QED corrections to $B_q \to \mu^+\mu^-$

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Implications of LHCb measurements and future prospects
CERN

Outline

- Ultra-soft photons – general considerations
- Power-enhanced, structure-dependent QED corrections to $B_q \to \mu^+\mu^-$
- Numerical impact of QED corrections on $B_q \to \mu^+\mu^-$
- Summary

See: Martin Beneke, Christoph Bobeth, Robert Szafron
arXiv:1908.07011 and arXiv:1708.09152

QED in Flavor Physics

ΔE – cut on photon energy (e.g. due to detector resolution)
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  Based on eikonal approximation, well understood, under the assumption that $\Delta E \ll \Lambda_{\mathrm{QCD}}$
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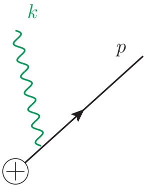
Both effects are important - even with strong cut on real photons ΔE,
the virtual photons can resolve the structure of the meson!
Virtual photons can couple to initial and final state and may have wave-lengths smaller than the typical meson size $\sim 1/\Lambda_{\text{QCD}}$

We refer to photons with energy $k \sim \Lambda_{\text{QCD}}$ as soft
Photons with momentum $k \sim \Delta E$ are *ultra-soft*

Ultra-soft photons

- Numerically important, but very easy to compute
- Based on eikonal approximation: spin universal



$$\varepsilon_\mu(k)\bar{u}(p)\gamma^\mu \frac{\not{p}+\not{k}+m}{(k+p)^2-m^2} \to \frac{\varepsilon_\mu(k)p^\mu}{p\cdot k}\bar{u}(p)$$

Note $k^\mu \ll p^\mu, m$

- General all-order solution is well known
  [see e.g. S. Weinberg, The Quantum theory of fields. Vol. 1]

$$\Gamma_{\beta\alpha} \to \mathcal{F}(A(\alpha\to\beta))\left(\frac{\Delta E}{\Lambda}\right)^{A(\alpha\to\beta)} \Gamma^\Lambda_{\beta\alpha} \approx \left(\frac{\Delta E}{\Lambda}\right)^{A(\alpha\to\beta)} \Gamma_{\beta\alpha}$$

where $\Gamma_{\beta\alpha}$ is 'non-radiative' rate, $\mathcal{F}(A(\alpha\to\beta)) \approx 1$

Note that Λ should be at most Λ_{QCD} or m

$$A(\alpha\to\beta) = -\frac{1}{8\pi^2}\sum_{nm}\frac{e_n e_m \eta_n \eta_m}{\beta_{nm}} \ln\left(\frac{1+\beta_{nm}}{1-\beta_{nm}}\right)$$

Ultra-soft photons in practice

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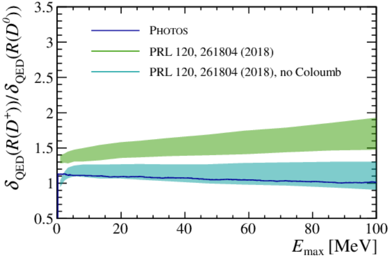
- Agreement with PHOTOS $\sim 1\%$ [S. Cali, S. Klaver, M. Rotondo, B. Sciascia, 1905.02702] – discrepancy due to the Coulomb effects (but not only!)



[1905.02702]

But to what extend can we trust Scalar QED and PHOTOS? What are their systematic shortcomings?

R_K and R_{K*}

Ultra-soft and collinear QED effects break lepton flavor universality for differential observables

$$\delta_{\rm QED} \sim \frac{\alpha}{\pi} \ln^2 \frac{m_B}{m_\ell}$$

These logs are connected with radiation from leptons – can be computed if we neglect radiation from mesons [M. Bordone, G. Isidori, A. Pattori, 1605.07633]

$B \to K \ell^+ \ell^-$	$\ell = e$	$\ell = \mu$
$m_B^{\rm rec} = 4.880$ GeV	-7.6%	-1.8%
$m_B^{\rm rec} = 5.175$ GeV	-16.9%	-4.6%
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for $q^2 \in [1, 6]$ GeV^2. This translates into $\Delta R_K = 3\%$

Agrees well with PHOTOS

One can expect that lepton-mass independent corrections will largely cancel in the ratio

$$R_K[q^2_{\rm min}, q^2_{\rm max}] = \frac{\int_{q^2_{\rm min}}^{q^2_{\rm max}} dq^2 \, \frac{d\Gamma(B \to K \mu^+ \mu^-)}{dq^2}}{\int_{q^2_{\rm min}}^{q^2_{\rm max}} dq^2 \, \frac{d\Gamma(B \to K e^+ e^-)}{dq^2}}$$

What about observables that are sensitive also to other types of logs?

What kind of logs should we expect?

This depends on the process and observables
For $B_s \to \mu^+\mu^-$ there are several relevant kinematical and dynamical scales

- m_B – the hard scale given by kinematics
- $m_b \sim m_B$ – heavy b quark mass – expansion parameter in HQET
- $\Lambda_{\rm QCD}$ – soft scale, typical momentum of the quarks in the meson (or inverse radius of the meson)
- $m_\mu \sim \Lambda_{\rm QCD}$ – collinear scale, muon mass acts as a regulator for collinear divergences

To compute corrections: *expand* the amplitude in $\lambda^2 = \frac{m_\mu}{m_B} \sim \frac{\Lambda_{\rm QCD}}{m_b}$
We need a more systematic approach than eikonal (soft) expansion!
Different logarithms appear

$$\ln \frac{m_\mu}{\Delta E} \sim 2.5; \qquad \ln \frac{m_B}{m_\mu} \sim 4; \qquad \ln \frac{m_B}{\Lambda_{\rm QCD}} \sim 3; \qquad \dots$$

Expansion parameter is $\frac{\alpha_{\rm em}}{\pi} \times \log^2$ rather than just $\frac{\alpha_{\rm em}}{\pi}$. Mixed QED-QCD logs are essential!

How to go beyond ultra-soft photon approximation in a systematic way?

Beyond ultra-soft photon approximation

- **Heavy quark expansion** works well for inclusive observables (see e.g. $\overline{B} \to X_s \ell^+ \ell^-$ [T. Huber, E. Lunghi, M. Misiak, D. Wyler, hep-ph/0512066; T. Huber, T. Hurth, E. Lunghi, 1503.04849])

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- Compute QED corrections on the **lattice** [N. Carrasco, V. Lubicz, G. Martinelli, C.T. Sachrajda, N. Tantalo, C. Tarantino, M. Testa, 1502.00257; M. Di Carlo, D. Giusti, V. Lubicz, G. Martinelli, C.T. Sachrajda, F. Sanfilippo, S. Simula, N. Tantalo, 1904.08731] – currently only light mesons

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- Use scale separation to our advantage and employ **effective field theory** approach

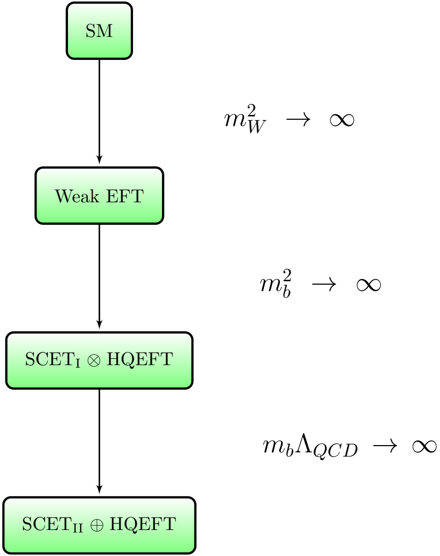
We will consider $B_s \to \mu^+ \mu^-$ and perform *power expansion* in

$$m_\mu \sim \Lambda_{\rm QCD} \ll m_B$$

EFT approach allows for a resummation of QED and QCD corrections, i.e. we can work to all orders in α_s and $\alpha_{\rm em}$ but to a fixed order in

$$\lambda^2 \equiv m_\mu / m_B \sim \Lambda_{\rm QCD} / m_B$$

Tower of EFTs



EFT approach to systematically integrate-out different scales

- Operatorial definitions allow separating non-perturbative input from perturbative corrections

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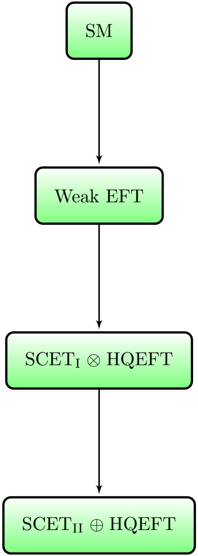

$$m_W^2 \rightarrow \infty$$

$$m_b^2 \rightarrow \infty$$

$$m_b \Lambda_{QCD} \rightarrow \infty$$

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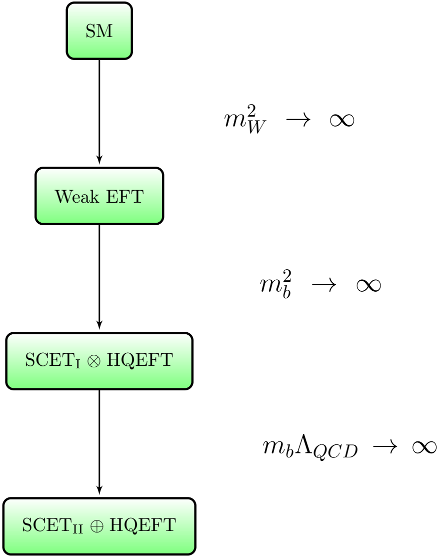

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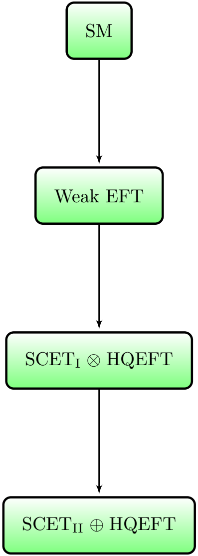

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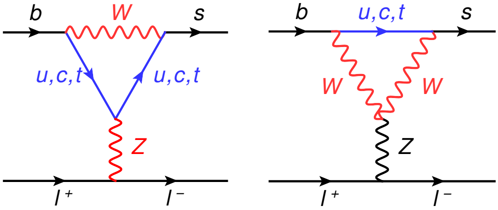
- Operatorial definitions allow separating non-perturbative input from perturbative corrections
- Renormalization Group technique can be used to perform resummation
- Objects have well-defined counting in λ and their computation is typically simpler than in the full theory
- It is more intuitive and simpler than the full theory

$B_s \to \mu^+\mu^-$

In the SM the process is

- loop suppressed (FCNC)



$$Br(B_s \to \mu^+\mu^-) \quad = \quad \frac{G_F^2 \alpha^2}{64\pi^3} f_{B_s}^2 \tau_{B_s} m_{B_s}^3 |V_{tb} V_{ts}^*|^2 \sqrt{1 - \frac{4m_\mu^2}{m_{B_s}^2}} \times \left| \frac{2m_\mu}{m_{B_s}} C_{10} \right|^2$$

see e.g. [C. Bobeth, M. Gorbahn, T. Hermann, M. Misiak, E. Stamou, M. Steinhauser, 1311.0903]

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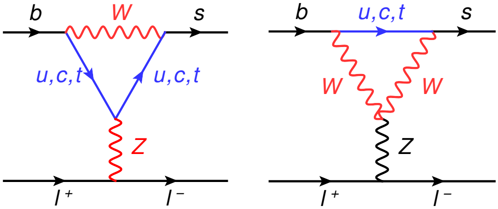

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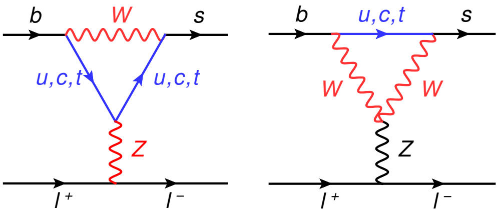

- helicity suppressed (scalar meson decaying into energetic muons, vector interaction)
- purely leptonic final state allows for a precise SM prediction, QCD contained in the meson decay constant f_{B_s} (in the absence of QED)

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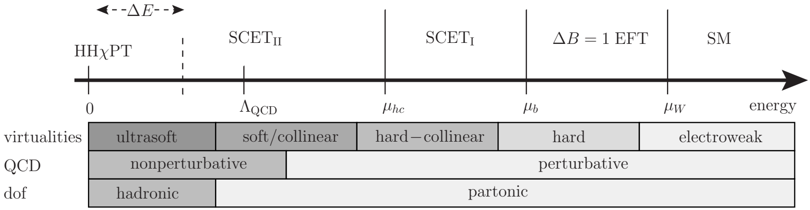
Modes in $B_s \to \mu^+\mu^-$



In addition to the 'standard' modes (such as collinear, hard, ultra-soft) we also have hard-collinear modes

- in perturbative region – we can exactly compute their contribution
- lead to enhancement of the QED corrections – exchange of hard-collinear photon can relax helicity suppression
- purely virtual – modify the 'non-radiative' decay rate

Note: this correction cannot be computed using ultra-soft photon approximation! It is sensitive to the meson structure!

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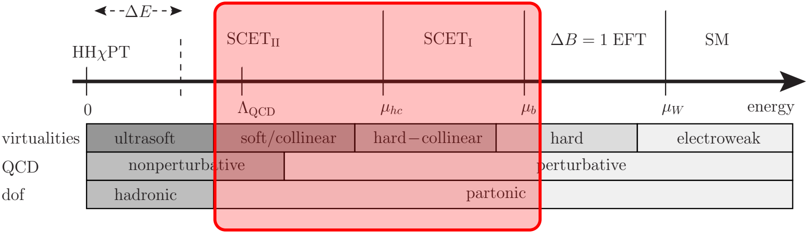

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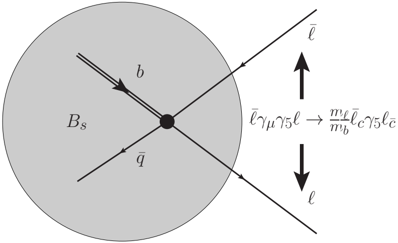
Helicity suppression

Can the helicity suppression be relaxed?



For $m_\ell \to 0$ the amplitude has to vanish

Annihilation and helicity flip take place at the same point $r \lesssim \frac{1}{m_b}$

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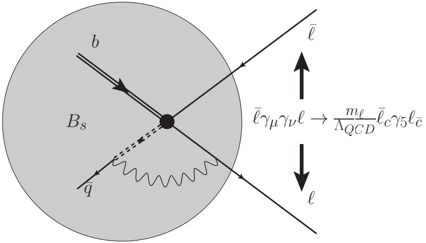


Annihilation and helicity flip can be separated by $r \sim \frac{1}{\sqrt{m_b \Lambda_{\mathrm{QCD}}}}$

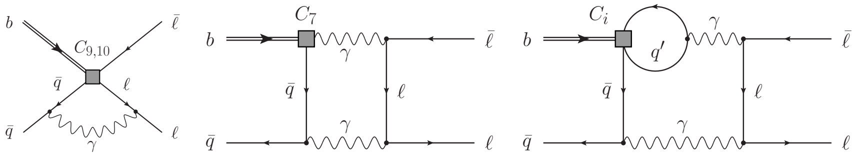
It is still a short distance effect since the size of the meson is $r \sim \frac{1}{\Lambda_{\mathrm{QCD}}}$

"Non-local annihilation"

For $m_\ell \to 0$ the amplitude still vanishes

The correction at amplitude level

[M. Beneke, C. Bobeth, R.S., 1708.09152]



$$
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& \times \Bigg\{ \int_0^1 du\,(1-u)\, C_9^{\rm eff}(u m_b^2)\, \int_0^\infty \frac{d\omega}{\omega}\,\phi_{B+}(\omega) \left[\ln\frac{m_b\omega}{m_\ell^2} + \ln\frac{u}{1-u}\right] \\
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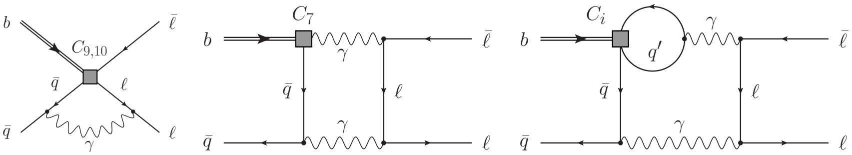

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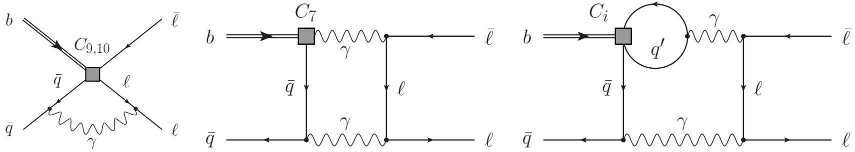

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- QED correction: Helicity suppression × power enhancement factor

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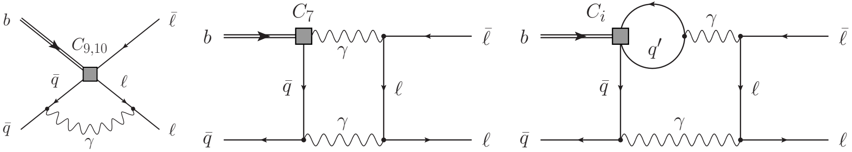

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- Convolution – short distance part

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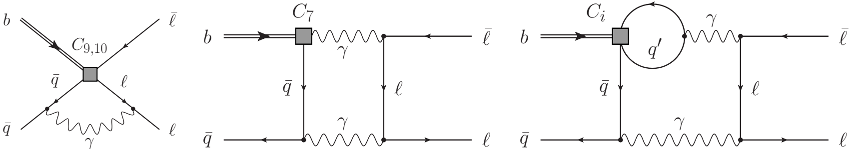

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- Convolution with the light-cone distribution function – structure dependent

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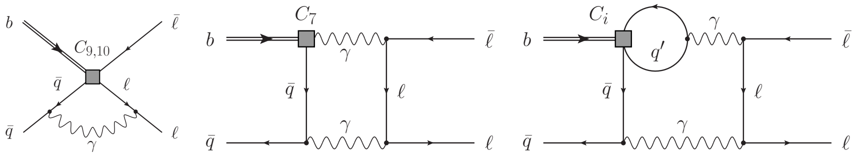


$$
\begin{aligned}
i\mathcal{A} \quad = \quad & m_\ell f_{B_q} \mathcal{N}\, C_{10}\, \bar{\ell}\gamma_5 \ell + \frac{\alpha_{\rm em}}{4\pi} Q_\ell Q_q\, m_\ell m_B\, f_{B_q}\mathcal{N}\, \bar{\ell}(1+\gamma_5)\ell \\
& \times \Bigg\{ \int_0^1 du\,(1-u)\, C_9^{\rm eff}(u m_b^2) \int_0^\infty \frac{d\omega}{\omega}\,\phi_{B+}(\omega) \left[\ln\frac{m_b\omega}{m_\ell^2} + \ln\frac{u}{1-u} \right] \\
& - Q_\ell C_7^{\rm eff} \int_0^\infty \frac{d\omega}{\omega}\,\phi_{B+}(\omega) \left[\ln^2\frac{m_b\omega}{m_\ell^2} - 2\ln\frac{m_b\omega}{m_\ell^2} + \frac{2\pi^2}{3} \right] \Bigg\}
\end{aligned}
$$

- Double logarithmic enhancement due to endpoint singularity

Factorization and Resummation in Effective Field Theory

Main idea: split a complicated object into a product (convolution) of simpler objects which can be systematically defined in terms of QFT operators

Factorization of the amplitude:

$$[\text{hard}] \times [\text{hard} - \text{collinear}] \times [\text{anti} - \text{hard} - \text{collinear}] \times [\text{soft}]$$

Each object in the factorization formula fulfills evolution equation which allows resummation of large corrections.

For example, the "hard function" fulfills RGE at LL

$$\frac{d}{d\ln\mu} H_m(\mu) = \frac{\alpha_{\rm em}}{\pi} 2Q_\ell^2 \times \ln\frac{m_{B_q}}{\mu} H_m(\mu)$$

Similar cusp anomalous dimension appears also for different objects

$$H_m(\mu) = H_m(\mu_b)\, \exp\left[-\frac{\alpha_{\rm em}}{\pi} Q_\ell^2 \ln^2\frac{\mu_b}{\mu}\right]$$

[M. Beneke, C. Bobeth, R.S., 1908.07011]

Numerical implications

$$\overline{\text{Br}}^{(0)}_{s\mu} = 3.677 \cdot 10^{-9} \times \left(1 - 0.0166\, S_9 + 0.0105\, S_7\right) = 3.660 \cdot 10^{-9}$$

$$\overline{\text{Br}}^{(0)}_{d\mu} = 1.031 \cdot 10^{-10} \times \left(1 - 0.0155\, S_9 + 0.0103\, S_7\right) = 1.027 \cdot 10^{-10}$$

Resummation decreases the QED effects by about 20%

$$S_9 \equiv \frac{\lambda_B(\mu_0)}{\lambda_B(\mu_{hc})} e^{S_q(\mu_b,\, \mu_{hc})} \in [0.77, 0.82]$$

(no resummation, only one loop QED means $S_9 = S_7 = 1$) Neglecting QED resummation, but using QCD resummation, $S_9 = S_7$

μ_{hc} [GeV]	$\frac{\lambda_B(\mu_0)}{\lambda_B(\mu_{hc})} e^{S_q(\mu_b,\, \mu_{hc})}$ QCD+QED	only QCD
1.0	0.815	0.817
1.5	0.815	0.817
2.0	0.769	0.769

QCD resummation is important! QED can be safely neglected

Error budget

[M. Beneke, C. Bobeth, R.S., 1908.07011]

$$\overline{\mathrm{Br}}_{s\mu}^{(0)} = \begin{pmatrix} 3.599 \\ 3.660 \end{pmatrix} \Big[1 + \begin{pmatrix} 0.032 \\ 0.011 \end{pmatrix}_{f_{B_s}} + 0.031|_{\mathrm{CKM}} + 0.011|_{m_t} + 0.006|_{\mathrm{pmr}} + 0.012|_{\text{non-pmr}} {}^{+0.003}_{-0.005}|_{\mathrm{LCDA}} \Big] \cdot 10^{-9},$$

$$\overline{\mathrm{Br}}_{d\mu}^{(0)} = \begin{pmatrix} 1.049 \\ 1.027 \end{pmatrix} \Big[1 + \begin{pmatrix} 0.045 \\ 0.014 \end{pmatrix}_{f_{B_d}} + 0.046|_{\mathrm{CKM}} + 0.011|_{m_t} + 0.003|_{\mathrm{pmr}} + 0.012|_{\text{non-pmr}} {}^{+0.003}_{-0.005}|_{\mathrm{LCDA}} \Big] \cdot 10^{-10},$$

i) main parametric long-distance f_{B_q}, $N_f = \begin{pmatrix} 2+1 \\ 2+1+1 \end{pmatrix}$ [FLAG, 1902.08191] and short-distance (CKM and m_t)

ii) remaining non-QED parametric (Γ_q, α_s) and non-QED non-parametric (μ_W, μ_b and higher order)

iii) from the B-meson LCDA parameters entering the QED correction

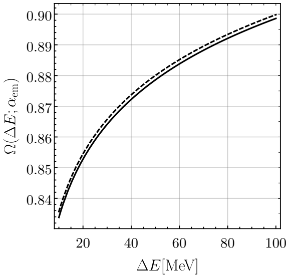
Ultra-soft photons

[M. Beneke, C. Bobeth, R.S., 1908.07011]

$$\overline{\mathrm{Br}}_{q\mu}(\Delta E) \equiv \overline{\mathrm{Br}}_{q\mu}^{(0)} \times \Omega(\Delta E; \alpha_{\mathrm{em}})$$

with radiative factor (remember $\Delta E \ll \Lambda_{\mathrm{QCD}}$)

$$\Omega(\Delta E; \alpha_{\mathrm{em}}) \equiv \left(\frac{2\Delta E}{m_{B_q}}\right)^{-\frac{2\alpha_{\mathrm{em}}}{\pi}\left(1+\ln\frac{m_\mu^2}{m_{B_q}^2}\right)}$$



Agrees with [A. Buras, J. Girrbach, D. Guadagnoli, G. Isidori, 1208.0934] without ad hoc assumptions about the choice of scales

Rate asymmetries in $B_q \to \mu^+\mu^-$

[M. Beneke, C. Bobeth, R.S., 1908.07011]

Measurement of the time-dependent rate asymmetry gives access to additional observables

$$\frac{\Gamma[B_q(t) \to \mu^+_\lambda \mu^-_\lambda] - \Gamma[\overline{B}_q(t) \to \mu^+_\lambda \mu^-_\lambda]}{\Gamma[B_q(t) \to \mu^+_\lambda \mu^-_\lambda] + \Gamma[\overline{B}_q(t) \to \mu^+_\lambda \mu^-_\lambda]} = \frac{C_q^\lambda \cos(\Delta m_{B_q} t) + S_q^\lambda \sin(\Delta m_{B_q} t)}{\cosh(y_q\, t/\tau_{B_q}) + A_q^\lambda \sinh(y_q\, t/\tau_{B_q})},$$

$C_q \equiv \frac{1}{2}(C_q^L + C_q^R)$ and $S_q \equiv \frac{1}{2}(S_q^L + S_q^R)$ are CP-odd

$\Delta C_q \equiv \frac{1}{2}(C_q^L - C_q^R)$ and $\Delta S_q \equiv \frac{1}{2}(S_q^L - S_q^R)$ are CP-even

In SM at LO in QED: $C_q^\lambda = S_q^\lambda = 0$ and $A_q^\lambda = 1$

QED induces a small deviation

$$C_s^\lambda = +\eta_\lambda\, 0.6\% \qquad S_s^\lambda = -\eta_\lambda\, 0.1\% \qquad A_s = 1 - 2.0 \cdot 10^{-5}$$

$$C_d = -0.08\% \qquad S_d = +0.03\% \qquad A_d^L = 1 - 1.4 \cdot 10^{-5}$$

$$\Delta C_d = +0.60\% \qquad \Delta S_d = -0.13\% \qquad A_d^R = 1 - 2.4 \cdot 10^{-5}$$

Not a 'null test' anymore, but the deviation is tiny

Summary and outlook

- Mesons are not point-like, the eikonal approximation is not enough because of large virtual corrections. We need to include QED corrections, which depend on the structure of the meson
- EFT is needed to define hadronic matrix elements properly and perform resummation (both QED and QCD)
- Systematic study of QED corrections and mixed QED – QCD effects is necessary to achieve good precision
- Dedicated studies are needed to validate MC and include virtual, structure-dependent corrections
- First step: $B_q \to \mu^+\mu^-$
  - leading log resummation in SCET
  - B-meson decay constant and LCDA $\to$ implications for lattice
  - establish QED factorization theorem

  The same should be done for other processes
- Our prediction should be compared with PHOTOS for small ΔE

It is of uttermost importance that theoretical community provides rigorous Standard Model predictions before we can have "fun with anomalies"

Exciting time for Standard Model physics!