Lattice results on exotics with hidden charm and bottom

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Implications of LHCb measurements and future prospects
16.10.2019
Hadrons with hidden charm and bottom; focus on exotic and lattice results:

- states well below strong decay threshold or treated as strongly stable \textit{“doable”}
- states above or just below one threshold \textit{“more difficult, but doable”}
- state above several threshold \((Z_c, Z_b, P_{c, ...}) \textit{“challenging”}

- lattice predictions of yet undiscovered exotic hadrons (but with different flavor than indicated above)
Lattice QCD

\[ L_{QCD} = -\frac{1}{4} G_{\mu\nu}^a G_{\mu\nu}^a + \sum_{q=u,d,s,c,b,t} \bar{q} i \gamma_\mu (\partial^\mu + i g_s G_{a}^\mu T^a) q - m_q \bar{q} q \]

\[ \langle C \rangle = \int \mathcal{D}G \mathcal{D}q \mathcal{D}\bar{q} \mathcal{C} e^{-S_{QCD}/\hbar} \]

discretized finite Euclidian space-time

Determine energies of eigenstates \( E_n \) and overlaps

charmonium: \( J^{PC} : \bar{c} \Gamma c , \ (\bar{c} \Gamma_1 u)(\bar{u} \Gamma_2 c) = D\bar{D} , \ [\bar{c} \Gamma_3 \bar{u}][c \Gamma_4 u] \)

\[ C_{ij}(t) = \langle 0 | \Theta_i(t) \Theta_j^+(0) | 0 \rangle = \sum_n \langle 0 | \Theta_i | n \rangle e^{-E_n t} \langle n | \Theta_j^* | 0 \rangle \]

\( J^{PC} = 1^{--} : \ E_1(\vec{p} = 0) = m_{J/\psi} \)

\( \bar{c} \bar{c} \) and \( bb \) annihilation omitted for all result in this talk.

Then hadrons below \( DD \) or \( BB \) are strongly stable

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\[ E_n(\vec{p} = 0) = m_n \]

States well below thresholds 
or 
treated as strongly stable

“doable”
Excited bottomonia, bottomonium hybrids

$b\bar{b}$ hybrids
$m_{\text{hybrid}} \geq 10.9 \text{ GeV}$


lattice QCD: $m_{\pi} = 400 \text{ MeV}$
relativistic $b$-quark:
main challenge $a_{mb}$ errors
states above $B\bar{B}$ threshold treated as strongly stable
most of states below $B\bar{B}$ experimentally discovered

previous lattice results on excited $b\bar{b}$ spectrum
[Wurtz, Lewis, Woloshyn, 1505.04410, PRD]

EFT+lattice prediction of hybrids [Brambilla, Lai, Segovia, Castella, Vario, 1805.07713, PRD 2019]
charmonium hybrids: backup slides
Non-existence of strongly stable fully beautiful tetraquark

Lattice QCD: No indication for strongly stable state (below threshold) with

\[ J^{PC} = 0^{++}, 1^{-+}, 2^{++} \]

threshold \( \eta_b \eta_b, \eta_b \Upsilon, \Upsilon \Upsilon \)

[Hughes, Eichten, Davies, HPQCD, 1710.03236, PRD 2018]
**Discovery of $B_c^*(2S)$ & confirmation of $B_c(2S)$**

$m[B_c(2S)]$:

$6872.1(1.3)(0.1)(0.8)$ MeV

$B_c^*(2S)$ peak at $M=m[B_c(2S)] - \Delta M$

$6841.2(0.6)(0.1)(0.8)$ MeV

agrees with [CMS, 1902.00571, PRL]

$B_c^*(2S) \rightarrow B_c^* \pi^+ \pi^-$, $B_c^* \rightarrow B_c \gamma$ photon undetected

**Lattice QCD:**

[HPQCD, 1207.5149, Lytle, talk at QWG19]

c: relativistic

b: NRQCD

$\Delta M = \{m[B_c^*] - m[B_c]\}$

$- \{m[B_c^*(2S)] - m[B_c(2S)]\}$

$m[B_c^*(2S)]$ determined using

$\Delta M$ from experiment and

$m(B_c^*)$-$m(B_c)$ from lattice (HPQCD)
States above or slightly below one threshold

“more difficult, but doable”
All experimentally discovered exotic hadrons are strongly decaying resonances!

**Hadronic resonances and shallow bound states from lattice (near/above one threshold)**

\[ \mathcal{O} \]

\[ E(H_1 H_2) \]

\[ m \]

\[ \Gamma \]

\[ \sigma(E) \propto |T(E)|^2 \]

\[ T_B(E) \propto \frac{1}{E^2 - m_B^2} \]

\[ T_R(E) = \frac{-m_R \Gamma}{E^2 - m_R^2 + i m_R \Gamma} \]

\[ T_B(E = m_B) = \infty \]

energy of eigenstate

scattering matrix for real E

analytic relation: Luscher 1991

continuation to complex E

location of poles in complex E plane

\[ E = E_{cm} \]

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Charmonia with $J^{PC}=3^{−}$ and $1^{−}$

LHCb 2019, 1903.12240, JHEP 2019


widths of resonances:
- $\psi(3770)$
  \[ \Gamma = \frac{g^2}{6\pi} \frac{p^3}{s} \]

<table>
<thead>
<tr>
<th>g</th>
<th>lat</th>
<th>exp</th>
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<td></td>
<td>16.0$^{+2.1}_{-0.2}$</td>
<td>18.7 ± 0.9</td>
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- $X(3842)$
  to narrow to resolve in this lat. sim.
Aim: look for poles in $D D^*$ scattering matrix

- Lattice QCD
  - first evidence [S.P., Leskovec, 1307.5172, PRL 2013]
  - Fock components: [Padmanath, Lang, S.P., 1503.03257, PRD 2015]

  crucial: $D \bar{D}^*$, $\bar{c}c$, less important: $(\bar{c}q)(cq)$

  no charged partner found up to $m=4.2$ GeV (in agreement with exp)
  unfortunately, no other published lattice paper on $X(3872)$ till now

- Dyson-Schwinger / Bethe-Salpeter approach
  [Wallbott, Eichmann, Fischer, 1905.02615, PRD 2019]

  location of pole in the scattering matrix

  - pole for X found although $\overline{c}c$ Fock component omitted, $qg$ annihilation omitted
    (in contrary: lattice studies find that $\overline{c}c$ is crucial for getting pole related to X)

- exp evidence that X is not completely molecular:
  ideal combination of $I=0,1$ (molecule) would lead to completely dominant rate to $J/\psi \rho$
  (since $J/\psi \omega$ is 7 MeV above and $\omega$ is very narrow), while exp rates are comparable

  $\overline{c}c$: $(I=0)$
  molecule: $(I=0) + (I=1)$
  $X \rightarrow J/\psi \omega, J/\psi \rho$

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Hadrons that strongly decay to several final states
Scattering in two or more channels
“challenging”

examples: all experimentally discovered Zc, Zb, Pc
Extracting scattering matrix from lattice

**Resonance above one threshold**

\[ R \rightarrow H_1 H_2 \quad T(E) \quad \leftarrow E_n \]

Lattice simulation of one-channel scattering via Luscher’s method: doable

**Resonance above two or more thresholds**

\[ R \rightarrow H_1 H_2, \quad H_1' H_2' \]

\[
T(E) = \begin{pmatrix}
T_{aa}(E) & T_{ab}(E) \\
T_{ba}(E) & T_{bb}(E)
\end{pmatrix}
\]

\[ \leftarrow E_n \]

Lattice simulation of coupled-channel scattering via Luscher’s method: challenging

- several coupled channels studied in the light-quark sector (Hadron Spectrum collaboration)
- only simulations for hadrons with heavy quarks
  - excited D mesons [Moir, Peardon, Ryan, Thomas, Wilson, 1607.07093, JHEP 2016]
  - \( Z_c \) channel [Chen et al., CLQCD, 1907.03371]
- final conclusions on many interesting states therefore not available (yet)
$P_c = \text{uud} \bar{c}c \rightarrow (\text{uud}) (\bar{c}c)$: $p J/\Psi$, ... \\

$\rightarrow (\text{udc}) (\bar{c}u)$: $\Sigma^+_c \bar{D}^0$, ....

Indications that $\Sigma^+_c \bar{D}^{(*)}$ molecular component is important:

- **experiment** finds them slightly below those thresholds

- supported by **phenomenological models** with $\rho/\omega$ exchange predicted 2010-2012 [Wu, Molina, Oset, Zou, 1007.0573, PRL; Wu et al., 1202.1036. PRC, Yang et al, 1105.2901, Wang et al, 1101.0453, PRC]

- **Lattice QCD** addressed simplified question:

  Do $P_c$ resonances appear in one-channel $p \ J/\psi \rightarrow P_c \rightarrow p \ J/\psi$ scattering if it is decoupled from other channels?

  Answer: No [Skerbis, S. P., 1811.02285, PRD 2019]

  $T(E)\approx0$ within large errors, small interaction, no resonance

  This indicates that coupling of $p J/\psi$ channel with other two-hadron channels is likely responsible for $P_c$ in experiment (in line with LHCb result)

  LHCb preliminary

  data  
  total fit  
  background 

  $P_c(4312)^*$  $P_c(4440)^*$  $P_c(4457)^*$ 

  $\Sigma_c(\frac{1}{2}^+) \bar{D}(0^-) \rightarrow J^p = \frac{1}{2}^-$  $\Sigma_c(\frac{1}{2}^+) \bar{D}^*(1^-) \rightarrow J^p = \frac{1}{2}^-, \frac{3}{2}^-$
Consensus on the nature of Zc(3900) has not been achieved

- re-analysis of all experimental data is compatible with several scenarios
  - resonance pole above threshold, bound state, virtual bound state,
  - kinematical enhancement via triangular diagram

  [Pilloni et al., 1612.06490, PLB 2017]

- Lattice QCD:
  - extract scattering matrix for coupled channel scattering $J/\psi \pi$, $D\bar{D}^*$
  - Zc*(3900) coupled-channel effect due to sizable $J/\Psi \pi$ and $D\bar{D}^*$ coupling, not genuine resonances (i.e. pole on the unphysical sheet above $D\bar{D}^*$ threshold)

  [Ikeda et al., HALQCD, 1602.03465, PRL]

  [Chen et al., CLQCD, 1907.03371]

- Luscher’s method: $T(E) \approx$ small, small interaction
  - no narrow resonance behavior found near $D\bar{D}^*$ threshold
  - in line with previous lattice study that did not extract the scattering matrix

**Z_{b}^{+}(10610), Z_{b}^{+}(10650)**

\[ Z_{b}^{+} \rightarrow \Upsilon(1S,2S,3S)\pi^{+}, \quad h_{b}(1S,2S)\pi^{+}, \quad B^{(*)}\bar{B}^{*} \]

Indications that molecular B B* in \( Z_{b}^{+}(10610) \) is crucial:

- lies near \( m_B+m_{B^*} \) threshold
- \( \text{Br}(B B^*)=85 \% \) although this mode is phase space suppressed
- molecule \( (S_{bb}=1) \otimes (S_{qq}=0) \leq (S_{bb}=0) \otimes (S_{qq}=1) \)
  this makes it natural that \( Z_{b} \) decays comparably to \( \Upsilon \) (\( S_{bb}=1 \)) and \( h_{b} \) (\( S_{bb}=0 \))

- Exploratory (!) lattice study of \( (S_{bb}=1) \otimes (S_{qq}=0) \) component with static b-quarks
  [S.P., Bahtiyar, Petkovic, 1909.02356], inspired by [Peters, Bicudo, Wagner, 1602.07621]

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**Strong attraction between B and \( \bar{B}^{*} \)**

for \( r=0.1-0.4 \text{ fm} \).

Might be responsible for existence of \( Z_{b} \)

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**Fock components**
Solving Schrödinger equation for $B B^*$ system with this $V(r)$.

Observed attraction leads to virtual $B B^*$ bound state slightly below threshold

$$\text{Re}[E_{Z_b}] = -32^{+29}_{-5} \text{ MeV}$$

This pole leads to peak in $N_{B B^*}$ above threshold (similar to $\exp$)

- Virtual bound state consistent with reanalysis of $\exp$ data

- So far $Z_b$ found only by Belle

- Could LHCb search for $Z_b$ in inclusive final state with $B B^*$ ?

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Lattice results on exotics with hidden bottom and charm
Lattice predictions of yet unobserved hadrons

• there are no reliable lattice predictions for yet-unobserved $\bar{Q}Q\bar{q}q$, $\bar{Q}Qqq$ ($Q = c, b$) since these states likely lie above several thresholds (very challenging)

• Instead, I list predictions of interesting states with different quark content that lie below strong threshold (doable)
Scalar $B_{s0}$ and axial $B_{s1}$

partner of scalar $D_{s0}(2317)$

bound state $B_{s0}^*$ found

lattice QCD, taking into account effects of $BK^{(*)}$ threshold

[C. Lang, D. Mohler, S.P., R. Woloshyn: 1501.0164, PLB2015]
Strongly stable doubly bottom tetraquarks

- $\bar{b}b\bar{d}d$ ($J^P = 1^+, I = 0$)
- $\bar{b}b\bar{u}u$ ($J^P = 1^+$)

**Diagram:**

1. This work [LAT]
2. Rosner, et al [23]
3. Francis, et al [25] [LAT]
4. Bicudo, et al [60] [LAT]
6. Cheung, et al [80] [LAT]

**Graph:**

- $\Delta E$ (MeV) vs. quark combinations

**Legend:**

- $BB^*$
- $B_sB^*$

**Notes:**

- Most firm prediction of a manifestly exotic hadron from lattice and other approaches.

- Taken from Junnarkar, Mathur, Padmanath [1810.12285]
Strongly and EM stable di-baryons

lattice QCD: Junnarkar, Mathur, [1906.06054, PRL 2019]

\[ \Omega_{ccc}^{J=3/2} \quad \Omega_{bhb}^{J=3/2} \quad \Omega_{sss}^{J=3/2} \quad \Omega_{bhb}^{J=3/2} \quad \Omega_{css}^{J=1/2} \quad \Omega_{ccs}^{J=1/2} \]

\[ J^P=1^+ \]

deuterium-like

strong decay threshold
Conclusions

• Compliments to experimental colleagues for discovering a number of conventional and unconventional hadrons!

• Masses of ground and excited hadrons: lattice results and exp agree well

• Lattice QCD can extract scattering matrices for scattering of hadrons: their poles give information on resonances, bound states and virtual bound states

• predictions for many yet undiscovered hadrons

• understanding conventional and exotic states above several thresholds requires extraction of coupled-channel scattering matrices from lattice ...
  Challenging, but hopefully forthcoming
Backup

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Lattice results on exotics with hidden bottom and charm
Charmonium resonances in DD from LHCb: first discovery of charmonium with $J=3$

$m_{X(3842)} = 3842.71 \pm 0.16 \pm 0.12 \text{ MeV}/c^2$,  
$\Gamma_{X(3842)} = 2.79 \pm 0.51 \pm 0.35 \text{ MeV}$,

$LHCb$ 2019

1903.12240

JHEP 2019

$J^{PC}$ not experimentally measured

$LHCb$ paper:

"The narrow natural width and the mass of the $X(3842)$ state suggest the interpretation as charmonium state with $J^{PC} = 3^{-+}$" 

Quark model quantum numbers:

$$n^{2s+1}l_J = 1^3 D_3$$
$Z_b^+(10610), Z_b^+(10650)$

- Lattice study, continued
  [S.P., Bahtiyar, Petkovic, 1909.02356]

$V(r)$ between $B$ and $B^*$
- $a=0.12$ fm
- $V(r<0.1 \text{ fm}) = ?$

Solving Schrodinger equation for $BB^*$ system with this $V(r)$.

Observed attraction leads to

a virtual bound state just below threshold
\[ \text{Re}[E_{Zb}] = -32^{+29}_{-5} \text{ MeV} \]

and also to a deep bound state
\[ \text{Re}[E_{Zb}] = -403 \pm 70 \text{ MeV} \]

- Could LHCb search for $Z_b$ in inclusive final state with $B\bar{B}^*$ ?

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Belle PRD 91 (2015) 072003
nothing claimed by Belle;
significant “bump” could perhaps
emerge at higher statistics

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LaNce study, continued
[S.P., Bahyar, Petkovic, 1909.02356]

\[ R[E_{Zb}] = -32^{+29}_{-5} \text{ MeV} \]
Excited charmonia, charmonium hybrids

Part of
$G$-wave

Red and blue are candidates for hybrids with excited glue

Most of these states ($J=3,4$ or exotic $J^{PC}=1^{+},2^{+},...$) yet to be experimentally discovered!!

Masses of hybrids in rough agreement with EFT+lattice

[ Brambilla, Lai, Segovia, Castella, Vario, 1805.07713, PRD 2019 ]

Prediction also for $bb$ hybrids

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