B_d^0 and B_s^0 mixing parameters from Lattice QCD

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for the RBC-UKQCD Collaborations

Based on arXiv:1812.08791

CERN, Geneva
Implications of LHCb measurements and future prospects

16 October 2019

CP3

SDU
DEPARTMENT OF MATHEMATICS
AND COMPUTER SCIENCE

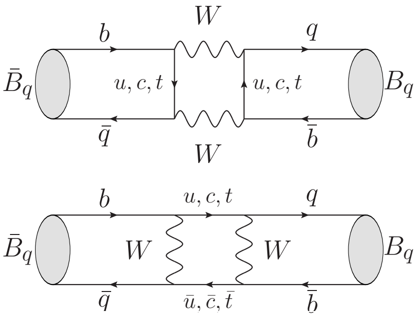
THE UNIVERSITY of EDINBURGH

Outline

1. Introduction
2. $SU(3)$ breaking ratios for $D_{(s)}$ and $B_{(s)}$ mesons [arXiv:1812.08791]
3. Beyond $SU(3)$ breaking ratios - individual bag parameters
4. Conclusion and Outlook

Neutral $B_{(s)}$ meson mixing - background

Neutral mesons oscillate:



where $q = d, s$

mass eigenstate $\neq$ flavour eigenstate

$$|B_{L,H}\rangle = p\,|B_q^0\rangle \pm q\,|\bar{B}_q^0\rangle$$

- $\Delta m_q \equiv m_H - m_L$
- $\Delta\Gamma_q \equiv \Gamma_L - \Gamma_H$
- $\Gamma_q \equiv (\Gamma_L + \Gamma_H)/2$

Time dependence:

$$|B_q^0(t)\rangle = g_+(t)\,|B_q^0\rangle + \frac{q}{p} g_-(t)\,|\bar{B}_q^0\rangle$$

$$|\bar{B}_q^0(t)\rangle = g_+(t)\,|\bar{B}_q^0\rangle + \frac{p}{q} g_-(t)\,|B_q^0\rangle$$

Neutral $B_{(s)}$ Meson Mixing - experiment

$$|g_\pm(t)|^2 = \frac{e^{-\Gamma_q t}}{2}\left[\cosh\left(\frac{\Delta\Gamma_q}{2}t\right) \pm \cos\left(\Delta m_q t\right)\right]$$

Δm can be measured very precisely as a frequency!

B_d^0: Many results

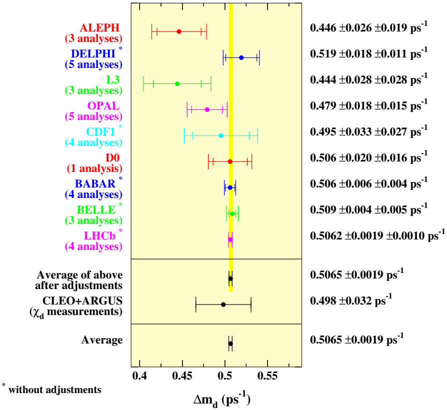
B_s^0: "Only" CDF and LHCb

$$\Delta m_d = 0.5065(19)\mathrm{ps}^{-1}$$

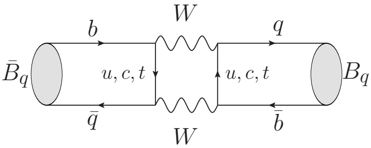
$$\Delta m_s = 17.757(21)\mathrm{ps}^{-1}$$

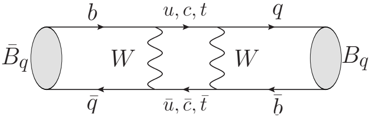
Well below per cent level!

[HFLAV]



Neutral $B_{(s)}$ Meson Mixing - theory





$$\Delta m \propto \underbrace{\left\langle B^0_{(s)} \middle| \mathcal{H}^{\Delta b=2} \middle| \bar{B}^0_{(s)} \right\rangle}_{\text{Short distance}} + \underbrace{\sum_n \frac{\left\langle B^0_{(s)} \middle| \mathcal{H}^{\Delta b=1} \middle| n \right\rangle \left\langle n \middle| \mathcal{H}^{\Delta b=1} \middle| \bar{B}^0_{(s)} \right\rangle}{E_n - M_{B_{(s)}}}}_{\text{Long distance}}$$

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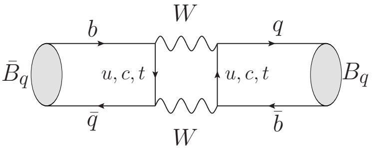

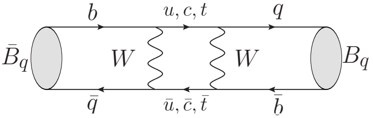

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$$\text{short distance} \propto \left| \sum_{q'=u,c,t} \frac{m_{q'}^2}{M_W^2} V_{q'b} V^*_{q'q} \right|^2 \approx \frac{m_t^4}{M_W^4} \left| V_{tb} V^*_{tq} \right|^2$$

SD: Top enhanced: $m_t^2 V_{tb} V_{tq}{}^* \gg m_c^2 V_{cb} V^*_{cq} \gg m_u^2 V_{ub} V^*_{uq}$

LD: Only m_c, m_u in intermediate states: no top + CKM suppressed

$\Rightarrow$ **Short distance dominated**.

Operator Product Expansion

Two scale problem: $\Lambda_{\mathrm{QCD}} \sim 1\,\mathrm{GeV} \ll m_{EW} \sim 100\,\mathrm{GeV}$:

$\Rightarrow$ OPE factorises this into

- Perturbative model-dependent Wilson coefficients $C_i(\mu)$
- **Non-perturbative model-independent matrix elements**

$$\left\langle B^0_{(s)} \middle| \mathcal{H}^{\Delta b=2} \middle| \bar{B}^0_{(s)} \right\rangle = \sum_i C_i(\mu) \left\langle B^0_{(s)} \middle| \mathcal{O}_i^{\Delta b=2}(\mu) \middle| \bar{B}^0_{(s)} \right\rangle$$

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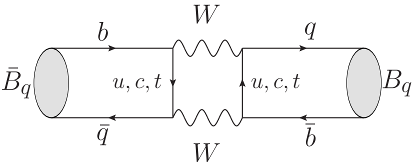
- 5 independent (parity even) operators $\mathcal{O}_i$.
- Only $\mathcal{O}_1$ is relevant for Δm:

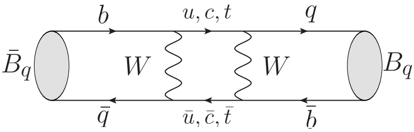
$$\mathcal{O}_1 = \left(\bar{b}_a \gamma_\mu \left(\mathbb{1} - \gamma_5\right) q_a\right) \left(\bar{b}_b \gamma_\mu \left(\mathbb{1} - \gamma_5\right) q_b\right) = \mathcal{O}_{VV+AA}$$

- Define bag parameters: $B_i = \left\langle \bar{B}^0_q \middle| \mathcal{O}_i \middle| B^0_q \right\rangle / \left\langle \bar{B}^0_q \middle| \mathcal{O}_i \middle| B^0_q \right\rangle_{VSA}$

Flavour Physics and CKM - neutral meson mixing

Experiment $\approx$ *CKM* $\times$ non-perturbative $\times$ (PT+kinematics)





$$\Delta m_P = |V_{tb}^* V_{tq}| \times f_P^2 \hat{B}_P \times m_P \frac{G_F^2 m_W^2}{6\pi^2} \mathcal{K}$$

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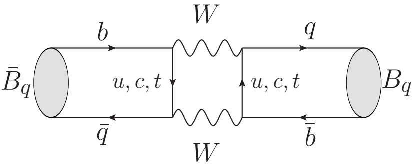
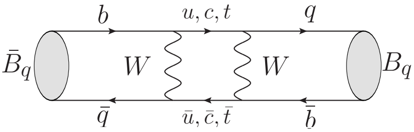


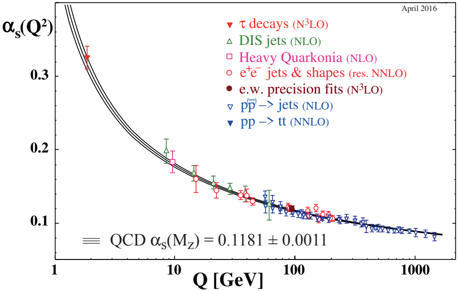


$$\Delta m_P = |V_{tb}^* V_{tq}| \times f_P^2 \hat{B}_P \times m_P \frac{G_F^2 m_W^2}{6\pi^2} \mathcal{K}$$

Computing ξ gives access to $|V_{td}/V_{ts}|$

$$\xi^2 \equiv \frac{f_{B_s}^2 \hat{B}_{B_s}}{f_B^2 \hat{B}_B} = \left|\frac{V_{td}}{V_{ts}}\right|^2 \frac{\Delta m_s}{\Delta m_d} \frac{m_B}{m_{B_s}}$$

Non-Perturbative Physics and Lattice QCD



Source: PDG

(former) BG/Q in Edinburgh

$\Rightarrow$ Large scale computing facilities

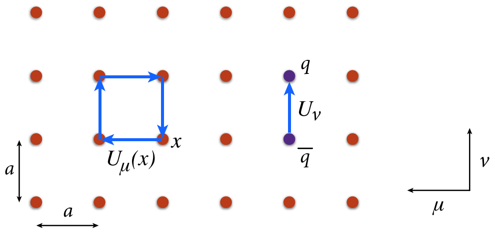
- At *low energy scales* perturbative methods fail
- Lattice QCD simulations provide **first principle precision predictions** for phenomenology
- Calculations need to be improved for observables where the error is dominated by **non-perturbative physics**...

Lattice QCD methodology

Wick rotate ($t \to i\tau$) Path Integral to Euclidean space:

$$\langle \mathcal{O} \rangle_E = \frac{1}{Z} \int \mathcal{D}[\psi, \overline{\psi}]\, \mathcal{D}[U]\, \mathcal{O}[\psi, \overline{\psi}, U]\, e^{-S_E[\psi,\overline{\psi},U]}$$

Introducing lattice renders PI large **but finite** dimensional.



PDG

- Finite lattice spacing a
  $\Rightarrow$ UV regulator
- Finite Box of length L
  $\Rightarrow$ IR regulator

$\Rightarrow$ Calculate PI **explicitly** via Monte Carlo sampling:

A Lattice Computation

Lattice vs Continuum

We simulate:

- at finite lattice spacing a
- in finite volume L^3
- lattice regularised
- Some bare input quark masses am_l, am_s, am_h
  In general: $m_\pi \neq m_\pi^{\text{phys}}$

We want:

- $a = 0$
- $L = \infty$
- some continuum scheme
- $m_l = m_l^{\text{phys}}$
- $m_s = m_s^{\text{phys}}$
- $m_h = m_c^{\text{phys}}, m_b^{\text{phys}}$

$\Rightarrow$ Need to control all limits!

$\rightarrow$ particularly simultaneously control FV and discretisation

$\Rightarrow$ Decide on a fermion action:

Wilson, Staggered, Twisted Mass, **Domain Wall fermions**, $\cdots$

Multiple scale problem: back of the envelope

Control IR (Finite Size Effects) and UV (discretisation) effects

$$m_\pi L \gtrsim 4$$
$$a^{-1} \gg \text{Mass scale of interest}$$

For $m_\pi = m_\pi^{\rm phys} \sim 140\,\mathrm{MeV}$ and $m_b \approx 4.2\,\mathrm{GeV}$:

$$L \gtrsim 5.6\,\mathrm{fm}$$
$$a^{-1} \sim 4.2\,\mathrm{GeV} \approx (0.05\,\mathrm{fm})^{-1}$$

Requires $N \equiv L/a \gtrsim 120 \Rightarrow N^3 \times (2N) \gtrsim 4 \times 10^8$ lattice sites.

VERY EXPENSIVE to satisfy both constraints simultaneously.

Choice of fermion actions (for *b*-physics)

"Relativistic actions"

- Wilson, twisted mass
- **Domain Wall Fermions**
- Overlap
- Staggered (asqtad, HISQ)

"Effective actions" for *b*

- Static quarks
- Non-Relativistic QCD
- Fermilab action
- Relativistic heavy quarks

Different properties:

- computational cost
- chirality
- tuning errors
- systematic errors
- discretisation effects
- renormalisation

Two main "paths":

1. Effective action for *b* - tune to *b* but systematic errors
2. Relativistic action for *b* - Theoretically cleaner but need extrapolation in heavy mass (for now)

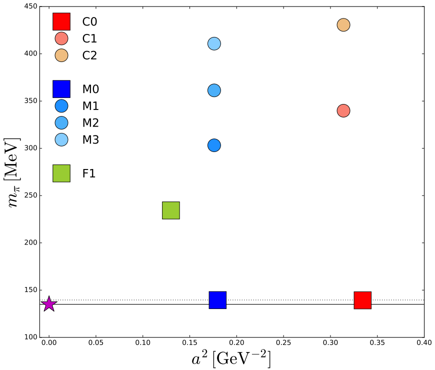
$SU(3)$ breaking ratios for $D_{(s)}$ and $B_{(s)}$ mesons

[1812.08791]

Edinburgh - Southampton - Boulder - BNL (RBC/UKQCD)

Peter Boyle, Luigi Del Debbio, Nicolas Garron, Andreas Jüttner, Amarjit Soni, JTT, Oliver Witzel

RBC/UKQCD $N_f = 2+1$ ensembles



- Iwasaki gauge action
- Domain Wall Fermion action
  $\Rightarrow N_f = 2+1$ flavours in the sea
  $\Rightarrow M_5 = 1.8$ for light and strange
- **2 ensembles with physical pion masses** [PRD **93** (2016) 074505]
- 3 Lattice spacings [JHEP **12** (2017) 008]
- Heavier m_π ensembles guide small chiral extrapolation of F1

Chiral Fermions:

$\Rightarrow$ $O(a)$ improved

$\Rightarrow$ Multiplicative renormalisation

Lattice set-up

Light and strange

- Unitary light quark mass
- Physical strange quark mass
- DWF parameters same between sea and valence
- Gaussian source (sink) smearing for better overlap with ground state

Heavy (charm and beyond)

- Möbius DWF
- $M_5 = 1.0,\ L_s = 12$
- Stout smeared (3 hits, $\rho = 0.1$)
- Range of quark masses from below charm to $\sim m_b/2$ on finest ensemble

$\Rightarrow$ **All DWF** mixed action set-up

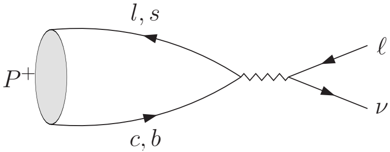
$\Rightarrow$ $\mathbb{Z}_2$-noise sources (volume average) on every 2nd time slice

$\Rightarrow$ Increased heavy quark reach compared to [JHEP **04** (2016) 037,JHEP **12** (2017) 008]

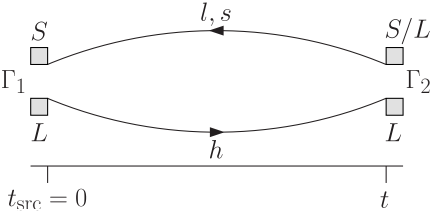
$\rightarrow$ extrapolation towards b

Measurement strategy

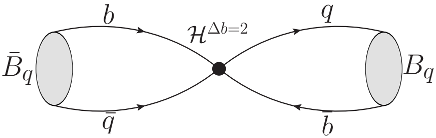
Leptonic decays:



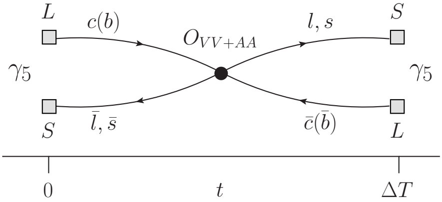
$$\mathcal{Z}_A \left\langle 0 \right| \overline{b}\gamma_4\gamma_5 q \left| B_q(0) \right\rangle = f_{B_q} m_{B_q}$$



$P^0 - \bar{P}^0$-mixing

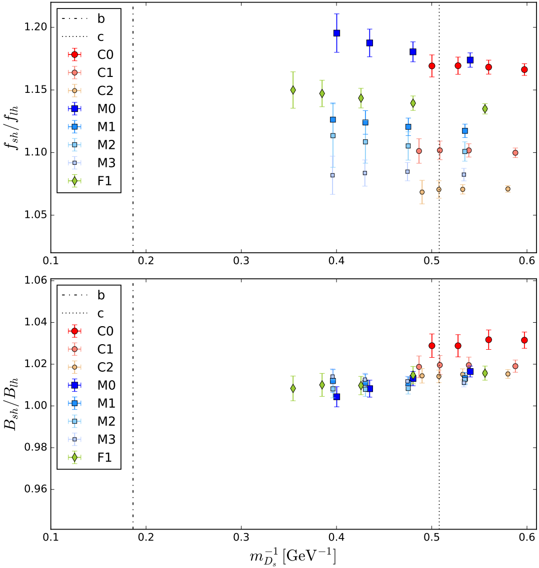


$$B_P = \frac{\left\langle \bar{P}^0 \middle| O_{VV+AA} \middle| P^0 \right\rangle}{8/3 f_P^2 m_P^2}$$



Many source-sink separations ΔT for 4-quark operator

Results of correlator fits - $SU(3)$ breaking ratios



- Renormalisation constants cancel
- Mild behaviour with (inverse) heavy meson mass
- Small discretisation effects: C0 and M0 differ in a
- Statistical error: 0.4 - 1.0 %

Global fit: ansatz

Base fit

$$O(a, m_\pi, m_H) = O(0, m_\pi^{\rm phys}, m_H^{\rm phys}) + C_{CL} a^2 + C_\chi \Delta m_\pi^2 + C_H \Delta m_H^{-1}$$

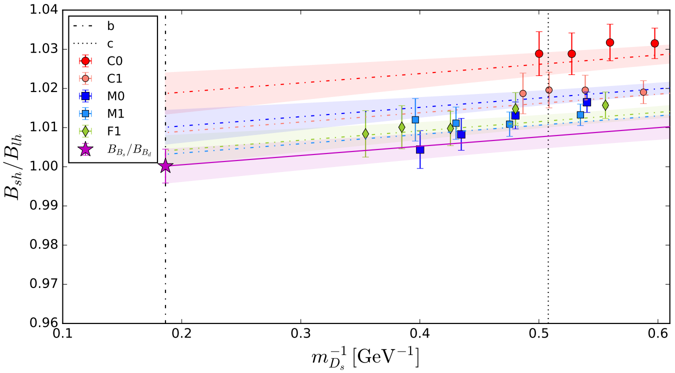
Assess systematic errors by

- varying cuts on pion mass
- using $m_H = m_B$, m_{B_s} and m_{η_b}
- varying inclusion/exclusion of heaviest data points
- varying inclusion/exclusion of fit parameters
- including/estimating higher order terms (a^4, $\left(\Delta m_\pi^2\right)^2$, $\left(\Delta m_H^{-1}\right)^2$)

$\Rightarrow$ Global fits are fully correlated.

Global fit: ratio of bag parameters

$$O(a, m_\pi, m_H) = \frac{B_{B_s}}{B_B} + C_{CL}a^2 + C_\chi \Delta m_\pi^2 + C_H \Delta m_H^{-1}$$



Ratio of bag parameters for $m_\pi \leq 350\,\mathrm{MeV}$

Global fit: ratio of bag parameters

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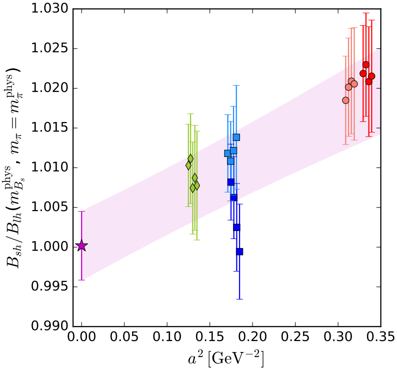
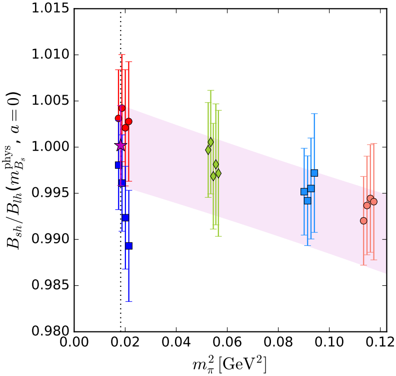




Data points projected to physical b-mass

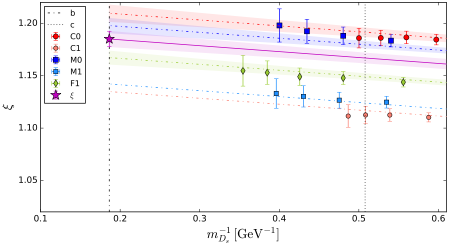
Global fit results - ratio of bag parameters and ξ

Recall:

$$\xi \equiv f_{B_s}/f_B \times \sqrt{B_{B_s}/B_B}$$

1. chiral-CL of product of ratios
2. product of chiral-CL of ratios.

$\xi(a, m_\pi, m_H)$



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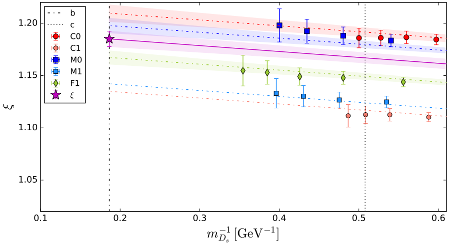


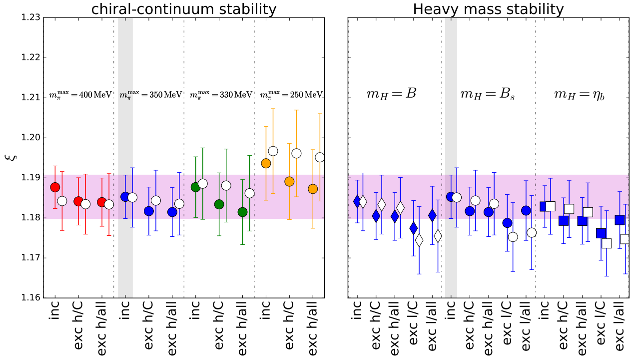
$$\lim_{a\to 0; m_q \to \text{phys}} \left[f_{hs}/f_{hl} \sqrt{B_{hs}/B_{hl}} \right] (a, m_\pi, m_H) = 1.1851(74)_{\text{stat}}$$

$$[f_{B_s}/f_B]_{\text{phys}} \times \sqrt{[B_{B_s}/B_B]_{\text{phys}}} = 1.1853(54)_{\text{stat}}$$

chiral continuum limit of individual ratios gives better signal

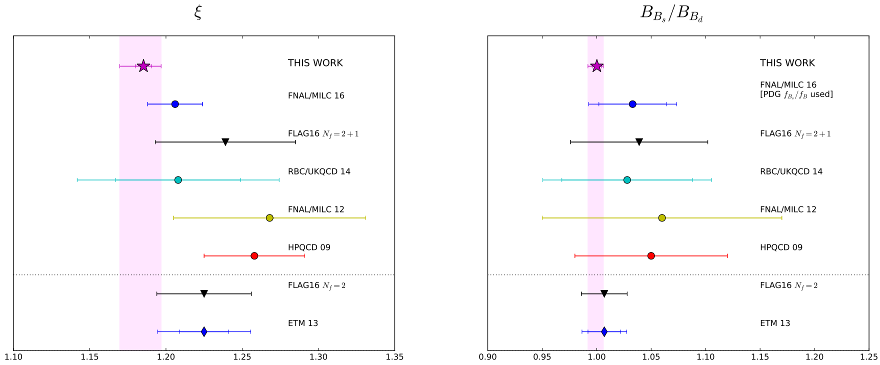
Systematic Errors - variations of cuts to data for ξ

- Global fits all correlated with satisfying p-values.
- sys error: includes chiral-CL (left), heavy mass (right), H.O. terms, $m_u \neq m_d$ and FV.



$$\xi = 1.1853(54)_{\mathrm{stat}} \binom{+116}{-156}_{\mathrm{sys}}$$

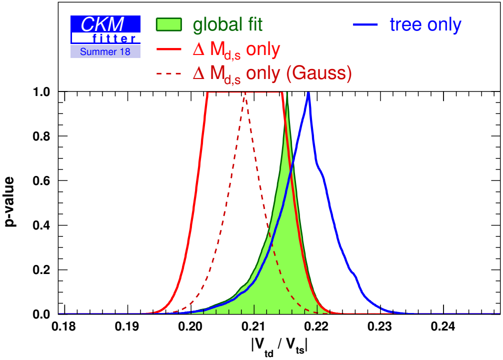
Comparison to literature - ratio of mixing parameters



- Complimentary - no effective action needed for b
- Complimentary - **no operator mixing**!
- **First time with physical pion masses**
- New results: HPQCD'19, King et al. '19

Results for $|V_{td}/V_{ts}|$

$$|V_{td}/V_{ts}| = \begin{cases} 0.2088(^{+16}_{-30}) & \text{CKMfitter (Summer '18)} \\ 0.211(3) & \text{UTfit (Summer '18)} \end{cases}$$



- Slight "discrepancy" between tree-only and loop determinations

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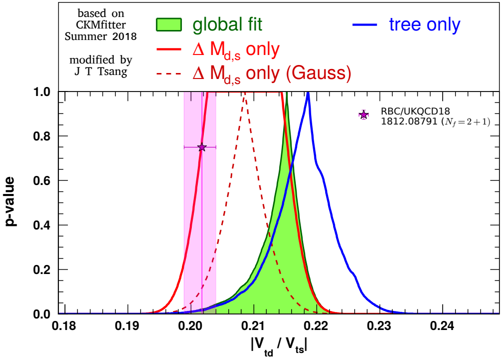

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- Requires more work!

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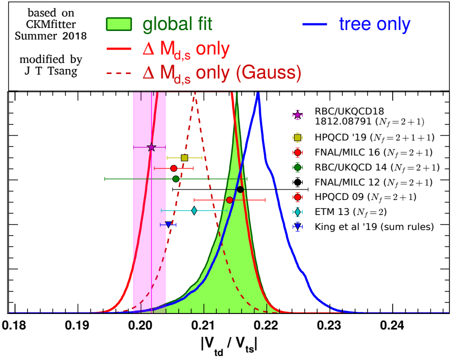
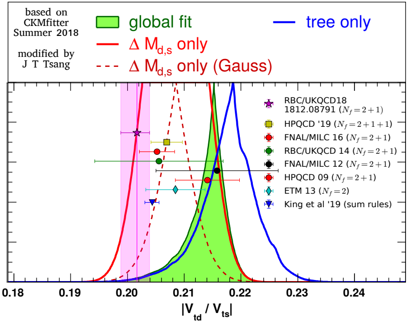


- Slight "discrepancy" between tree-only and loop determinations
- Error still dominated by theory
- Requires more work!
- Many groups are active
- Next target: V_{td}, V_{ts}

Results for $|V_{td}/V_{ts}|$ - lattice action details

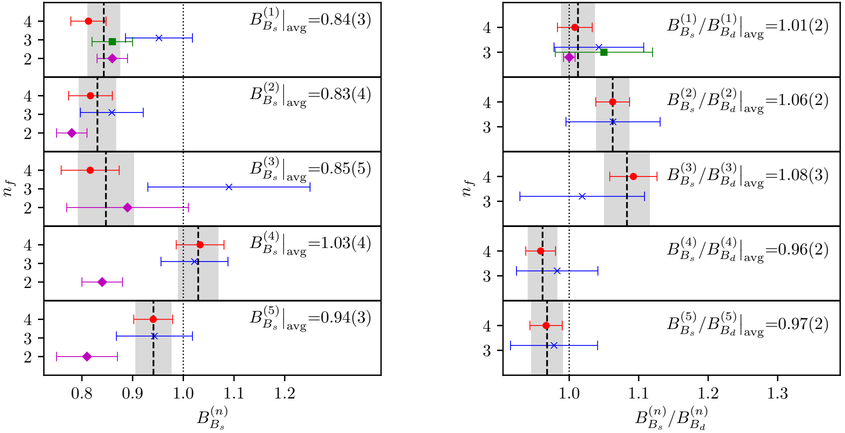
- 2+1 FNAL/MILC'16: asqtad light, Fermilab heavy
- 2+1 RBC/UKQCD'14: DWF light, static heavy
- 2+1 FNAL/MILC'12: asqtad light, Fermilab heavy
- 2+1 HPQCD'09: asqtad light, NRQCD heavy
- 2 EMT'13: twisted mass Wilson light and heavy: using *ratio method*



- 2+1+1 HPQCD'19: HISQ light, NRQCD heavy
- 2+1 RBC/UKQCD'18: DWF light, DWF heavy: mass extrapolation

Beyond $SU(3)$ breaking ratios - individual bag parameters

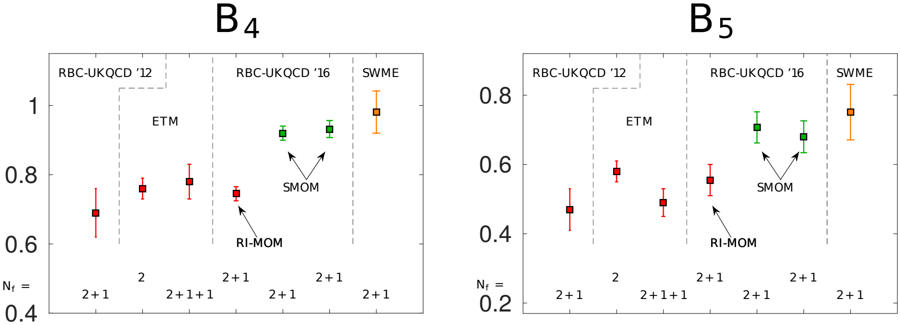
Plots taken from HPQCD'19 - 1907.01025



HPQCD'19, FNAL/MILC'16, *left*: ETM'13, *right*: RBC/UKQCD'18, HPQCD'09

Tension in $B_{B_s}^{(4)}$ and $B_{B_s}^{(5)}$ - but not in ratio

Very reminiscent of neutral Kaon mixing for the same operators:



(Plots produced by N. Garron, taken from J. Kettle's talk at Lattice 2018)

- Issue might be renormalisation (arXiv:1609.03334, arXiv:1708.03552)
- We are in the process of finalising this for $K - \bar{K}$
- We are computing full set of bag parameters for B_d and B_s too!

Next steps: Decay constants and bag parameters

1. Mixed action renormalisation:
   Different choice of (domain wall) action between light/strange and heavy quarks leads to a mixed action
   Mixed action renormalisation constants cancel for appropriate ratios (f_{B_s}/f_B, B_{B_s}/B_B), but are needed for individual decay constants and bag parameters

$\Rightarrow$ Need to carry out the fully non-perturbative mixed action renormalisation as outlined in JHEP **12** (2017) 008.

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$\Rightarrow$ analogous to RBC/UKQCD's $K - \bar{K}$ study (1812.04981, in preparation)

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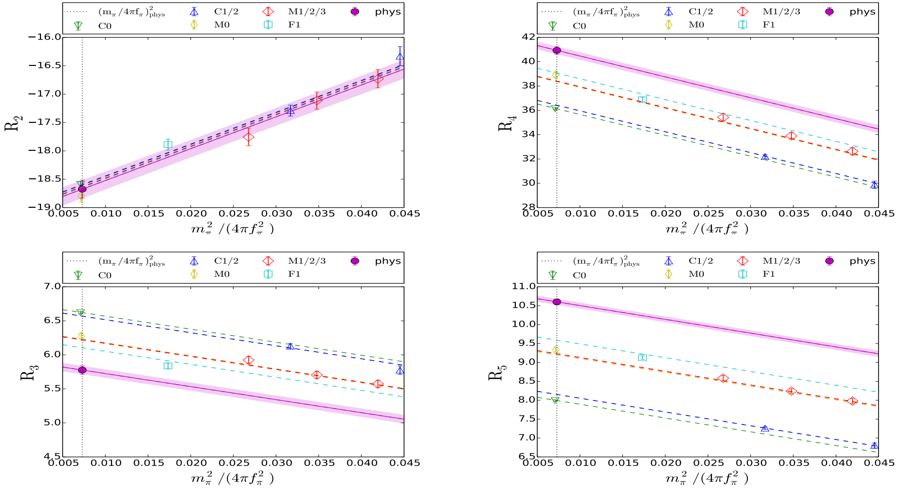
3. Supplement data set with JLQCD ensemble
   (in collaboration with S. Hashimoto and T. Kaneko)

$\Rightarrow$ further reach in m_H due to finer lattice spacing

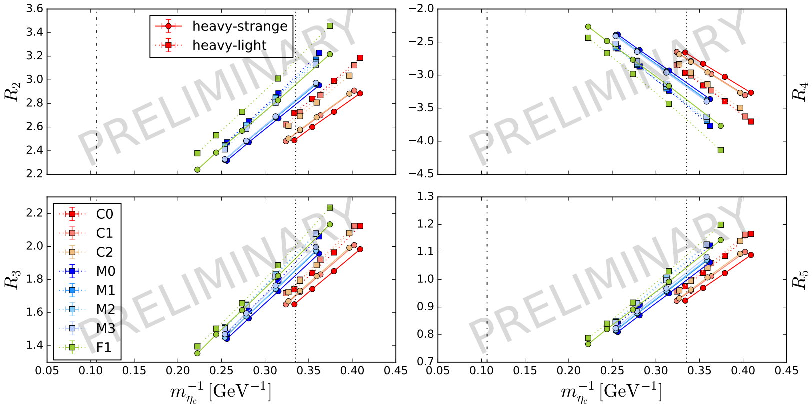
preliminary $K^0 - \bar{K}^0$ results [1812.04981,in preparation]

$$R_i \equiv \langle \bar{P}^0 | \mathcal{O}_i | P^0 \rangle / \langle \bar{P}^0 | \mathcal{O}_1 | P^0 \rangle$$



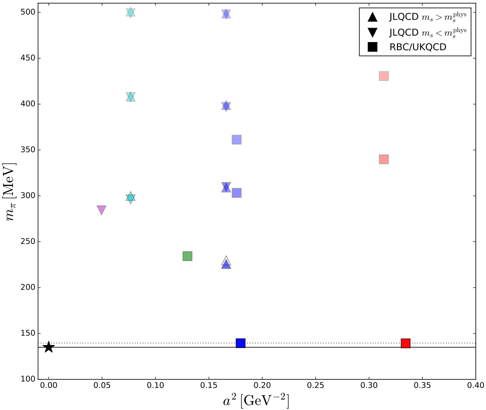
PRELIMINARY RESULTS in $\overline{MS}$ at 3 GeV

$B^0_{(s)} - \bar{B}^0_{(s)}$ (and $D^0 - \bar{D}^0$) **PRELIMINARY and BARE**



- "quite linear" in m_H^{-1}
- similar slopes for h-l and h-s $\Rightarrow SU(3)$ breaking rat's?
- renormalisation to be done (mixed action + op mixing)
- analogous analysis to $K - \bar{K}$ paper + m_H dependence

Increased set of ensembles



JLQCD (triangles)

Fine lattices: $a^{-1} = 2.4 - 4.5\,\mathrm{GeV}$

UKQCD (squares)

+RBC Physical Pion masses

Both: $N_f = 2 + 1$ DWF

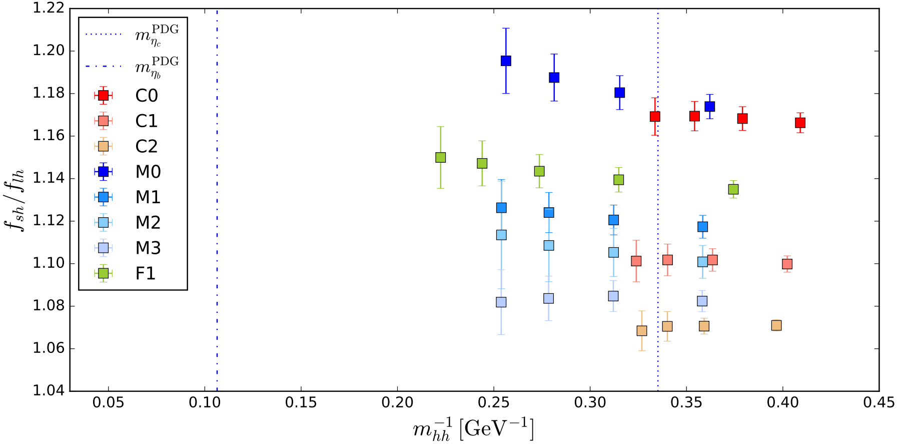
3+3 Lattice Spacings

$\Rightarrow$ Fine lattices: Further heavy quark reach on JLQCD ensembles

$\Rightarrow$ Chiral extrapolation stabilised by $m_\pi^{\rm phys}$ ensembles

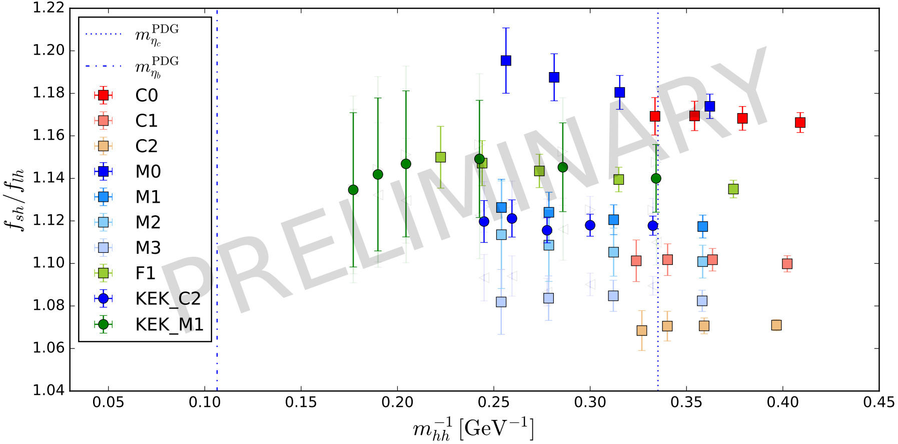
$\Rightarrow$ **Combined physics analysis with S. Hashimoto and T. Kaneko**

JLQCD + RBC/UKQCD data: ratio of decay constants I



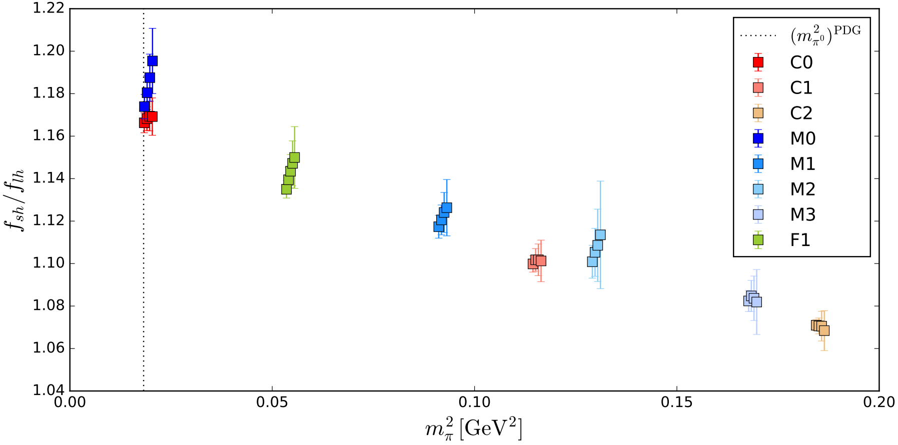
RBC-UKQCD data set from arXiv:1812.08791

JLQCD + RBC/UKQCD data: ratio of decay constants I

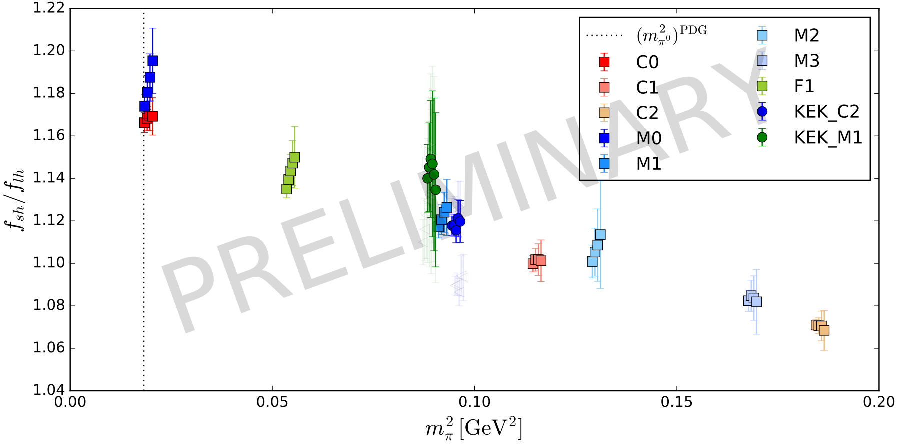


Increased reach in the heavy-mass. The fine KEK ensemble is yet to come!

JLQCD + RBC/UKQCD data: ratio of decay constants II



JLQCD + RBC/UKQCD data: ratio of decay constants II



Conclusions and Outlook

Since last year

- **2 new lattice results**: including m_π^{phys} ensembles
- error on $|V_{td}/V_{ts}|$ still theory dominated: need more work!
- Individual bag parameter uncertainties $\sim 4 - 7\%$ Ratios are more precise!
- Many different actions and methods on the market
- First result without use of effective action: extrapolate in m_h from heavier than charm region to m_b

Ongoing within RBC/UKQCD

- Supplementing dataset with very fine JLQCD ensembles

$\Rightarrow$ Data production ongoing

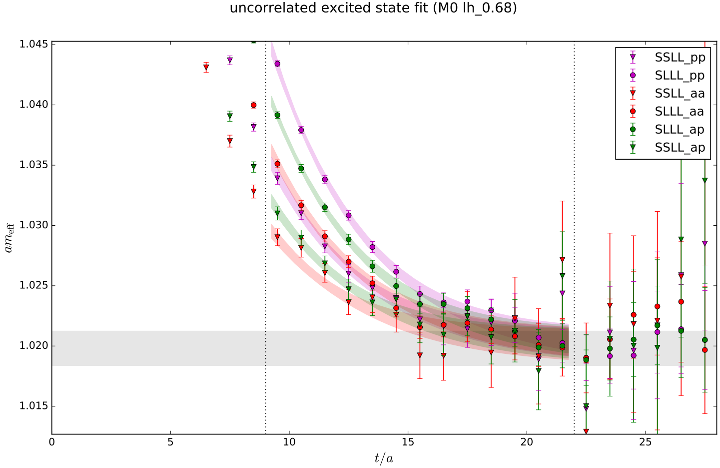
- Combined fit with universality constraint
- Address RI/MOM vs RI/SMOM in $K - \bar{K}$ mixing
- Mixed action renormalisation underway

$\Rightarrow$ Determine $f_{B_{(s)}}$, $f_{D_{(s)}}$

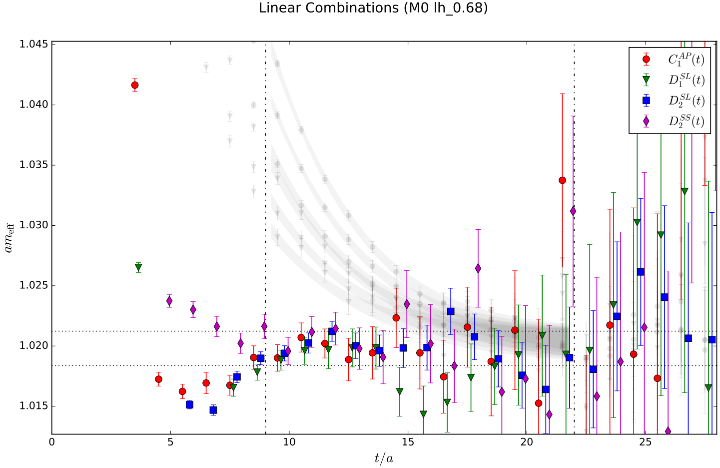
$\Rightarrow$ Full mixing operator basis for B_d and B_s.

ADDITIONAL SLIDES

Correlator fitting: two-points

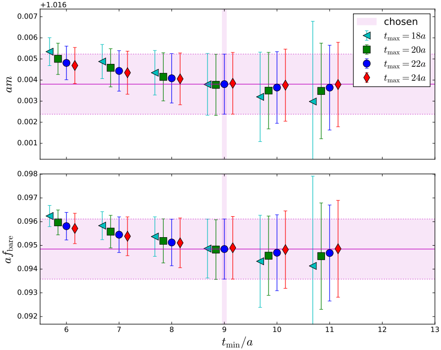


Correlator fitting: two-points

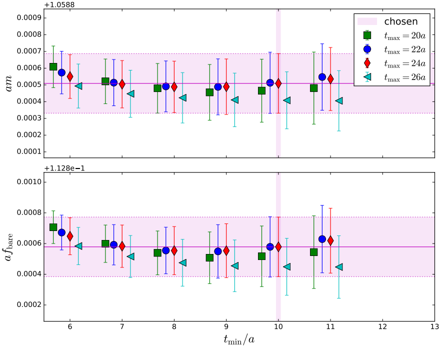


Correlator fitting: stability (2-point functions)

heavy-light on M0 ($am_h = 0.68$)

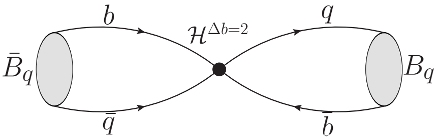


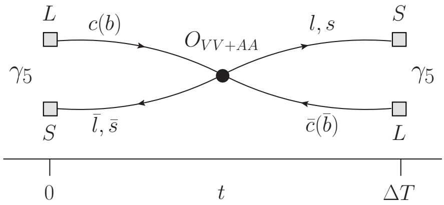
heavy-strange on M0 ($am_h = 0.68$)



Stability under variation of fit ranges

Correlator fitting of 4-quark operators: strategy





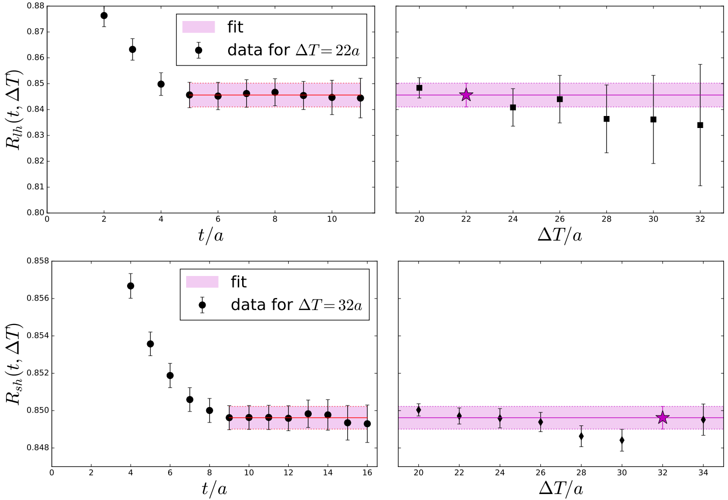
$$C_3(t, \Delta T) \equiv \left\langle P(\Delta T) O_{VV+AA}(t) \bar{P}^\dagger(0) \right\rangle$$

$$R(t, \Delta T) = \frac{C_3(t, \Delta T)}{8/3 C_{PA}(\Delta T - t) C_{AP}(t)} \to B_P \quad \text{for} \quad t, \Delta T \gg 0$$

- Expect $R(t, \Delta T)$ to plateau for large t
- Check stability of plateaux value by varying ΔT

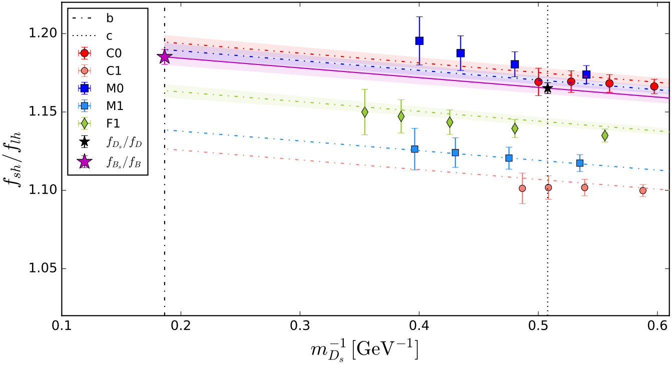
Correlator Fitting of 4-quark operators II

Ex: $am_h = 0.68$ on M0



Global fit: ratio of decay constants

$$O(a, m_\pi, m_H) = \frac{f_{B_s}}{f_B} + C_{CL} a^2 + C_\chi \Delta m_\pi^2 + C_H \Delta m_H^{-1}$$



Ratio of decay constants for $m_\pi \leq 350\,\mathrm{MeV}$

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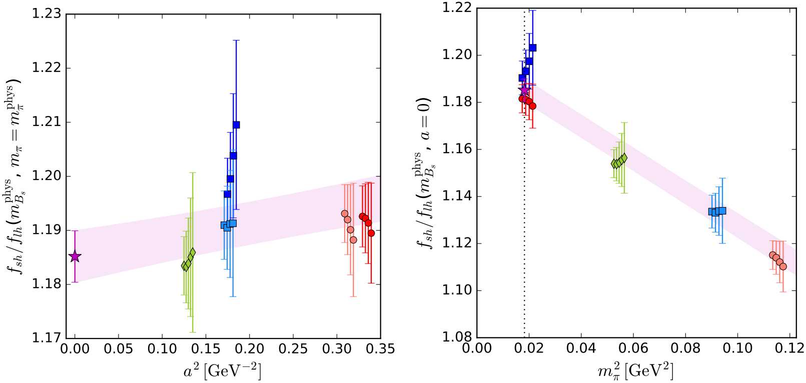

Ratio of decay constants for $m_\pi \leq 350\,\mathrm{MeV}$

Global fit results for ξ

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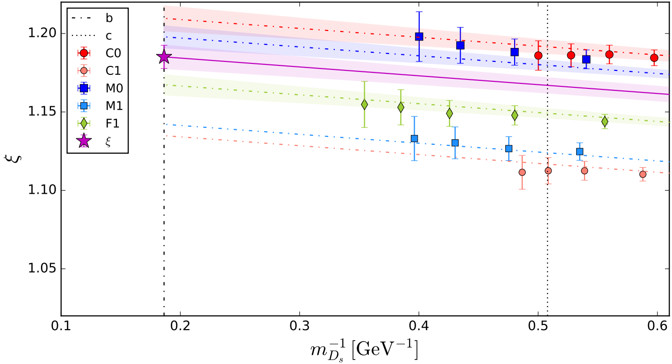

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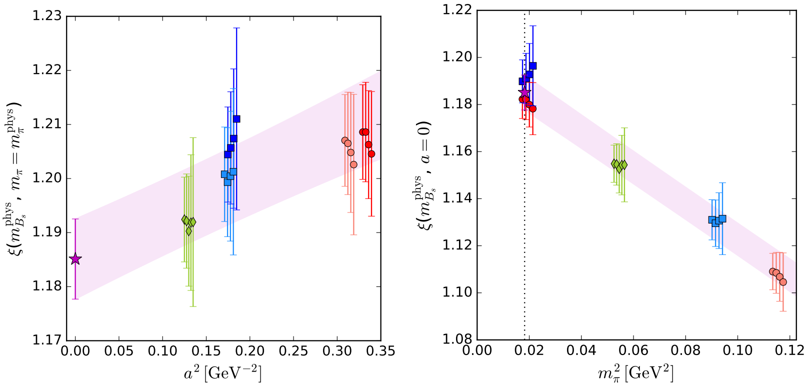
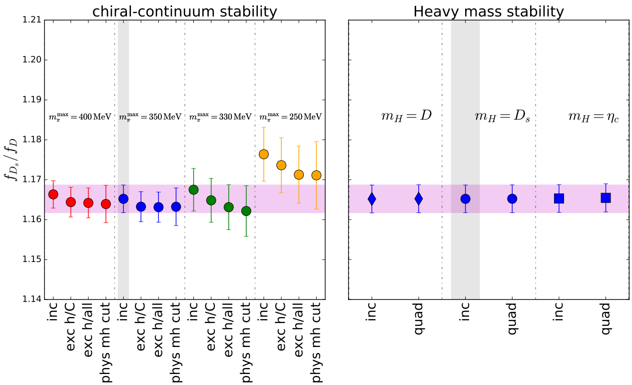


Ratio of decay constants for $m_\pi \leq 350\,\mathrm{MeV}$

Systematic Errors - variations of cuts to data for f_{D_s}/f_D

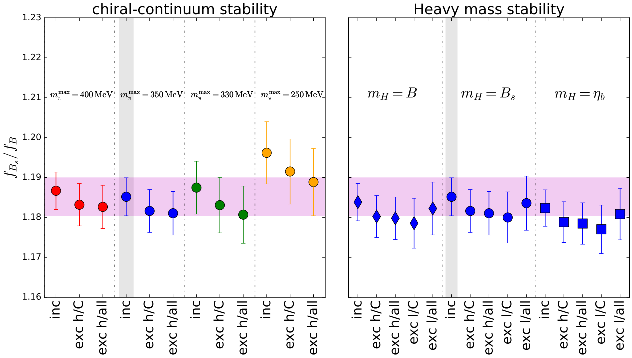
- Global fits all correlated with satisfying p-values.
- sys error: includes chiral-CL (left), heavy mass (right), H.O. terms, $m_u \neq m_d$ and FV.



$$f_{D_s}/f_D = 1.1652(35)_{\mathrm{stat}} \binom{+120}{-\ \ 52}_{\mathrm{sys}}$$

Systematic Errors - variations of cuts to data for f_{B_s}/f_B

- Global fits all correlated with satisfying p-values.
- sys error: includes chiral-CL (left), heavy mass (right), H.O. terms, $m_u \neq m_d$ and FV.



$$f_{B_s}/f_B = 1.1852(48)_{\rm stat}\,\binom{+134}{-145}_{\rm sys}$$

Systematic Errors - variations of cuts to data for B_{B_s}/B_B

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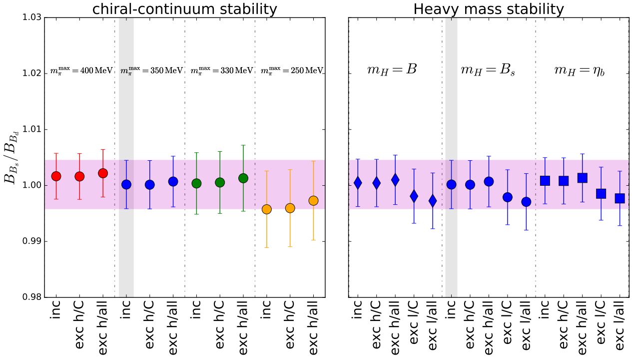

$$B_{B_s}/B_B = 1.0002(43)_{\mathrm{stat}} \left({}^{+60}_{-82}\right)_{\mathrm{sys}}$$

Systematic Errors - variations of cuts to data for ξ

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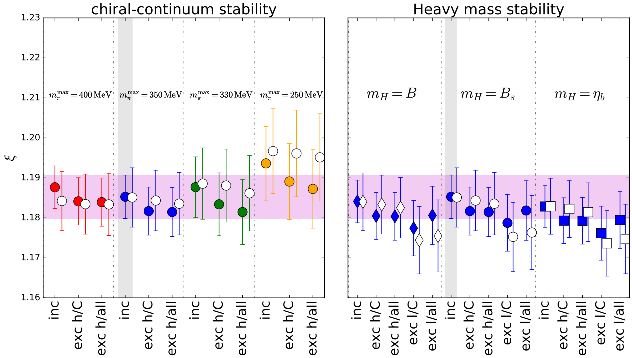


$$\xi = 1.1853(54)_{\mathrm{stat}} \left(\begin{smallmatrix}+116 \\ -156\end{smallmatrix}\right)_{\mathrm{sys}}$$

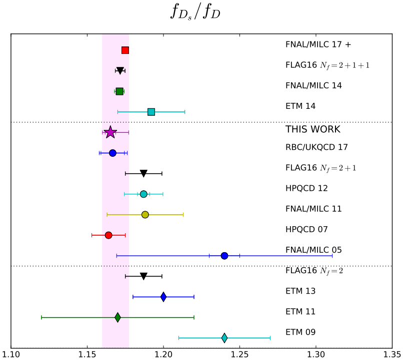
Error budget of 1812.04981

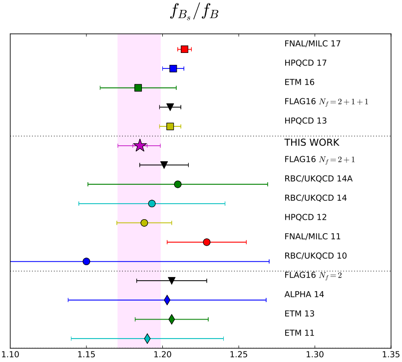
Experimental precision on $\Delta m_s \sim 0.1\%$ and $\Delta m_d \sim 0.4\%$.
Theoretical precision on $\xi \sim 1.3\%$

	f_{D_s}/f_D		f_{B_s}/f_B		ξ		B_{B_s}/B_{B_d}	
	absolute	relative	absolute	relative	absolute	relative	absolute	relative
central	1.1652		1.1852		1.1853		1.0002	
stat	0.0035	0.30%	0.0048	0.40%	0.0054	0.46%	0.0043	0.43%
fit chiral-CL	$^{+0.0112}_{-0.0031}$	$^{+0.96}_{-0.26}\%$	$^{+0.0110}_{-0.0045}$	$^{+0.93}_{-0.38}\%$	$^{+0.0084}_{-0.0038}$	$^{+0.71}_{-0.32}\%$	$^{+0.0020}_{-0.0044}$	$^{+0.20}_{-0.44}\%$
fit heavy mass	$^{+0.0003}_{-0.0000}$	$^{+0.02}_{-0.00}\%$	$^{+0.0000}_{-0.0081}$	$^{+0.00}_{-0.69}\%$	$^{+0.0000}_{-0.0091}$	$^{+0.00}_{-0.77}\%$	$^{+0.0012}_{-0.0031}$	$^{+0.12}_{-0.31}\%$
H.O. heavy	0.0000	0.00%	0.0054	0.45%	0.0049	0.41%	0.0021	0.21%
H.O. disc.	0.0009	0.07%	0.0009	0.07%	0.0021	0.18%	0.0016	0.16%
$m_u \neq m_d$	0.0009	0.08%	0.0009	0.07%	0.0010	0.08%	0.0001	0.01%
finite size	0.0021	0.18%	0.0021	0.18%	0.0021	0.18%	0.0018	0.18%
total systematic	$^{+0.0114}_{-0.0039}$	$^{+0.98}_{-0.34}\%$	$^{+0.0125}_{-0.0137}$	$^{+1.06}_{-1.16}\%$	$^{+0.0102}_{-0.0146}$	$^{+0.86}_{-1.24}\%$	$^{+0.0041}_{-0.0070}$	$^{+0.41}_{-0.70}\%$
total sys+stat	$^{+0.0120}_{-0.0052}$	$^{+1.03}_{-0.45}\%$	$^{+0.0134}_{-0.0145}$	$^{+1.13}_{-1.22}\%$	$^{+0.0116}_{-0.0156}$	$^{+0.97}_{-1.32}\%$	$^{+0.0060}_{-0.0082}$	$^{+0.60}_{-0.82}\%$

$\Rightarrow$ **Systematically Improvable** with finer lattices at (near) physical m_π.

Comparison to literature - ratio of decay constants





- Self consistent with RBC/UKQCD17: JHEP **12** (2017) 008
- Complimentary to (most) literature - no effective action for b.
- One of few results with physical pion masses.

$$|V_{cd}/V_{cs}| = 0.2148(56)_{\text{exp}} \left({}^{+22}_{-10}\right)_{\text{lat}}$$

Non-Perturbative Renormalisation of mixed action

SMOM ren. conds. relates amputated vertex functions to Z factors.

$$\begin{aligned} 1 &= \lim_{\bar{m}\to 0} \frac{1}{12q^2} \mathrm{Tr}\left[\left(q \cdot \Lambda_A^{\mathrm{ren}}\right) \gamma_5 \not{q}\right] \Big|_{\mathrm{sym}} \\ &= \frac{Z_A}{Z_q} \lim_{\bar{m}\to 0} \frac{1}{12q^2} \mathrm{Tr}\left[\left(q \cdot \Lambda_A^{\mathrm{bare}}\right) \gamma_5 \not{q}\right] \Big|_{\mathrm{sym}} \\ &\equiv \frac{Z_A}{Z_q} \mathcal{P}[\Lambda_A^{\mathrm{bare}}] \end{aligned}$$

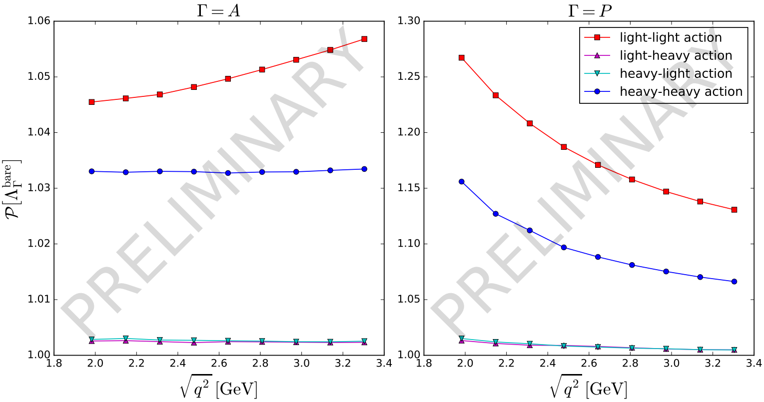
So for actions i,j

$$\frac{\mathcal{P}[\Lambda_A^{\mathrm{bare}}]^{ii} \mathcal{P}[\Lambda_A^{\mathrm{bare}}]^{jj}}{\left(\mathcal{P}[\Lambda_A^{\mathrm{bare}}]^{ij}\right)^2} = \frac{(Z_A^{ij})^2}{Z_A^{ii} Z_A^{jj}}$$

But for non-mixed actions we can determine Z_A^{ii} from conserved current.

Preliminary mixed action renormalisation

First study on single configuration



$\Rightarrow$ mixed NPR is feasible

$\Rightarrow$ need to compute Z_A^{hh} from conserved current to obtain Z_A^{hl}