Comprehensive study of $\tau(B_s)/\tau(B_d)$

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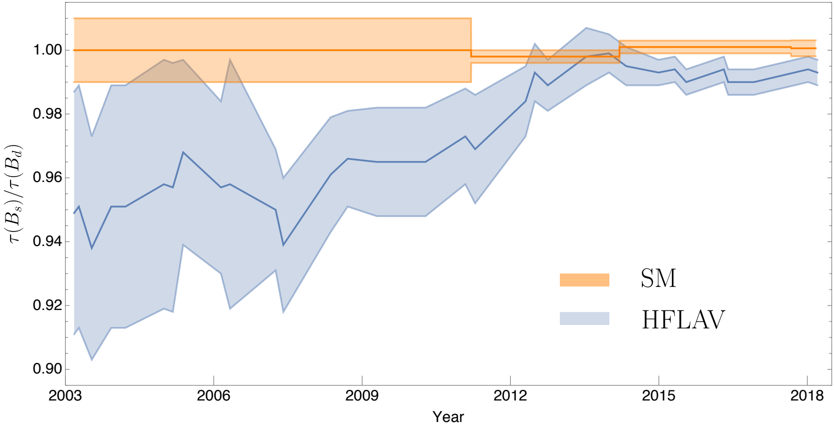
History of $\tau(B_s)/\tau(B_d)$

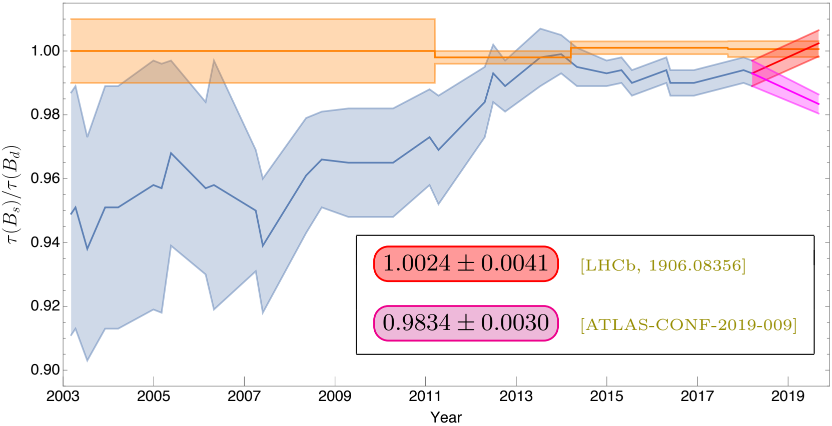
Credit M. Kirk



Motivation

★ Current experimental status:

Credit M. Kirk



Motivation

★ Current theoretical status:

- ◇ In the Heavy Quark Expansion (HQE) framework

[Shifman, Voloshin '85]

$$\frac{\tau(B_s)}{\tau(B_d)} = \frac{\Gamma_b + \delta\Gamma_{B_d}}{\Gamma_b + \delta\Gamma_{B_s}} \approx 1 - \underbrace{\Gamma_b^{-1}(\delta\Gamma_{B_s} - \delta\Gamma_{B_d})}_{-0.0006 \pm 0.0025}$$

[Kirk, Lenz, Rauh '17]

  - ∗ Γ_b - leading contribution
  - ∗ $\delta\Gamma_{B_q}$ - subleading effects

- ◇ Multiple astonishing cancellations arise
- ◇ Unique possibility
  - ∗ to compete with increasing experimental precision
  - ∗ to validate HQE expansion
  - ∗ to test for BSM scenarios and search for invisible decays

Motivation

⋆ BSM contributions appear in the lifetime ratio as:

$$\underbrace{\frac{\tau(B_s)}{\tau(B_d)}}_{\text{exp.}} \approx 1 - \underbrace{\Gamma_b^{-1}\left(\delta\Gamma_{B_s}^{\text{SM}} - \delta\Gamma_{B_d}^{\text{SM}}\right)}_{\text{theory}}$$

$$- \underbrace{\left[\text{BR}(B_s \to X)^{\text{BSM}} - \text{BR}(B_d \to X)^{\text{BSM}}\right]}_{\text{indirectly constrained}}$$

- NP could affect differently $b \to s$ and $b \to d$ transitions
- Possibility to constrain BSM contributions at permille level
- Hints for NP in $b \to s\mu^+\mu^-$ and $b \to c\tau^-\bar{\nu}_\tau$ processes might point towards large effects in $b \to s\tau^+\tau^-$ [Capdevila et al. '18; Bordone et al. '19]

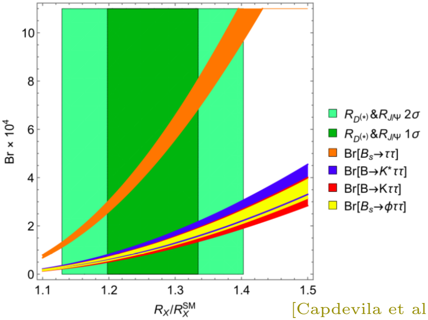
[See also talk of M. König]

The $B_s \to \tau^+\tau^-$ decay

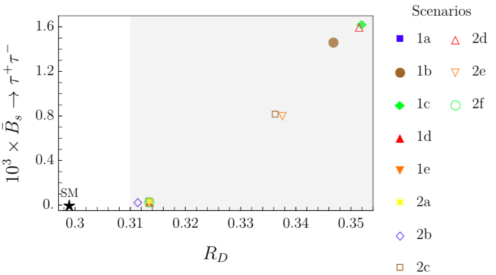
◇ Suppressed in the SM and experimentally very challenging:

$$\text{Br}(B_s \to \tau^+\tau^-) \begin{cases} \stackrel{\text{SM}}{=} (7.73 \pm 0.49) \times 10^{-7} & \text{[Bobeth et al. '14]} \\ \stackrel{\text{EXP}}{<} 6.8 \times 10^{-3} & \text{[LHCb, 1703.02508]} \end{cases}$$

◇ Correlations between $R_{D^{(*)}}$, $R_{J/\psi}$ and $B_s \to \tau^+\tau^-$, ...



[Capdevila et al. '18]



[Bordone et al. '19]

◇ These big effects might be visible in the $\tau(B_s)/\tau(B_d)$ lifetime ratio

The theoretical framework

⋆ From the optical theorem:

$$\Gamma_{B_q} = \frac{1}{2m_{B_q}} \mathrm{Im} \langle B_q | \, i \int d^4x \, \mathcal{T}\{\mathcal{L}_{\rm eff}(x), \mathcal{L}_{\rm eff}(0)\} \, |B_q\rangle$$

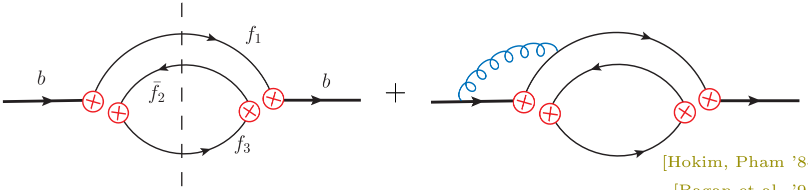
⋆ Expand in inverse power of m_b: $\quad p_b^\mu = m_b v^\mu + k^\mu$

$$\Gamma_{B_q} = \underbrace{\Gamma_0 \langle \mathcal{O}_3 \rangle}_{\Gamma_b} + \underbrace{\Gamma_2 \frac{\langle \mathcal{O}_5 \rangle}{m_b^2} + \Gamma_3 \frac{\langle \mathcal{O}_6 \rangle}{m_b^3} + \ldots + 16\pi^2 \left[\tilde{\Gamma}_3 \frac{\langle \tilde{\mathcal{O}}_6 \rangle}{m_b^3} + \tilde{\Gamma}_4 \frac{\langle \tilde{\mathcal{O}}_7 \rangle}{m_b^4} + \ldots \right]}_{\delta\Gamma_{B_q}}$$

- $\Gamma_i, \tilde{\Gamma}_i$ - short distance coefficients
- $\mathcal{O}_d, \tilde{\mathcal{O}}_d$ - local quark operator of dimension d
- $\dfrac{\delta\Gamma_{B_q}^{(d)}}{\Gamma_b} \sim \left(\dfrac{k}{m_b}\right)^{d-3} \sim \left(\dfrac{1\,\mathrm{GeV}}{4.5\,\mathrm{GeV}}\right)^{d-3}$ - small parameter

The leading contribution

- ⋆ Free b-quark decay, independent of the spectator quark



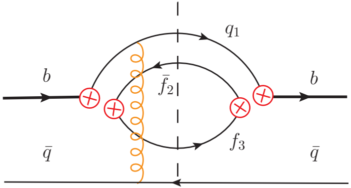
f_i - all possible fermions

[Hokim, Pham '84]
[Bagan et al. '94]
[Lenz, Nierste, Ostermaier '97]
[Greub, Liniger '00, '01]
[Krinner, Lenz, Rauh '13]

- ⋆ $\Gamma_b = \frac{G_F^2 m_b^5}{192\pi^3} |V_{cb}|^2 \, c_{3,b}$
- ⋆ $c_{3,b} = c_{3,b}^{c\bar{c}s} + c_{3,b}^{c\bar{u}d} + c_{3,b}^{ce\bar{\nu}_e} + c_{3,b}^{c\mu\bar{\nu}_\mu} + c_{3,b}^{c\tau\bar{\nu}_\tau} + \ldots$
- ⋆ NLO corrections are found to have sizeable effect [Review by Lenz, '15]
  - ◇ Numerical update in progress

The 2-loop power corrections

- ⋆ Include spectator quark effects:



- ⋆ Key ideas:
  - ◇ Background field method [Novikov et al. '84]

$$iS(x,y) = \int d^4z \, iS^{(0)}(x-z) i\not{A}(z) \, iS^{(0)}(z-y) + \dots$$

  - ◇ Heavy Quark Expansion [Shifman, Voloshin '85]

- ⋆ Non-perturbative parameters arise:
  - ◇ $\mu_\pi^2(B_q), \mu_G^2(B_q)$ (at $1/m_b^2$)
  - ◇ $\rho_D^3(B_q), \rho_{LS}^3(B_q)$ (at $1/m_b^3$)

The 2-loop power corrections

- $\star$ At order $1/m_b^2$ coefficients known for both SL and NL decays

  [Bigi et al. '92, Blok, Shifman '93]

  - $\diamond$ We have recomputed them for two arbitrary masses

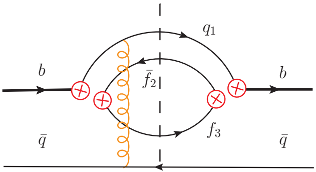
- $\star$ At order $1/m_b^3$ coefficients known only for SL decays

  [Gremm, Kapustin '96]

  - $\diamond$ From naive estimate:

$$\frac{\delta\Gamma_{B_s}^{(6)} - \delta\Gamma_{B_d}^{(6)}}{\Gamma_b} \sim \mathcal{O}(10^{-3})$$

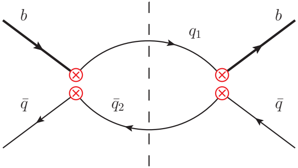
  - $\diamond$ Possible large effect in $\tau(B_s)/\tau(B_d)$



- $\star$ Computation of $1/m_b^3$ corrections for NL decays in progress

  [Lenz, MLP, Rusov]

⋆ Spectator quark effects: another type of contributions



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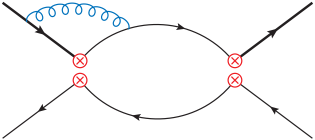


[Uraltsev '96; Neubert, Sachrajda '96] [Franco et al. '02]

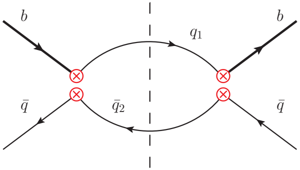
⋆ 1-loop, factor of $16\pi^2$ enhancement (compared to previous diagram)

⋆ Appear first at $1/m_b^3$

- ◇ Additional non-perturbative input $B_{1,2}(B_q)$ and $\epsilon_{1,2}(B_q)$:
  - ∗ known for B_d meson [Kirk, Lenz, Rauh '17]
  - ∗ in progress for B_s meson [King, Lenz, Rauh]

The 1-loop power corrections

⋆ Expanding further leads to $1/m_b^4$ corrections



⋆ More non-perturbative parameters $\rho_i(B_q)$, $\sigma_i(B_q)$ $\quad (i = 1, \ldots, 6)$

- ◇ Not determined yet

  [Kirk, Lenz, Rauh '17]
- ◇ Use vacuum insertion approximation: $\rho_i = 1 \pm 1/12, \quad \sigma_i = 0 \pm 1/6$
- ◇ First calculation for B_s-mixing by Lattice QCD [HPQCD, 1910.00970]
  - ∗ Confirmation of vacuum insertion approximation

(Very) preliminary numerical result

$$\tau(B_s)/\tau(B_d) \;=\; \boxed{0.9988} \pm \underbrace{0.0002}_{\text{par.}} \pm \underbrace{0.0006}_{\mu_\pi^2,\,\mu_G^2} \pm \underbrace{0.0013}_{\mu} \pm \underbrace{\substack{0.0024 \\ 0.0023}}_{\rho_i,\sigma_i} \pm \underbrace{\substack{0.0028 \\ 0.0020}}_{\mathrm{B}_i,\epsilon_i} \pm \ldots$$

$$\downarrow$$

$$\boxed{1 + \underbrace{0.0033}_{\Delta\Gamma_5^{(LO)}} - \underbrace{0.0046}_{\Delta\tilde{\Gamma}_6^{(NLO)}} + \underbrace{0.0001}_{\Delta\tilde{\Gamma}_7^{(LO)}} \ldots}$$

- 10% $SU(3)_f$ violation (upper values: 0.0024, 0.0028)
- Exact $SU(3)_f$ symmetry (lower values: 0.0023, 0.0020)

⋆ Peculiarities of 1-loop $1/m_b^3$ contributions

$$\delta\tilde{\Gamma}_{B_q}^{(6)} \sim \left\{ \underbrace{\left(\frac{C_2^2}{3} + 2\,C_1C_2 + 3\,C_1^2\right)}_{\approx 10^{-2}} \left((\underbrace{B_2^q}_{\approx 1} - \underbrace{B_1^q}_{\approx 1}) + \underbrace{\mathcal{O}\left(\frac{m_c^2}{m_b^2}\right)}_{\approx 0.05} \right) + \underbrace{2C_2^2}_{\approx 2}\; \underbrace{f(\epsilon_2,\epsilon_1)}_{\text{color suppr.}} \right\}$$

- Strong suppression despite loop enhancement
- 2-loop $1/m_b^3$ corrections might have crucial effect

Conclusion

- The lifetime ratio $\tau(B_s)/\tau(B_d)$ is theoretically understood at the permille level - even higher experimentally precision is desirable
- In the SM, multiple cancellations arise, leading to sensitivity to higher order corrections
- Potential to indirectly constrain some BSM models
- Improve current SM prediction computing:
  - Bag parameters for B_s meson at order $1/m_b^3$
  - 2-loop spectator quark effects at order $1/m_b^3$

Thanks for the attention

Backup slides

Quark Hadron Duality

Experiment at hadron level, calculation at quark-gluon level

- ⋆

$$\text{QHD violation} \equiv \begin{cases} 1/m_Q \text{ corrections in } \Gamma \\ \text{oscillatory terms in } \Gamma \end{cases}$$

- ⋆ In the '90s appears discrepancy:

$$\frac{\tau(\Lambda_b)}{\tau(B_d)} = \begin{cases} \sim 0.96 & \text{[Shifman, Voloshin '86]} \\ 0.798 \pm 0.034 & \text{[HFAG '03]} \end{cases}$$

- ⋆ 2019 status:

$$\frac{\tau(\Lambda_b)}{\tau(B_d)} = \begin{cases} 0.935 \pm 0.054 & \text{[Lenz '14]} \\ 0.969 \pm 0.006 & \text{[HFLAV '19]} \end{cases}$$

  - ◇ Shift of $4.9\,\sigma$

Quark Hadron Duality

Experiment at hadron level, calculation at quark-gluon level

- Compare HQE with experiments:
  - No sign of any significant deviation
  - $\Delta\Gamma_s$ highly sensitive (fewer states, smaller phase space)
    - Good agreement
- Simplified models of QCD:
  - SV limit: no duality violation for SL and NL decays

    [Boyd, Grinstein, Manohar '95; Grinstein, Savrov '03]
  - 'tHooft model: no $1/m_Q$ corrections, tiny oscillatory terms

    [Grinstein, Lebed '97, '98, '01]

    [Bigi, Shifman, Uraltsev, Vainshtein '98, '99, '00]