Amplitude analysis tools

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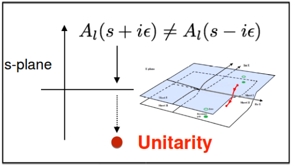
Tools in hadron spectroscopy

General principles of the scattering theory

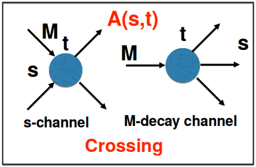
- Lorentz invariance = independence of the reference frame, known behavior under boosts and rotations
- Unitarity = constraint to imaginary part of scattering amplitude
- Analyticity = implementation of relevant, closest singularities
- Crossing = decay and scattering regions are analytically connected

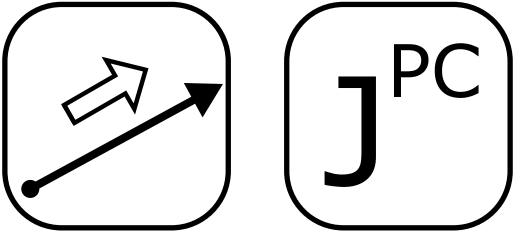


$$A(s,t) = \sum_l A_l(s) P_l(z_s)$$

Analyticity

$$A_l(s) = \lim_{\epsilon \to 0} A_l(s+i\epsilon)$$





Quantum numbers and polarization

- investigation of the angular analysis formalism (LHCb+JPAC)
  [EPJ C78 (2018) 727 (JPAC), EPJ C78 (2018) 229 (JPAC), MM et al.(JPAC) arXiv:1910.04566]

Conventional helicity approach

Complicated cases: particles with spin in isobar model [Herndon(1975)], [Hansen (1974)]

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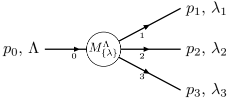
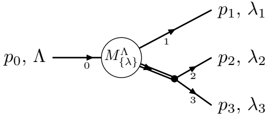
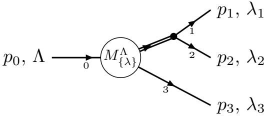
$$M^{\Lambda}_{\{\lambda\}} = M^{\Lambda}_{1,\{\lambda\}} + M^{\Lambda}_{2,\{\lambda\}} + M^{\Lambda}_{3,\{\lambda\}}$$

$$= \underbrace{H_1 D(\phi_1, \theta_1, 0)\, D(\phi_{23}, \theta_{23}, 0) W_1(\dots)}+ \underbrace{H_3 D(\phi_3, \theta_3, 0)\, D(\phi_{12}, \theta_{12}, 0) W_3(\dots)}$$

$$\underbrace{M^{\Lambda}_{\{\lambda\}}}$$

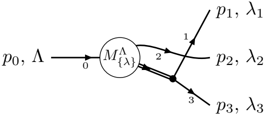






$$+ \underbrace{H_2 D(\phi_2, \theta_2, 0)\, D(\phi_{31}, \theta_{31}, 0) W_2(\dots)}$$

- A special set of angles for every decay chain



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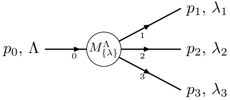

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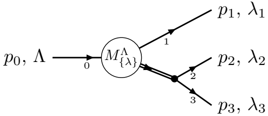

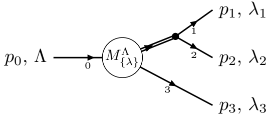

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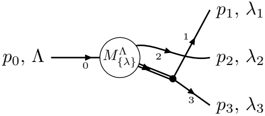
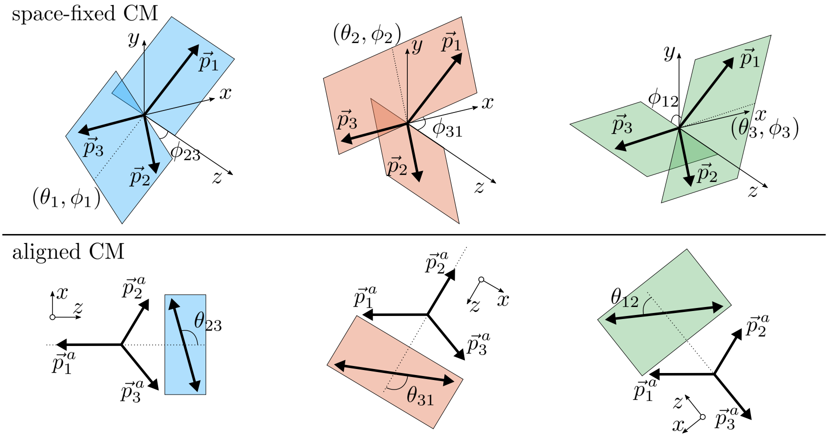


- A special set of angles for every decay chain
- Consistently of quantization direction – **Wigner rotations**

$$W_i(\dots) = D^{j_1}(\tilde{\phi}^i_1, \tilde{\theta}^i_1, 0)\, D^{j_2}(\tilde{\phi}^i_2, \tilde{\theta}^i_2, 0)\, D^{j_3}(\tilde{\phi}^i_3, \tilde{\theta}^i_3, 0)$$

Different angles for decay chains

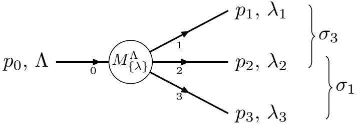


$$M^{\Lambda}_{\{\lambda\}} = H_1 D(\phi_1, \theta_1, 0)\, D(\phi_{23}, \theta_{23}, 0) W_1(\dots) + H_3 D(\phi_3, \theta_3, 0)\, D(\phi_{12}, \theta_{12}, 0) W_3(\dots)$$
$$+ H_2 D(\phi_2, \theta_2, 0)\, D(\phi_{31}, \theta_{31}, 0) W_2(\dots)$$

The Dalitz-Plot decomposition

[MM et al.(JPAC), arXiv:1910.04566]

Reformulation of the helicity approach



$$= \sum_{\nu} \underbrace{D^{J*}_{\Lambda\nu}(\phi_1, \theta_1, \phi_{23})}_{\text{Decay-plane orientation}} \times \underbrace{O^{\nu}_{\{\lambda\}}(\{\sigma\})}_{\text{Dalitz-plot function}}$$

Model-independent factorization of the overall rotation:

- Exploits properties of the Lorentz group (orientation – just three Euler angles)
- Dalitz-plot function depends entirely on 2 variables, $\{\sigma\} \equiv \{\sigma_1, \sigma_2, \sigma_3\}$
- No azimuthal phase factors in $O^{\nu}_{\{\lambda\}}$.

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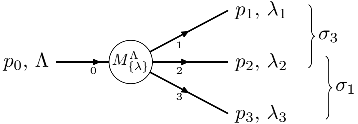

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Gives significant benefits to

- Pentaquark analysis, Λ_b/Λ_c polarionation measurements, Baryonic decay chains,...

Dalitz-Plot function

[MM et al.(JPAC), arXiv:1910.04566]

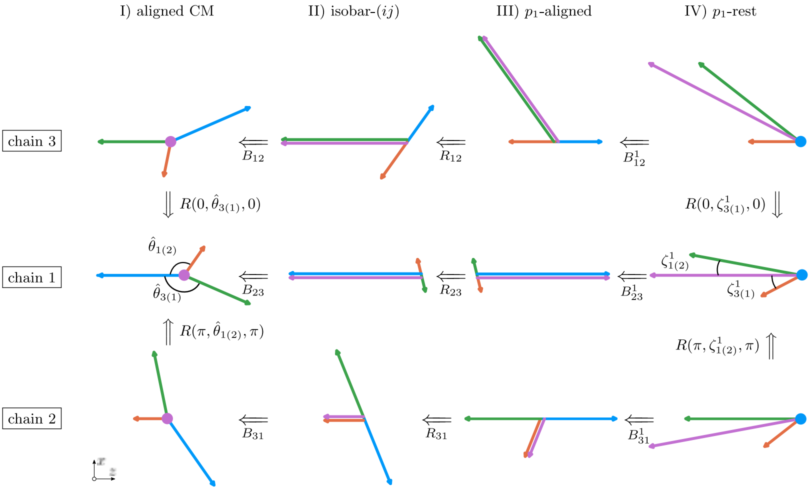
Master formula $0 \to 1\,2\,3$ decay with arbitrary spins

$$O^{\nu}_{\{\lambda\}}(\{\sigma\}) = \sum_{(ij)k} \sum_{s}^{(ij)\to i,j} \sum_{\tau} \sum_{\{\lambda'\}} n_J n_s \, d^{J}_{\nu,\tau-\lambda'_k}(\hat{\theta}_{k(1)}) X_s^{\tau,\lambda'_k;\lambda'_i,\lambda'_j}(\sigma_k) \, d^{s}_{\tau,\lambda'_i-\lambda'_j}(\theta_{ij})$$
$$\times \, d^{j_1}_{\lambda'_1,\lambda_1}(\zeta^1_{k(0)}) \, d^{j_2}_{\lambda'_2,\lambda_2}(\zeta^2_{k(0)}) \, d^{j_3}_{\lambda'_3,\lambda_3}(\zeta^3_{k(0)}),$$

- Three decay chains, $(ij)k \in \{(12)3, (23)1, (31)2\}$.
- $\theta_{ij} = \theta_{ij}(\{\sigma\})$ is an isobar decay angle
- $\hat{\theta}_{k(1)} = \hat{\theta}_{k(1)}(\{\sigma\})$ is the particle-0 Wigner angle
- $\zeta^i_{k(0)} = \zeta^i_{k(0)}(\{\sigma\})$ is the particle-i Wigner angle
- $X_s^{\tau,\lambda'_k;\lambda'_i,\lambda'_j}(\sigma_k) \Rightarrow X_s^{LS;l's'}(\sigma_k)$ is the only model-dependent input (lineshape functions)

Origin of the Wigner angle

[MM et al.(JPAC), arXiv:1910.04566]



$\Lambda_c \to pK\pi$ polarization studies

- proposal for the electromagnetic dipole moments of charmed baryons
  [EPJC 77 (2017) 181, arXiv:1708.08483]
- polarization information for complex decay chains

$$\boxed{M_\lambda^\Lambda = \sum_\nu D_{\Lambda\nu}^{1/2*}(\phi_1, \theta_1, \phi_{23}) O_\lambda^\nu(\sigma_1, \sigma_3),}$$

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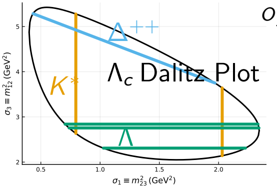
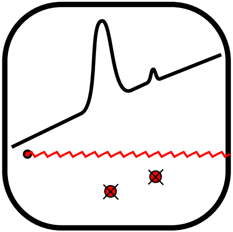
$$M_\lambda^\Lambda = \sum_\nu D_{\Lambda\nu}^{1/2*}(\phi_1, \theta_1, \phi_{23}) O_\lambda^\nu(\sigma_1, \sigma_3),$$

- Resonances in all channels, Λ^0, K^{*0}, Δ^{++}
- Possible Triangle Singularity near $\Lambda\eta$ threshold [Liu, Xiao-Hai et al., PRD100 (2019)]



$$\begin{aligned} O_\lambda^\nu(\sigma_1, \sigma_3) = &\sum_s^{K^*\to K\pi} \sum_\tau d_{\nu,\tau-\lambda}^{1/2}(0)\, X_s^{\tau,\lambda}(\sigma_1)\, d_{\tau,0}^s(\theta_{23}) \\ &+ \sum_s^{\Delta\to\pi p} \sum_{\tau,\lambda'} d_{\nu,\tau}^{1/2}(\hat\theta_{2(1)})\, X_s^{\tau,\lambda'}(\sigma_2)\, d_{\tau,-\lambda'}^s(\theta_{31})\, d_{\lambda',\lambda}^{1/2}(\tilde\theta_{2(1)}^1) \\ &+ \sum_s^{\Lambda\to pK} \sum_{\tau,\lambda'} d_{\nu,\tau}^{1/2}(\hat\theta_{3(1)})\, X_s^{\tau,\lambda'}(\sigma_3)\, d_{\tau,\lambda'}^s(\theta_{12})\, d_{\lambda',\lambda}^{1/2}(\tilde\theta_{3(1)}^1). \end{aligned}$$



Analytic properties of the amplitude

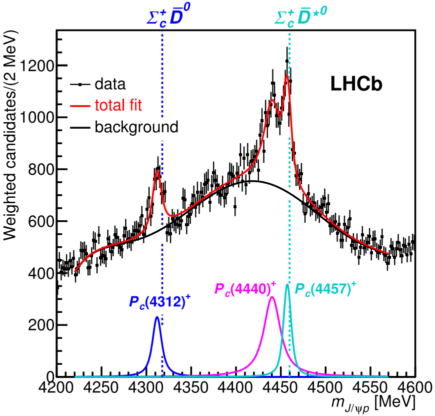
- Botton-up approach to the nature of the states
  [C. Fernandez, et al.(JPAC), PRL123 (2019)]

Extracting resonance properties

[LHCb, PRL122 (2019)]

1-dim. fit and extensive systematic studies:

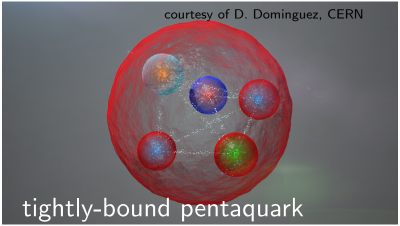
- Three different projection methods
- Several background parametrization
- Interference effects
- Procedure is validated using 6-dim. MC

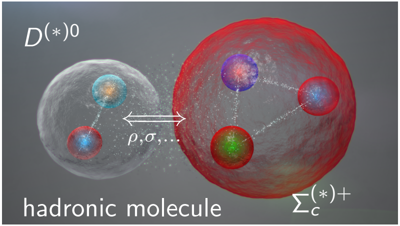


Mass and width of the peaks

State	M [MeV]	Γ [MeV]	(95% CL)
$P_c(4312)^+$	$4311.9 \pm 0.7^{+6.8}_{-0.6}$	$9.8 \pm 2.7^{+\ 3.7}_{-\ 4.5}$	(< 27)
$P_c(4440)^+$	$4440.3 \pm 1.3^{+4.1}_{-4.7}$	$20.6 \pm 4.9^{+\ 8.7}_{-10.1}$	(< 49)
$P_c(4457)^+$	$4457.3 \pm 0.6^{+4.1}_{-1.7}$	$6.4 \pm 2.0^{+\ 5.7}_{-\ 1.9}$	(< 20)
$P_c(4380)^+$	inconclusive with 1-dim. analysis		

Two main interpretations of P_c states





- The state should have high probably to disintegrate to $J/\psi(c\bar{c})\ p(uud)$
- Diquark picture with a potential barrier [Maiani et al., PLB778, 247 (2018)]
- Does not relate appearance to the thresholds

[see Ref. in arXiv:1904.03947]

- Masses are near threshold of $\overline{D}^{0(*)}\Sigma_c^+$,
- Natural mechanism to separate charm quarks,
- Suggest importance of ρ/σ exchanges.

[W. L. Wang et al., Phys. Rev. C84 (2011) 015203]

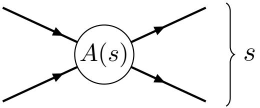
[Z.-C. Yang et al., Chin. Phys. C36 (2012) 6]

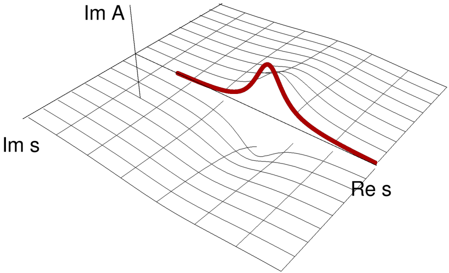
[J.-J. Wu et al., Phys. Rev. C85 (2012) 044002]

Basics of amplitude analysis

aka *S*-matrix theory

[Gribov, Collins, Martin-Spearman]





Scattering PW-amplitude

$A(s)$ is a transition amplitude

- Complex analytic function,
- Can be analytically continued to the complex energy plane, i.e. $A(\mathrm{Re}\, s + i\, \mathrm{Im}\, s)$
- The observed projection, $A(\mathrm{Re}\, s + i0)$

- All structures in the energy spectum have origin in the complex plane
- Hadronic resonances - complex **poles**!
- Production thresholds - branching points.

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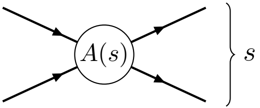

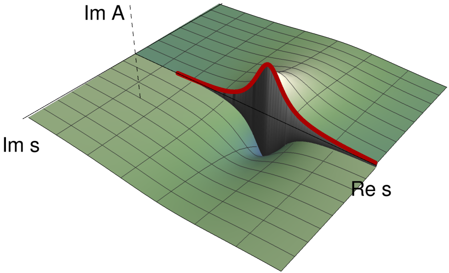

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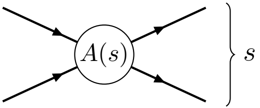

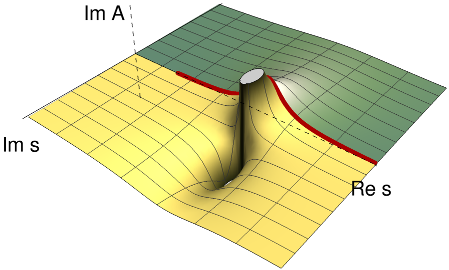

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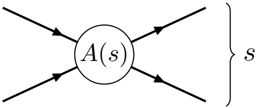

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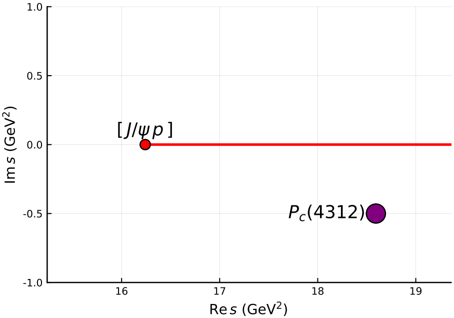

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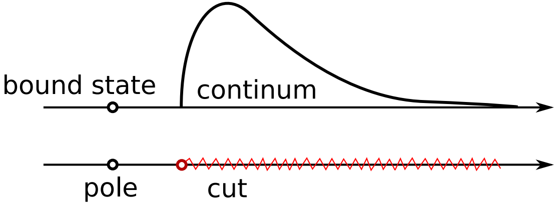
Classical bound-state (molecular) picture

Complex energy plane

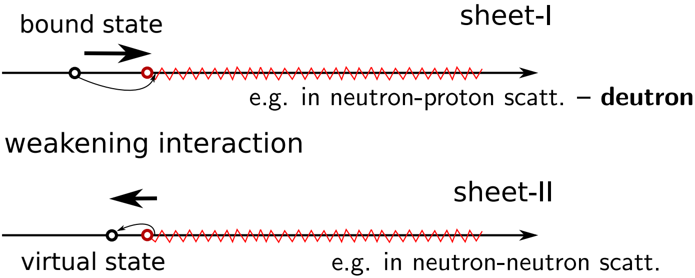
Structures of the complex scattering amplitude correspond to physical



- bound state - pole of the complex scattering amplitude
- continuum - free particles above elastic threshold (branching of the complex plane)

Strength of interaction

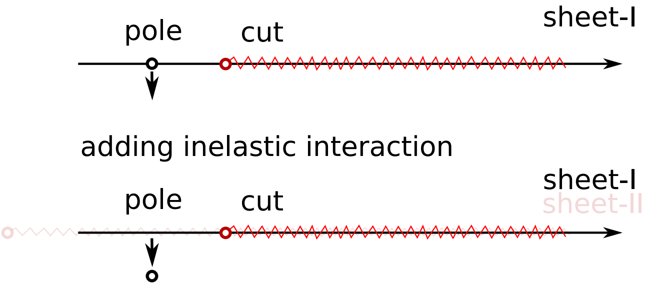
Molecular bound-state vs virtual state



- as weaker the binding as closer the pole to the threshold
- at some point moves to the unphysical sheet and leaves to $-\infty$.

Influence of the inelastic channels

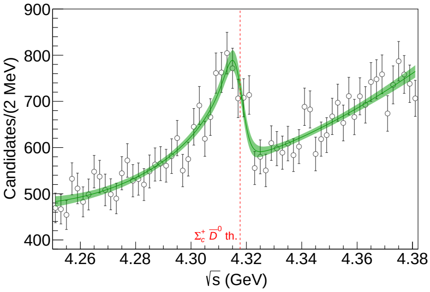
Width of the molecular state



- probability to move to continuum of other channels leads to a shift of the pole
- lower threshold introduces the branching point and cut
  $\Rightarrow$ the pole is still on the physical sheet, causality is not violated.

$P_c(4312)$ in molecular picture

Fit to the LHCb data [C. Fernandez, et al.(JPAC), PRL123 (2019)]



Two channels model: $\Sigma_c^+ \bar{D}^0$ and $J/\psi p$.

- Parameter m_{12} sets the coupling between two channels

Scattering-length approximation

$$T_{ij}^{-1} = m_{ij} - ik_i\delta_{ij},$$
$$k_i = \sqrt{s - s_i}$$

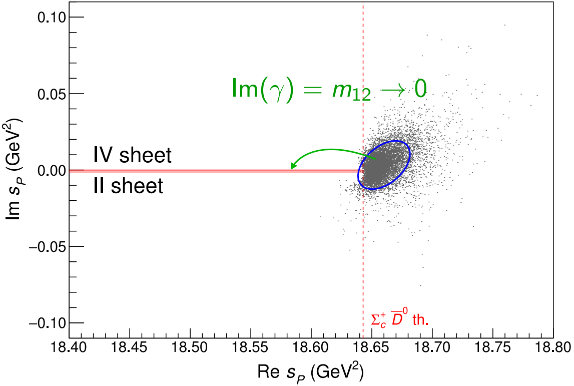
Intensity of the data

$$I(s) = \rho(s)(|\, T_{11}(s)\, p(s)|^2 + b(s)),$$

- $p(s)$ is a production vector
- $b(s)$ is a background term
- $\rho(s)$ is a phase-space factor.

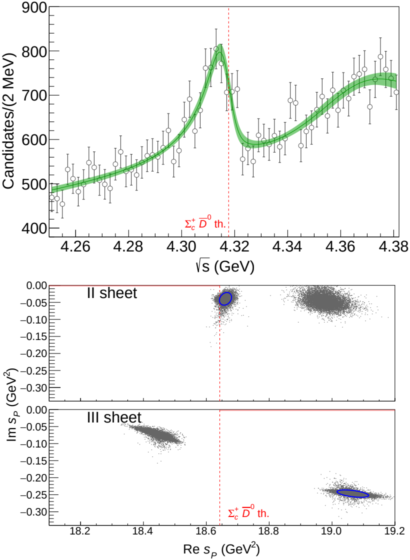
Pole position of the $P_c(4312)$ state

[C. Fernandez, et al.(JPAC), PRL123 (2019)]



- Uncertainties to the pole position due to statistical uncertainties (bootstrap sample)
- When coupling m_{12} is turned off, the pole ends up on the unphysical sheet (**virtual state**)

Case B: effective-range model



Two channels: $\Sigma_c^+ \bar{D}^0$ and $J/\psi p$.

Effective-range approximation

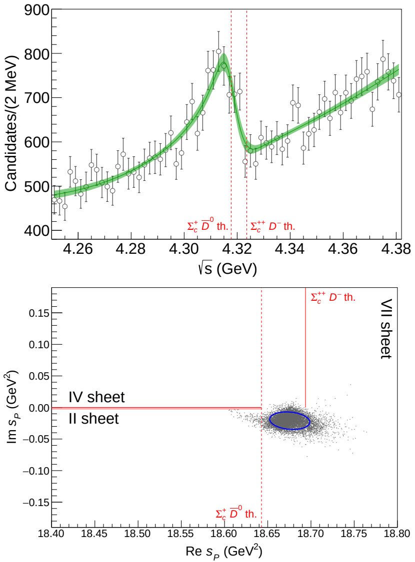
$$T_{ij}^{-1} = m_{ij} + c_{ij} s - i k_i \delta_{ij},$$
$$k_i = \sqrt{s - s_i}$$

Fit parameters

- m_{11}, m_{22}, $m_{12} = m_{21}$, c_{11}, c_{22}
- p_0, p_1, b_0, and b_1.

⇒ A consistent result for the pole position

Three channels with the effective-range approximation



Three channels: $\Sigma_c^+\bar{D}^0$, $\Sigma_c^{++}\bar{D}^-$ and $J/\psi p$.

Effective-range model

$$T_{ij}^{-1} = m_{ij} + c_{ij}s - ik_i\delta_{ij},$$
$$k_i = \sqrt{s - s_i}$$

Fit parameters

- m_{11}, m_{22}, $m_{12} = m_{13}$, m_{23}, c_{11}, $c_{22} = c_{33}$, m is symmetric.
- p_0, p_1, b_0, and b_1.

$\Rightarrow$ A consistent result for the pole position

Conclusion

General S-**matrix principles** and **symmetries** are the main reliable tools for dealing with non-perturbative theory

Dalitz-Plot decomposition

- Requirement of Lorentz Invariance
- Explicit separation of degrees of freedom

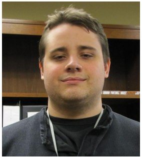
$\Rightarrow$ significant simplification of amplitude construction

Pentaquark lineshape studies

- Imposing correct analytic structure ($\Sigma_c^+ \bar{D}^0$ branch cut)
- Pole position is established, consistent for different models
- $\Sigma_c^+ \bar{D}^0 - J/\psi p$ decoupling, the pole moves to become the virtual state.
- Outlook – extend the studies to $P_c(4440)$ and $P_c(4457)$

Thank you for attention

Thanks to my collaborators (JPAC group):



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Vincent Mathieu

Miguel Albaladejo

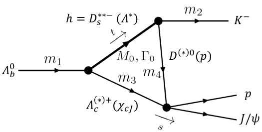
Cesar Fernandez

Adam Szczepaniak

Also:

- Lukasz Bibrzycki, Daniel Winney,
- Sara Mitchell, Marco Pappagallo, Tomasz Skwarnicki

Rescattering interpretation



- There are many thresholds around P_c peaks
  - $\Lambda_c \bar{D}^0$, $\Sigma_c \bar{D}^0$, $\chi_c N^*$ with different exchanges as suggested in [Guo et al.(PRD92 (2015) 071502), U.-G. Meißner et al. (PLB751 (2015) 59), X.-H. Liu et al. (PLB757 (2016) 231), MM (arXiv:1507.06552)]
- An appropriate Triangle Singularity can be found for all peaks
- BUT, as soon as **width** of exchange particle is taken into account

$\Rightarrow$ no acceptable description in rescattering picture have been found