -15.58 \\
E_{sym} (MeV) & 39.9 \\
K_0 & 295.77 \\
\hline
\end{array}
\]
For PSRJ0740+6620

\[ M_{max} = 2.17^{+0.11}_{-0.10} M_\odot \]

for the binary neutron star merger GW170817

\( R(1.6M_\odot) > 10.7 \text{ km} \)

&

\( R(1.4M_\odot) < 13.6 \text{ km} \)
Hyperon Puzzle
Phase Transition From Hyper Nuclear Matter to Deconfined Quark Matter as a Solution to the Hyperon Puzzle

Initiation of a new collaboration that joins different domains of state-of-the-art expertise

- **LOCV**
  For hadronic phase

- **nl-NJL**
  For quark phase
Hamiltonian of nuclear matter: \( H = \sum_i \frac{p_i^2}{2m_i} + \sum_{i \neq j} V(ij) \)

Trial wave function: \( \Psi(1 \ldots A) = F(1 \ldots A) \Phi(1 \ldots A) \)

\[
E = \langle H \rangle = \frac{1}{N} \frac{\langle \Psi | H | \Psi \rangle}{\langle \Psi | \Psi \rangle} = E_1 + E_{MB} \approx E_1 + E_2
\]

- A pure variational method in configuration space
- Generalized to finite temperature
- Calculation of correlation functions
- Using both central and tensor correlation functions
- Energy per baryon and correlation functions are state-dependent
- Using normalization condition as the only constraint
nl-NJL model
Nonlocal Nambu–Jona-Lasinio model

Characteristics

- Nonlocal covariant extension of the NJL model
- Quark fields interact via nonlocal (momentum dependent) vertices
- Nonlocal interactions regularize the model in such a way there is not need to introduce sharp cutoffs

nl-NJL model

- Constant coefficients (model A)
- Density-dependent coefficients (model B)

First Order Phase Transition (PT) by a Maxwell construction

\[\mu_H = \mu_Q = \mu_c\]
\[T_H = T_Q = T_c\]
\[P_H(\mu_B, \mu_e) = P_Q(\mu_B, \mu_e) = p_c\]
Model B
Symmetric Matter
Replacement Interpolation Method (RIM)
A Mixed Phase Approach

\[ P_M(\mu) = a(\mu - \mu_c)^2 + b(\mu - \mu_c) + P_c + \Delta P. \]

\[ P_M(\mu_c) = P_c + \Delta P = P_M. \]

\[ P_M(\mu_{cH}) = P_H(\mu_{cH}) = P_H, \]
\[ P_M(\mu_{cQ}) = P_Q(\mu_{cQ}) = P_Q, \]
\[ n_M(\mu_{cH}) = n_H(\mu_{cH}), \]
\[ n_M(\mu_{cQ}) = n_Q(\mu_{cQ}). \]
RIM for Model A
Main Results

1. Model B with density dependent parameters allows for an intermediate hypernuclear matter phase in the hybrid star, between the nuclear and color superconducting quark matter phase, while in model A such a phase cannot be realized because the phase transition onset is at low densities, before the hyperon threshold density is passed.

2. For model A the cases with a sufficiently strong repulsive vector mean field ($\eta > 0.12$) which produce reasonable hybrid star EoS have also a phase transition under isospin-symmetric conditions. For $\eta = 0.12$ the critical density is $n_c = 0.79 \, fm^{-3}$ and for $\eta = 0.15$ it is $n_c = 0.98 \, fm^{-3}$. For the less repulsive vector mean fields ($\eta \leq 0.11$) there is no deconfinement transition in symmetric matter!

3. For model B in which a density-dependent bag pressure serves as a confining mechanism at low densities, a deconfinement phase transition under isospin-symmetric conditions is predicted for all considered parametrizations (set 1 - set 4) at densities between $2.2 \, n_0$ and $2.7 \, n_0$.

4. Using the RIM, for model A the low density problem (no confinement) is cured and model B is now more realistic with a mixed phase.
\[ f(ij) = \sum_{\alpha p=1}^{3} f_{\alpha}^{p}(ij) O_{\alpha}^{p}(ij) \]

\[ \alpha = \{J, L, S, T, T_z\} \]

\[ O_{\alpha}^{p}(ij) = 1, \quad \frac{1}{6} (S_{12} + 4P_t), \quad \frac{1}{6} (2P_t - S_{12}) \]

\[ S_{12} = 3 (\sigma_1 \cdot \hat{r})(\sigma_2 \cdot \hat{r}) - \sigma_1 \cdot \sigma_2 \]

**LOCV Method:** Lowest Order
Constrained Variational Method
The only constraint in LOCV method is renormalization condition of wave functions.

\[ |ij\rangle = |k_1, 1/2, m_{\sigma_1}, 1/2, m_{\tau_1}, k_2, 1/2, m_{\sigma_2}, 1/2, m_{\tau_2}\rangle \]

\[ \langle \Psi | \Psi \rangle = 1 - \sum_{ij} \langle ij | F_p^2 - F^2 | ij - ji \rangle : \chi = \frac{1}{N} \sum_{ij} \langle ij | F_p^2 - F^2 | ij - ji \rangle = 0 \]

\[ F_p = \begin{cases} \left( 1 - \frac{9}{2} \left( \frac{I_1(K_0r)}{K_0r} \right)^2 \right)^{-\frac{1}{2}} & T_z = \pm 1 \\ 1 & T_z = 0 \end{cases} \]

\[ E_2 = \int dr \left[ G(f^2(r)) + S(f(r)) - \lambda(f(r)) \right] = \int dr \left[ L(f'(r), f(r)) \right], \delta E_2 = 0 \]

\[ \frac{\partial \mathcal{L}}{\partial f} - \frac{\partial}{\partial r} \frac{\partial \mathcal{L}}{\partial f'} = 0 \]