$V_{cb}$ and form factors in $B \rightarrow D^{(*)} \ell\nu$

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Talk at the workshop
“Implications of LHCb measurements and future prospects”
17th of October 2019
Importance of (semi-)leptonic hadron decays

In the Standard Model:
- Tree-level, $\sim |V_{ij}|^2 G_F^2 F F^2$
- Determination of $|V_{ij}|$ (7/9)

Beyond the Standard Model:
- Leptonic decays $\sim m_l^2$
  - Large relative NP influence possible (e.g. $H^{\pm}$)
- NP in semi-leptonic decays small/moderate
  - Need to understand the SM very precisely!
    - e.g. radiative corrections [see R. Szafron’s talk]

Key advantages:
- Large rates
- Minimal hadronic input $\Rightarrow$ systematically improvable
- Differential distributions $\Rightarrow$ large set of observables
Lepton-non-Universality in $b \to c \tau \nu$ 2019

- $R(D^{(*)})$: BaBar, Belle, LHCb
  - average $3.\times\sigma$ from SM
- Other $b \to c$ observables:
  - largely SM-like
    - ($R(J/\psi)$ large even w/ NP)
- $R(K^{(*)})$: $\sim 4\sigma$ from SM
  - between light leptons
- $b \to s\ell\ell$ global fit: $\sim 5\sigma$

contours 68%, filled 95% CL

\[ \sim 15\% \text{ of a SM tree decay } \sim V_{cb}: \text{ This is a huge effect!} \]
- Need contribution of $\sim 5 - 10\%$ (w/ interference)
  - or $\gtrsim 40\%$ (w/o interference) of SM

- Check SM prediction (main topic of this talk)
- NP analyses [talks by D.Guadagnoli, O.Sumensari, Soni, M.König, A.Peñuelas]
- Require form factors independent of data!
Form factor parametrizations:
FFs central non-perturbative input in semileptonic decays, e.g.

\[
\langle D(p_D)|\bar{c}\gamma^\mu b|\bar{B}(p_B)\rangle = f_+(q^2) \left[ (p_B^\mu + p_D^\mu) - \frac{M_B^2 - M_D^2}{q^2} q^\mu \right] + f_0(q^2) \frac{M_B^2 - M_D^2}{q^2} q^\mu
\]

“BGL parametrization”:
- Analytic structure: account for cuts and poles explicitly
  - remainder can be expanded in simple power series in $z$
- Use quark-hadron-duality (+crossing sym., unitarity)
  - Absolute bounds on coefficients, rapid convergence
  - Efficient expansion of individual FFs with few coefficients

“HQE parametrization” (→ CLN)
- Exploit heavy-quark spin-flavour symmetry for $m_{b,c} \to \infty$
  - All $B(*) \to D(*)$ FFs given by Isgur-Wise function $\xi(z)$
  - Systematic expansion in $1/m_{b,c}$ and $\alpha_s$
  - Also $z$ expansion, no bounds on individual coefficients
  - Less parameters in total, FFs related
Puzzling $V_{cb}$ results

The $V_{cb}$ puzzle has been around for 20+ years...

- $\sim 3\sigma$ between exclusive (mostly $B \to D^*\ell\nu$) and inclusive $V_{cb}$
- Inclusive determination: includes $\mathcal{O}(1/m_b^3, \alpha_s/m_b^2, \alpha_s^2)$
  - Excellent theoretical control, $|V_{cb}| = 42.00 \pm 0.64$
- Exclusive determinations: $B \to D^{(*)}\ell\nu$, using CLN (fixed!)
  - CLN: HQE @ $\mathcal{O}(1/m_{c,b}, \alpha_s) +$ slope-curvature relation in $\xi$

Recent developments:

- Unfolded differential measurements made available by Belle
  - Different parametrizations possible
- Lattice calculations for $B \to D$ FFs at non-zero recoil
  - Agreement of analyses for $B \to D$
- $B \to D^*$ FFs: several analyses ongoing [see O. Witzel’s talk]
  - So far: only one FF at zero recoil
  - Larger differences in theory analyses
  - Specifically treatment of $1/m_c^2$ important
- New LCSR analysis of all $B \to D^{(*)}$ FFs [Gubernari/Kokulu/vDyk’18]
$V_{cb}$ from $B \rightarrow D$

2015: Unfolded $B \rightarrow D\ell\nu$ spectra [Belle] + finite recoil LQCD [HPQCD,MILC]

BGL analysis by Bigi/Gambino:

- Improved unitarity constraints
- Lattice data “contradict” CLN (sensitivity to higher $1/m$ orders)

$|V_{cb}| = 40.49(96) \times 10^{-3}$, compatible with $V_{cb}^{incl}$ and $B \rightarrow D^*$

HQE analysis w/ partial $1/m_c^2$ works [Bernlochner+’17,MJ/Straub’18]
\( V_{cb} \) from \( B \rightarrow D^* \) 2017

2017: Prel. unfolded spectrum (4 variables) from Belle

However, in this case no finite-recoil FFs available from lattice

w/ Belle results SM fit in BGL possible (including lattice (+LCSR))

Results: [Bigi+, Grinstein+]

- Both CLN and BGL yield excellent fits
  
  \[ |V_{cb}^{\text{CLN}}| = 38.2(15) \times 10^{-3} \]
  
  \[ |V_{cb}^{\text{BGL}}| = 41.7(21)[40.4(17)] \times 10^{-3} \text{ (w/ or w/o LCSR)} \]

- BGL 1 – 2\( \sigma \) higher, larger difference than expected!

- Intriguing result, but requires confirmation exp. + lattice

Uncertainties due to parametrization were underestimated
- Using BGL, no indication of a \( V_{cb} \) puzzle in 2017 data

- Lattice data will give additional insights

N.B.: This discussion relates to SM \( R(D, D^*) \) predictions
\( V_{cb} + R(D^*) \) w/ data + lattice + unitarity [Gambino/MJ/Schacht’19]

(see also [Fajfer+, Nierste+, Bernlochner+, Bigi+, Grinstein+, Nandi+... ] )

Recent untagged analysis by Belle with 4 1D distributions [1809.03290]

“Tension with the \( (V_{cb}) \) value from the inclusive approach remains”

Analysis of 2017+2018 Belle data with BGL form factors:

- Datasets roughly compatible
- d’Agostini bias + syst. important
- All FFs to \( z^2 \) to include uncertainties
- 2018: no parametrization dependence

\[
|V_{cb}^{D^*}| = 39.6^{+1.1}_{-1.0} \times 10^{-3}
\]

\[
R(D^*) = 0.254^{+0.007}_{-0.006}
\]
Theory determination of $b \rightarrow c$ Form Factors

SM: BGL fit to data + FF normalization $\rightarrow |V_{cb}|$

NP: can affect the $q^2$-dependence, introduces additional FFs

To determine general NP, FF shapes needed from theory

In [MJ/Straub’18,Bordone/MJ/vDyk’19], we use all available theory input:

• Unitarity bounds (using results from [BGL,Bigi/Gambino(/Schacht)'16'17])

• LQCD for $f_{+,0}(q^2) (B \rightarrow D), h_{A_1}(q^2_{\text{max}}) (B \rightarrow D^*)$
  [HPQCD’15,’17,Fermilab/MILC’14,’15]

• LCSR for all FFs but $f_T$ [Gubernari/Kokulu/vDyk’18]

• Consistent HQET expansion to $\mathcal{O}(\alpha_s, 1/m_b, 1/m_c^2)$

improved description

[Struyve/Straub/18]

FFs under control;
$R(D^*) = 0.247(6)$
$V_{cb} = 40.0(11) \times 10^{-3}$
Robustness of the HQE expansion up to $1/m_c^2$

[Bordone/MJ/vDyk’19]

Testing FFs by comparing to data and fits in BGL parametrization:

- Fits 3/2/1 and 2/1/0 are **theory-only fits(!)**
- $k/l/m$ denotes orders in $z$ at $O(1, 1/m_c, 1/m_c^2)$
- $w$-distribution yields information on FF shape $\rightarrow V_{cb}$
- Angular distributions more strongly constrained by theory, only
  - Predicted shapes perfectly confirmed by $B \rightarrow D^{(*)}\ell\nu$ data
  - $V_{cb}$ from Belle’17 compatible between HQE and BGL!
Robustness of the HQE expansion up to $1/m_c^2$

[Bordone/MJ/vDyk'19]

Testing FFs by comparing to data and fits in BGL parametrization:

- $B \to D^*$ BGL coefficient ratios from:
  1. Data (Belle’17+’18) + weak unitarity (yellow)
  2. HQE theory fit 2/1/0 (red)
  3. HQE theory fit 3/2/1 (blue)

Again compatibility of theory with data

2/1/0 underestimates the uncertainties massively

For $b_i, c_i \rightarrow f, \mathcal{F}_1$ data and theory complementary
An example NP analysis: $b \to c\ell\nu (\ell = e, \mu)$ [MJ/Straub’18]

NP in left-handed vector current:

$$\tilde{V}^\ell_{cb} = V_{cb} \left[ |1 + C_V^\ell|^2 + \sum_{\ell' \neq \ell} |C_V^{\ell\ell'}|^2 \right]^{1/2}$$

Only subset of data usable

$B \to D, D^*$ in agreement

No sign of LFNU

$\downarrow$ constrained to be $\lesssim \% \times V_{cb}$

NP in right-handed vector current:

Usual suspect for excl. vs. incl.

[e.g. Voloshin’97]

Test of SMEFT [Catá/MJ’15]

Full $B \to D^{(*)}$ data usable

$B \to D^*$: Qualitative change

No constraint over SM
Conclusions

$b \to c$ transitions important in the SM and beyond; various puzzles

- Recent developments allow for qualitatively new analyses
  - BGL analyses of $B \to D(\ast)$ reduce $V_{cb}$ puzzle
  - 2017+2018 data: still $V_{cb} \sim 1.9\sigma$ from $V_{cb}^{\text{incl.}}$
  - Large parametrization dependence in Belle 2017 data

- For NP theory determination of form factors required
  - First analysis at $1/m_c^2$ provides all FFs
  - Parametrization dependence in Belle data removed
  - Excellent agreement w/ data, $1/m_c$ expansion works

- Averaging $B \to (D, D^*, X_c)\ell\nu$: “Tension” at $1.\times\sigma$
- NP in $b \to c\ell\nu$: strong constraints, qualitative progress for $V_R$

Exciting times ahead in semileptonic decays!
Claim in 2018 [Chavez-Saab/Toledo] : \( R(D_\pi) \sim 0.275, \) “Closing the gap” . . .

This was wrong, erratum: 0.253 (in line w/ others)

Erratum due to numerical issue; here: conceptual issue

The amplitudes for the decay chain are written as

\[
\langle D^*(k, \eta) | \bar{c} \gamma^\mu (1 - \gamma_5) b | \bar{B}(k + q) \rangle \equiv \eta^*_\alpha(k) M^{\mu\alpha} \\
\langle D_\pi | \mathcal{L}_{QCD} | D^*(k, \eta) \rangle = \eta'_{\alpha'}(k) M^{\alpha'}
\]

- \( M^{\mu\alpha} \) is then parametrized in a standard way by FFs
- The polarization sum in narrow width approximation yields

\[
\sum_{\lambda=\pm 1,0} \eta(\lambda)_{\alpha} \eta^*(\lambda)_{\alpha'} = - \left( g_{\alpha\alpha'} - \frac{k_{\alpha} k_{\alpha'}}{M^2_D} \right)
\]

For \( k_{\alpha} k_{\alpha} = M^2_{D^*} \), a contribution \( \sim k^\alpha \) in \( M^{\alpha\mu} \) vanishes!
Allowing for a propagating off-shell $D^*$: Additional terms have to be suppressed by $\Gamma_{D^*/|k_{D^*}|}$!

Why does that not happen in [Chavez-Saab/Toledo’18]?

- $\mathcal{M}^{\alpha\mu}$ has to fulfill on-shell-condition $k_\alpha \mathcal{M}^{\alpha\mu} = 0$ for on-shell $D^*$!
- The standard FF parametrization does not fulfill this
  - Usually irrelevant due to the narrow-width approximation
  - Off-shell $D^*$: $k_\alpha \mathcal{M}^{\alpha\mu} = 0$ must be ensured modifying FFs

$$q^\mu \mapsto q^\mu - \frac{(q \cdot k)}{k^2} k^\mu,$$

$$g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2} \mapsto g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2} - \frac{k^\mu k^\nu}{k^2} + \frac{(q \cdot k) k^\mu q^\nu}{k^2 q^2}.$$ 

Result: expected suppression of off-shell contributions
  - Tiny, can be safely neglected
BR measurements and isospin violation [MJ 1510.03423]

Detail due to high precision and small NP

- Relevant for $\sigma_{\text{BR}}/\text{BR} \sim O(\%)$

Branching ratio measurements require normalization...

- $B$ factories: depends on $\Upsilon \rightarrow B^+ B^-$ vs. $B^0 \bar{B}^0$
- LHCb: normalization mode, usually obtained from $B$ factories

Assumptions entering this normalization:

- PDG: assumes $r_{+0} \equiv \Gamma(\Upsilon \rightarrow B^+ B^-)/\Gamma(\Upsilon \rightarrow B^0 \bar{B}^0) \equiv 1$
- LHCb: assumes $f_u \equiv f_d$, uses $r_{+0}^{\text{HFAG}} = 1.058 \pm 0.024$

Both approaches problematic:

- Potential large isospin violation in $\Upsilon \rightarrow BB$ [Atwood/Marciano’90]
- Measurements in $r_{+0}^{\text{HFAG}}$ assume isospin in exclusive decays
  - This is one thing we want to test!
- Avoiding this assumption yields $r_{+0} = 1.035 \pm 0.038$
  (potentially subject to change, in contact with Belle members)
Higgs EFT(s) - relating cc and nc processes

Apparent gap between EW and NP scales:
- EFT approach at the electroweak scale:
  - ✓ SM particle content
  - ✓ SM gauge group
  - ? Embedding of $h$
  - ? Power-counting
- 🔵 Formulate NLO

Linear embedding of $h$:
- $h$ part of doublet $H$
- Appropriate for weakly-coupled NP
- Power-counting: dimensions
  - Finite powers of fields
- LO: SM

Non-linear embedding of $h$:
- $h$ singlet, $U$ Goldstones
- Appropriate for strongly-coupled NP
- Power-counting: loops ($\sim \chi_{\text{PT}}$)
  - Arbitrary powers of $h/v, \phi$
- LO: SM + modified Higgs-sector

E
- NP (>few TeV)

EW (h,t,Z,W)

B (~5 GeV)

QCD (<1 GeV)
Implications of the Higgs EFT for flavour EFT [Cata/MJ’15]

At scales $\mu \ll \mu_{EW}$: remove top + heavy gauge bosons
- Construct EFT from “light” fermions + QCD, QED
- Gauge group: $SU(3)_C \times U(1)_{em}$

Example: $b \to c\tau\nu$ transitions:

$$\mathcal{L}_{\text{eff}}^{b \to c\tau\nu} = -\frac{4G_F}{\sqrt{2}} V_{cb} \sum_j C_j \mathcal{O}_j$$

- All operators present already in the linear EFT
- However: Relations between different transitions:
  - $C_{V_R}$ is lepton-flavour universal [see also Cirigliano+’09]
  - Relations between charged- and neutral-current processes, e.g.
  $$\sum_{U=u,c,t} \lambda_{Us} C_{S_R}^{(U)} = -\frac{e^2}{8\pi^2} \lambda_{ts} C_{S}^{(d)}$$ [see also Cirigliano+’12, Alonso+’15]
- These relations are absent in the non-linear EFT
- Flavour physics can distinguish between Higgs embeddings!
Two types of contributions:

1. Operators already present at the EW scale → identification
2. Tree-level contributions of HEFT operators with SM ones
   - e.g. HEFT $\bar{b}sZ$ vertex with $Z \rightarrow \ell\ell$
   - Both of the same order

Previous work (linear EFT) e.g. [D'Ambrosio+'02, Cirigliano+'09, Alonso+'14]

A word of caution: flavour hierarchies have to be considered!
- Mostly relevant when SM is highly suppressed, e.g. for EDMs
Implications of the Higgs EFT for flavour [Cata/MJ’15]

$q \rightarrow q’\ell\ell$:

- Tensor operators absent in linear EFT for $d \rightarrow d’\ell\ell$ [Alonso+’14]
  - Present in general! (already in linear EFT for $u \rightarrow u’\ell\ell$)
- Scalar operators: linear EFT $C_S^{(d)} = -C_P^{(d)}$, $C_S’^{(d)} = C_P’^{(d)}$ [Alonso+’14]
  - Analogous for $u \rightarrow u’\ell\ell$, but no relations in general!

$q \rightarrow q’\ell\nu$:

- All operators are independently present already in the linear EFT
- However: Relations between different transitions:
  - $C_{VR}$ is lepton-flavour universal [see also Cirigliano+’09]
  - Relations between charged- and neutral-current processes, e.g.
    \[
    \sum_{U=u,c,t} \lambda_U s C_{SR}^{(U)} = -\frac{e^2}{8\pi^2} \lambda_{ts} C_S^{(d)}
    \] [see also Cirigliano+’12, Alonso+’15]
- These relations are again absent in the non-linear EFT

Flavour physics sensitive to Higgs embedding!
- Surprising, since no Higgs is involved
- Difficult differently [e.g. Barr+, Azatov+’15]
Experimental analyses used

<table>
<thead>
<tr>
<th>Decay</th>
<th>Observable</th>
<th>Experiment</th>
<th>Comment</th>
<th>Year</th>
</tr>
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<tbody>
<tr>
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Different categories of data:

- Only total rates vs. differential distributions
- $e, \mu$-averaged vs. individual measurements
- Correlation matrices given or not
- Sometimes presentation prevents use in non-universal scenarios 😞
- Recent Belle analyses (mostly) exemplary 😊
Scalar operators

For $m_\ell \to 0$, no interference with SM

- For fixed $V_{cb}$, scalar NP increases rates

Close to $q^2 \to q^2_{\text{max}}$ in the SM:  
$$\frac{d\Gamma(B \to D\ell\nu)}{dq^2} \propto f_+^2 (q^2 - q^2_{\text{max}})^{3/2}$$

With scalar contributions:  
$$\frac{d\Gamma(B \to D\ell\nu)}{dq^2} \propto f_0^2 |C_{SR} + C_{SL}|^2 (q^2 - q^2_{\text{max}})^{1/2}$$

- Endpoint very sensitive to scalar contributions! \[\text{[see also Nierste+’08]}\]

Scalar contributions ruled out by the distributions ($\Gamma_1 = \Gamma_2$):

![Graph showing the distributions of scalar contributions](image_url)
Scalar operators

For $m_\ell \to 0$, no interference with SM

For fixed $V_{cb}$, scalar NP increases rates

Close to $q^2 \to q_{\text{max}}^2$ in the SM:

$$\frac{d\Gamma(B \to D_{\ell}\nu)}{dq^2} \propto f_+^2 \left(q^2 - q_{\text{max}}^2\right)^{3/2}$$

With scalar contributions:

$$\frac{d\Gamma(B \to D_{\ell}\nu)}{dq^2} \propto f_0^2 |C_{SR} + C_{SL}|^2 \left(q^2 - q_{\text{max}}^2\right)^{1/2}$$

Endpoint very sensitive to scalar contributions! [see also Nierste+’08]

Fit with scalar couplings (generic $C_{S_{L,R}}$):

Slightly favours large contributions in muon couplings with $C_{SR}^\mu \approx -2C_{SL}^\mu$
Scalar operators

For $m_\ell \to 0$, no interference with SM

- For fixed $V_{cb}$, scalar NP increases rates

Close to $q^2 \to q^2_{\text{max}}$ in the SM:

$$\frac{d\Gamma(B \to D\ell\nu)}{dq^2} \propto f_+^2 (q^2 - q^2_{\text{max}})^{3/2}$$

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- Endpoint very sensitive to scalar contributions! [see also Nierste+’08]

Also for LQ $U_1$ (or $V_2$): $B \to D$ stronger than $B \to D^*$, $X_c$:

Possible large contribution in $C_{SR}^\mu$ excluded by $B \to D$
Tensor operators

For $m_\ell \to 0$, no interference with SM

- For fixed $V_{cb}$, tensor contributions increase rates

Close to $q^2 \to q^2_{\text{min}}$:

$$\frac{d\Gamma_T(B \to D^*\ell\nu)}{dq^2} \propto q^2 C_{V_L}^2 (A_1(0)^2 + V(0)^2) + 16m_B^2 C_T^2 T_1(0)^2 + O\left(\frac{m_{D^*}^2}{m_B^2}\right)$$

- Endpoint ($q^2 \sim 0$) very sensitive to tensor contributions!

Tensor contributions ruled out by the distributions ($\Gamma_1 = \Gamma_2$):

![Graph showing the relationship between $q^2$ and the differential branching fraction with SM and tensor contributions. The graph includes data points from Belle and lines for $C_T = 0.1$ and $C_T = 0$.](image)
Tensor operators

For $m_\ell \to 0$, no interference with SM

- For fixed $V_{cb}$, tensor contributions increase rates

  Close to $q^2 \to q^2_{\text{min}}$:

  $$\frac{d\Gamma_T(B \to D^{\ast}\ell\nu)}{dq^2} \propto q^2 C_{V_L}^2 \left( A_1(0)^2 + V(0)^2 \right) + 16m_B^2 C_T^2 T_1(0)^2 + O \left( \frac{m_{D^{\ast}}^2}{m_B^2} \right)$$

- Endpoint ($q^2 \sim 0$) very sensitive to tensor contributions!

Fit for generic $C_{S_L}$ and $C_T$ (including LQs $S_1$ and $R_1$):

$B \to D^\ast$ favours large contributions in $C_{S_L}^{e,\mu}$, ruled out by $B \to D$