Pentaquark states

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Meson baryon interaction. SU(3) chiral Lagrangians Local hidden gauge approach Extension to the charm sector Ω_c states Hidden charm pentaquark states Ξ_{cc} states Predictions for Ω_b and hidden charm strange states

Observation of Five New Narrow Ω_c^0 States Decaying to $\Xi_c^+ K^-$



The $\Xi_c \text{ K}$ -mass spectrum is studied with a sample of pp collision data by LHCb , PRL 017

Five clean narrow peaks are obtained $\Omega_c(3000)^0$, $\Omega_c(3050)^0$, $\Omega_c(3066)^0$, $\Omega_c(3090)^0$, and $\Omega_c(3119)^0$

| Resonance | Mass (MeV) | Γ (MeV) |
|------------------------|--|-----------------------|
| $\Omega_c(3000)^0$ | $3000.4 \pm 0.2 \pm 0.1^{+0.3}_{-0.5}$ | $4.5\pm0.6\pm0.3$ |
| $\Omega_c(3050)^0$ | $3050.2 \pm 0.1 \pm 0.1 \substack{+0.3 \\ -0.5}$ | $0.8\pm0.2\pm0.1$ |
| | | <1.2 MeV, 95% C.L. |
| $\Omega_c(3066)^0$ | $3065.6 \pm 0.1 \pm 0.3^{+0.3}_{-0.5}$ | $3.5 \pm 0.4 \pm 0.2$ |
| $\Omega_{c}(3090)^{0}$ | $3090.2 \pm 0.3 \pm 0.5 \substack{+0.3 \\ -0.5}$ | $8.7 \pm 1.0 \pm 0.8$ |
| $\Omega_{c}(3119)^{0}$ | $3119.1 \pm 0.3 \pm 0.9^{+0.3}_{-0.5}$ | $1.1\pm0.8\pm0.4$ |
| | 010 | <2.6 MeV 95% C L |

Observation of a Narrow Pentaquark State, $P_c(4312)^+$ and of the Two-Peak Structure of the $P_c(4450)^+$



[35] J.-J. Wu, R. Molina, E. Oset, and B. S. Zou PRL 2010

[36] J.-J. Wu, R. Molina, E. Oset, and B. S. Zou PRC 2011

[37] C. W. Xiao, J. Nieves, and E. Oset

PRD 2013

| State | M [MeV] | Γ [MeV] |
|---------------|--------------------------------|-------------------------------|
| $P_c(4312)^+$ | $4311.9 \pm 0.7^{+6.8}_{-0.6}$ | $9.8 \pm 2.7^{+3.7}_{-4.5}$ |
| $P_c(4440)^+$ | $4440.3 \pm 1.3^{+4.1}_{-4.7}$ | $20.6 \pm 4.9^{+8.7}_{-10.1}$ |
| $P_c(4457)^+$ | $4457.3 \pm 0.6^{+4.1}_{-1.7}$ | $6.4\pm2.0^{+5.7}_{-1.9}$ |

LHCb, PRL 2019

bound states [32–34]. The masses of the $P_c(4312)^+$ and $P_c(4457)^+$ states are approximately 5 MeV and 2 MeV below the $\Sigma_c^+ \bar{D}^0$ and $\Sigma_c^+ \bar{D}^{*0}$ thresholds, respectively, as illustrated in Fig. 6, making them excellent candidates for bound states of these systems. The $P_c(4440)^+$ could be the second $\Sigma_c \bar{D}^*$ state, with about 20 MeV of binding energy, since two states with $J^P = 1/2^-$ and $3/2^-$ are possible. In fact, several papers on hidden-charm states created dynamically by charmed meson-baryon interactions [35–37] were published well before the first observation of the P_c^+ structures [1], and some of these predictions for $\Sigma_c^+ \bar{D}^0$ and $\Sigma_c^+ \bar{D}^{*0}$ states [32–34] are consistent with the observed narrow P_c^+ states. Such an interpretation of the $P_c(4312)^+$

Meson baryon interaction:

Chiral Lagrangian

$$\mathcal{L}^{B} = \frac{1}{4f_{\pi}^{2}} \langle \bar{B}i\gamma^{\mu} [(\Phi \partial_{\mu} \Phi - \partial_{\mu} \Phi \Phi) B - B(\Phi \partial_{\mu} \Phi - \partial_{\mu} \Phi \Phi)] \rangle$$

$$\Phi = \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^{0} + \frac{1}{\sqrt{6}}\eta & \pi^{+} & K^{+} \\ \pi^{-} & -\frac{1}{\sqrt{2}}\pi^{0} + \frac{1}{\sqrt{6}}\eta & K^{0} \\ K^{-} & \bar{K}^{0} & -\frac{2}{\sqrt{6}}\eta \end{pmatrix} \qquad \qquad B = \begin{pmatrix} \frac{1}{\sqrt{2}}\Sigma^{0} + \frac{1}{\sqrt{6}}\Lambda & \Sigma^{+} & p \\ \Sigma^{-} & -\frac{1}{\sqrt{2}}\Sigma^{0} + \frac{1}{\sqrt{6}}\Lambda & n \\ \Xi^{-} & \Xi^{0} & -\frac{2}{\sqrt{6}}\Lambda \end{pmatrix}$$

Coupled channels:

SU(3)

$$K^{-}p \ \bar{K}^{0}n, \pi^{0}\Lambda, \pi^{0}\Sigma^{0}, \pi^{+}\Sigma^{-}, \pi^{-}\Sigma^{+}, \eta\Lambda, \eta\Sigma^{0}, K^{+}\Xi^{-}, K^{0}\Xi^{0}$$
$$V_{ij} = -C_{ij}\frac{1}{4f^{2}}(k^{0} + k'^{0})$$

$$\frac{k}{p} + \frac{k}{p} + \frac{k}{p} + \frac{k}{p} + \frac{T}{p} + \frac{T}$$



Equivalent method:
Local hidden gauge
Approach. M. Bando,
Phys. Rept 1988

$$\mathcal{L}_{VPP} = -ig\langle [\Phi, \partial_{\mu}\Phi]V^{\mu} \rangle, \qquad \mathcal{L}_{BBV} = g\left(\langle \bar{B}\gamma_{\mu}[V^{\mu}, B] \rangle + \langle \bar{B}\gamma_{\mu}B \rangle \langle V^{\mu} \rangle \right)$$

$$g=M_{V}/2 f_{\pi}$$

$$\Phi = \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^{0} + \frac{1}{\sqrt{6}}\eta & \pi^{+} & K^{+} \\ \pi^{-} & -\frac{1}{\sqrt{2}}\pi^{0} + \frac{1}{\sqrt{6}}\eta & K^{0} \\ K^{-} & \bar{K}^{0} & -\frac{2}{\sqrt{6}}\eta \end{pmatrix} \qquad \qquad B = \begin{pmatrix} \frac{1}{\sqrt{2}}\Sigma^{0} + \frac{1}{\sqrt{6}}\Lambda & \Sigma^{+} & p \\ \Sigma^{-} & -\frac{1}{\sqrt{2}}\Sigma^{0} + \frac{1}{\sqrt{6}}\Lambda & n \\ \Xi^{-} & \Xi^{0} & -\frac{2}{\sqrt{6}}\Lambda \end{pmatrix}$$

$$V_{\mu} = \begin{pmatrix} \frac{1}{\sqrt{2}}\rho^{0} + \frac{1}{\sqrt{2}}\omega & \rho^{+} & K^{*+} \\ \rho^{-} & -\frac{1}{\sqrt{2}}\rho^{0} + \frac{1}{\sqrt{2}}\omega & K^{*0} \\ K^{*-} & \bar{K}^{*0} & \phi \end{pmatrix}_{\mu}$$

$$K^{-} \qquad K^{-} \qquad K^{-} \qquad \pi^{-} \qquad K^{+} \qquad K^{+}$$

$$\downarrow \rho, \omega \qquad \downarrow K^{*} \qquad \downarrow \rho, \omega, \phi$$

$$\downarrow p \qquad p \qquad p \qquad \Sigma^{+} \qquad \Sigma^{-} \qquad \Sigma^{-}$$
(a) (b) (c)

Instead of using the Lagrangians, one can use the meson or baryon wave functions with the suitable operators

PPV vertex (Sakai, Roca, E. O, PRD 96 (2107))

 $g' \frac{1}{\sqrt{2}} (u\bar{u} - d\bar{d}), \text{ for } \rho^0 \text{ exchange}, \\ g' \frac{1}{\sqrt{2}} (u\bar{u} + d\bar{d}), \text{ for } \omega \text{ exchange}, \\ for \ \omega \text{ exchange}, \\ g' \frac{1}{\sqrt{2}} (u\bar{u} + d\bar{d}), \text{ for } \omega \text{ exchange}, \\ g' \frac{1}{\sqrt{2}} (u\bar{u} + d\bar{d}), \text{ for } \omega \text{ exchange}, \\ g' \frac{1}{\sqrt{2}} (u\bar{u} + d\bar{d}), \text{ for } \omega \text{ exchange}, \\ g' \frac{1}{\sqrt{2}} (u\bar{u} + d\bar{d}), \text{ for } \omega \text{ exchange}, \\ g' \frac{1}{\sqrt{2}} (u\bar{u} + d\bar{d}), \text{ for } \omega \text{ exchange}, \\ g' \frac{1}{\sqrt{2}} (u\bar{u} + d\bar{d}), \text{ for } \omega \text{ exchange}, \\ g' \frac{1}{\sqrt{2}} (u\bar{u} + d\bar{d}), \text{ for } \omega \text{ exchange}, \\ g' \frac{1}{\sqrt{2}} (u\bar{u} + d\bar{d}), \text{ for } \omega \text{ exchange}, \\ g' \frac{1}{\sqrt{2}} (u\bar{u} + d\bar{d}), \text{ for } \omega \text{ exchange}, \\ g' \frac{1}{\sqrt{2}} (u\bar{u} + d\bar{d}), \text{ for } \omega \text{ exchange}, \\ g' \frac{1}{\sqrt{2}} (u\bar{u} + d\bar{d}), \text{ for } \omega \text{ exchange}, \\ g' \frac{1}{\sqrt{2}} (u\bar{u} + d\bar{d}), \text{ for } \omega \text{ exchange}, \\ g' \frac{1}{\sqrt{2}} (u\bar{u} + d\bar{d}), \text{ for } \omega \text{ exchange}, \\ g' \frac{1}{\sqrt{2}} (u\bar{u} + d\bar{d}), \text{ for } \omega \text{ exchange}, \\ g' \frac{1}{\sqrt{2}} (u\bar{u} + d\bar{d}), \text{ for } \omega \text{ exchange}, \\ g' \frac{1}{\sqrt{2}} (u\bar{u} + d\bar{d}), \text{ for } \omega \text{ exchange}, \\ g' \frac{1}{\sqrt{2}} (u\bar{u} + d\bar{d}), \text{ for } \omega \text{ exchange}, \\ g' \frac{1}{\sqrt{2}} (u\bar{u} + d\bar{d}), \text{ for } \omega \text{ exchange}, \\ g' \frac{1}{\sqrt{2}} (u\bar{u} + d\bar{d}), \text{ for } \omega \text{ exchange}, \\ g' \frac{1}{\sqrt{2}} (u\bar{u} + d\bar{d}), \text{ for } \omega \text{ exchange}, \\ g' \frac{1}{\sqrt{2}} (u\bar{u} + d\bar{d}), \text{ for } \omega \text{ exchange}, \\ g' \frac{1}{\sqrt{2}} (u\bar{u} + d\bar{d}), \text{ for } \omega \text{ exchange}, \\ g' \frac{1}{\sqrt{2}} (u\bar{u} + d\bar{d}), \text{ for } \omega \text{ exchange}, \\ g' \frac{1}{\sqrt{2}} (u\bar{u} + d\bar{d}), \text{ for } \omega \text{ exchange}, \\ g' \frac{1}{\sqrt{2}} (u\bar{u} + d\bar{d}), \text{ for } \omega \text{ exchange}, \\ g' \frac{1}{\sqrt{2}} (u\bar{u} + d\bar{d}), \text{ for } \omega \text{ exchange}, \\ g' \frac{1}{\sqrt{2}} (u\bar{u} + d\bar{d}), \text{ for } \omega \text{ exchange}, \\ g' \frac{1}{\sqrt{2}} (u\bar{u} + d\bar{d}), \text{ for } \omega \text{ exchange}, \\ g' \frac{1}{\sqrt{2}} (u\bar{u} + d\bar{d}), \text{ for } \omega \text{ exchange}, \\ g' \frac{1}{\sqrt{2}} (u\bar{u} + d\bar{d}), \text{ for } \omega \text{ exchange}, \\ g' \frac{1}{\sqrt{2}} (u\bar{u} + d\bar{d}), \text{ for } \omega \text{ exchange}, \\ g' \frac{1}{\sqrt{2}} (u\bar{u} + d\bar{d}), \text{ for } \omega \text{ exchange}, \\ g' \frac{1}{\sqrt{2}} (u\bar{u} + d\bar{d}), \text{ for } \omega \text{ e$

Example in the charm sector:

$$D^+ = c\bar{d}$$

$$\begin{split} &-\langle c\bar{d}|g'\frac{1}{\sqrt{2}}((u\partial_{\mu}\bar{u}-\partial_{\mu}u\bar{u})-(d\partial_{\mu}\bar{d}-\partial_{\mu}d\bar{d}))|c\bar{d}\rangle\\ &=-g'\frac{1}{\sqrt{2}}(ip_{\mu}+ip'_{\mu}) \end{split}$$

One does not need to use SU(4). Yet, it is practical to evaluate it using $\mathcal{L}_{\text{VPP}} = -ig\langle [\Phi, \partial_{\mu}\Phi]V^{\mu} \rangle$, and the SU(4) matrices

$$P = \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^{0} + \frac{1}{\sqrt{3}}\eta + \frac{1}{\sqrt{6}}\eta' & \pi^{+} & K^{+} & \bar{D}^{0} \\ \pi^{-} & -\frac{1}{\sqrt{2}}\pi^{0} + \frac{1}{\sqrt{3}}\eta + \frac{1}{\sqrt{6}}\eta' & K^{0} & D^{-} \\ K^{-} & \bar{K}^{0} & -\frac{1}{\sqrt{3}}\eta + \sqrt{\frac{2}{3}}\eta' & D^{-}_{s} \\ D^{0} & D^{+} & D^{+}_{s} & \eta_{c} \end{pmatrix} \qquad V = \begin{pmatrix} \frac{1}{\sqrt{2}}\rho^{0} + \frac{1}{\sqrt{2}}\omega & \rho^{+} & K^{*+} & \bar{D}^{*0} \\ \rho^{-} & -\frac{1}{\sqrt{2}}\rho^{0} + \frac{1}{\sqrt{2}}\omega & K^{*0} & \bar{D}^{*-} \\ K^{*-} & \bar{K}^{*0} & \phi & D^{*-}_{s} \\ D^{*0} & D^{+} & D^{+}_{s} & \eta_{c} \end{pmatrix}$$





$$\langle p|g\rho^{0}|p\rangle \equiv \frac{1}{\sqrt{2}} \frac{1}{\sqrt{2}} \langle \phi_{\rm MS} \chi_{\rm MS} + \phi_{\rm MA} \chi_{\rm MA} |g \frac{1}{\sqrt{2}} (u\bar{u} - d\bar{d})|$$
$$\times \phi_{\rm MS} \chi_{\rm MS} + \phi_{\rm MA} \chi_{\rm MA} \rangle,$$
(10)

TABLE I. J = 1/2 states chosen and threshold mass in MeV.

| States | $\Xi_c \bar{K}$ | $\Xi_c' \bar{K}$ | ΞD | $\Omega_c \eta$ | ΞD^* | $\Xi_c \bar{K}^*$ | $\Xi_c' \bar{K}^*$ |
|-----------|-----------------|------------------|---------|-----------------|-----------|-------------------|--------------------|
| Threshold | 2965 | 3074 | 3185 | 3243 | 3327 | 3363 | 3472 |

TABLE II. J = 3/2 states chosen and threshold mass in MeV.

| States | $\Xi_c^* \bar{K}$ | $\Omega_c^*\eta$ | ΞD^* | $\Xi_c \bar{K}^*$ | Ξ^*D | $\Xi_c' \bar{K}^*$ |
|-----------|-------------------|------------------|-----------|-------------------|----------|--------------------|
| Threshold | 3142 | 3314 | 3327 | 3363 | 3401 | 3472 |

Back to Ω_c states

Molecular Ω_c states generated from coupled meson-baryon channels

Debastiani, Dias, Liang, Oset, PRD 2018

BARYON WAVE FUNCTIONS

 $\Xi_c^+: \frac{1}{\sqrt{2}}c(us - su)$, and the spin wave function is the mixed antisymmetric, χ_{MA} , for the two light quarks.

 Ξ_c^0 : the same as Ξ_c^+ , changing $(us - su) \rightarrow (ds - sd)$.

 $\Xi_c^{\prime+}$: $\frac{1}{\sqrt{2}}c(us + su)$, and now the spin wave function for the three quarks is the mixed symmetric, χ_{MS} , in the last two quarks,

 $\Xi_c^{\prime 0}$: the same as Ξ_c^{\prime} , changing $(us + su) \rightarrow (ds + sd)$.

 Ω_c^0 : *css*, and the spin wave function χ_{MS} in the last two quarks, like that for Ξ_c' .

Note that we do not use SU(4) baryon wave functions, the heavy quark is singled out and flavor-spin symmetry is demanded for the light quarks.



FIG. 3. Diagrams in the $\overline{K}\Xi_c \rightarrow \overline{K}\Xi_c$ transition.

Upper vertex

$$\mathcal{L}_{\text{VPP}} = -ig\langle [\Phi, \partial_{\mu}\Phi] V^{\mu} \rangle \qquad -it_{K^{-} \to K^{-}} \begin{pmatrix} \rho^{0} \\ \omega \\ \phi \end{pmatrix} = gV_{\mu}(-ip^{\mu} - ip'^{\mu}) \begin{pmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \\ -1 \end{pmatrix},$$
$$-it_{K^{-} \to \bar{K}^{0}\rho^{-}} = g\rho^{+\mu}(-ip^{\mu} - ip'^{\mu}),$$

g=m_v/2 f, f=93 MeV

Lower vertex

$$\frac{1}{\sqrt{2}}\langle (us - su) | \begin{pmatrix} g\frac{1}{\sqrt{2}}(u\bar{u} - d\bar{d}) \\ g\frac{1}{\sqrt{2}}(u\bar{u} + d\bar{d}) \\ gs\bar{s} \end{pmatrix} | \frac{1}{\sqrt{2}}(us - su) \rangle$$
$$= \begin{pmatrix} \frac{1}{\sqrt{2}}g \\ \frac{1}{\sqrt{2}}g \\ g \end{pmatrix}.$$

No need to invoke SU(4)

With light vector exchange the heavy quarks are spectators. Nothing depends upon them. Heavy quark symmetry is automatically implemented

$$V_{ij} = D_{ij} \frac{1}{4f_{\pi}^2} (p^0 + p'^0)$$

The exchange of heavy vectors is penalized

$$\frac{1}{(q^0)^2 - |\mathbf{q}|^2 - m_{D_s^*}^2} \approx \frac{1}{(m_D - m_K)^2 - m_{D_s^*}^2},$$

 $\lambda \equiv \frac{-m_V^2}{(m_D - m_K)^2 - m_{D_*^*}^2} \approx 0.25$

TABLE III. D_{ij} coefficients of Eq. (23) for the meson-baryon states coupling to $J^P = 1/2^-$ in *s*-wave.

| J = 1/2 | $\Xi_c \bar{K}$ | $\Xi_c' \bar{K}$ | ΞD | $\Omega_c \eta$ | ΞD^* | $\Xi_c \bar{K}^*$ | $\Xi_c' \bar{K}^*$ |
|-------------------|-----------------|------------------|------------------------------|-----------------------------|-----------|------------------------------|-----------------------------|
| $\Xi_c \bar{K}$ | -1 | 0 | $-\frac{1}{\sqrt{2}}\lambda$ | 0 | 0 | 0 | 0 |
| $\Xi_c' \bar{K}$ | | -1 | $\frac{1}{\sqrt{6}}\lambda$ | $-\frac{4}{\sqrt{3}}$ | 0 | 0 | 0 |
| ΞD | | | -2 | $\frac{\sqrt{2}}{3}\lambda$ | 0 | 0 | 0 |
| $\Omega_c \eta$ | | | | 0 | 0 | 0 | 0 |
| ΞD^* | | | | | -2 | $-\frac{1}{\sqrt{2}}\lambda$ | $\frac{1}{\sqrt{6}}\lambda$ |
| $\Xi_c \bar{K}^*$ | | | | | | -1 | 0 |
| $\Xi_c' ar{K}^*$ | | | | | | | -1 |

$$T = [1 - VG]^{-1}V, \qquad G_l^{II} = G_l^I + i\frac{2M_l q}{4\pi\sqrt{s}}, \qquad T_{ij} = \frac{g_i g_j}{\sqrt{s} - z_R}$$

Exp (MeV) TABLE VI. The coupling constants to various channels for the poles in the $J^P = 1/2^-$ sector, with $q_{\text{max}} = 650$ MeV, and $g_i G_i^{II}$ in MeV.

| МГ | MeV. | | | | | | | |
|-----------|-----------------------|--------------------------------|--------------------------------|--------------------------------|---------------------------------------|-----------|-------------------|--------------------|
| 3050, 0.8 | 3054.05 + i0.44 | $\Xi_c \bar{K}$ | $\Xi_c' \bar{K}$ | ΞD | $\Omega_c \eta$ | ΞD^* | $\Xi_c \bar{K}^*$ | $\Xi_c' \bar{K}^*$ |
| | $g_i \\ g_i G_i^{II}$ | -0.06 + i0.14 -1.40 - i3.85 | 1.94 + i0.01 -34.41 - i0.30 | -2.14 + i0.26 9.33 - i1.10 | $\frac{1.98 + i0.01}{-16.81 - i0.11}$ | 0 0 | 0 0 | 0 0 |
| 3090, 8.7 | 3091.28 + i5.12 | $\Xi_c ar{K}$ | $\Xi_c' \bar{K}$ | ED 🔨 | $\Omega_c \eta$ | ΞD^* | $\Xi_c \bar{K}^*$ | $\Xi_c' \bar{K}^*$ |
| | $g_i \\ g_i G_i^{II}$ | 0.18 - i0.37 5.05 + i10.19 | 0.31 + i0.25 -9.97 - i3.67 | 5.83 - i0.20 -29.82 + i0.31 | 0.38 + i0.23 -3.59 - i2.23 | 0 0 | 0 0 | 0 0 |

TABLE VIII. The coupling constants to various channels for the poles in the $J^P = 3/2^-$ sector, with $q_{\text{max}} = 650$ MeV, and $g_i G_i^{II}$ in MeV.

| 3119, 1.1 | 3124.84 | $\Xi_c^* \bar{K}$ | $\Omega_c^*\eta$ | ΞD^* | $\Xi_c \bar{K}^*$ | Ξ*D / | $\Xi_c' \bar{K}^*$ |
|-----------|-----------------------|-------------------------------|-------------------------------|-----------|-------------------|--------------------------------|--------------------|
| | $g_i \\ g_i G_i^{II}$ | 1.95 - 35.65 | 1.98 -16.83 | 0 0 | 0 0 | -0.65 1.93 | 0 0 |
| | 3290.31 + i0.03 | $\Xi_c^* \bar{K}$ | $\Omega_c^*\eta$ | ΞD^* | $\Xi_c \bar{K}^*$ | Ξ*D | $\Xi_c' \bar{K}^*$ |
| | $g_i \\ g_i G_i^{II}$ | 0.01 + i0.02 -0.62 - i0.18 | 0.31 + i0.01 -5.25 - i0.18 | 0 3 0 | 0 0 | 6.22 - i0.04 -31.08 + i0.20 | 0 0 |

We get three states in very good agreement with experiment, both mass and width

Related work:

- [15] J. Hofmann and M. F. M. Lutz, Nucl. Phys. A763, 90 (2005).
- [16] C. E. Jimenez-Tejero, A. Ramos, and I. Vidana, Phys. Rev. C 80, 055206 (2009).
- [17] O. Romanets, L. Tolos, C. Garcia-Recio, J. Nieves, L. L. Salcedo, and R. G. E. Timmermans, Phys. Rev. D 85, 114032 (2012).

Revisions made after experiment to fit some parameter [41] G. Montaña, A. Feijoo, and A. Ramos, Eur. Phys. J. A **54**, 64 (2018).

Uses SU(4) : matrix elements exchanging light vectors are equal. Results similar to ours, but only two states, since they study $1/2^{-}$ states only

J.~Nieves, R.~Pavao and L.~Tolos, Omega _c excited states within a SU(6)}_ HQSS model, Eur. Phys. J. C 78 114 (2018)

Better results than in [17] but the widths and the positions not so good as in the works of Montaña and present work .

Heavy quark spin symmetric molecular states from $\bar{D}^{(*)}\Sigma_c^{(*)}$ and other coupled channels in the light of the recent LHCb pentaquarks

C. W. Xiao,¹ J. Nieves,² and E. Oset^{2,3} PRD (2019)

$$I = 1/2, \ \eta_c N, \ J/\psi N, \ \bar{D}\Lambda_c, \ \bar{D}\Sigma_c, \ \bar{D}^*\Lambda_c, \ \bar{D}^*\Sigma_c, \ \bar{D}^*\Sigma_c^* \text{ for spin parity } J^P = 1/2^-$$
$$J/\psi N, \ \bar{D}^*\Lambda_c, \ \bar{D}^*\Sigma_c, \ \ \bar{D}\Sigma_c^*, \ \bar{D}^*\Sigma_c^* \text{ for } J^P = 3/2^-$$
$$T = [1 - V G]^{-1} V_c$$

HQSS tells that the interaction cannot depend on the spin of the heavy quarks. Then one rewrites the physical states in terms of a basis of states where the spin of the light quarks and the heavy ones are separated.

One uses the Wigner Eckart theorem to write matrix elements in terms of a few reduced matrix elements

This produces symmetries in the matrix elements of the interaction.

•
$$J = 1/2, I = 1/2$$

I = 1/2

•
$$J = 3/2, I = 1/2$$

$$J/\psi N \ \bar{D}^* \Lambda_c \quad \bar{D}^* \Sigma_c \qquad \bar{D} \Sigma_c^* \qquad \bar{D} \Sigma_c^* \qquad \bar{D}^* \Sigma_c^*$$

$$\begin{pmatrix} \mu_1 & \mu_{12} & \frac{\mu_{13}}{3} & -\frac{\mu_{13}}{\sqrt{3}} & \frac{\sqrt{5}\mu_{13}}{3} \\ \mu_{12} & \mu_2 & \frac{\mu_{23}}{3} & -\frac{\mu_{23}}{\sqrt{3}} & \frac{\sqrt{5}\mu_{23}}{3} \\ \frac{\mu_{13}}{3} & \frac{\mu_{23}}{3} & \frac{1}{9} (8\lambda_2 + \mu_3) & \frac{\lambda_2 - \mu_3}{3\sqrt{3}} & \frac{1}{9} \sqrt{5} (\mu_3 - \lambda_2) \\ -\frac{\mu_{13}}{\sqrt{3}} & -\frac{\mu_{23}}{\sqrt{3}} & \frac{\lambda_2 - \mu_3}{3\sqrt{3}} & \frac{1}{3} (2\lambda_2 + \mu_3) & \frac{1}{3} \sqrt{\frac{5}{3}} (\lambda_2 - \mu_3) \\ \frac{\sqrt{5}\mu_{13}}{3} & \frac{\sqrt{5}\mu_{23}}{3} & \frac{1}{9} \sqrt{5} (\mu_3 - \lambda_2) & \frac{1}{3} \sqrt{\frac{5}{3}} (\lambda_2 - \mu_3) & \frac{1}{9} (4\lambda_2 + 5\mu_3) \end{pmatrix}_{I=1/2}$$

•
$$J = 5/2, I = 1/2$$

 $\bar{D}^* \Sigma_c^* : (\lambda_2)_{I=1/2}$

The different terms are evaluated using an extension of the local hidden gauge approach, with the exchange of vector mesons.

$$\mu_{1} = 0, \quad \mu_{23} = 0, \quad \lambda_{2} = \mu_{3}, \quad \mu_{13} = -\mu_{12},$$

$$\mu_{2} = \frac{1}{4f^{2}}(k^{0} + k'^{0}), \quad \mu_{3} = -\frac{1}{4f^{2}}(k^{0} + k'^{0}),$$

$$\mu_{12} = -\sqrt{6} \frac{m_{\rho}^{2}}{p_{D^{*}}^{2} - m_{D^{*}}^{2}} \frac{1}{4f^{2}} (k^{0} + k'^{0}),$$

f= f_{π} =93 MeV, k^0 , k'^0 are the energies of the external mesons

The only free parameter is the subtraction constant in the regularization of the meson baryon loops. We take it such that the average mass of our states agrees with experiment.

TABLE I. Dimensionless coupling constants of the $(I = 1/2, J^P = 1/2^-)$ poles found in this work to the different channels.

| (4306. | 38 + i7.62) MeV | | | | | | |
|----------|-------------------|--------------|--------------------|-------------------|-----------------------|-------------------------|------------------------|
| | $\eta_c N$ | $J/\psi N$ | $ar{D}\Lambda_c$ | $\bar{D}\Sigma_c$ | $\bar{D}^* \Lambda_c$ | $\bar{D}^*\Sigma_c$ | $\bar{D}^* \Sigma_c^*$ |
| g_i | 0.67 + i0.01 | 0.46 - i0.03 | 0.01 - i0.01 | 2.07 - i0.28 | 0.03 + i0.25 | 0.06 - i0.31 | 0.04 - i0.15 |
| $ g_i $ | 0.67 | 0.46 | 0.01 | 2.09 | 0.25 | 0.31 | 0.16 |
| (4452.9) | 96 + i11.72) MeV | | | | | | |
| | $\eta_c N$ | $J/\psi N$ | $\bar{D}\Lambda_c$ | $\bar{D}\Sigma_c$ | $\bar{D}^*\Lambda_c$ | $\bar{D}^*\Sigma_c$ | $\bar{D}^*\Sigma_c^*$ |
| g_i | 0.24 + i0.03 | 0.88 - 0.11 | 0.09 - i0.06 | 0.12 - i0.02 | 0.11 - i0.09 | $1.97 - \mathrm{i}0.52$ | 0.02 + i0.19 |
| $ g_i $ | 0.25 | 0.89 | 0.11 | 0.13 | 0.14 | 2.03 | 0.19 |
| (4520.4) | 45 + i11.12) MeV | | | | | | |
| | $\eta_c N$ | $J/\psi N$ | $\bar{D}\Lambda_c$ | $\bar{D}\Sigma_c$ | $\bar{D}^*\Lambda_c$ | $\bar{D}^*\Sigma_c$ | $\bar{D}^*\Sigma_c^*$ |
| g_i | 0.72 - i0.10 | 0.45 - i0.04 | 0.11 - i0.06 | 0.06 - i0.02 | 0.06 - i0.05 | 0.07 - i0.02 | 1.84 - i0.56 |
| $ g_i $ | 0.73 | 0.45 | 0.13 | 0.06 | 0.08 | 0.08 | 1.92 |

| | r | TABLE II. S | ame as Table | I for $J^P = 3$ | $s/2^{-}$. | |
|------|------------------------|--------------|----------------------|---------------------|---------------------|-----------------------|
| ???? | (4374.33 + i6.87) MeV | $J/\psi N$ | $ar{D}^*\Lambda_c$ | $\bar{D}^*\Sigma_c$ | $\bar{D}\Sigma_c^*$ | $\bar{D}^*\Sigma_c^*$ |
| | g_i | 0.73 - i0.06 | 0.11 - i0.13 | 0.02 - i0.19 | 1.91-i0.31 | 0.03 - i0.30 |
| | $ g_i $ | 0.73 | 0.18 | 0.19 | 1.94 | 0.30 |
| | (4452.48 + i1.49) MeV | $J/\psi N$ | $\bar{D}^*\Lambda_c$ | $\bar{D}^*\Sigma_c$ | $\bar{D}\Sigma_c^*$ | $\bar{D}^*\Sigma_c^*$ |
| | g_i | 0.30 - i0.01 | 0.05 - i0.04 | 1.82 - i0.08 | 0.08 - i0.02 | 0.01 - i0.19 |
| | $ g_i $ | 0.30 | 0.07 | 1.82 | 0.08 | 0.19 |
| | (4519.01 + i6.86) MeV | $J/\psi N$ | $\bar{D}^*\Lambda_c$ | $\bar{D}^*\Sigma_c$ | $\bar{D}\Sigma_c^*$ | $\bar{D}^*\Sigma_c^*$ |
| | g_i | 0.66 - i0.01 | 0.11 - i0.07 | 0.10 - i0.3 | 0.13 - i0.02 | 1.79-i0.36 |
| | $ g_i $ | 0.66 | 0.13 | 0.10 | 0.13 | 1.82 |

 $|g_i| \qquad 0.66 \qquad 0.13 \qquad 0.10 \qquad 0.13 \qquad 1.82$ And a state at : 4519.23 MeV and a zero width for the single channel $\bar{D}^*\Sigma_c^*$ with $J = 5/2^-$ TABLE III. Identification of some of the I = 1/2 resonances found in this work with experimental states.

| $Mass \ [MeV]$ | Width [MeV] | Main channel | J^P | Experimental state |
|----------------|-------------|---------------------|-----------|--------------------|
| 4306.4 | 15.2 | $\bar{D}\Sigma_c$ | $1/2^{-}$ | $P_{c}(4312)$ |
| 4453.0 | 23.4 | $\bar{D}^*\Sigma_c$ | $1/2^{-}$ | $P_{c}(4440)$ |
| 4452.5 | 3.0 | $\bar{D}^*\Sigma_c$ | $3/2^{-}$ | $P_{c}(4457)$ |



M.Z. Liu, Y.W. Pang, F.Z. Peng, M. Sanchez-Sanchez, L.S. Geng, A. Hosaka, M. Pavon-Valderrama Arxiv 1903.11560, PRL 2019 Similar conclusions based on single channels.

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Prediction of hidden charm strange molecular baryon states with To be observed heavy quark spin symmetry C.W. Xiao, J. Nieves and E. Oset $\ln J/\psi \Lambda = \eta_c \Lambda, J/\psi \Lambda, \bar{D}\Xi_c, \bar{D}_s \Lambda_c, \bar{D}\Xi'_c, \bar{D}^*\Xi_c, \bar{D}^*\Xi'_c, \bar{D}^*\Xi'_c, \bar{D}^*\Xi'_c = 1/2^ J/\psi \Lambda, \bar{D}^*\Xi_c, \bar{D}^*_s \Lambda_c, \bar{D}^*\Xi'_c, \bar{D}^*\Xi^*_c = 1/2^-$

TABLE I. Dimensionless coupling constants of the $(I = 0, J^P = 1/2^-)$ poles found in this work.

| | $\eta_c \Lambda$ | $J/\psi\Lambda$ | $\bar{D}\Xi_c$ | $\bar{D}_s \Lambda_c$ | $\bar{D}\Xi_c'$ | $\bar{D}^* \Xi_c$ | $\bar{D}_s^* \Lambda_c$ | $\bar{D}^* \Xi_c'$ | $\bar{D}^* \Xi_c^*$ |
|---------|------------------|-----------------|----------------|-----------------------|-----------------|-------------------|-------------------------|--------------------|---------------------|
| 42' | 76.59 + i7.67 | | | | | | | | |
| g_i | 0.17 - i0.03 | 0.29 - i0.07 | 2.93 + i0.08 | 0.76 + i0.31 | 0.00 + i0.01 | 0.01 + i0.02 | 0.01 + i0.04 | 0.01 - i0.02 | 0.01 - i0.03 |
| $ g_i $ | 0.17 | 0.30 | 2.93 | 0.82 | 0.01 | 0.02 | 0.05 | 0.02 | 0.03 |
| 442 | 29.84 + i7.92 | | | | | | | | |
| g_i | 0.29 - i0.11 | 0.17 - i0.07 | 0.00 - i0.00 | 0.00 - i0.00 | 0.15 - i0.26 | 2.78 + i0.01 | 0.66 + i0.32 | 0.01 + i0.05 | 0.01 + i0.03 |
| $ g_i $ | 0.31 | 0.18 | 0.00 | 0.00 | 0.30 | 2.78 | 0.73 | 0.05 | 0.04 |
| 44: | 36.70 + i1.17 | | | | | | | | |
| g_i | 0.24 + i0.03 | 0.14 + 0.01 | 0.00 - i0.00 | 0.00 - i0.00 | 1.72 - i0.04 | 0.22 - i0.31 | 0.06 - i0.01 | 0.01 - i0.04 | 0.01 - i0.03 |
| $ g_i $ | 0.24 | 0.14 | 0.00 | 0.00 | 1.72 | 0.38 | 0.07 | 0.04 | 0.03 |
| 458 | 80.96 + i2.44 | | | | | | | | |
| g_i | 0.12 - i0.00 | 0.37 - i0.04 | 0.02 - i0.01 | 0.02 - i0.01 | 0.03 - i0.00 | 0.02 - i0.02 | 0.03 - i0.02 | 1.57 - i0.17 | 0.00 + i0.02 |
| $ g_i $ | 0.12 | 0.37 | 0.02 | 0.02 | 0.03 | 0.03 | 0.03 | 1.58 | 0.02 |
| 46 | 50.86 + i2.59 | | | | | | | | |
| g_i | 0.32 - i0.05 | 0.19 - i0.03 | 0.02 - i0.01 | 0.03 - i0.02 | 0.02 - i0.00 | 0.01 - i0.01 | 0.02 - i0.01 | 0.01 - i0.00 | 1.41 - i0.23 |
| $ g_i $ | 0.32 | 0.19 | 0.03 | 0.04 | 0.02 | 0.02 | 0.02 | 0.02 | 1.43 |

| | | TABLE] | II. Same as T | able I for J^P = | $= 3/2^{-}.$ | |
|--|--|--------------------------------------|--------------------------------------|--------------------------------------|--------------------------------------|--------------------------------------|
| | $J/\psi\Lambda$ | $\bar{D}^* \Xi_c$ | $\bar{D}_s^* \Lambda_c$ | $\bar{D}^* \Xi_c'$ | $\bar{D}\Xi_c^*$ | $\bar{D}^* \Xi_c^*$ |
| 442 | 29.52 + i7.67 | | | | | |
| g_i | 0.31 - i0.10 | 2.77 - i0.02 | 0.67 + i0.32 | 0.00 + i0.0.02 | 0.00 - i0.06 | 0.00 + i0.0.04 |
| $ g_i $ | 0.32 | 2.77 | 0.74 | 0.02 | 0.06 | 0.04 |
| 450 | 06.99 + i1.03 | | | | | |
| g_i | 0.27 - i0.02 | 0.02 - i0.03 | 0.02 - i0.02 | 0.00 - i0.03 | 1.56 - i0.07 | 0.00 - i0.05 |
| $ g_i $ | 0.27 | 0.03 | 0.03 | 0.03 | 1 56 | 0.05 |
| | 0.2. | 0.05 | 0.05 | 0.05 | 1.00 | 0.09 |
| 458 | 30.96 + i0.34 | 0.05 | 0.05 | 0.05 | 1.50 | 0.00 |
| $\frac{458}{g_i}$ | 80.96 + i0.34 0.14 - i0.01 | 0.03 0.01 - i0.01 | 0.03 0.01 - i0.01 | 1.54 – i0.02 | 0.02 - i0.00 | 0.00 - i0.04 |
| $\frac{458}{g_i}$ $ g_i $ | 30.96 + i0.34 0.14 - i0.01 0.14 | 0.03 0.01 - i0.01 0.01 | 0.03 0.01 - i0.01 0.02 | 1.54 – i0.02 1.54 | 0.02 - i0.00 0.02 | 0.00 - i0.04 0.04 |
| $ \begin{array}{c} 458 \\ g_i \\ \hline g_i \\ \hline 465 \end{array} $ | $ \frac{60.21}{80.96 + i0.34} \\ 0.14 - i0.01 \\ 0.14 \\ 50.58 + i1.48 $ | 0.03 0.01 - i0.01 0.01 | 0.03 0.01 - i0.01 0.02 | 1.54 – i0.02 1.54 | 0.02 - i0.00 0.02 | 0.00 - i0.04 0.04 |
| $ \begin{array}{c} 458 \\ g_i \\ g_i \\ \hline g_i \\ \hline g_i \\ \hline g_i \\ \hline g_i \end{array} $ | $ \begin{array}{r} $ | 0.01 - i0.01 0.01 0.02 - i0.01 | 0.01 - i0.01 0.02 0.03 - i0.02 | 1.54 - i0.02 1.54 0.03 - i0.01 | 0.02 - i0.00 0.02 0.03 - i0.00 | 0.00 - i0.04 0.04 1.40 - i0.13 |

Conclusions

Extension of chiral unitary theory to the heavy sector, using the exchange of vectors in the hidden gauge approach, together with unitarity in coupled channels leads to neat predictions for molecular states

The important terms in the interaction come from light vector Exchange -> 1) heavy quaks are spectators and matrix elements do not depend upon them HEAVY QUARK SYMMETRY AUTOMATICALLY FULFILLED. 2) One does not have to invoke SU(4) to evaluate matrix elements.

Recent results for Ω_c states and the new pentaquarks states of hidden charm, are giving support to these molecular pictures.

Reliable predictions are made for many states likely to be observed in the near future. LHCb has the key to them

LOWEST ORDER HQSS CONSTRAINTS

HQSS predicts that all types of spin interactions vanish for infinitely massive quarks: The dynamics is unchanged under arbitrary transformations of the spin of the heavy quark (Q). The spin-dependent interactions are proportional to the chromomagnetic moment of the heavy quark, and hence they are of the order of $1/m_0$. The total angular momentum \vec{J} of the hadron is always a conserved quantity, but in this case the spin of the heavy quark \vec{S}_O is also conserved in the $m_Q \rightarrow \infty$ limit. Consequently, the spin of the light degrees of freedom $\vec{S}_l = \vec{J} - \vec{S}_O$ is a conserved quantity in that limit. Thus, heavy hadrons come in

$$|l_{1}s_{1}j_{1}; l_{2}s_{2}j_{2}; JM\rangle = \sum_{S,L} [(2S+1)(2L+1)(2j_{1}+1)(2j_{2}+1)]^{1/2} \begin{cases} l_{1} & l_{2} & L \\ s_{1} & s_{2} & S \\ j_{1} & j_{2} & J \end{cases} |l_{1}l_{2}L; s_{1}s_{2}S; JM\rangle,$$
generic: $l_{1} \qquad l_{2} \qquad s_{1} \quad s_{2} \qquad j_{1} \qquad j_{2} \qquad L \qquad S \qquad J$

HQSS:
$$\ell_M\left(\frac{1}{2}\right) = \ell_B = \frac{1}{2} - \frac{1}{2} = J_M(0, 1) = J_B\left(\frac{1}{2}, \frac{3}{2}\right) = \mathcal{L} = S_{c\bar{c}} = J\left(\frac{1}{2}, \frac{3}{2}, \frac{5}{2}\right).$$



 $_{(\ell'_{M'}\ell'_{B})}\langle S'_{c\bar{c}}, \mathcal{L}'; J', \alpha'| H^{\text{QCD}}| S_{c\bar{c}}, \mathcal{L}; J, \alpha\rangle_{(\ell_{M},\ell_{B})} = \delta_{\alpha\alpha'}\delta_{JJ'}\delta_{S'_{c\bar{c}}}S_{c\bar{c}}}\delta_{\mathcal{L}\mathcal{L}'}\langle \ell'_{M}\ell'_{B}\mathcal{L}; \alpha|| H^{\text{QCD}}||\ell_{M}\ell_{B}\mathcal{L}; \alpha\rangle_{(\ell_{M},\ell_{B})}$

(i)
$$|S_{c\bar{c}} = 0, \mathcal{L} = \frac{1}{2}; J = \frac{1}{2}\rangle_{(\ell_{M}=0,\ell_{B}=\frac{1}{2})}, |S_{c\bar{c}} = 0, \mathcal{L} = \frac{1}{2}; J = \frac{1}{2}\rangle_{(\ell_{M}=1/2,\ell_{B}=0)}, |S_{c\bar{c}} = 0, \mathcal{L} = \frac{1}{2}; J = \frac{1}{2}\rangle_{(\ell_{M}=0,\ell_{B}=\frac{1}{2})}, |S_{c\bar{c}} = 1, \mathcal{L} = \frac{1}{2}; J = \frac{1}{2}\rangle_{(\ell_{M}=0,\ell_{B}=\frac{1}{2})}, |S_{c\bar{c}} = 1, \mathcal{L} = \frac{1}{2}; J = \frac{1}{2}\rangle_{(\ell_{M}=1/2,\ell_{B}=0)}, |S_{c\bar{c}} = 1, \mathcal{L} = \frac{1}{2}; J = \frac{3}{2}\rangle_{(\ell_{M}=0,\ell_{B}=\frac{1}{2})}, |S_{c\bar{c}} = 1, \mathcal{L} = \frac{1}{2}; J = \frac{3}{2}\rangle_{(\ell_{M}=0,\ell_{B}=\frac{1}{2})}, |S_{c\bar{c}} = 1, \mathcal{L} = \frac{1}{2}; J = \frac{3}{2}\rangle_{(\ell_{M}=1/2,\ell_{B}=1)}, |S_{c\bar{c}} = 1, \mathcal{L} = \frac{1}{2}; J = \frac{3}{2}\rangle_{(\ell_{M}=0,\ell_{B}=\frac{3}{2})}, |S_{c\bar{c}} = 0, \mathcal{L} = \frac{3}{2}; J = \frac{3}{2}\rangle_{(\ell_{M}=0,\ell_{B}=\frac{3}{2})}, |S_{c\bar{c}} = 1, \mathcal{L} = \frac{3}{2}; J = \frac{3}{2}\rangle_{(\ell_{M}=1/2,\ell_{B}=1)}, |V||||S_{c\bar{c}} = 1, \mathcal{L} = \frac{3}{2}; J = \frac{3}{2}\rangle_{(\ell_{M}=1/2,\ell_{B}=1)}, |V|||S_{c\bar{c}} = 1, \mathcal{L} = \frac{3}{2}; J = \frac{3}{2}\rangle_{(\ell_{M}=0,\ell_{B}=\frac{3}{2})}, |S_{c\bar{c}} = 1, \mathcal{L} = \frac{3}{2}; J = \frac{3}{2}\rangle_{(\ell_{M}=0,\ell_{B}=\frac{3}{2})}, |S_{c\bar{c}} = 1, \mathcal{L} = \frac{3}{2}; J = \frac{3}{2}\rangle_{(\ell_{M}=0,\ell_{B}=\frac{3}{2})}, |S_{c\bar{c}} = 1, \mathcal{L} = \frac{3}{2}; J = \frac{3}{2}\rangle_{(\ell_{M}=0,\ell_{B}=\frac{3}{2})}, |S_{c\bar{c}} = 1, \mathcal{L} = \frac{3}{2}; J = \frac{3}{2}\rangle_{(\ell_{M}=0,\ell_{B}=\frac{3}{2})}, |S_{c\bar{c}} = 1, \mathcal{L} = \frac{3}{2}; J = \frac{3}{2}\rangle_{(\ell_{M}=0,\ell_{B}=\frac{3}{2})}, |S_{c\bar{c}} = 1, \mathcal{L} = \frac{3}{2}; J = \frac{3}{2}\rangle_{(\ell_{M}=0,\ell_{B}=\frac{3}{2})}, |S_{c\bar{c}} = 1, \mathcal{L} = \frac{3}{2}; J = \frac{3}{2}\rangle_{(\ell_{M}=0,\ell_{B}=\frac{3}{2})}, |S_{c\bar{c}} = 1, \mathcal{L} = \frac{3}{2}; J = \frac{3}{2}\rangle_{(\ell_{M}=0,\ell_{B}=\frac{3}{2})}, |S_{c\bar{c}} = 1, \mathcal{L} = \frac{3}{2}; J = \frac{3}{2}\rangle_{(\ell_{M}=0,\ell_{B}=\frac{3}{2})}, |S_{c\bar{c}} = 1, \mathcal{L} = \frac{3}{2}; J = \frac{3}{2}\rangle_{(\ell_{M}=0,\ell_{B}=\frac{3}{2})}, |S_{c\bar{c}} = 1, \mathcal{L} = \frac{3}{2}; J = \frac{3}{2}\rangle_{(\ell_{M}=0,\ell_{B}=\frac{3}{2})}, |S_{c\bar{c}} = 1, \mathcal{L} = \frac{3}{2}; J = \frac{3}{2}\rangle_{(\ell_{M}=0,\ell_{B}=\frac{3}{2})}, |S_{c\bar{c}} = 1, \mathcal{L} = \frac{3}{2}; J = \frac{3}{2}\rangle_{(\ell_{M}=0,\ell_{B}=\frac{3}{2})}, |S_{c\bar{c}} = 1, \mathcal{L} = \frac{3}{2}; J = \frac{3}{2}\rangle_{(\ell_{M}=0,\ell_{B}=\frac{3}{2})}, |S_{c\bar{c}} = 1, \mathcal{L} = \frac{3}{2}; J = \frac{3}{2}\rangle_{(\ell_{M}=0,\ell_{B}=\frac{3}{2})}, |S_{c\bar{c}} = 1, \mathcal{L} = \frac{3}{2}; J =$$