

Form factors from lattice QCD

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Implications of LHCb measurements and future prospects
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Lattice QCD

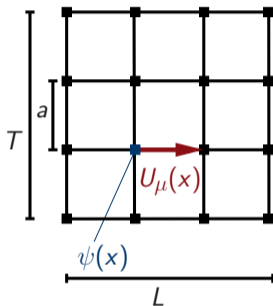
Lattice calculation

- ▶ Wick-rotate to Euclidean time $t \rightarrow i\tau$
- ▶ Discretize space-time and set up a hypercube of finite extent $L^3 \times T$ and spacing a
- ▶ Use path integral formalism

$$\langle \mathcal{O} \rangle_E = \frac{1}{Z} \int \mathcal{D}[\psi, \bar{\psi}] \mathcal{D}[U] \mathcal{O}[\psi, \bar{\psi}, U] e^{-S_E[\psi, \bar{\psi}, U]}$$

⇒ Large but finite dimensional path integral

- ▶ Finite volume of length $L \rightarrow$ IR regulator
 - Study physics in a finite box of volume $(aL)^3$
 - Strongly prefer decays with 1 (QCD-stable) hadronic final state (narrow width approximation)
- ▶ Finite lattice spacing $a \rightarrow$ UV regulator
 - Quark masses need to obey $am < 1$



Simulating charm and bottom (schematic)

$$a^{-1} > 1.5 \text{ GeV}$$

charm: RHQ; extrapolations of fully relativistic actions (?)

bottom: HQET, NRQCD, RHQ

$$a^{-1} > 2.2 \text{ GeV}$$

charm: fully relativistic action

bottom: (guided) extrapolation of fully relativistic action

$$a^{-1} > 4.6 \text{ GeV}$$

bottom: fully relativistic action

HQET: static limit, relatively noisy

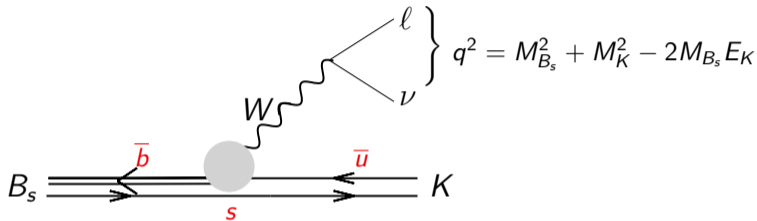
NRQCD: non-relativistic QCD, no continuum limit

RHQ or Fermilab: relativistic heavy quark action, complicated discretization errors

(heavy) HISQ or (heavy) MDWF: fully relativistic, clean nonperturbative renormalization

$$B_s \rightarrow K l \nu$$

$|V_{ub}|$ from exclusive semileptonic $B_s \rightarrow K\ell\nu$ decay



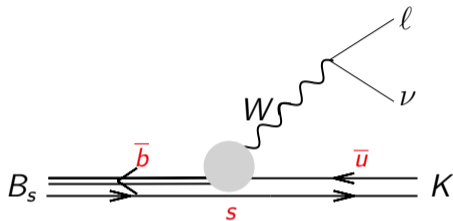
► Conventionally parametrized by (B_s meson at rest)

$$\frac{d\Gamma(B_s \rightarrow K\ell\nu)}{dq^2} = \frac{G_F^2 |V_{ub}|^2}{24\pi^3} \frac{(q^2 - m_\ell^2)^2 \sqrt{E_K^2 - M_K^2}}{q^4 M_{B_s}^2}$$

$$\times \left[\left(1 + \frac{m_\ell^2}{2q^2}\right) M_{B_s}^2 (E_K^2 - M_K^2) |f_+(q^2)|^2 + \frac{3m_\ell^2}{8q^2} (M_{B_s}^2 - M_K^2)^2 |f_0(q^2)|^2 \right]$$

nonperturbative input

$|V_{ub}|$ from exclusive semileptonic $B_s \rightarrow K\ell\nu$ decay



► $f_+(q^2)$ and $f_0(q^2)$

- Parametrizes interactions due to the (nonperturbative) strong force
- Use operator product expansion (OPE) to identify short distance contributions
- Calculate the flavor changing currents as point-like operators using lattice QCD

► Conventionally parametrized by (B_s meson at rest)

$$\frac{d\Gamma(B_s \rightarrow K\ell\nu)}{dq^2} = \frac{G_F^2 |V_{ub}|^2}{24\pi^3} \frac{(q^2 - m_\ell^2)^2 \sqrt{E_K^2 - M_K^2}}{q^4 M_{B_s}^2}$$

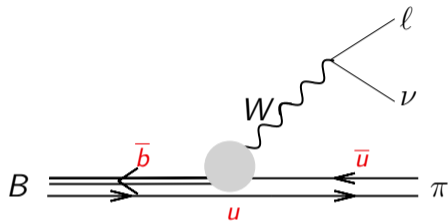
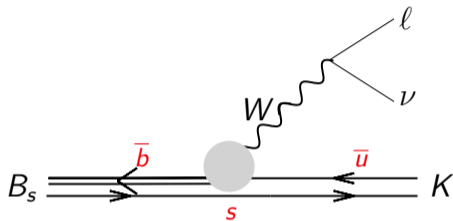
experiment

CKM

known

$$\times \left[\left(1 + \frac{m_\ell^2}{2q^2}\right) M_{B_s}^2 (E_K^2 - M_K^2) |f_+(q^2)|^2 + \frac{3m_\ell^2}{8q^2} (M_{B_s}^2 - M_K^2)^2 |f_0(q^2)|^2 \right]$$

nonperturbative input

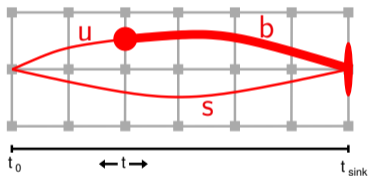
$|V_{ub}|$ from exclusive semileptonic $B_s \rightarrow K \ell \nu$ decay

- ▶ Compared to $B \rightarrow \pi \ell \nu$ only spectator quark differs
 - Lattice QCD prefers s quark over u quark: statistically more precise, computationally cheaper
 - B factories run mostly at $\Upsilon(4s)$ threshold $\Rightarrow B$ mesons
 - LHC collisions create many B and B_s mesons which decay \Rightarrow LHCb

$B_s \rightarrow K\ell\nu$ form factors

- ▶ Parametrize the hadronic matrix element for the flavor changing vector current V^μ in terms of the form factors $f_+(q^2)$ and $f_0(q^2)$

$$\langle K | V^\mu | B_s \rangle = f_+(q^2) \left(p_{B_s}^\mu + p_K^\mu - \frac{M_{B_s}^2 - M_K^2}{q^2} q^\mu \right) + f_0(q^2) \frac{M_{B_s}^2 - M_K^2}{q^2} q^\mu$$



- ▶ Calculate 3-point function by
 - Inserting a quark source for a strange quark propagator at t_0
 - Allow it to propagate to t_{sink} , turn it into a sequential source for a b quark
 - Use a “light” quark propagating from t_0 and contract both at t with $t_0 \leq t \leq t_{sink}$

$B_s \rightarrow K\ell\nu$ form factors

- ▶ Parametrize the hadronic matrix element for the flavor changing vector current V^μ in terms of the form factors $f_+(q^2)$ and $f_0(q^2)$

$$\langle K | V^\mu | B_s \rangle = f_+(q^2) \left(p_{B_s}^\mu + p_K^\mu - \frac{M_{B_s}^2 - M_K^2}{q^2} q^\mu \right) + f_0(q^2) \frac{M_{B_s}^2 - M_K^2}{q^2} q^\mu$$

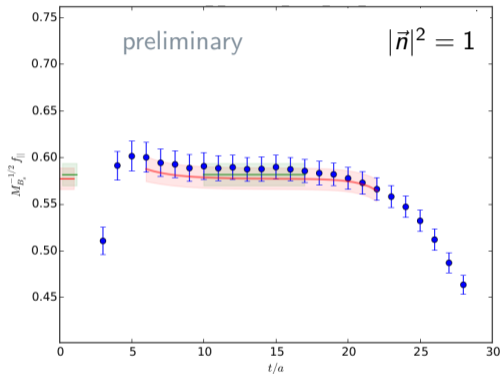
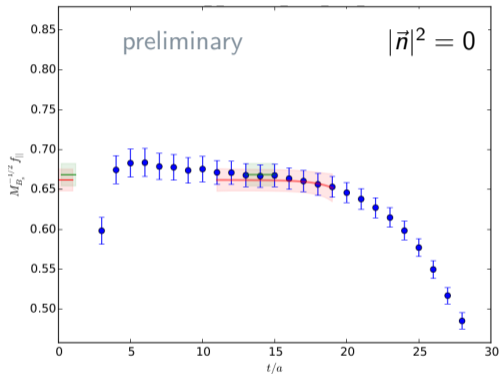
- ▶ On the lattice we prefer to compute

$$f_{\parallel}(E_K) = \langle K | V^0 | B_s \rangle / \sqrt{2M_{B_s}} \quad \text{and} \quad f_{\perp}(E_K) p_K^i = \langle K | V^i | B_s \rangle / \sqrt{2M_{B_s}}$$

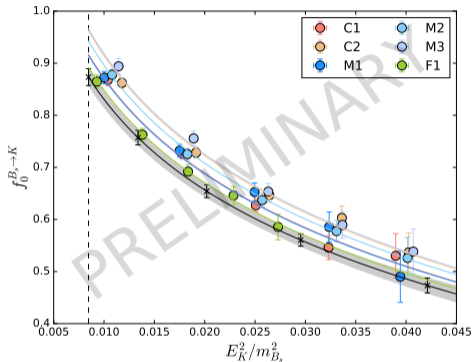
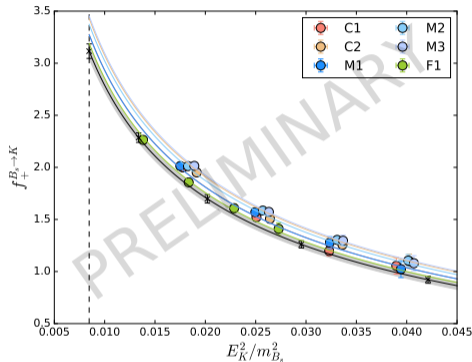
- ▶ Both are related by

$$f_0(q^2) = \frac{\sqrt{2M_{B_s}}}{M_{B_s}^2 - M_K^2} \left[(M_{B_s} - E_K) f_{\parallel}(E_K) + (E_K^2 - M_K^2) f_{\perp}(E_K) \right]$$

$$f_+(q^2) = \frac{1}{\sqrt{2M_{B_s}}} \left[f_{\parallel}(E_K) + (M_{B_s} - E_K) f_{\perp}(E_K) \right]$$

$B_s \rightarrow K\ell\nu$ form factors: F1 ensemble

- Comparison of fit to the ground state only with fit including one excited state term for K and B_s

Chiral-continuum extrapolation using SU(2) hard-kaon χ PT

- ▶ Updating calculation [Flynn et al. PRD 91 (2015) 074510] with improved values for a^{-1} and RHQ parameters

- ▶ $f_{pole}(M_K, E_K, a^2) = \frac{1}{E_K + \Delta} c^{(1)} \times \left[1 + \frac{\delta f}{(4\pi f)^2} + c^{(2)} \frac{M_\pi^2}{\Lambda^2} + c^{(3)} \frac{E_K}{\Lambda} + c^{(4)} \frac{E_K^2}{\Lambda^2} + c^{(5)} \frac{a^2}{\Lambda^2 a_{32}^4} \right]$

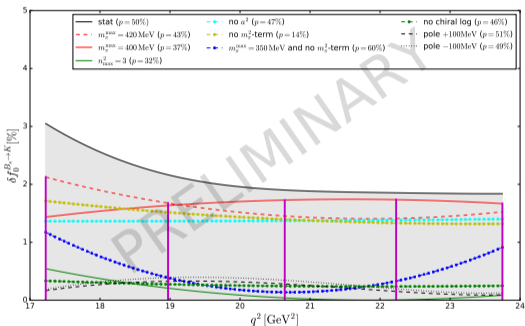
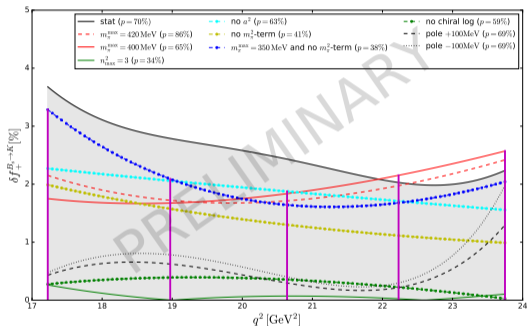
- ▶ δf non-analytic logs of the kaon mass and hard-kaon limit is taken by $M_K/E_K \rightarrow 0$

Estimate systematic errors due to

- ▶ Chiral-continuum extrapolation
 - Use alternative fit functions, vary pole mass, etc.
 - Impose different cuts on the data
- ▶ Discretization errors of light and heavy quarks
 - Estimate via power-counting
- ▶ Uncertainty of the renormalization factors
 - Estimate effect of higher loop corrections
- ▶ Finite volume, iso-spin breaking, ...
- ▶ Uncertainty due to RHQ parameters and lattice spacing (a^{-1})
 - Carry out additional simulations to test effects on form factors
- ▶ Uncertainty of strange quark mass
 - Repeat simulation with different valence quark mass

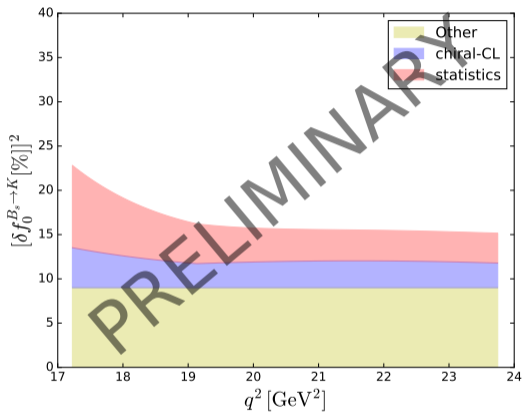
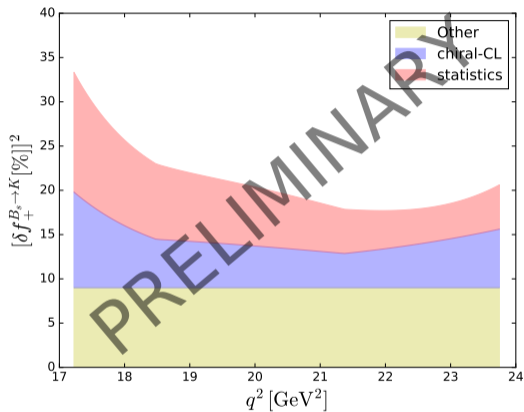
⇒ full error budget

PRELIMINARY error budget $B_S \rightarrow K\ell\nu$



$$\blacktriangleright \delta f = |f_{\text{variation}} - f_{\text{central}}| / f_{\text{central}}$$

PRELIMINARY error budget $B_s \rightarrow K\ell\nu$



- ▶ “Other”: 3% placeholder to cover higher order corrections, lattice spacing, finite volume, ...

Kinematical extrapolation (z-expansion)

- ▶ Map q^2 to z with minimized magnitude in the semi-leptonic region: $|z| \leq 0.146$

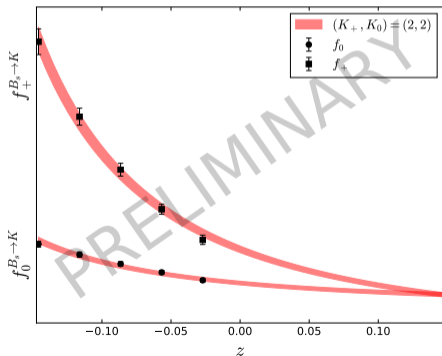
$$z(q^2, t_0) = \frac{\sqrt{1-q^2/t_+} - \sqrt{1-t_0/t_+}}{\sqrt{1-q^2/t_+} + \sqrt{1-t_0/t_+}} \quad \text{with}$$

$$t_{\pm} = (M_B \pm M_{\pi})^2$$

$$t_0 \equiv t_{\text{opt}} = (M_B + M_{\pi})(\sqrt{M_B} - \sqrt{M_{\pi}})^2$$

[Boyd, Grinstein, Lebed, PRL 74 (1995) 4603]

[Bourelly, Caprini, Lellouch, PRD 79 (2009) 013008]



- ▶ Express f_+ as convergent power series
- ▶ f_0 is analytic, except for B^* pole
- ▶ BCL with poles $M_+ = B^* = 5.33$ GeV and $M_0 = 5.63$ GeV
- ▶ Exploit kinematic constraint $f_+ = f_0 \Big|_{q^2=0}$
- Include HQ power counting to constrain size of f_+ coefficients
- ▶ Systematic errors subject to changes!

Kinematical extrapolation (z-expansion)

- ▶ Map q^2 to z with minimized magnitude in the semi-leptonic region: $|z| \leq 0.146$

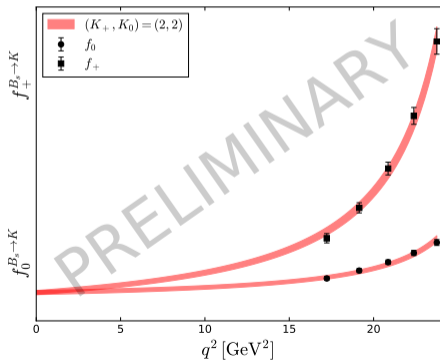
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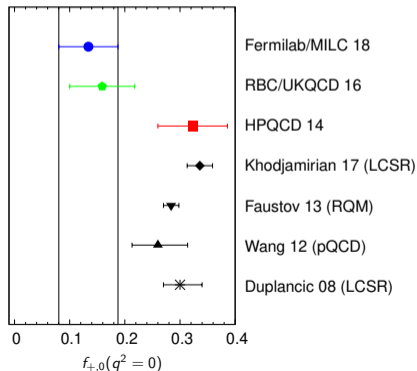
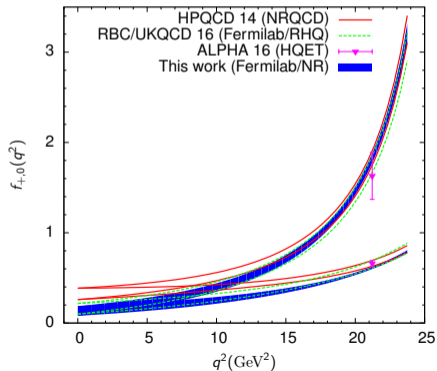
- ▶ Allows to compare shape of form factors
 - Obtained by other lattice calculations
 - [Bouchard et al. PRD 90 (2014) 054506]
 - [Bazavov et al. arXiv:1901.02561]
 - Predicted by QCD sum rules and alike
- ▶ Combination with experiment leads to the overall normalization: $|V_{ub}|$
- ▶ Systematic errors subject to changes!

$B_s \rightarrow K\ell\nu$

- ▶ HPQCD, RBC-UKQCD, ALPHA

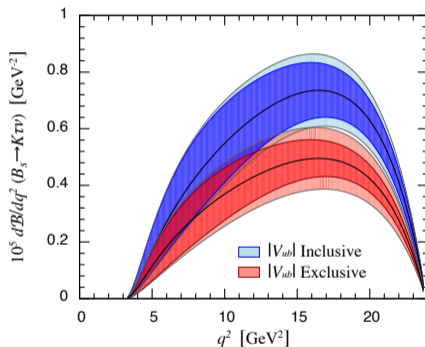
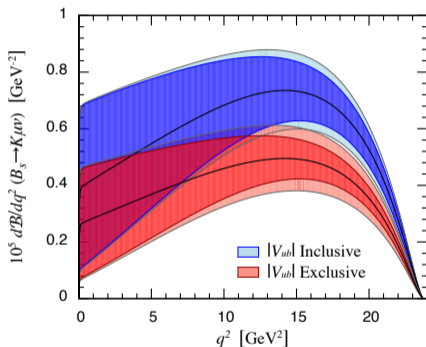
[Bouchard et al. PRD90(2014)054506] [Flynn et al. PRD91(2015)074510] [Bahr et al. PLB757(2016)473]

- ▶ New 2019: Fermilab/MILC [Bazavov et al. PRD100(2019)034501]



Phenomenological predictions

- Predict SM differential branching fractions using $|V_{ub}|$ as input for lepton = μ or τ

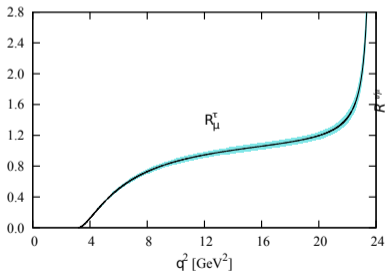


[Flynn et al. PRD 91 (2015) 074510]

Phenomenological predictions

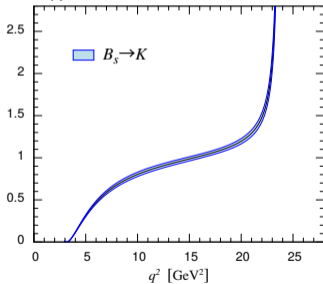
- ▶ Predict SM differential branching fractions using $|V_{ub}|$ as input for lepton = μ or τ
- ▶ Predict ratio of branching fractions \rightsquigarrow LFUV

$$R_K^{\tau/\mu} = 0.695(50)$$

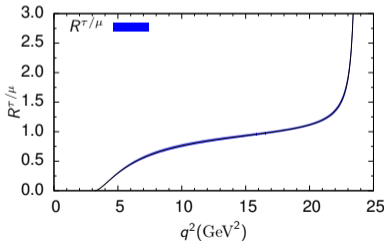


[Bouchard et al. PRD90(2014)054506]

$$R_K^{\tau/\mu} = 0.77(12)$$



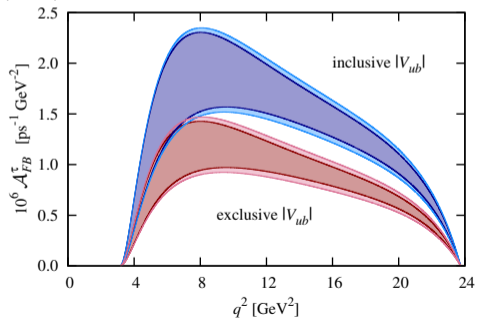
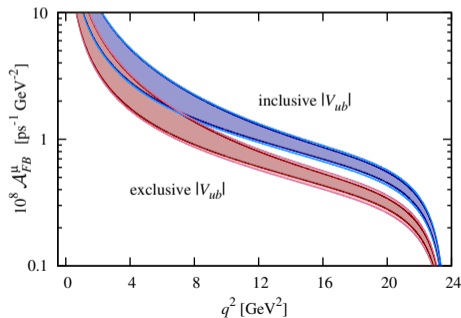
[Flynn et al. PRD 91 (2015) 074510]



[Bazavov et al. PRD100(2019)034501]

Phenomenological predictions

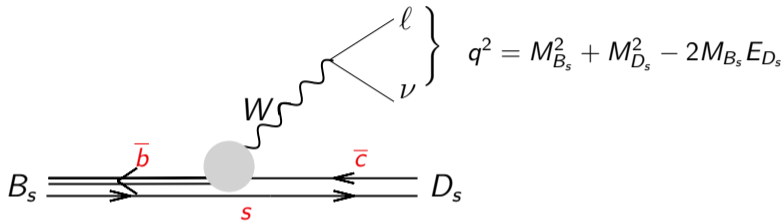
- ▶ Predict SM differential branching fractions using $|V_{ub}|$ as input for lepton = μ or τ
- ▶ Predict ratio of branching fractions \rightsquigarrow LFUV
- ▶ Predict forward-backward asymmetries using $|V_{ub}|$ as input for lepton = μ or τ



[Bouchard et al. PRD90(2014)054506]

$$B_s \rightarrow D_s l \nu$$

$|V_{cb}|$ from exclusive semileptonic $B_s \rightarrow D_s\ell\nu$ decay



► Conventionally parametrized by (B_s meson at rest)

► Accommodate **charm quarks**

$$\frac{d\Gamma(B_s \rightarrow D_s\ell\nu)}{dq^2} = \frac{G_F^2 |V_{ub}|^2}{24\pi^3} \frac{(q^2 - m_\ell^2)^2 \sqrt{E_{D_s}^2 - M_{D_s}^2}}{q^4 M_{B_s}^2}$$

experiment
CKM
known

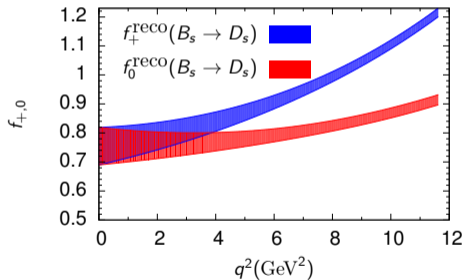
$$\times \left[\left(1 + \frac{m_\ell^2}{2q^2}\right) M_{B_s}^2 (E_{D_s}^2 - M_{D_s}^2) |f_+(q^2)|^2 + \frac{3m_\ell^2}{8q^2} (M_{B_s}^2 - M_{D_s}^2)^2 |f_0(q^2)|^2 \right]$$

nonperturbative input

$B_s \rightarrow D_s\ell\nu$

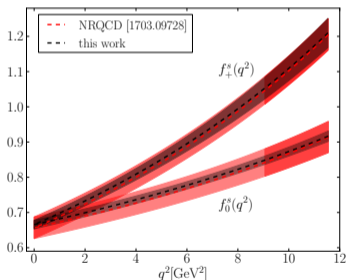
- ▶ z-expansion enforcing kinematical constraint $f_0^{B_s \rightarrow D_s}(0) = f_+^{B_s \rightarrow D_s}(0)$
- ▶ Fermilab/MILC 2019
- ▶ HPQCD 2017 and 2019
- ▶ RBC-UKQCD (preliminary)

[Bazavov et al. PRD100(2019)034501 (reco)]



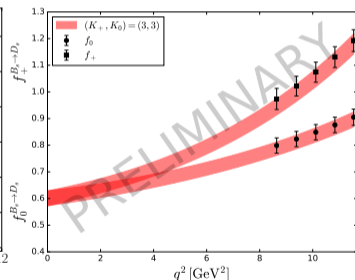
- ▶ Reconstructed from $[f_{+,0}(B \rightarrow D)]^{2015}$ and $[f_{+,0}(B_s \rightarrow D_s)/f_{+,0}(B \rightarrow D)]^{2012}$

[Monahan et al. PRD95(2017)114506]
[McLean et al. arXiv:1906.00701]



- ▶ $R(D_s) = [0.301(6)]^{2017}, [0.299(5)]^{2019}$
- ▶ Modified BCL $M_+ = B_c^* = 6.33$ GeV and $M_0 = 6.42$ GeV [2017]

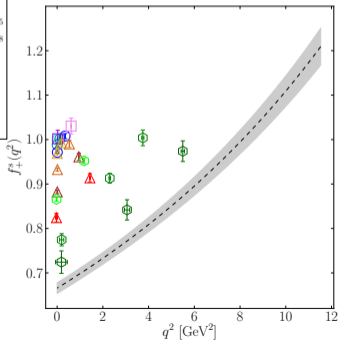
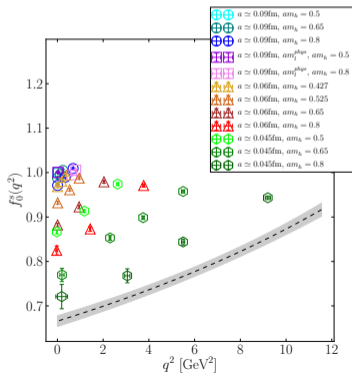
[OW Lattice X IF 2019]



- ▶ Incomplete error budget!
- ▶ BCL: poles $M_+ = B_c^* = 6.33$ GeV and $M_0 = 6.42$ GeV

$B_s \rightarrow D_s\ell\nu$: HPQCD 2019 [McLean et al. arXiv:1906.00701]

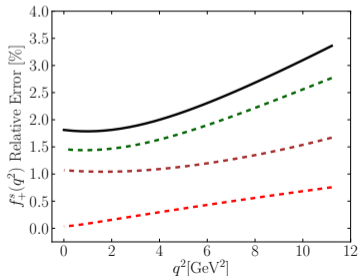
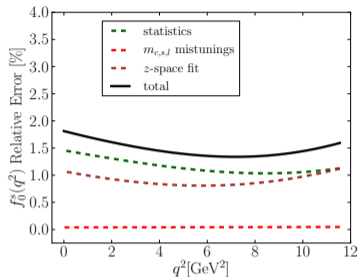
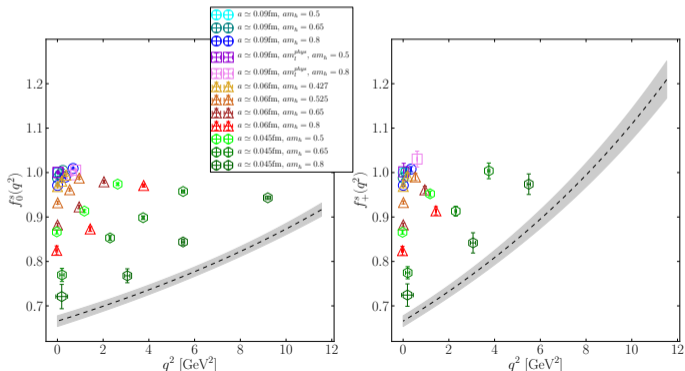
- ▶ Gauge fields: 2+1+1 flavor HISQ ensembles generated by MILC
- ▶ u, d, s, c valence HISQ; **bottom**: “heavy” HISQ \rightsquigarrow fully nonperturbative renormalization
 - Simulate array of “lighter” b -quarks and extrapolate
 - Cover full q^2 range using twisted BC



- ▶ Modified BCL z-expansion
 - Physical final state (D_s); lighter than physical initial state H_s
 - Requires to reconsider poles for unphysical H_{c0} and H_c^*
- ▶ $R(D_s) = 0.2987(46)$

$B_s \rightarrow D_s\ell\nu$: HPQCD 2019 [McLean et al. arXiv:1906.00701]

- ▶ Gauge fields: 2+1+1 flavor HISQ ensembles generated by MILC
- ▶ u, d, s, c valence HISQ; **bottom**: “heavy” HISQ
 - Simulate array of “lighter” b -quarks and extrapolate
 - Cover full q^2 range using twisted BC



$$B \rightarrow D^* l \nu$$

$B \rightarrow D^*\ell\nu$ form factors

- ▶ Vector final state with narrow width approximation

$$\begin{aligned} \langle D^*(k, \lambda) | \bar{c}\gamma^\mu b | B(p) \rangle &= f_V \frac{2i\epsilon^{\mu\nu\rho\sigma} \epsilon_\nu^* k_\rho p_\sigma}{M_B + M_{D^*}} \\ \langle D^*(k, \lambda) | \bar{c}\gamma^\mu \gamma_5 b | B(p) \rangle &= f_{A_0}(q^2) \frac{2M_{D^*} \epsilon^* \cdot q}{q^2} q^\mu \\ &\quad + f_{A_1}(q^2) (M_B + M_{D^*}) \left[\epsilon^{*\mu} - \frac{\epsilon^* \cdot q}{q^2} q^\mu \right] \\ &\quad - f_{A_2}(q^2) \frac{\epsilon^* \cdot q}{M_B + M_{D^*}} \left[k^\mu + p^\mu - \frac{M_B^2 - M_{D^*}^2}{q^2} q^\mu \right] \end{aligned}$$

- ▶ Commonly the HQET variable $w = v \cdot v' > 1$ is used with $v = p/M_B$ and $v' = k/M_{D^*}$

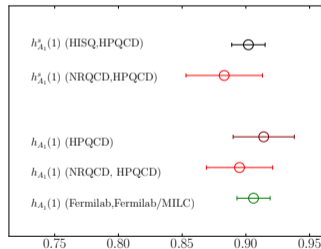
$$B \rightarrow D_{(s)}^{(*)}\ell\nu$$

► Still only results at zero recoil [FLAG 2019]

Collaboration	Ref.	N_f	publication status	continuum extrapolation	chiral extrapolation	finite volume	renormalization	heavy-quark treatment	$w = 1$ form factor / ratio
HPQCD 15, HPQCD 17	[614, 616]	2+1	A	○	○	○	○	✓	$\mathcal{G}^{B \rightarrow D}(1)$ 1.035(40)
FNAL/MILC 15C	[613]	2+1	A	★	○	★	○	✓	$\mathcal{G}^{B \rightarrow D}(1)$ 1.054(4)(8)
Atoui 13	[610]	2	A	★	○	★	—	✓	$\mathcal{G}^{B \rightarrow D}(1)$ 1.033(95)
HPQCD 15, HPQCD 17	[614, 616]	2+1	A	○	○	○	○	✓	$\mathcal{G}^{B_s \rightarrow D_s}(1)$ 1.068(40)
Atoui 13	[610]	2	A	★	○	★	—	✓	$\mathcal{G}^{B_s \rightarrow D_s}(1)$ 1.052(46)
HPQCD 17B	[618]	2+1+1	A	○	★	★	○	✓	$\mathcal{F}^{B \rightarrow D^*}(1)$ 0.895(10)(24)
FNAL/MILC 14	[612]	2+1	A	★	○	★	○	✓	$\mathcal{F}^{B \rightarrow D^*}(1)$ 0.906(4)(12)
HPQCD 17B	[618]	2+1+1	A	○	★	★	○	✓	$\mathcal{F}^{B_s \rightarrow D_s^*}(1)$ 0.883(12)(28)
HPQCD 15, HPQCD 17	[614, 616]	2+1	A	○	○	○	○	✓	$R(D)$ 0.300(8)
FNAL/MILC 15C	[613]	2+1	A	★	○	★	○	✓	$R(D)$ 0.299(11)

► New 2019: HPQCD $B_s \rightarrow D_s^*\ell\nu$ at zero recoil

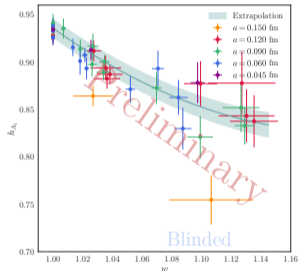
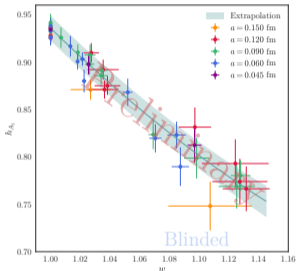
[McLean et al. PRD99(2019)114512]



[Atoui et al. EPJC74(2014)2861] [Bailey et al. PRD89(2014)114504] [Bailey et al. PRD92(2015)034506]
[Na et al PRD92(2015)054510] [Monahan et al. PRD95(2017)114506] [Harrison et al. PRD97(2018)054502]

Update Fermilab/MILC: $B \rightarrow D^*\ell\nu$ [A. Vaquero Lattice X IF 2019]

Results: Chiral-continuum fits



- **Left** Old fit, **Right** New fit. Preliminary blinded results.
- Both plots differ on the accounting of discretization effects, which seem to be large at large recoil

▶ Old double ratio

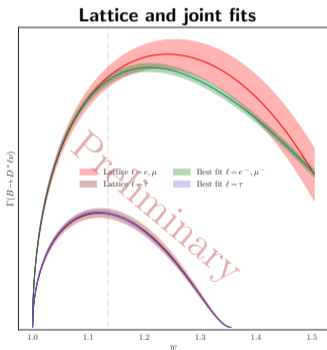
$$\frac{C_{B \rightarrow D^*}^{3pt, A_j}(p_\perp, t, T) C_{D^* \rightarrow B}^{3pt, A_j}(p_\perp, t, T)}{C_{D^* \rightarrow D^*}^{3pt, V^4}(0, t, T) C_{B \rightarrow B}^{3pt, V^4}(0, t, T)} = \frac{M_{D^*}}{E_{D^*}(p_\perp)} \frac{Z_{D^*}^2(p_\perp)}{Z_{D^*}^2(0)} e^{-(E_{D^*}(p_\perp) - M_{D^*})T} \left(\frac{1+w}{2} h_{A_1}(w) \right)^2$$

▶ New ratio

$$\frac{C_{B \rightarrow D^*}^{3pt, A_1}(p_\perp, t, T)}{C_{B \rightarrow D^*}^{3pt, A_1}(0, t, T)} \rightarrow \frac{C_{B \rightarrow D^*}^{3pt, A_1}(p_\perp, t, T)}{C_{B \rightarrow D^*}^{3pt, A_1}(0, t, T)} \times \sqrt{\frac{C_{D^*}^{2pt}(0, t)}{C_{D^*}^{2pt}(p_\perp, t)}}$$

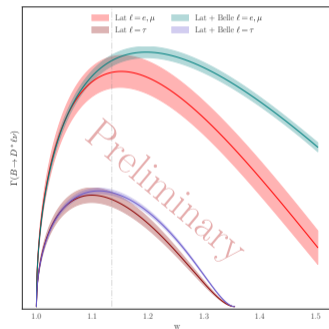
Update Fermilab/MILC: $B \rightarrow D^*\ell\nu$ [A. Vaquero Lattice X IF 2019]

Results: $R(D^*)$



► Old analysis

[Vaquero et al. arXiv:1906.01019]

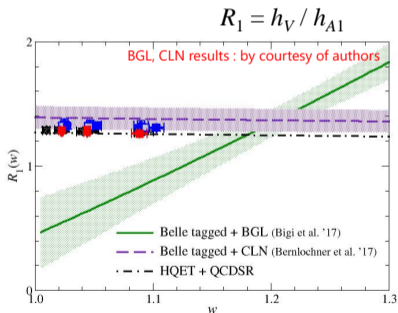


Update JLQCD: $B \rightarrow D^*\ell\nu$ [T. Kaneko Lattice X IF 2019]

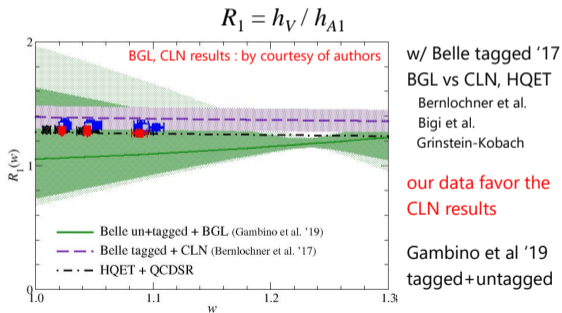
▶ Ratio method:
$$\frac{\langle D^* | V_\mu^{\text{lat}} | B \rangle}{\langle D^* | A_\mu^{\text{lat}} | B \rangle} \rightarrow \frac{h_V(w)}{h_{A_1}(w)}$$

⇒ renormalization factors Z_A, Z_V cancel
[Hashimoto et al. PRD61(1999)014502]

LQCD vs BGL vs CLN vs HQET



LQCD vs BGL vs CLN vs HQET

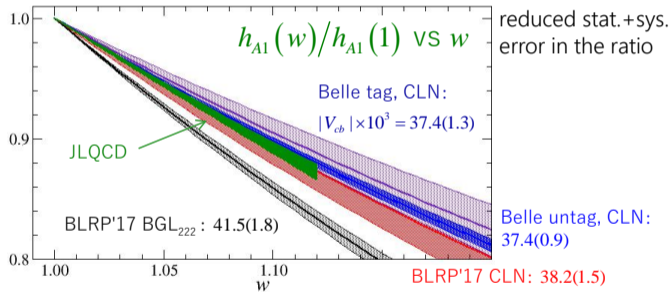


consistency among LQCD, BGL, CLN, HQET

Update JLQCD: $B \rightarrow D^* \ell \nu$ [T. Kaneko Lattice X IF 2019]

LQCD vs BGL vs CLN

shape of h_{A1}

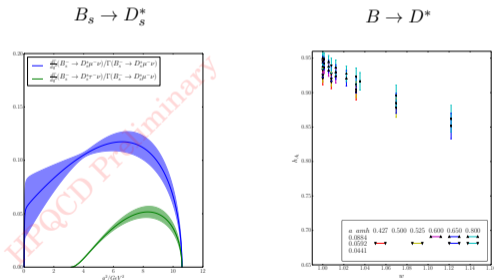


$$h_{A1}(w)/h_{A1}(1) = 1 - 8\rho^2 z + (53\rho^2 - 15)z^2 + (231\rho^2 - 91)z^3$$

Further updates: $B \rightarrow D^*\ell\nu$

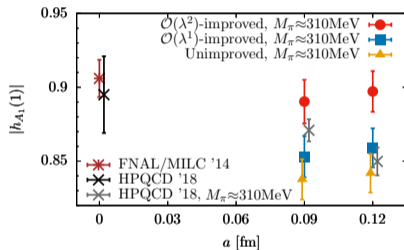
- ▶ HPQCD form factors for $B_{(s)} \rightarrow D_{(s)}^*\ell\nu$
[Plenary talk A. Lytle Lattice 2019]

HPQCD $B_{(s)} \rightarrow D_{(s)}^*$



Figs. courtesy Judd Harrison

- ▶ LANL/SWME form factors at zero recoil
[PoS Lattice2018 283]



summary

Summary

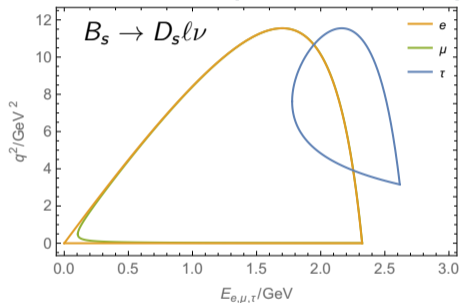
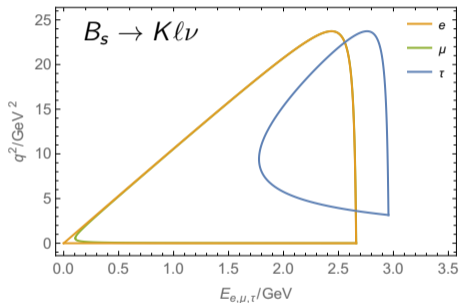
- ▶ Many calculation for exclusive decays are in progress [OW overview talk Beauty 2019]
 - Calculations are hard, tedious, and take time
 - ↪ Not easy to gain by considering the ratio $B_s \rightarrow K\mu\nu/B_s \rightarrow D_s\mu\nu$

- ▶ New ideas to compute inclusive decays using lattice techniques
[Hashimoto PTEP 2017 (2017) 053B03] [Hansen, Meyer, Robaina arXiv:1704.08993] [Bailas Lattice 2019]
- ▶ Interesting new ideas for radiative decays
[Kane et al. arXiv:1907.00279] [Martinelli Lattice 2019] [Sachrajda Lattice 2019]
- ▶ Not covered: B_c decays, $R(J/\psi)$, etc.
[Colquhoun et al. PoS Lattice2016 281][Harrison Poster Beauty 2019]
- ▶ Not covered: exclusive baryonic decays
 - $\Lambda_b \rightarrow \Lambda_c\ell\nu$ and $\Lambda_b \rightarrow p\ell\nu \Rightarrow |V_{cb}|/|V_{ub}|$ [Detmold, Lehner, Meinel, PRD92(2015)034503]
 - $\Lambda_b \rightarrow \Lambda_c\tau\nu$ [Datta et al. JHEP08(2017)131]
 - $\Lambda_c \rightarrow \Lambda\ell\nu$ [Meinel PRL118(2017)082001]

Ratio $B_s \rightarrow K\mu\nu/B_s \rightarrow D_s\mu\nu$

- ▶ Phase space is quite different and our simulations are most precise near q_{\max}^2

[Plots courtesy by J. Flynn]

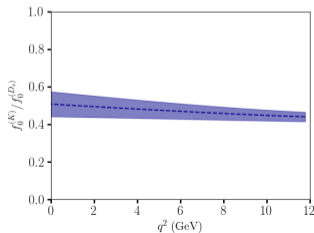
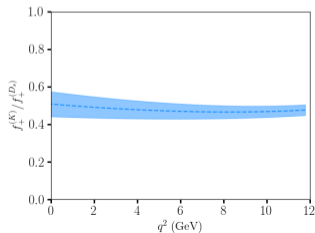


- ▶ Order of steps in the lattice calculation does not help
 - Chiral-continuum extrapolation and estimate of systematic uncertainties are performed **before** kinematical z -expansion
 - ↔ Modified z -expansion for ratio or simultaneous correlated fit of form factors

Ratio $B_s \rightarrow K_{\mu\nu} / B_s \rightarrow D_{s\mu\nu}$

▶ HPQCD [Monahan et al. PRD98(2018)114509]

- 2+1 flavor asqtad sea quark;
 u, d, s, c HISQ valence, NRQCD b
- Correlated and simultaneous fits
- Perturbative NRQCD-continuum matching
with significantly reduced uncertainty



▶ Fermilab/MILC [Bazavov et al. PRD100(2019)034501]

- 2+1 flavor asqtad sea and u, d, s valence quark;
Fermilab/RHQ c, b
- Combination z extrapolated form factors over
restricted q^2 range
- Neglecting statistical correlations

