Comprehensive study of $\tau(B_s)/\tau(B_d)$

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History of $\tau(B_s)/\tau(B_d)$
**Motivation**

Current experimental status:

![Graph showing 
\[ \frac{\tau(B_s)}{\tau(B_d)} \]
with data points at 2003: 0.94, 2005: 0.96, 2007: 0.98, 2009: 0.98, 2011: 1.00, 2013: 0.98, 2015: 0.97, 2017: 0.96, 2019: 0.95.]

- **LHCb**: $1.0024 \pm 0.0041$ [LHCb, 1906.08356]
- **ATLAS**: $0.9834 \pm 0.0030$ [ATLAS-CONF-2019-009]
Motivation

★ Current theoretical status:

◊ In the Heavy Quark Expansion (HQE) framework

\[
\frac{\tau(B_s)}{\tau(B_d)} = \frac{\Gamma_b + \delta\Gamma_{B_d}}{\Gamma_b + \delta\Gamma_{B_s}} \approx 1 - \Gamma_b^{-1}(\delta\Gamma_{B_s} - \delta\Gamma_{B_d}) - 0.0006 \pm 0.0025
\]

[Shifman, Voloshin ’85, Kirk, Lenz, Rauh ’17]

★ Γ_b - leading contribution ★ δΓ_{B_q} - subleading effects

◊ Multiple astonishing cancellations arise

◊ Unique possibility

★ to compete with increasing experimental precision

★ to validate HQE expansion

★ to test for BSM scenarios and search for invisible decays
BSM contributions appear in the lifetime ratio as:

$\frac{\tau(B_s)}{\tau(B_d)} \approx 1 - \frac{1}{\Gamma_b} \left( \delta \Gamma_{B_s}^{\text{SM}} - \delta \Gamma_{B_d}^{\text{SM}} \right)_{\text{theory}}$

- $\left[ \text{BR}(B_s \to X)^{\text{BSM}} - \text{BR}(B_d \to X)^{\text{BSM}} \right]_{\text{indirectly constrained}}$

- NP could affect differently $b \to s$ and $b \to d$ transitions

- Possibility to constrain BSM contributions at permille level

- Hints for NP in $b \to s\mu^+\mu^-$ and $b \to c\tau^-\bar{\nu}_\tau$ processes might point towards large effects in $b \to s\tau^+\tau^-$ [Capdevila et al. '18; Bordone et al. '19]

[See also talk of M. König]
The $B_s \to \tau^+ \tau^-$ decay

- Suppressed in the SM and experimentally very challenging:

$$\text{Br}(B_s \to \tau^+ \tau^-) \begin{cases} \text{SM} = (7.73 \pm 0.49) \times 10^{-7} & [\text{Bobeth et al. '14}] \\ \text{EXP} < 6.8 \times 10^{-3} & [\text{LHCb, 1703.02508}] \end{cases}$$

- Correlations between $R_D(\ast)$, $R_{J/\psi}$ and $B_s \to \tau^+ \tau^-$, ...

- These big effects might be visible in the $\tau(B_s)/\tau(B_d)$ lifetime ratio

[Capdevila et al. '18] [Bordone et al. '19]
The theoretical framework

★ From the optical theorem:

\[ \Gamma_{B_q} = \frac{1}{2m_{B_q}} \text{Im} \langle B_q | i \int d^4x \mathcal{T} \{ \mathcal{L}_{\text{eff}}(x), \mathcal{L}_{\text{eff}}(0) \} | B_q \rangle \]

★ Expand in inverse power of \( m_b \):

\[ p^\mu_b = m_b v^\mu + k^\mu \]

\[ \Gamma_{B_q} = \Gamma_0 \langle \mathcal{O}_3 \rangle + \Gamma_2 \frac{\langle \mathcal{O}_5 \rangle}{m_b^2} + \Gamma_3 \frac{\langle \mathcal{O}_6 \rangle}{m_b^3} + \ldots + 16\pi^2 \left[ \tilde{\Gamma}_3 \frac{\langle \tilde{\mathcal{O}}_6 \rangle}{m_b^3} + \tilde{\Gamma}_4 \frac{\langle \tilde{\mathcal{O}}_7 \rangle}{m_b^4} + \ldots \right] \]

\[ \delta \Gamma_{B_q} \]

★ \( \Gamma_i, \tilde{\Gamma}_i \) - short distance coefficients

★ \( \mathcal{O}_d, \tilde{\mathcal{O}}_d \) - local quark operator of dimension \( d \)

\[ \frac{\delta \Gamma_{B_q}^{(d)}}{\Gamma_b} \sim \left( \frac{k}{m_b} \right)^{d-3} \sim \left( \frac{1 \text{ GeV}}{4.5 \text{ GeV}} \right)^{d-3} \] - small parameter
The leading contribution

- Free b-quark decay, independent of the spectator quark

\[ \Gamma_b = \frac{G_F^2 m_b^5}{192\pi^3} |V_{cb}|^2 c_{3,b} \]

\[ c_{3,b} = c_{3,b}^{\bar{c}\bar{s}} + c_{3,b}^{\bar{c}\bar{u}d} + c_{3,b}^{\bar{c}e\bar{\nu}_e} + c_{3,b}^{\bar{c}\mu\bar{\nu}_\mu} + c_{3,b}^{\bar{c}\tau\bar{\nu}_\tau} + \ldots \]

- NLO corrections are found to have sizeable effect [Review by Lenz, ’15]

- Numerical update in progress
The 2-loop power corrections

★ Include spectator quark effects:

★ Key ideas:

○ Background field method [Novikov et al. '84]

\[
iS(x, y) = \int d^4 z \, iS^{(0)}(x - z) iA(z) iS^{(0)}(z - y) + \ldots
\]

○ Heavy Quark Expansion [Shifman, Voloshin '85]

★ Non-perturbative parameters arise:

○ \( \mu_\pi^2(B_q), \mu_G^2(B_q) \) (at \( 1/m_b^2 \))

○ \( \rho_D^3(B_q), \rho_{LS}^3(B_q) \) (at \( 1/m_b^3 \))
The 2-loop power corrections

★ At order $1/m_b^2$ coefficients known for both SL and NL decays

[Bigi et al. ’92, Blok, Shifman ’93]

◊ We have recomputed them for two arbitrary masses

★ At order $1/m_b^3$ coefficients known only for SL decays

[Gremm, Kapustin ’96]

◊ From naive estimate:

$$\frac{\delta \Gamma^{(6)}_{B_s} - \delta \Gamma^{(6)}_{B_d}}{\Gamma_b} \sim \mathcal{O}(10^{-3})$$

◊ Possible large effect in $\tau(B_s)/\tau(B_d)$

★ Computation of $1/m_b^3$ corrections for NL decays in progress

[Lenz, MLP, Rusov]
The 1-loop power corrections

★ Spectator quark effects: another type of contributions

★ 1-loop, factor of $16\pi^2$ enhancement (compared to previous diagram)

★ Appear first at $1/m_b^3$

◇ Additional non-perturbative input $B_{1,2}(B_q)$ and $\epsilon_{1,2}(B_q)$:

* known for $B_d$ meson [Kirk, Lenz, Rauh ’17]

* in progress for $B_s$ meson [King, Lenz, Rauh]
The 1-loop power corrections

★ Expanding further leads to $1/m_b^4$ corrections

★ More non-perturbative parameters $\rho_i(B_q), \sigma_i(B_q)$ \hspace{1cm} ($i = 1, \ldots, 6$)

- Not determined yet \hspace{1cm} [Kirk, Lenz, Rauh ’17]

- Use vacuum insertion approximation: $\rho_i = 1 \pm 1/12, \quad \sigma_i = 0 \pm 1/6$

- First calculation for $B_s$-mixing by Lattice QCD \hspace{1cm} [HPQCD, 1910.00970]

★ Confirmation of vacuum insertion approximation
(Very) preliminary numerical result

$$\frac{\tau(B_s)}{\tau(B_d)} = 0.9988 \pm 0.0002 \pm 0.0006 \pm 0.0013 \pm 0.0024 \pm 0.0023 \pm 0.0028 \pm \ldots$$

\[ 1 + 0.0033 - 0.0046 + 0.0001 \ldots \]

- 10% $SU(3)_f$ violation
- Exact $SU(3)_f$ symmetry

**Peculiarities of 1-loop $1/m_b^3$ contributions**

$$\delta \tilde{\Gamma}_{B_q}^{(6)} \sim \left\{ \left( \frac{C_2^2}{3} + 2 C_1 C_2 + 3 C_1^2 \right) \left( \frac{m_c^2}{m_b^2} \right) \right\} \approx 10^{-2}$$

- Strong suppression despite loop enhancement
- 2-loop $1/m_b^3$ corrections might have crucial effect
Conclusion

- The lifetime ratio $\tau(B_s)/\tau(B_d)$ is theoretically understood at the permille level - even higher experimentally precision is desirable.

- In the SM, multiple cancellations arise, leading to sensitivity to higher order corrections.

- Potential to indirectly constrain some BSM models.

- Improve current SM prediction computing:
  - Bag parameters for $B_s$ meson at order $1/m_b^3$.
  - 2-loop spectator quark effects at order $1/m_b^3$. 
Thanks for the attention
Backup slides
Quark Hadron Duality

Experiment at hadron level, calculation at quark-gluon level

★

\[ \text{QHD violation} \equiv \begin{cases} 1/m_Q \text{ corrections in } \Gamma \\ \text{osscillatory terms in } \Gamma \end{cases} \]

★ In the ’90s appears discrepancy:

\[ \frac{\tau(\Lambda_b)}{\tau(B_d)} = \begin{cases} \sim 0.96 & \text{[Shifman, Voloshin ’86]} \\ 0.798 \pm 0.034 & \text{[HFAG ’03]} \end{cases} \]

★ 2019 status:

\[ \frac{\tau(\Lambda_b)}{\tau(B_d)} = \begin{cases} 0.935 \pm 0.054 & \text{[Lenz ’14]} \\ 0.969 \pm 0.006 & \text{[HFLAV ’19]} \end{cases} \]

◇ Shift of 4.9 \sigma
Quark Hadron Duality

Experiment at hadron level, calculation at quark-gluon level

★ Compare HQE with experiments:

◊ No sign of any significant deviation

◊ $\Delta \Gamma_s$ highly sensitive (fewer states, smaller phase space)

☆ Good agreement

★ Simplified models of QCD:

◊ SV limit: no duality violation for SL and NL decays

[Boyd, Grinstein, Manohar '95; Grinstein, Savrov '03]

◊ ’tHooft model: no $1/m_Q$ corrections, tiny oscillatory terms

[Grinstein, Lebed ’97, ’98, ’01]
[Bigi, Shifman, Uraltsev, Vainshtein ’98, ’99, ’00]