Is GR Unique?

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Ref: arXiv:1708.03757 by CL, S. Mukohyama
arXiv: 1811.02467 by CL

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Overview

I. Introduction and Motivation

"Don't modify gravity, understand it!"
- - - a big guy

"We understand gravity by modifying it."
- - - C.L

Smash it, then understand it!

Is GR unique?

According to Lovelock theorem, the answer is YES if we assume

1. 3+1 dimensional space-time;
2. metricity;
3. 2nd order equation of motion;
4. space-time diffeomorphism $\rightarrow$ 8 first class constraints

However, if we assume

1. 3+1 dimensional space-time;
2. metricity;
3. 2nd order equation of motion;
4. 8 first class constraints

Is GR still unique?

The symmetries of space-time in GR

$$x^i \to x^i + \xi^i(t, \mathbf{x}), \qquad t \to t + \xi^0(t, \mathbf{x})$$

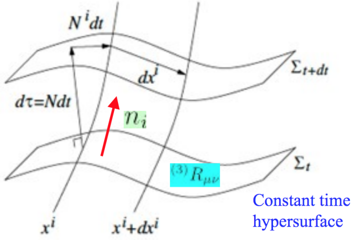
what if ...

$$\downarrow$$

$$x^i \to x^i + \xi^i(t, \mathbf{x}) \qquad + \qquad \text{internal gauge symmetry?}$$

ADM formalism

$$ds^2 = -N^2 dt^2 + h_{ij}\left(dx^i + N^i dt\right)\left(dx^j + N^j dt\right)$$



The Einstein-Hilbert action

$$S = \int d^4x\, N\sqrt{h}\left[\underbrace{K_{ij}K^{ij} - K^2}_{Kinetic\ \ term} + \underbrace{^{(3)}R}_{gradient\ \ term} + ...\right]$$

where $K_{ij} = \frac{1}{2N}\left(\dot{h}_{ij} - \nabla_i N_j - \nabla_j N_i\right)$

$^{(3)}R$: 3-d Ricci scalar

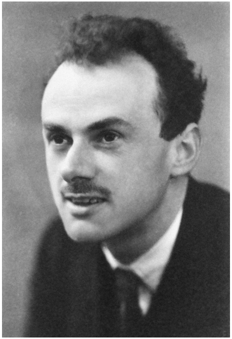
conjugate momenta $\pi^{ij} \equiv \partial\mathcal{L}/\partial\dot{h}_{ij}, \pi_N \equiv \partial\mathcal{L}/\partial\dot{N}$ and $\pi_i \equiv \partial\mathcal{L}/\partial\dot{N}^i$,
Legendre transformation

$$H = \int d^3x \left[N\mathcal{H}_0 + N^i\mathcal{H}_i\right] ,$$

All of constraints $\mathcal{H}_0 \approx 0$ and $\mathcal{H}_i \approx 0$ are first class!

Hamiltonian Analysis by P. Dirac @ 1950s∼1960s

“God created our universe !”



“I have an equation, do you have a one too ?”

P. A. Dirac, Can. J. Math. 2 129 (1950);
Proc. Roy. Soc. London, ser. A, 246, 326 (1958);

P. A. M. Dirac, Lectures on Quantum Mechanics (Yeshiva University, New York 1964).

1. Legendre transformation from action to Hamiltonian $\mathcal{L} \to H$
2. Primary constraint: $p^i \equiv \frac{\partial \mathcal{L}}{\partial \dot{q}_i} = 0$, include it in the Hamitonian
$$\tilde{H} = H + \lambda^i p_i,$$
3. Secondary constraint: constraint must hold throughout time evolution
$$\phi^j = \frac{dp^j}{dt} = \{p^j, \tilde{H}\} \approx 0\,, \qquad \{F, G\} \equiv \frac{\delta F}{\delta q_j}\frac{\delta G}{\delta p^j} - \frac{\delta F}{\delta p^j}\frac{\delta G}{\delta q_j}\,,$$
4. total Hamiltonian if no tertiary constraints are generated,
$$H_{tot} = H + \lambda^i p_i + \lambda^j \phi_j,$$
and a full set of constraints $\phi^A = \left(p^i, \phi_j\right)$
5. compute all of Poisson brackets $\{\phi^A, \phi^B\}$
   - ϕ^α is first class if all $\{\phi^\alpha, \phi^A\} \approx 0$,
   - The rest of constraints are second class.

Why does first class constraint matter?

- every constraint eliminates (at least) one degree in the phase space;
- a first class constraint eliminates two degrees in phase space, i.e. one degree in physical space-time;
- 1st class constraints generate **local gauge transformation**;

$$H_{tot} = ... + \lambda^{\alpha}\phi_{\alpha} \, ,$$

The consistency conditions

$$\frac{d\phi^{\alpha}}{dt} = \{\phi^{\alpha}, H_{tot}\} \approx 0$$

do not fix coefficients λ^{α} : a gauge redundant!

$$\delta F = \lambda_{\alpha}\{F, \phi^{\alpha}\}, \qquad \text{generator of gauge transformation.}$$

Degree of Freedom Counting

The number of degrees of freedom

$$= \quad N_{\text{canonical pairs}} - \frac{1}{2} \times N_{\text{2nd class}} - N_{\text{1st class}}$$

Example 1: $\mathcal{L} = \frac{1}{4} F_{\mu\nu} F^{\mu\nu}, \qquad A_\mu \to A_\mu + \partial_\mu \theta$

$$H = \frac{1}{2}\pi_i \pi_i + (\partial_i A_j \partial_i A_j - \partial_i A_j \partial_j A_i) - A_0 \partial_i \pi_i + \lambda_0 \pi_0,$$

$$\pi_i \equiv \frac{\partial \mathcal{L}}{\partial \dot{A}_i} = \dot{A}_i = E_i \qquad \text{electric field},$$

Primary $\pi_0 \approx 0$ and Secondary $\partial_i \pi_i \approx 0$ are first class: $U(1)$ symmetry!

Therefore, a photon has $4 - 2 = 2$ polarisations!

Example 2: General Relativity $S = \int d^4x\sqrt{-g}\left[\frac{M_p^2}{2}\mathcal{R} - \Lambda\right]$

$$S = \int d^4x \; N\sqrt{h}\left[\underbrace{K_{ij}K^{ij} - K^2}_{Kinetic\;\;term} + \underbrace{{}^{(3)}R}_{gradient\;\;term} + \Lambda \dots\right]$$

where $K_{ij} = \frac{1}{2N}\left(\dot{h}_{ij} - \nabla_i N_j - \nabla_j N_i\right)$

${}^{(3)}R$: 3-d Ricci scalar

Hamiltonian reads

$$H = \int d^3x \left[N\mathcal{H}_0 + N^i\mathcal{H}_i + \lambda_N\pi_N + \lambda^i\pi_i\right] ,$$

all these 8 constraints are 1st class $\rightarrow$ space-time diff invariance

$$x^i \rightarrow x^i + \xi^i(t, \mathbf{x}) , \qquad t \rightarrow t + \xi^0(t, \mathbf{x})$$

Therefore, a graviton has $10 - 8 = 2$ polarisations!

The symmetries of space-time in GR

$$x^i \to x^i + \xi^i(t, \mathbf{x}), \qquad t \to t + \xi^0(t, \mathbf{x})$$

what if ...

$\downarrow$

$$x^i \to x^i + \xi^i(t, \mathbf{x}) \qquad + \qquad \text{internal gauge symmetry?}$$

reformulate our question:

Are there any gravity theories which are as good as GR, in the sense that all constraints are 1st class?

II. a self-consistency condition

Hamiltonian analysis

We are interested in the theory linear in lapse,

$$S = \int dtd^3x\sqrt{h}NF\left(K_{ij}, R_{ij}, \nabla_i, h^{ij}, t\right) \ ,$$

where,

$$\det\left(\frac{\partial^2 F}{\partial K_{ij}\partial K_{kl}}\right) \neq 0 \, ,$$

To perform the Hamiltonian analysis a la Dirac, We need Q_{ij} and υ^{ij},

$$S = \int d^4x\sqrt{h}N\left[F\left(Q_{ij}, R_{ij}, \nabla_i, h^{ij}, t\right) + \upsilon^{ij}\left(Q_{ij} - K_{ij}\right)\right] \ .$$

the momenta conjugate to $(h_{ij},\ N,\ N^i,\ Q_{ij},\ \upsilon^{ij})$

$$\begin{aligned} \pi^{ij} &= \frac{\partial \mathcal{L}}{\partial \dot{h}_{ij}} = -\frac{1}{2}\sqrt{h}\upsilon^{ij}, \qquad \pi_N = \frac{\partial \mathcal{L}}{\partial \dot{N}} = 0, \qquad \pi_i = \frac{\partial \mathcal{L}}{\partial \dot{N}^i} = 0\, , \\ P^{ij} &= \frac{\partial \mathcal{L}}{\partial \dot{Q}_{ij}} = 0, \qquad U_{ij} = \frac{\partial \mathcal{L}}{\partial \dot{\upsilon}^{ij}} = 0\, . \end{aligned}$$

a self-consistency condition

$$H_{\rm tot} = \int d^3x \left[\lambda_c \mathcal{C} + \tilde{N}^i \mathcal{H}_i + \lambda_N \pi_N + \lambda^i \pi_i + \chi_{ij} P^{ij} + \varphi^{ij} U_{ij} + \lambda_{ij} \Psi^{ij} + \phi_{ij} \Phi^{ij}\right] ,$$

where

- $\pi_N \approx \pi_i \approx P^{ij} \approx U_{ij} \approx \Psi^{ij} \approx 0$ are 22 primary constraints;
- there are 10 secondary constraints,

$$\begin{aligned} 0 &\approx \frac{d\pi_N}{dt} = \{\pi_N, H\} = -\mathcal{C} , \\ 0 &\approx \frac{d\pi_i}{dt} = \{\pi_i, H\} = -\mathcal{H}_i , \\ 0 &\approx \frac{dP^{ij}}{dt} = \{P^{ij}, H\} = N\Phi^{ij} , \end{aligned}$$

where

$$\Phi^{ij} \equiv \sqrt{h} \left(\frac{\partial F}{\partial Q_{ij}} + v^{ij} \right) .$$

a self-consistency condition

- there are 7 first class constraints $\pi_N \approx 0$, $\pi_i \approx 0$ and

$$\bar{\mathcal{H}}_i^E[\lambda^i] = \int d^3x \left[\pi^{ij}\mathcal{L}_\lambda h_{ij} + P^{ij}\mathcal{L}_\lambda Q_{ij} + U_{ij}\mathcal{L}_\lambda \upsilon^{ij} + \pi_N \mathcal{L}_\lambda N\right] ,$$

where $\mathcal{L}_\lambda$: Lie derivative along the vector λ^i, and useful notation

$$\bar{\mathcal{O}}[\lambda] \equiv \int d^3x\lambda\mathcal{O} , \qquad \bar{\mathcal{O}}_i[\lambda^i] \equiv \int d^3x\lambda^i\mathcal{O}_i ,$$

- The complete set of other independent constraints is

$$\mathcal{C} \approx 0 , \qquad P^{ij} \approx 0 , \qquad U_{ij} \approx 0 , \qquad \Phi^{ij} \approx 0 , \qquad \Psi^{ij} \approx 0 .$$

In total, we have 25 remaining constraints at each point,

$$\phi_a \equiv \left(\mathcal{C}, P^{ij}, U_{ij}, \Phi^{ij}, \Psi^{ij}\right)$$

where $a = 1, \cdots, 25$ and $(ij) = (11), (22), (33), (12), (23), (31)$.

a self-consistency condition

key matrix $M_{ab}(x, y)$

the infinite dimensional matrix

$$M_{ab}(x,y) \equiv \{\phi_a(x), \phi_b(y)\} \approx \begin{pmatrix} 0 & 0_6^T & u_1^T & 0_6^T & \hat{u}_2^T \\ 0_6 & 0_{6,6} & 0_{6,6} & A_1 & 0_{6,6} \\ -u_1 & 0_{6,6} & 0_{6,6} & a\mathbf{1}_{6,6} & b\mathbf{1}_{6,6} \\ 0_6 & -A_1^T & -a\mathbf{1}_{6,6} & 0_{6,6} & \hat{A}_2 \\ -\hat{u}_2 & 0_{6,6} & -b\mathbf{1}_{6,6} & -\hat{A}_2 & A_3 \end{pmatrix},$$

is very crucial!

- If $\mathrm{Det} M_{ab}(x,y) \neq 0$, all of $\phi_a \equiv \left(\mathcal{C}, P^{ij}, U_{ij}, \Phi^{ij}, \Psi^{ij}\right)$ are second class,

$$44 - 2 \times 7 - 25 = 5$$

  the dimension of phase space is **odd!**

- We have to demand $\mathrm{Det} M_{ab}(x,y) \approx 0$.

A self-consistency condition

$$\mathrm{Det}\, M_{ab}(x,y) \approx 0$$

$$\downarrow$$

$$\int d^3x \left[\frac{\delta \bar{\mathcal{C}}[\beta]}{\delta h_{ij}(x)} Q_{ij}(x)\alpha(x) - \frac{\delta \bar{\mathcal{C}}[\alpha]}{\delta h_{ij}(x)} Q_{ij}(x)\beta(x) \right] \approx 0\,, \quad \text{for } {}^{\forall}\alpha(x)\,,\ {}^{\forall}\beta(x)\,.$$

$$\downarrow$$

$$\int d^3x \left\{ \frac{\tilde{\delta}\bar{F}[\sqrt{h}\alpha]}{\tilde{\delta}R_{kl}(x)} \cdot \nabla^j \nabla^i \left[\left(Q_{jl} h_{ik} - \frac{1}{2} Q_{kl} h_{ij} - \frac{1}{2} Q h_{ik} h_{jl} \right) \beta \right] - (\alpha \leftrightarrow \beta) \right\} \approx 0\,,$$

$$\downarrow$$

$$-\frac{\partial F}{\partial R_{kl}} \nabla^j \left(Q_{il} h_{jk} - \frac{1}{2} Q_{kl} h_{ij} - \frac{1}{2} Q h_{jk} h_{il} \right) + \nabla^j \left(\frac{\partial F}{\partial R_{kl}} \right) \cdot \left(Q_{jl} h_{ik} - \frac{1}{2} Q_{kl} h_{ij} - \frac{1}{2} Q h_{ik} h_{jl} \right) \approx 0\,.$$

$$\text{for} F = F\left(Q_{ij}, R_{ij}, h^{ij}, t \right)\,.$$

$\tilde{\delta}/\tilde{\delta}R_{ij}$: the functional derivative with respect to R_{ij} when $(R_{ij},\ h_{ij},\ \alpha,\ \beta)$ are considered as independent variables.

III. Theories with 2 d.o.f

Example I

Einstein's General Relativity

let us consider the simple ansatz,

$$F = f_1(\mathcal{Q}) + f_2(R)\,, \quad \text{where } \mathcal{Q} \equiv Q_{ij}Q^{ij} - Q^2\,,$$

Momentum constraints + Consistency condition

$$F = c_1(t)\mathcal{Q} + c_2(t)R - \Lambda(t)\,, \quad c_1(t) \neq 0\,,\ c_2(t) \neq 0\,.$$

It is Einstein gravity if $c_1,\ c_2$ and Λ are constant.

- If they are time dependent, then the theory is still equivalent to Einstein gravity if

$$c_1 c_2 = \text{constant}\,, \quad c_1\Lambda = \text{constant}$$

- If $c_1 c_2 \neq$ constant then $0 \approx \partial\mathcal{C}^E/\partial t = \partial\mathcal{C}/\partial t$ gives a tertiary constraint. Both of Hamiltonian constraint and the tertiary constraint are 2nd class.

Example II

A square root gravity

Let's take another ansatz,

$$F = f_1(\mathcal{Q})\, f_2(R) - \Lambda(t)\,, \quad f_1'(\mathcal{Q}) \neq 0\,,\; f_2'(R) \neq 0$$

Momentum constraints + Consistency condition

$$f_1 = \sqrt{A(t)\mathcal{Q} + B(t)}\,, \quad f_2 = \sqrt{C(t)R + D(t)}\,, \quad A(t) \neq 0\,,\; C(t) \neq 0$$

where A, B, C and D are integration "constants" that may depend on time.

- If all coefficients are constant, all constraints are 1st class $\rightarrow$ 2 d.o.f.
- If they are time dependent, then Hamiltonian constraint is still 1st class if

$$A = \frac{\text{constant} \cdot B}{\Lambda^2 - BD}\,, \qquad AC = \text{constant.}$$

- Otherwise, the time dependence generates a tertiary constraint (2nd class).

Example II

A square root gravity

it is illustrative to rewrite the action as

$$S = \int d^4x\sqrt{h}N\left[\xi M(t)^4\sqrt{\left(1+\frac{c_1(t)}{M(t)^2}\mathcal{K}\right)\left(1+\frac{c_2(t)}{M(t)^2}R\right)} - \Lambda(t)\right],$$

where $\mathcal{K} = K_{ij}K^{ij} - K^2$, $\xi = \pm 1$, $M = (BD)^{1/8}$, $c_1 = M^2 A/B$ and $c_2 = M^2 C/D$. In the weak field limit,

$$S \simeq \int d^4x\sqrt{h}N\left[\xi M^4 - \Lambda + \frac{\xi}{2}M^2(c_1\mathcal{K} + c_2 R) + ...\right].$$

The effective Planck mass and sound speed of GW

$$M_p^2 = \xi c_1 M^2, \quad c_g^2 = \frac{c_2}{c_1}, \quad \Lambda_{\rm eff} = \frac{\Lambda - \xi M^4}{\xi c_1 M^2},$$

We then demand that $\xi M^4 - \Lambda \simeq 0$ to cancel out the bare cosmological constant.

Example II

Matter coupling

- minimal coupling

$$S = \int d^4x\sqrt{h}N\left[\xi M(t)^4\sqrt{\left(1+\frac{c_1(t)}{M(t)^2}\mathcal{K}\right)\left(1+\frac{c_2(t)}{M(t)^2}R\right)} - \Lambda(t) - \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi)\right]$$

However, algebra is not closed, inconsistent (S. Liberati et. al, 1802.02537)

$$\{\mathcal{C}[\alpha], \mathcal{C}[\beta]\} \neq 0$$

solution: novel type of matter coupling, with all constraints still being first class. (arXiv:1811.02467, by CL.)

Example II

Matter coupling

- Novel matter coupling

$$\mathcal{C} \equiv -\xi_g \sqrt{h} B^{1/2} \left[CR + D - \frac{4}{Ah} \left(\pi^{ij}\pi_{ij} - \frac{1}{2}\pi^2 \right) \right]^{1/2} + \sqrt{h}\, \mathcal{U} \left(\frac{\pi_\phi^2}{h}, \nabla_i \phi \nabla^i \phi, \phi \right),$$

self-consistency condition

$$\{\mathcal{C}[\alpha], \mathcal{C}[\beta]\} \approx 0,$$

$$\downarrow$$

$$\mathcal{U} = \xi_m \left(\frac{B^2 C}{A} \right)^{1/4} \sqrt{\zeta \frac{\pi_\phi^2}{h} + \frac{1}{\zeta} \nabla_i \phi \nabla^i \phi + \Lambda},$$

$$\downarrow$$

$$\mathcal{L} = \sqrt{h} N \xi \left\{ M^4 \sqrt{\left(1 + \frac{c_1}{M^2} \mathcal{K} \right) \left(1 + \frac{c_2}{M^2} R \right)} - \sqrt{\left[\sqrt{\frac{c_2}{c_1}} M^4 - \frac{\zeta}{N^2} \left(\dot{\phi} - N^i \partial_i \phi \right)^2 \right] \left(\frac{1}{\zeta} \nabla_i \phi \nabla^i \phi + \Lambda \right)} \right\}.$$

Example II

The cosmology of square root gravity

We take FLRW ansatz,

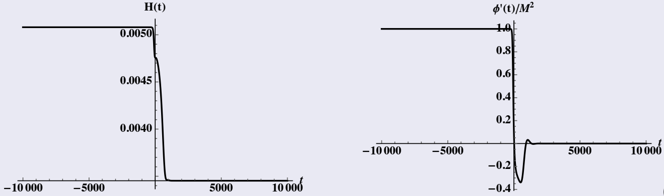
$$ds^2 = -N^2 dt^2 + a^2 d\mathbf{x}^2 .$$

we obtain the Friedmann equations of the form

$$3c_1 M^2 H^2 = \frac{\frac{1}{2}\dot{\phi}^2 + V(\phi)}{1 + V(\phi)/M^4}, \qquad 2c_1 M^2 \dot{H} = \frac{-\dot{\phi}^2 + \dot{\phi}^4/M^4}{1 + 2V(\phi)/M^4} .$$

and E.o.M for scalar field,

$$\left(1 + \frac{2V}{M^4}\right)\ddot{\phi} + 3H\left(1 + \frac{2V}{M^4}\right)\left(1 - \frac{\dot{\phi}^2}{M^4}\right)\dot{\phi} + \left(1 - \frac{\dot{\phi}^2}{M^4}\right)\frac{\partial V}{\partial \phi} = 0.$$



(on going, with D. Yeom)

Example II

Multi matter fields

- multi scalar fields

$$\mathcal{U} = \xi \left(\frac{B^2 C}{A}\right)^{1/4} \sqrt{\sum_I \left(\zeta_I \frac{\pi_I^2}{h} + \frac{1}{\zeta_I} \nabla_i \phi_I \nabla^i \phi_I\right) + \Lambda}.$$

$$\downarrow$$

$$\mathcal{L}_m = -\xi \sqrt{h} N \left[\sqrt{\frac{c_2}{c_1}} M^4 - \sum_I \frac{\zeta_I}{N^2} \left(\dot{\phi}_I - N^i \partial_i \phi_I\right)^2\right]^{1/2} \cdot \left(\sum_I \frac{1}{\zeta_I} \nabla_i \phi_I \nabla^i \phi_I + \Lambda\right)^{1/2}.$$

- Is it compatible with gauge symmetry in SM?

$$\mathcal{U} = \xi \sqrt{h} \left(\frac{B^2 C}{A}\right)^{1/4} \left[\frac{\pi_A^i \pi_A^j h_{ij}}{h} + \frac{1}{2} F_{ij} F^{ij} + \frac{\pi_1^2}{h} + \frac{\pi_2^2}{h} + 2eA_i \left(\phi_1 \nabla^i \phi_2 - \phi_2 \nabla^i \phi_1\right) + \nabla_i \phi_1 \nabla^i \phi_1 \right.$$

$$\left. + \nabla_i \phi_2 \nabla^i \phi_2 + \left(m^2 + e^2 A_i A^i\right)\left(\phi_1^2 + \phi_2^2\right) + \frac{\pi_\chi^2}{h} + \nabla_i \chi \nabla^i \chi + V(\chi) + \Lambda\right]^{1/2},$$

Scalar QED, all 8+2 constraints are 1st class

generalisation

- The explicit time dependences of parameters A, B, C, D and ζ satisfy

$$\frac{d\mathcal{C}}{dt} = \frac{\partial \mathcal{C}}{\partial t} + \{\mathcal{C}, H\} \approx \frac{\partial \mathcal{C}}{\partial t} \approx 0\,.$$

  This condition gives us

$$\frac{\partial \zeta}{\partial t} = 0, \qquad \frac{\partial (AC)}{\partial t} = 0, \qquad \frac{\partial \Lambda}{\partial t} = \frac{\partial \left(D\sqrt{A/C} \right)}{\partial t}.$$

- Generalise to the whole class of theories

$$\mathcal{C} \sim -\left[CR + D - \frac{4}{Ah}\left(\pi^{ij}\pi_{ij} - \frac{1}{2}\pi^2 \right) \right]^n + \left(\Lambda + \zeta \frac{\pi_\phi^2}{h} + \frac{1}{\zeta}\nabla^i\phi\nabla_i\phi \right)^n$$

  We recover the results of square root gravity if $n = 1/2$, and we recover GR if $n = 1$.

Example III

Exponential gravity

We take the following ansatz,

$$F = f_1(\mathcal{Q}) + \exp\left[c_1 R + f_2(\mathcal{Q})\right].$$

Momentum constraints + Consistency condition + Hamiltonian constraint

$$\begin{aligned} F &= 2\Lambda c_3 \mathcal{K} + \Lambda + c_2 \exp\left[c_1 R + c_3 \mathcal{K}\right] \\ &= \Lambda + c_2 + (2\Lambda c_3 + c_2 c_3)\,\mathcal{K} + c_1 c_2 R + \frac{1}{2} c_2 (c_1 R + c_3 \mathcal{K})^2 + \cdots . \end{aligned}$$

We then demand $\Lambda + c_2 \simeq 0$ to cancel out the bare cosmological constant.

- If all coefficients are constant, then Hamiltonian constraint is 1st class;
- If they are time dependent, the HC is still 1st class if

$$c_1 c_3 \Lambda^2 = \text{constant}, \quad \frac{\ln(-\Lambda/c_2)}{c_1} = \text{constant}.$$

- Otherwise, they generate a tertiary constraint (2nd class).

Example IV

Lapse independent term

The theories that we just found in previous subsections can be extended to

$$S = \int dtd^3x\sqrt{h}\left[NF + G\left(R_{ij}, \nabla_i, h^{ij}, t\right)\right],$$

where F is the theories satisfies our consistency condition.

- See Ref. arXiv:1711.10472 by CL, J. Quintin and R. H. Brandenberger, resolving two of main problems of a matter bounce senario.

Conclusion

remarks:

- Searching for the theory in which all constraints are 1st class is very interesting problem!
- self-consistency condition for the theory linear in lapse.
- examples:
    - Einstein's gravity;
    - a square root gravity;
    - an exponential gravity;
    - extension to the one with lapse independent term;
    - more solutions?

Thank you!

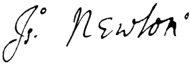
Appendix

A very basic question:

Law of physics $\longrightarrow$ EoM with no more than two time derivatives

$$e.g. \qquad F = m\ddot{x}$$

WHY? Have you ever thought about it?



Ostrogradsky theorem

Consider an action

$$S = \int dt \left[\ddot{q}^2 - V(q)\right]$$

The canonical variables

$$Q_1 = q, \qquad Q_2 = \dot{q},$$

and their conjugate momenta

$$P_1 = \frac{\partial L}{\partial \dot{q}} - \frac{d}{dt}\frac{\partial L}{\partial \ddot{q}}, \qquad P_2 = \frac{\partial L}{\partial \ddot{q}}.$$

The Hamiltonian

$$H = P_1 Q_2 + \frac{1}{2}P_1^2 + V(Q_1)$$

unbounded from below.

Mikhail Vasilyevich Ostrogradsky
Sept. 24, 1801 - Jan. 1, 1862
Ukrainian mathematician in the Russian Empire.