Firewall controversy and
Euclidean path integral approach

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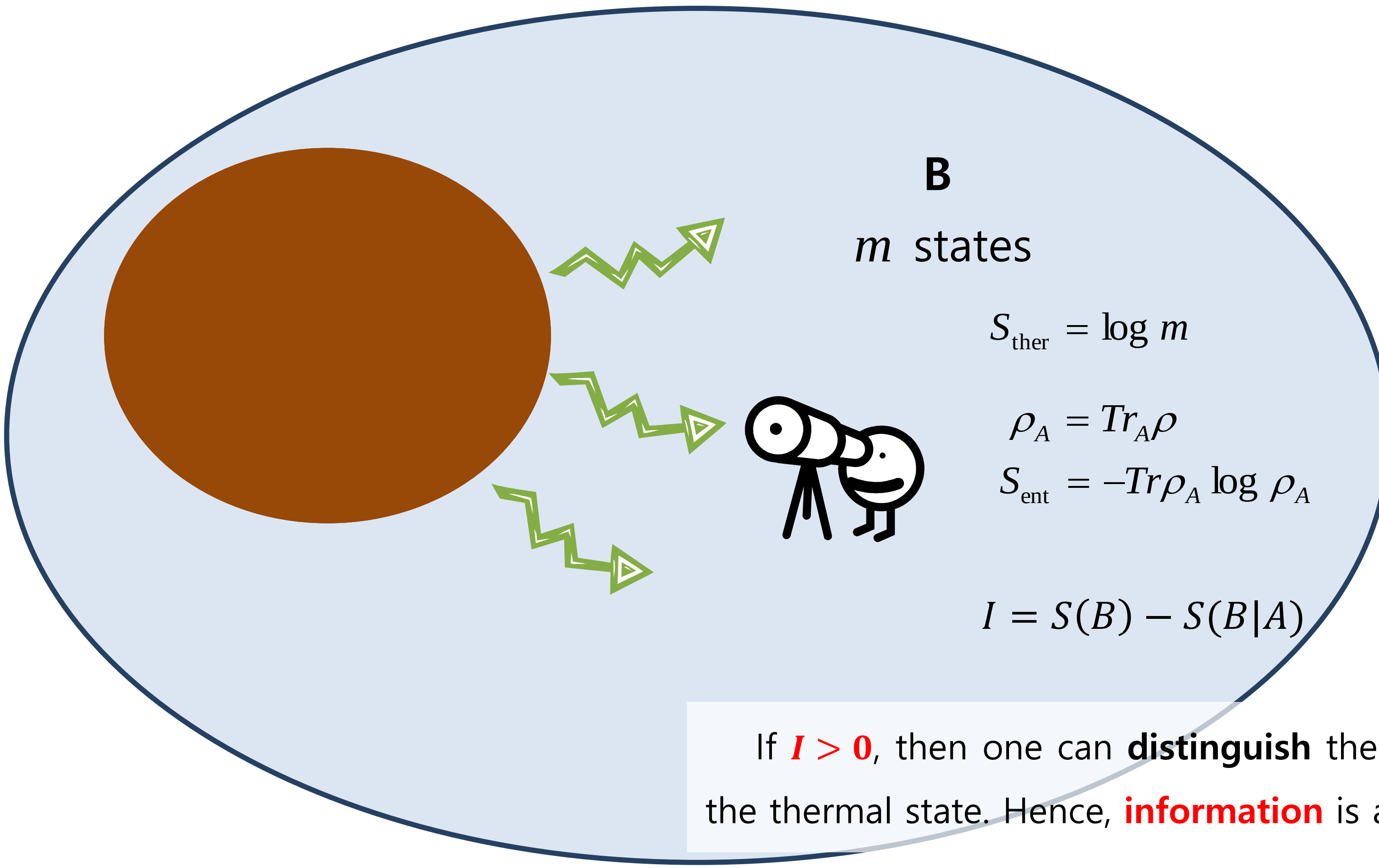
How can we define information of a black hole?

Preliminaries

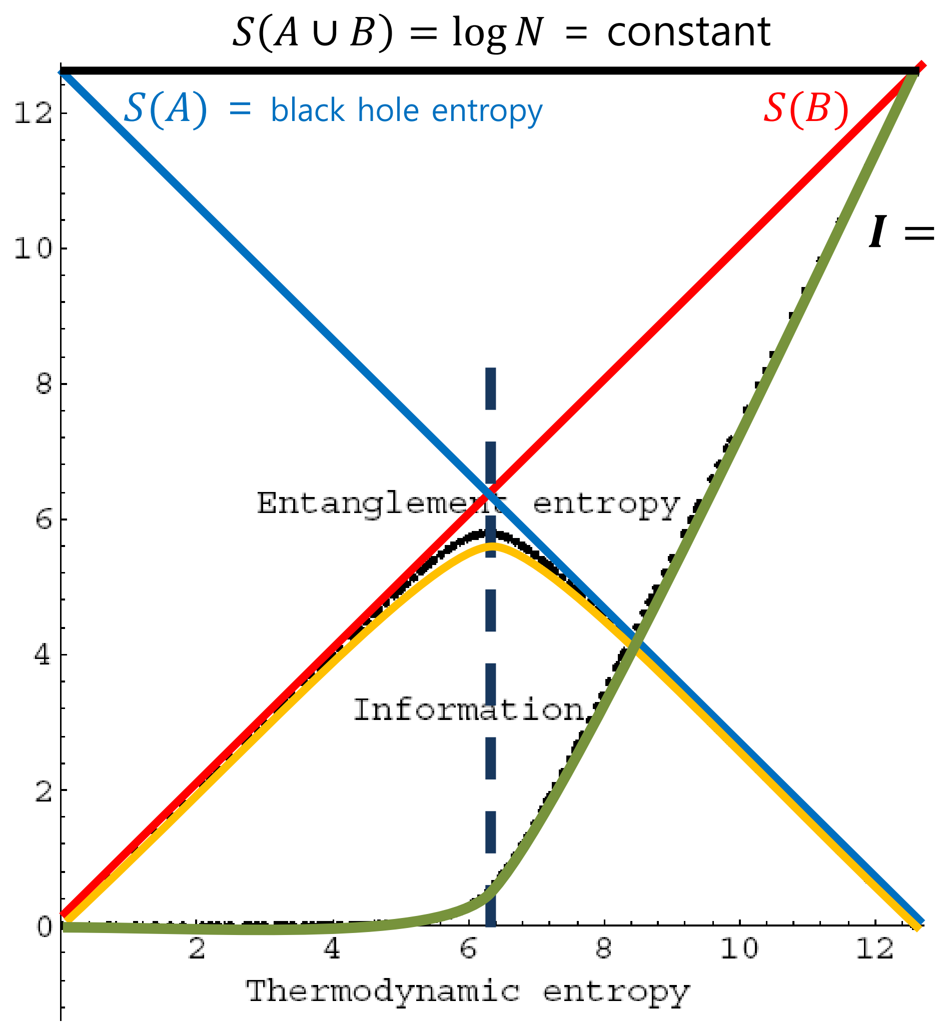
Preliminaries (Page, 1993)

$n \times m$ states



If $I > 0$, then one can **distinguish** the state from the thermal state. Hence, **information** is attached.

Page's argument (Page, 1993)



$$\boldsymbol{I = S(B) - S(B|A)}$$

$$S(B) = \log m$$

$$S(B|A) = \sum_{k=n+1}^{mn} \frac{1}{k} - \frac{m-1}{2n}$$

(For a pure and random system, conjectured by Page, 1993; proven by Sen, 1996.)

If $\boldsymbol{S(A)} \propto$ **Area**, then information will come out when the initial area decreases to its half value. This time is called the **information retention time**.

(Reformulated) information loss problem (Hawking, 1976)

Let us assume that **general relativity** and **quantum mechanics** are all true. Also, we assume that the **black hole area is proportional to the Boltzmann entropy**.

- According to the Page argument, after the information retention time, information should come out. Around the information retention time, the black hole is still large enough. Hence, Hawking radiation should carry information.
- However, according to Hawking's calculation, the radiation from the black hole is totally thermal.

How can we reconcile this tension?

Speculations toward black hole complementarity

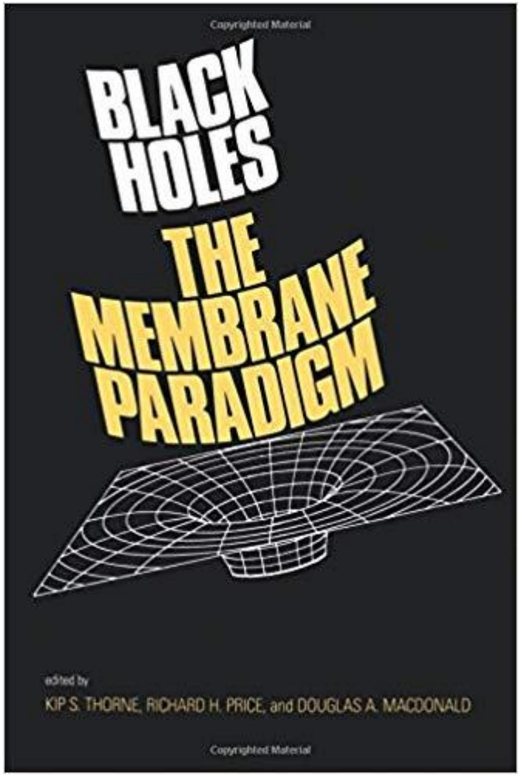
Let's believe that **everybody was right**.

Black hole complementarity ('t Hooft, Susskind, …)

The principle of black hole complementarity says that **no observer sees violation of natural laws**.

1. For **in-falling observer**, general relativity is a good description.
2. For **asymptotic observer**, unitary semi-classical (local) quantum field theory is a good description.
3. Both observers are **complementary**: for any observer, unitarity of quantum mechanics and its consequences (e.g., no-cloning theorem) should be satisfied.

Membrane paradigm/D-brane picture (Thorne, Callan, Maldacena, …)

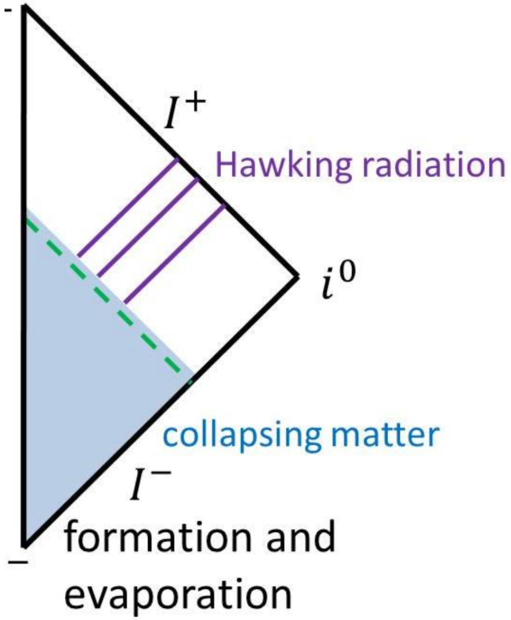


One strong motivation of black hole complementarity was the **membrane paradigm**.

Since it takes the infinite coordinate time to cross the event horizon, one may interpret that the outside observer never sees that the infalling matter crosses the horizon.

The infalling matter are attached by the **stretched horizon** outside the event horizon. This picture is consistent with the **D-brane picture**, where the interior of the black hole is identified to the D-branes which wrap the horizon.

Membrane paradigm/D-brane picture (Thorne, Callan, Maldacena, …)



Then can there be a black hole formation?

There have been some suggestions that there is no formation of a true apparent horizon nor a singularity, e.g., Kawai-Matsuo-Yokokura model or Baccetti-Mann-Terno model.

However, after the detailed calculations, one can show that **an apparent horizon as well as a singularity do form** **(Chen, Unruh, Wu and DY, 2018)**.

So, from the beginning, the physical ground of black hole complementarity was too poor.

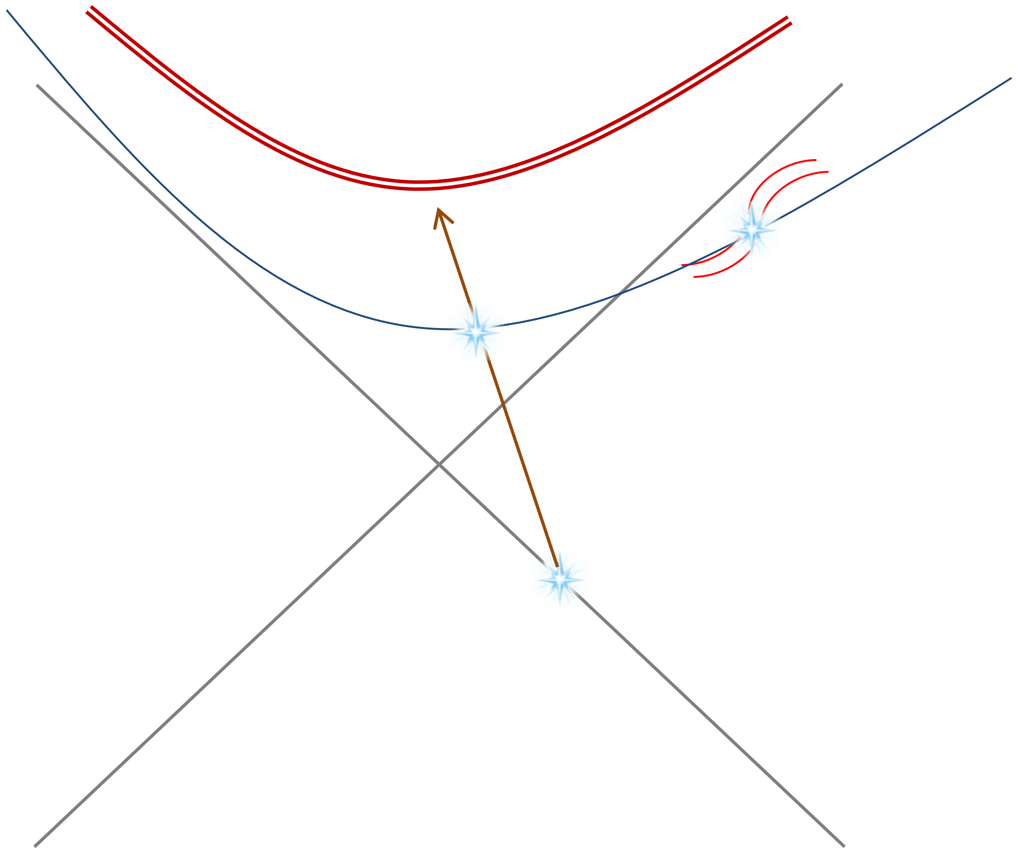
Black hole complementarity ('t Hooft, Susskind, …)

The principle of black hole complementarity says that **no observer sees violation of natural laws**.

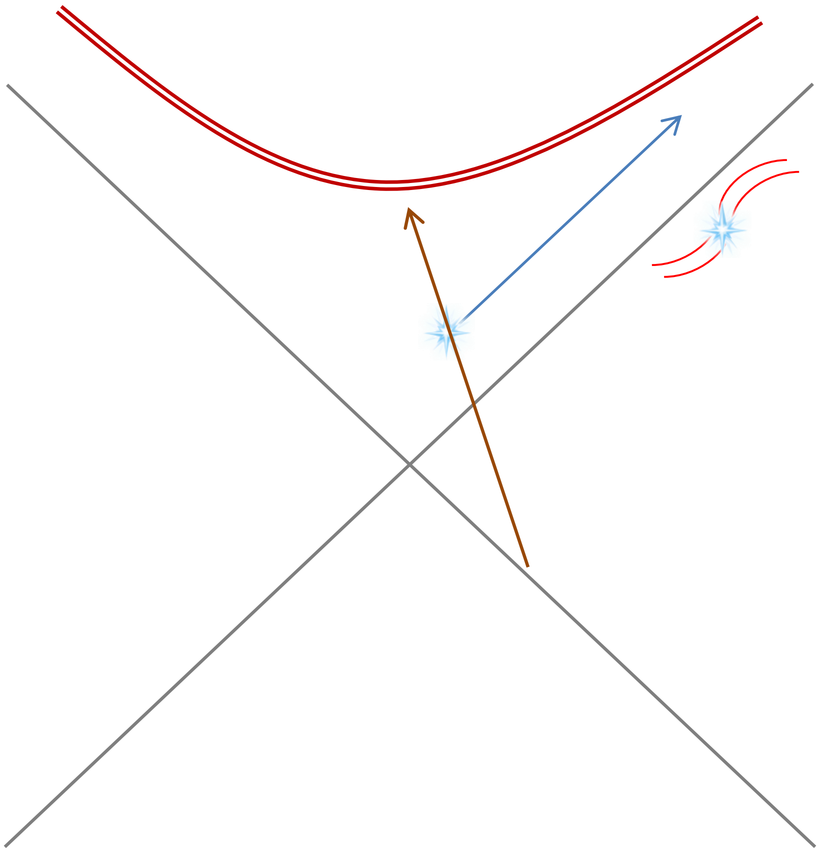
1. For **in-falling observer**, general relativity is a good description.
2. For **asymptotic observer**, unitary semi-classical (local) quantum field theory is a good description.
3. Both observers are **complementary**: for any observer, unitarity of quantum mechanics and its consequences (e.g., no-cloning theorem) should be satisfied.

Are they consistent?

Duplication experiment (Susskind and Thorlacius, 1994)

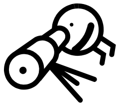


Duplication experiment (Susskind and Thorlacius, 1994)



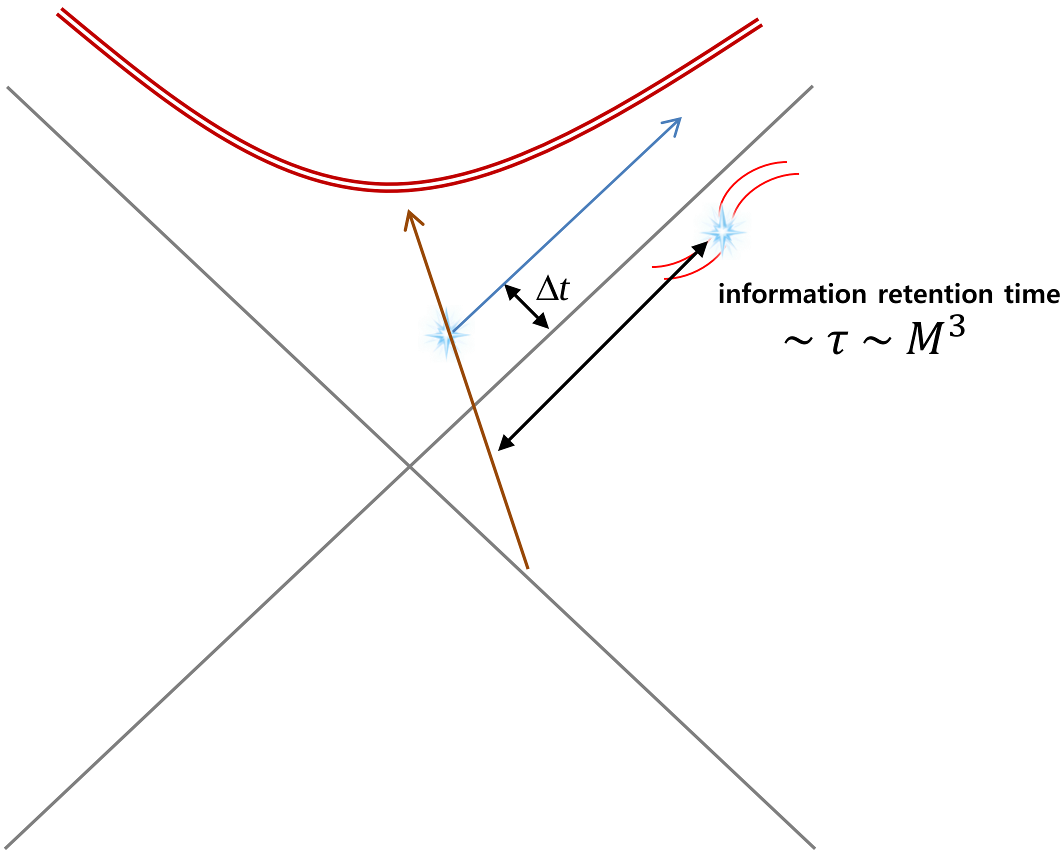
Information duplication?

$$U|\Psi>=|\Psi>|\Psi>$$



Duplication experiment (Susskind and Thorlacius, 1994)

Is a duplication experiment possible?



$$\Delta t \sim e^{-\frac{\tau}{r_0}} \sim e^{-M^2}$$

$$\Delta E \sim e^{M^2} > M$$

Safety condition of duplication

Scrambling time

Black holes as mirrors: quantum information in random subsystems

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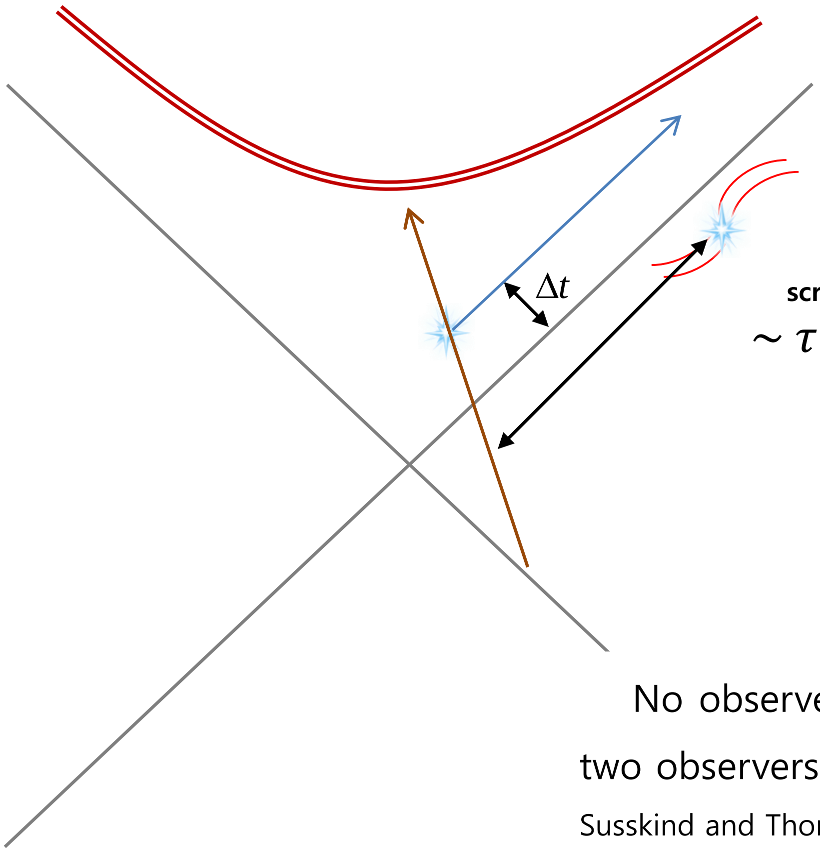
Abstract

We study information retrieval from evaporating black holes, assuming that the internal dynamics of a black hole is unitary and rapidly mixing, and assuming that the retriever has unlimited control over the emitted Hawking radiation. If the evaporation of the black hole has already proceeded past the "half-way" point, where half of the initial entropy has been radiated away, then additional quantum information deposited in the black hole is revealed in the Hawking radiation very rapidly. Information deposited prior to the half-way point remains concealed until the half-way point, and then emerges quickly. These conclusions hold because typical local quantum circuits are efficient encoders for quantum error-correcting codes that nearly achieve the capacity of the quantum erasure channel. Our estimate of a black hole's information retention time, based on speculative dynamical assumptions, is just barely compatible with the black hole complementarity hypothesis.

Duplication experiment



scrambling time

$$\sim \tau \sim M \log M$$

$$\Delta t \sim e^{-\frac{\tau}{r_0}} \sim 1/M$$

$$\Delta E \sim M \geq M$$

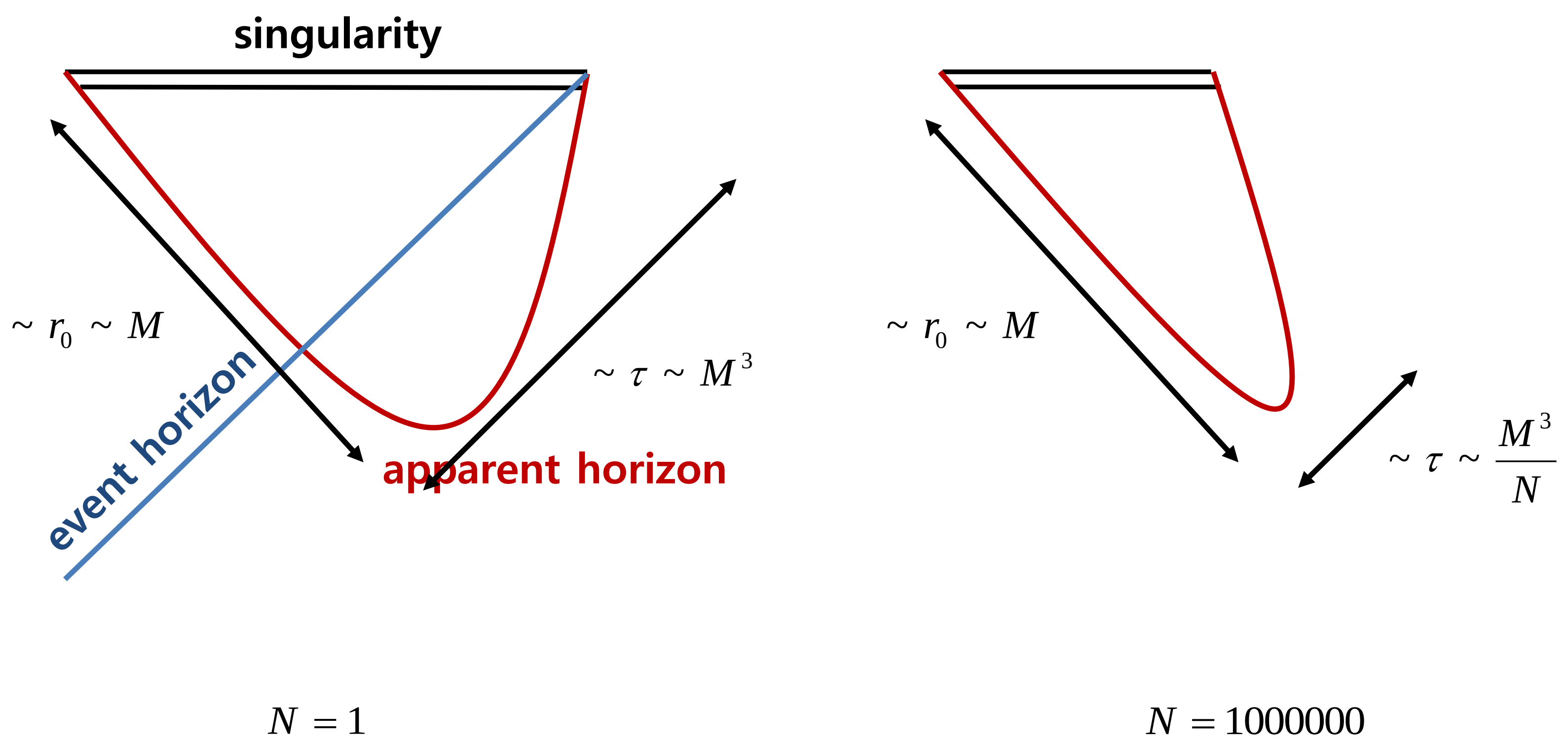
Marginally safe

No observer can see the duplication of information. Therefore, two observers are **complementary**. (Susskind, Thorlacius and Uglum, 1993; Susskind and Thorlacius, 1994)

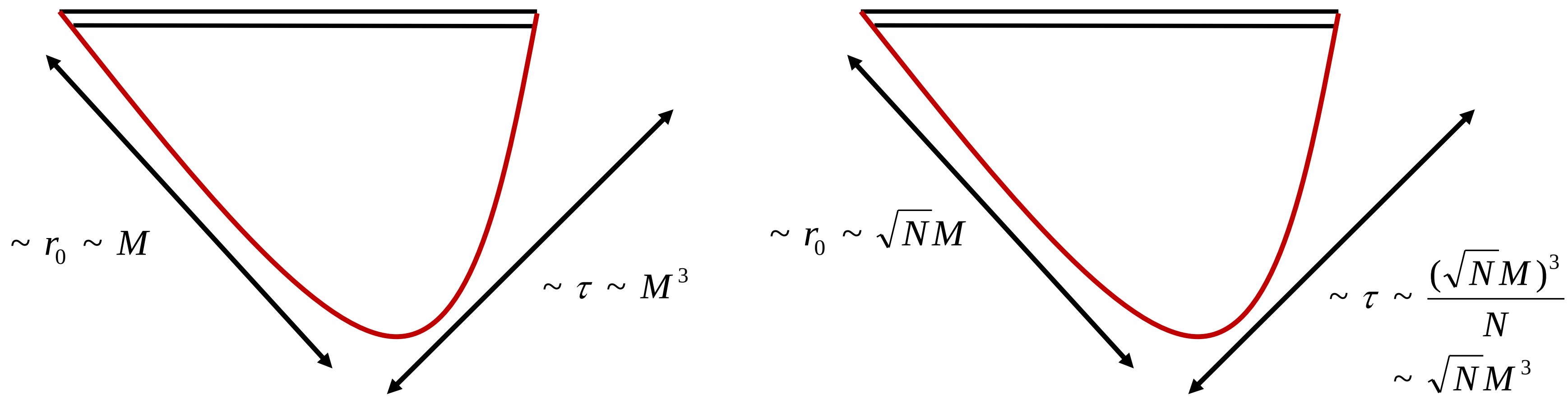
Two arguments against black hole complementarity

Large N rescaling and AMPS arguments

Large N rescaling (DY and Zoe, 2011)



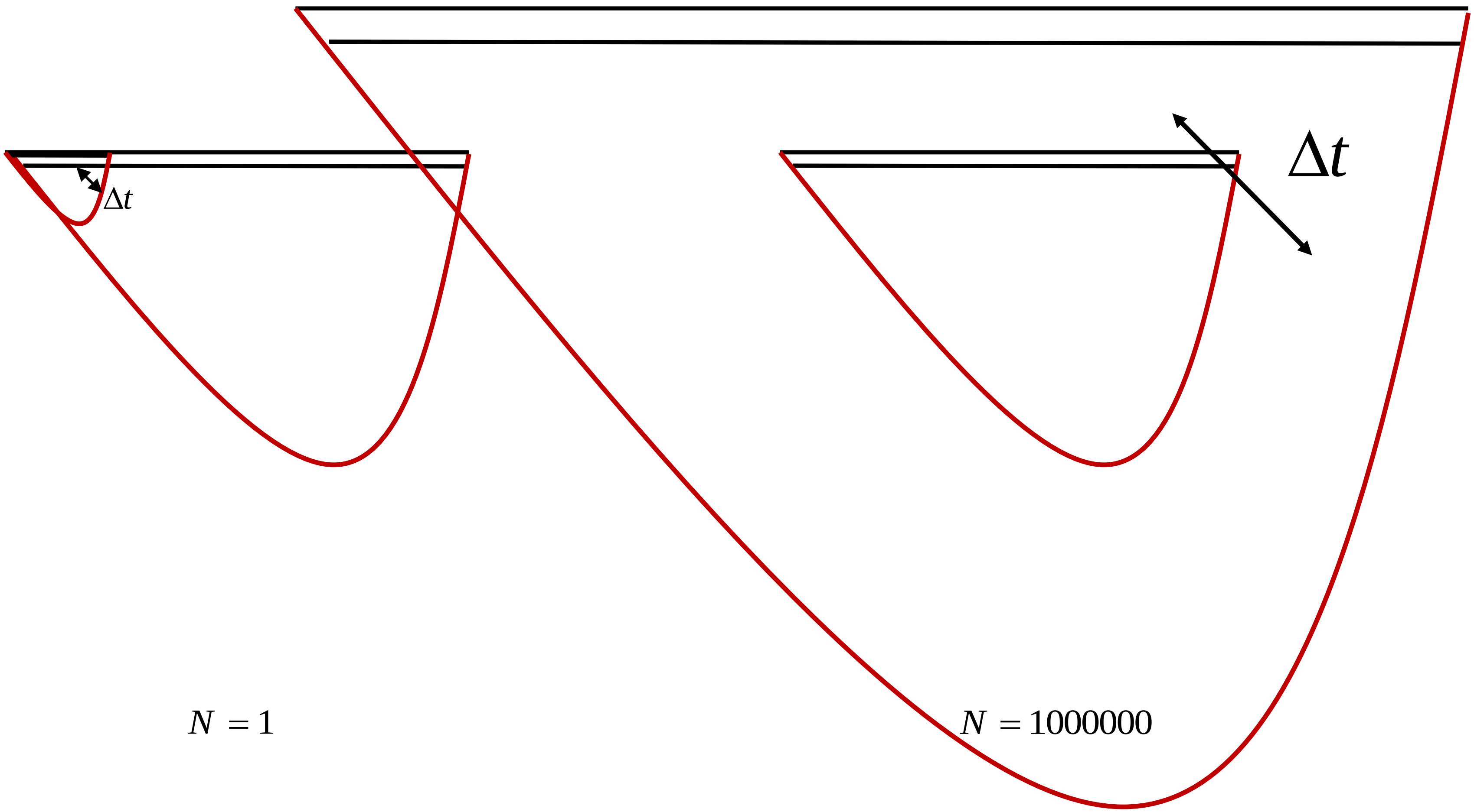
Large N rescaling (DY and Zoe, 2011)



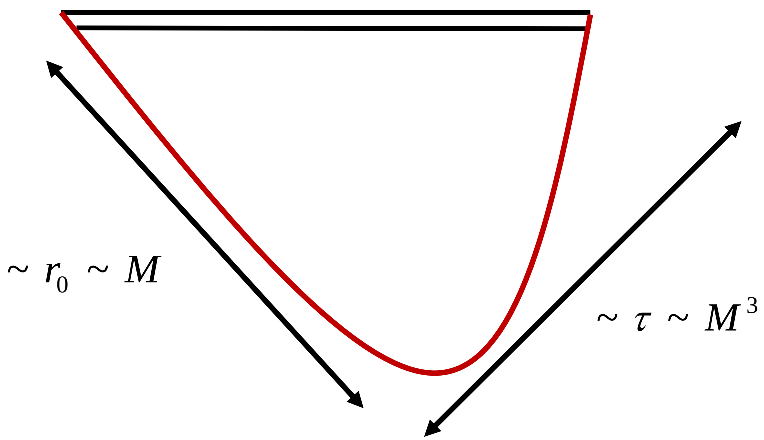
$N = 1$

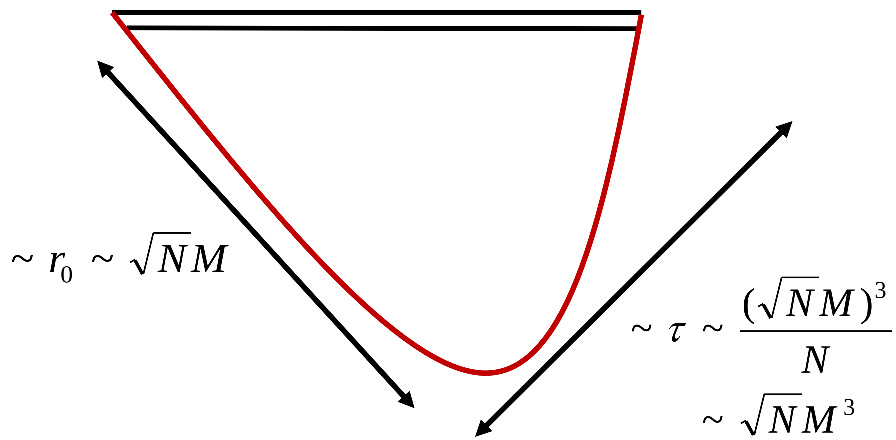
$N = 1000000$

Large N rescaling (DY and Zoe, 2011)



Large N rescaling (DY and Zoe, 2011)





$$\Delta t \sim \exp - \frac{\tau}{r_0} \sim \exp - \frac{\tau}{M}$$

$$N = 1$$

$$\Delta t \sim \sqrt{N} \exp - \frac{\tau}{M}$$

$$N = 1000000$$

Application to black hole complementarity

For the information retention time

$$\tau \sim M^3 \qquad \Delta t \sim \sqrt{N} \exp - \frac{\tau}{M}$$

$$\Delta E \sim \frac{1}{\sqrt{N}} \exp \frac{\tau}{M}$$

$$\Delta E > \sqrt{N} M \quad \rightarrow \quad \exp M^2 > N M$$

For the scrambling time

$$\tau \sim M \log M \qquad \Delta t \sim \sqrt{N} \exp \left(- \frac{M \log \sqrt{N} M}{M} \right) \sim \sqrt{N} \exp \left(- \log \sqrt{N} M \right)$$

$$\Delta E \sim \frac{1}{\sqrt{N}} \exp \log \sqrt{N} M$$

$$\Delta E > \sqrt{N} M \quad \rightarrow \quad M > \sqrt{N} M$$

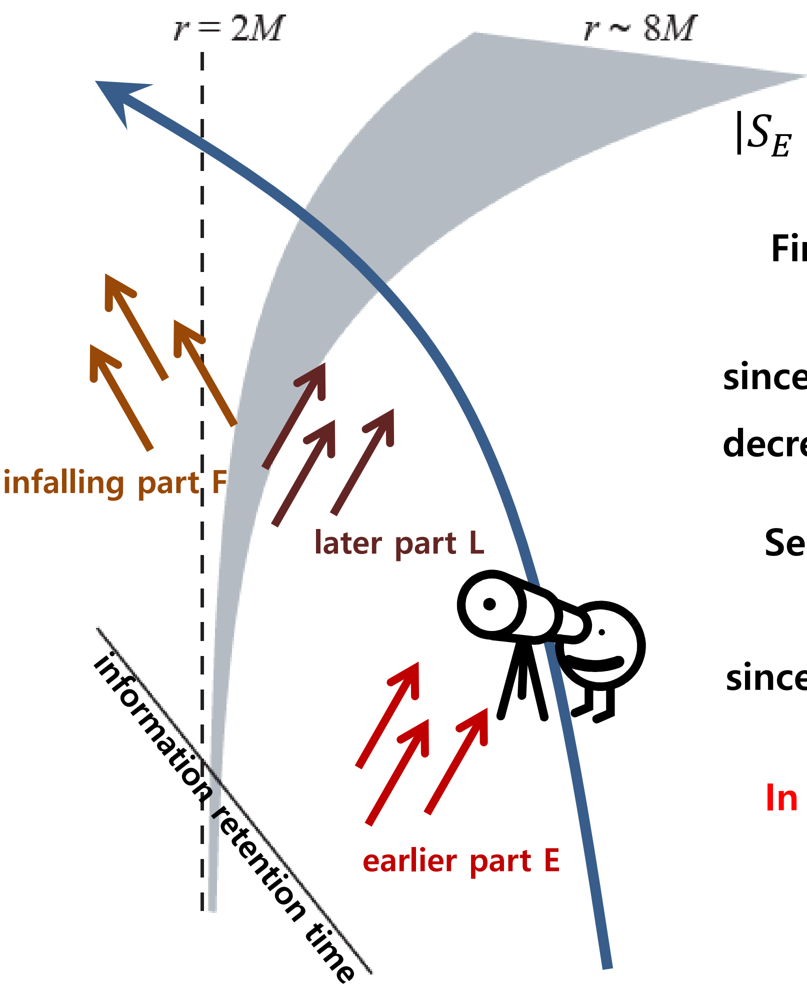
So what?

Duplication of information is observable, if there are many scalar fields within the semi-classical limit.

Regarding the information retention time, the required scalar field is exponentially large. However, regarding the scrambling time, the required scalar field can be sufficiently reduced.

Hence, black hole complementarity can be violated by duplication experiments.

AMPS arguments (Almheiri, Marolf, Polchinski and Sully, 2012)



$S_{EL} + S_{LF} \geq S_L + S_{ELF}$ **strong subadditivity**

$|S_E - S_{LF}| \leq S_{ELF} \leq S_E + S_{LF}$ **subadditivity**

First,

$$S_{EL} < S_E$$

since after the half-way point, outside entropy S_{EL} should decrease from its maximum S_E.

Second,

$$S_{LF} = 0$$

since nothing happens for free-falls.

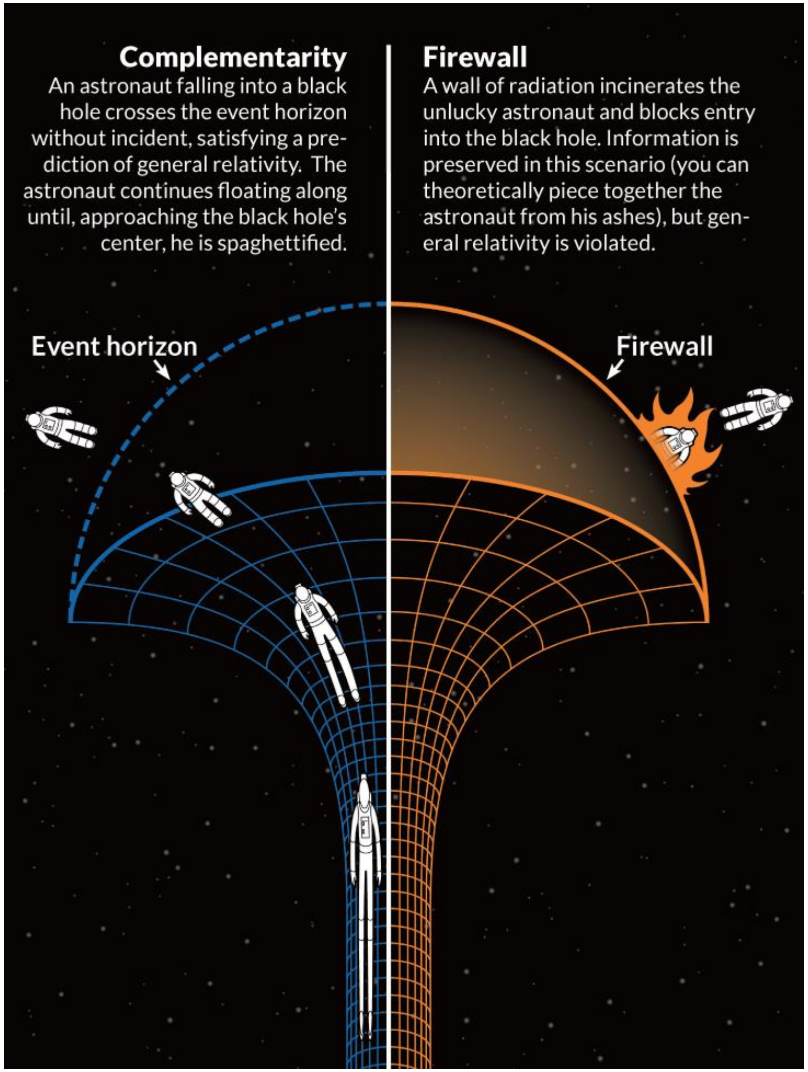
In the end, $S_L < 0$?

Firewall controversy

Which natural law should we throw out?

1. General relativity
2. Quantum mechanics
3. Bekenstein-Hawking formula

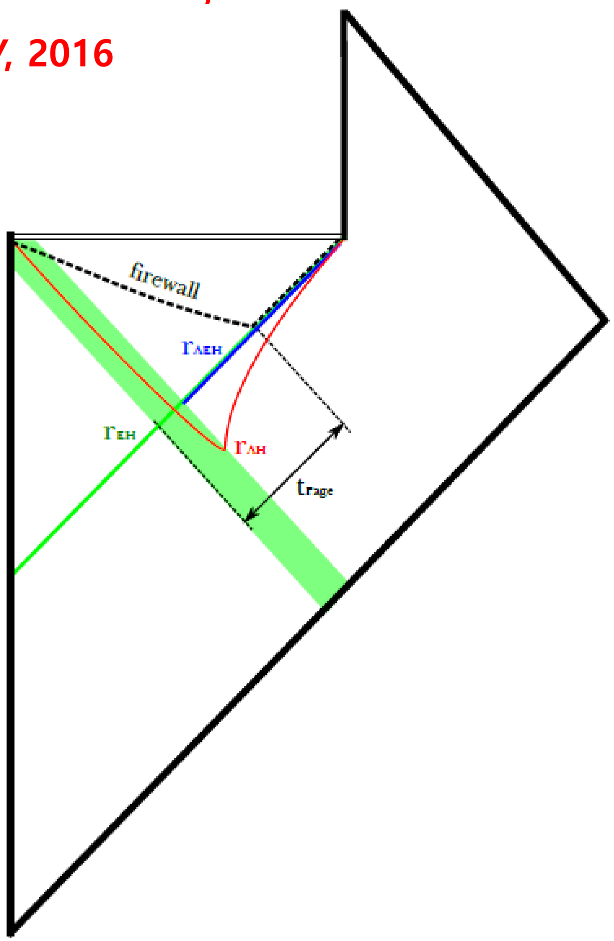
AMPS gave up GR: firewall

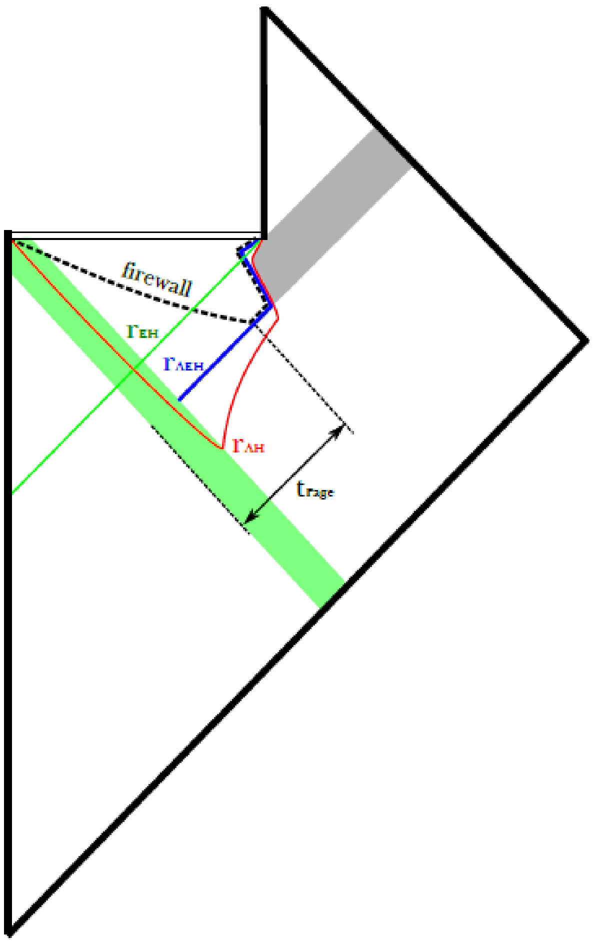


If there is a firewall, it can be naked!

Hwang, Lee and DY, 2013; Kim, Lee and DY, 2013

Chen, Ong, Page, Sasaki and DY, 2016





Can we construct a firewall?

Embed to string-inspired model?

Kang and DY, 2019 (appear tomorrow)

$$S = \int dx^4 \sqrt{-g} e^{-2\phi} \left(\mathcal{R} + 4\partial_\mu \phi \partial^\mu \phi - \frac{1}{12} H_{\mu\nu\rho} H^{\mu\nu\rho} \right)$$

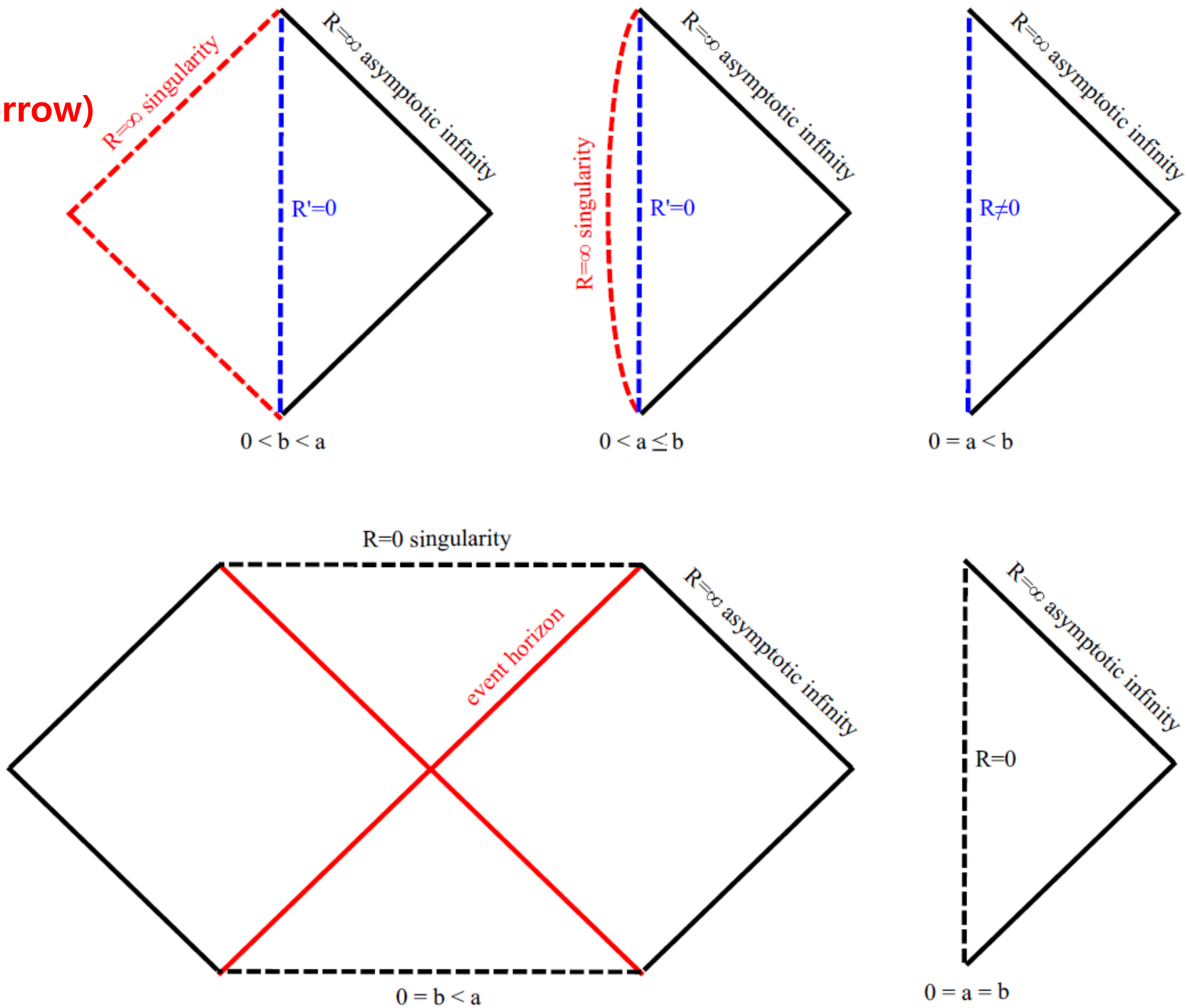
$$ds^2 = e^{2\phi(r)} \left(-A(r) dt^2 + \frac{1}{A(r)} dr^2 + \frac{C(r)}{A(r)} d\Omega^2 \right)$$

$$\begin{aligned}
e^{2\phi} &= \gamma_+ \left(\frac{r-\alpha}{r+\beta} \right)^{\frac{b}{\sqrt{a^2+b^2}}} + \gamma_- \left(\frac{r-\alpha}{r+\beta} \right)^{-\frac{b}{\sqrt{a^2+b^2}}}, \\
A(r) &= \left(\frac{r-\alpha}{r+\beta} \right)^{\frac{a}{\sqrt{a^2+b^2}}}, \\
C(r) &= (r-\alpha)(r+\beta), \\
\alpha &\equiv \frac{a}{a+b} \sqrt{a^2+b^2}, \\
\beta &\equiv \frac{b}{a+b} \sqrt{a^2+b^2}, \\
\gamma_\pm &\equiv \frac{1}{2} \left(1 \pm \sqrt{1 - \frac{h^2}{b^2}} \right).
\end{aligned}$$

Can we construct a firewall?

Embed to string-inspired model?

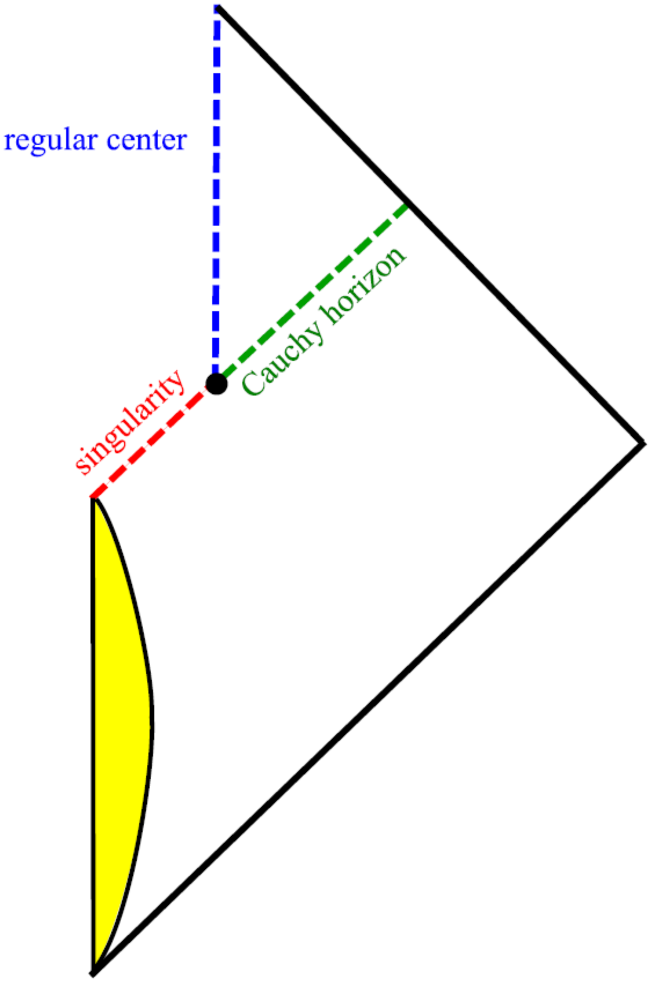
Kang and DY, 2019 (appear tomorrow)



More on naked firewall

Weak-coupling limit:

Manifestly, there is no interior.



More on naked firewall

Strong-coupling limit:

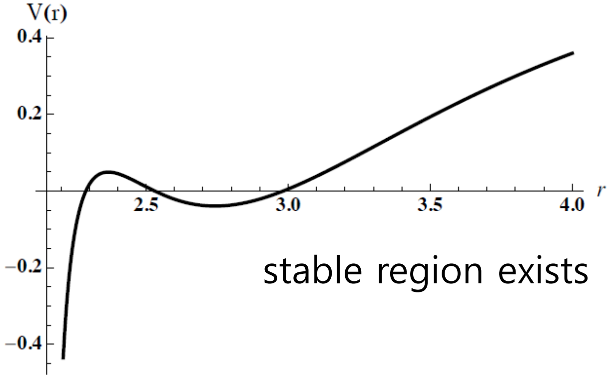
Eventually, it rapidly disappears.

Junction equation for star-like interior

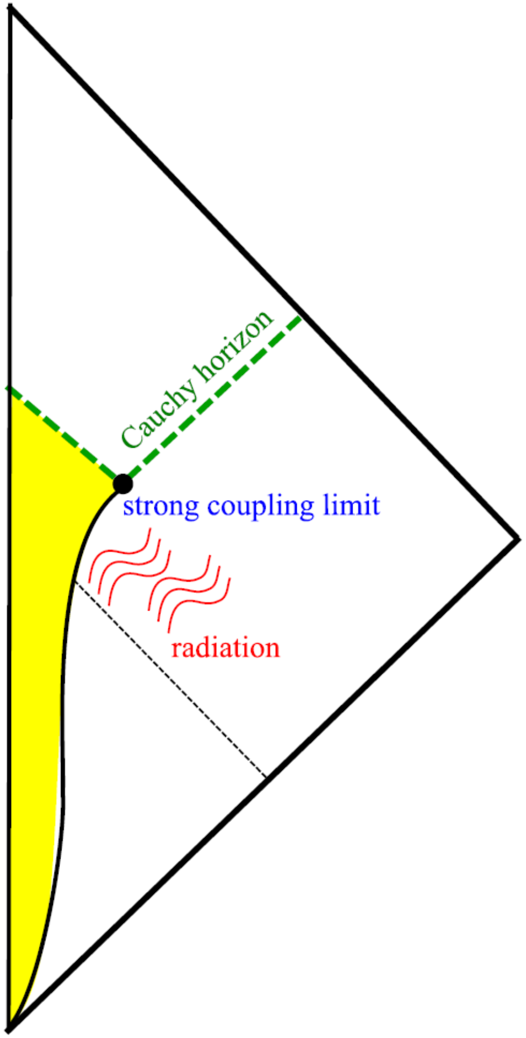
$$\epsilon_{-}\sqrt{f_{-}(R)+\dot{R}^{2}}-\epsilon_{+}\sqrt{f_{+}(R)+\dot{R}^{2}}=4\pi\sigma(R)R$$

Condition for w = -1

$$2\frac{d\phi}{dr}-\frac{d^{2}R/dr^{2}}{dR/dr}=4\pi R\frac{d\sigma}{dr}$$

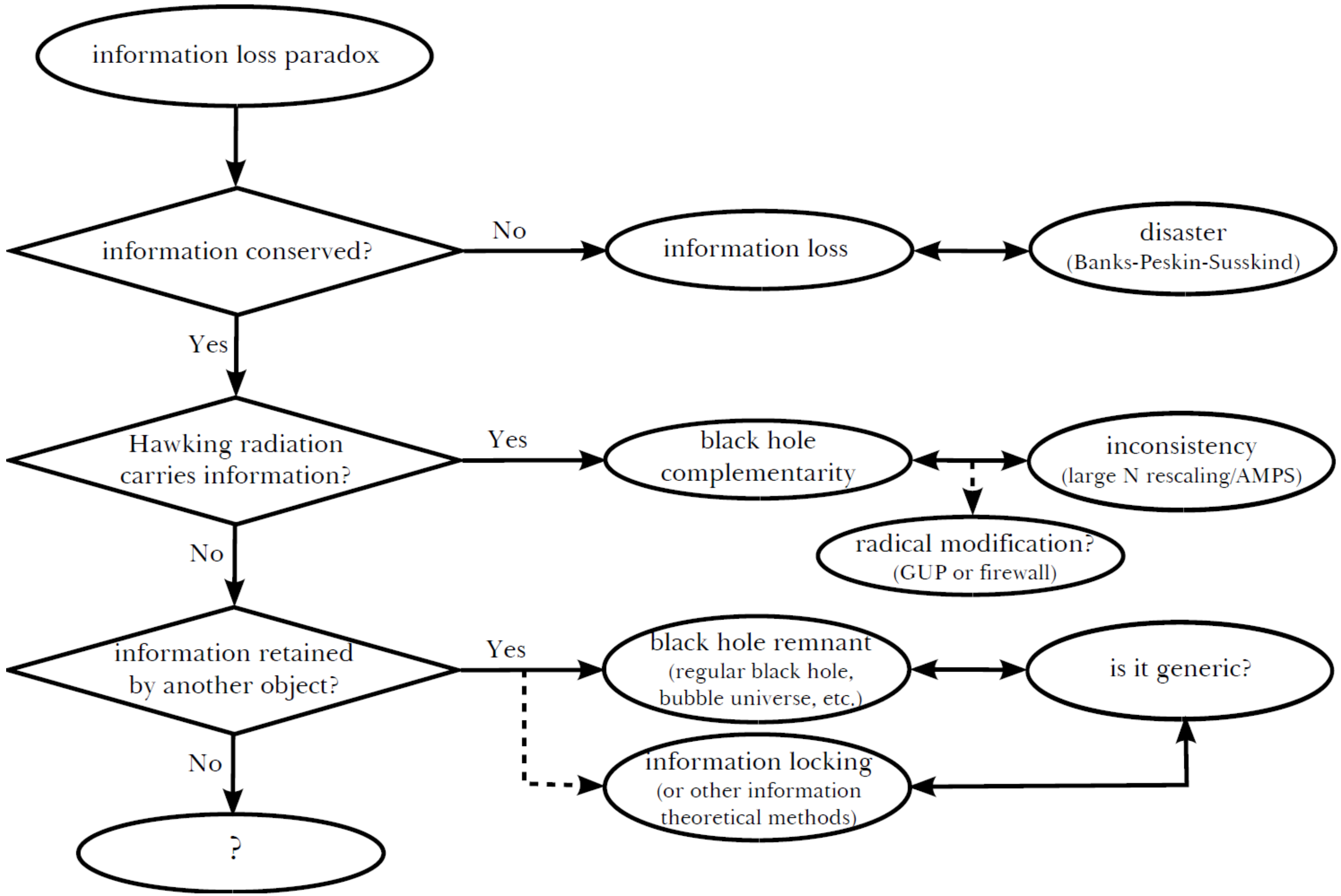


stable region exists

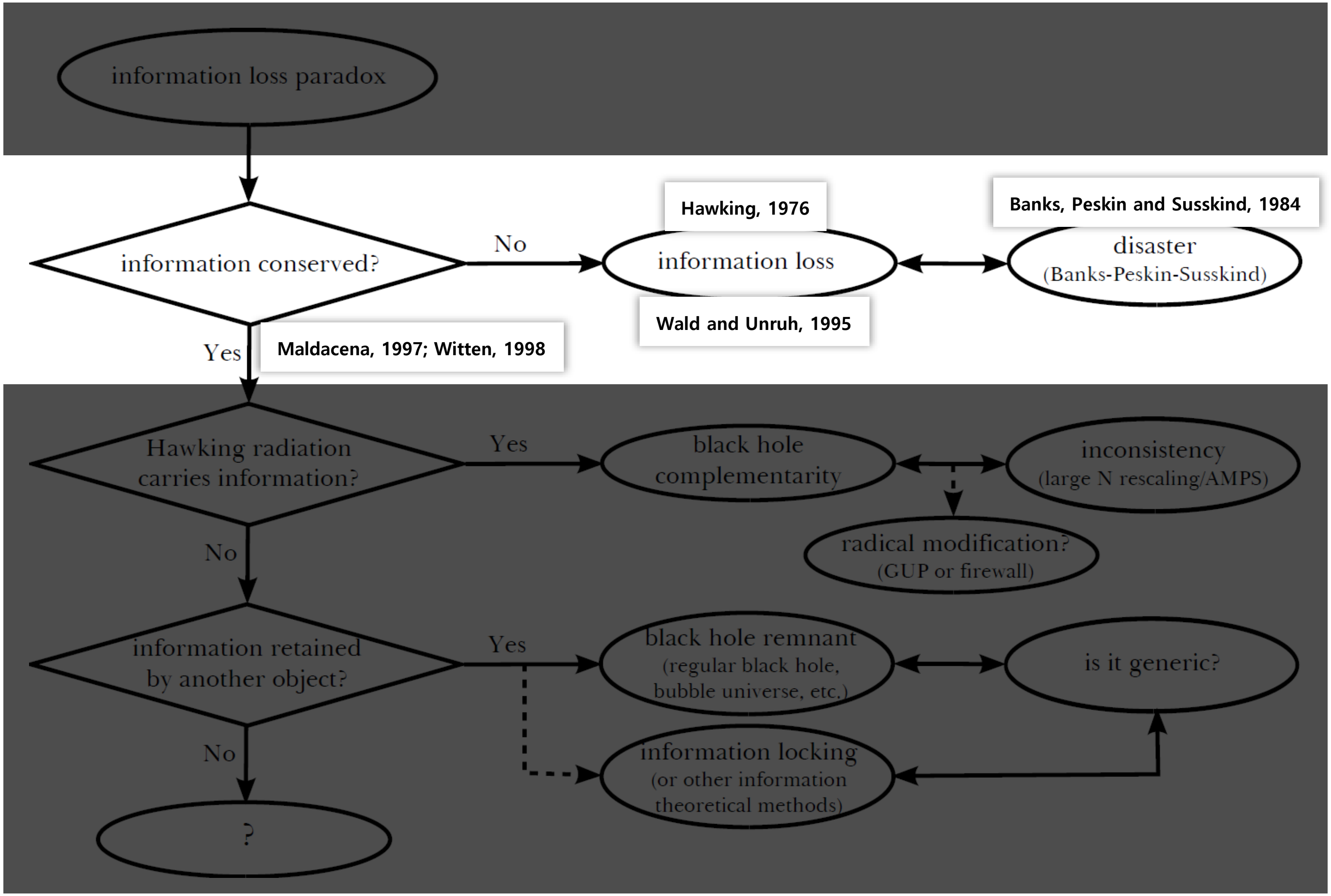


A bird's-eye view for candidate resolutions

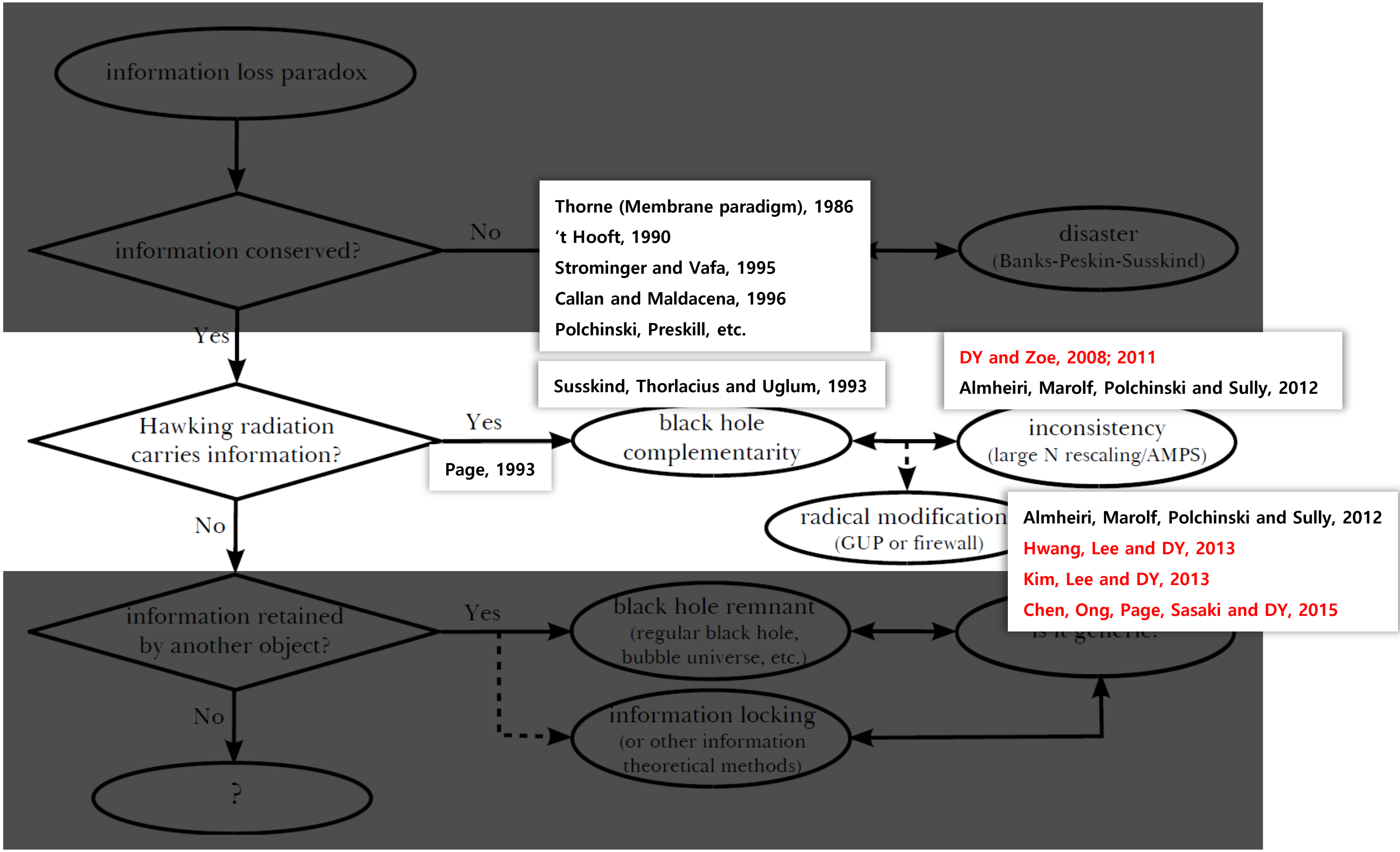
A bird's-eye view (from Chen, Ong and DY, 2014)



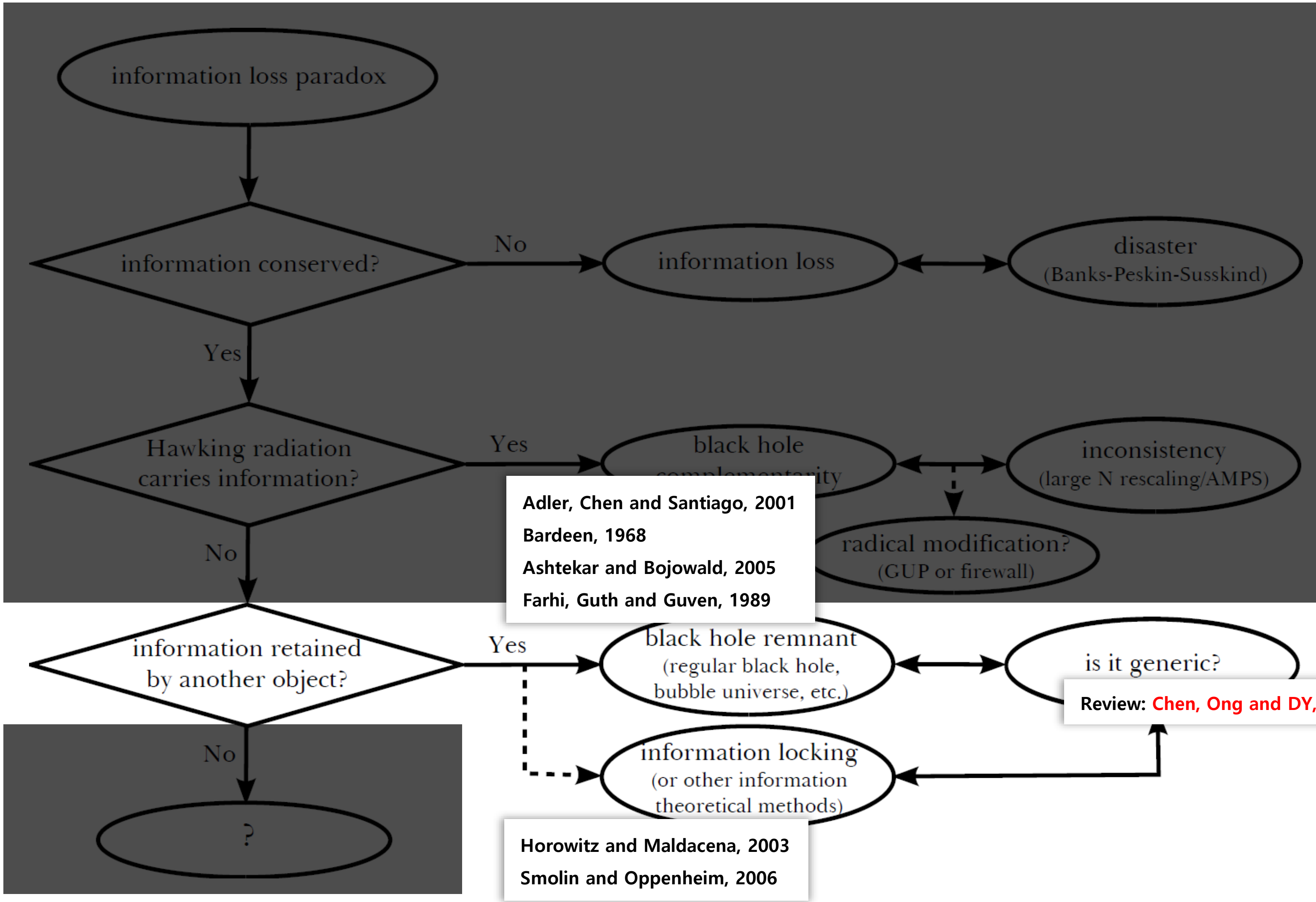
A bird's-eye view: Information loss vs. unitarity



A bird's-eye view: Black hole complementarity controversy



A bird's-eye view: Remnant picture

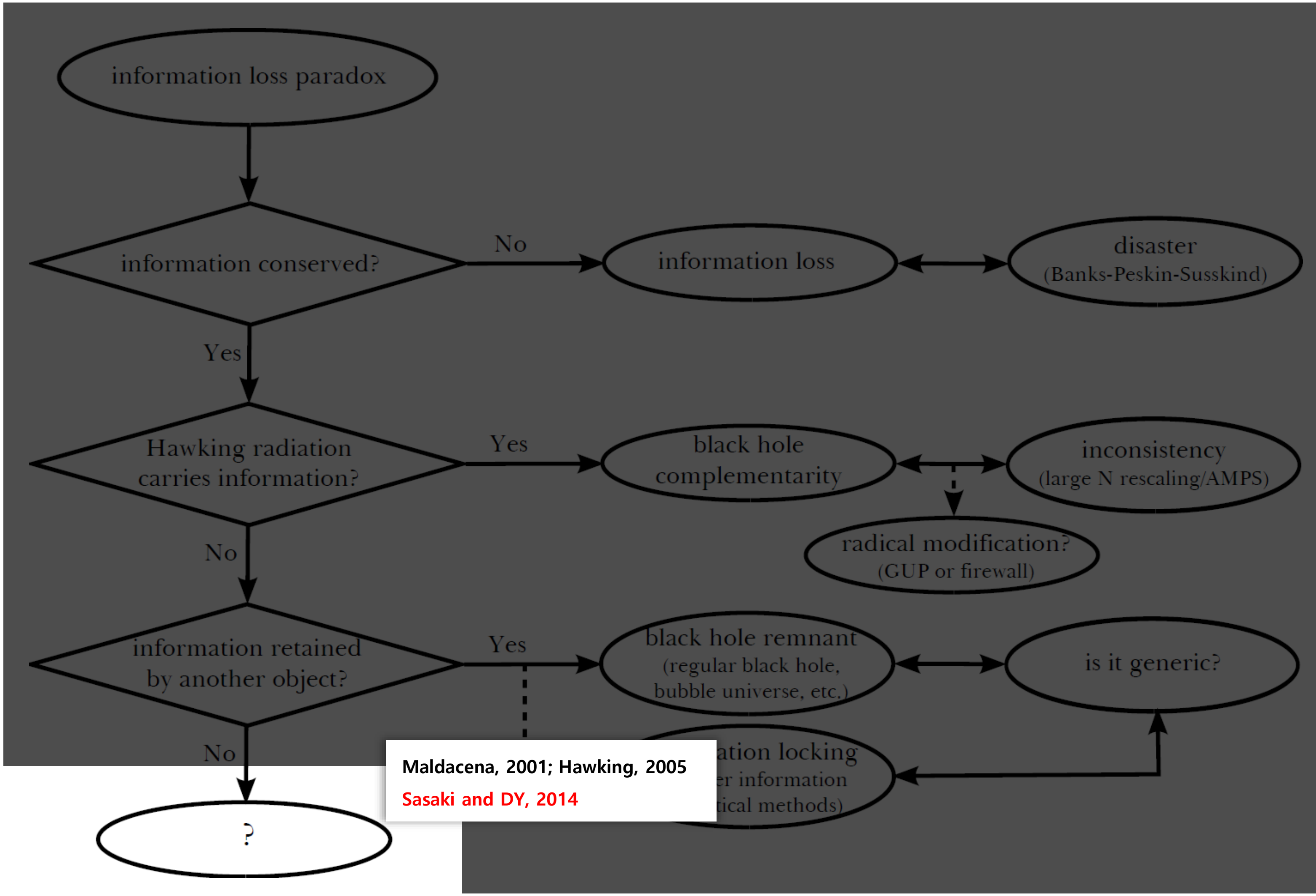


Adler, Chen and Santiago, 2001
Bardeen, 1968
Ashtekar and Bojowald, 2005
Farhi, Guth and Guven, 1989

Review: Chen, Ong and DY, 2014

Horowitz and Maldacena, 2003
Smolin and Oppenheim, 2006

A bird’s-eye view



Maldacena, 2001; Hawking, 2005

Sasaki and DY, 2014

Euclidean path integral approach and the resolution of the information loss problem

Maldacena, 2001; Hawking, 2005

Sasaki and DY, 2014

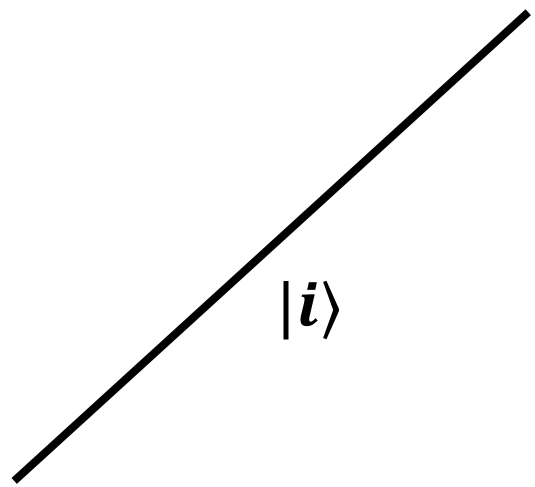
Lee, Lee and DY, 2015

Chen, Domenech, Sasaki and DY, 2015; 2017

Chen, Hu and DY, 2015

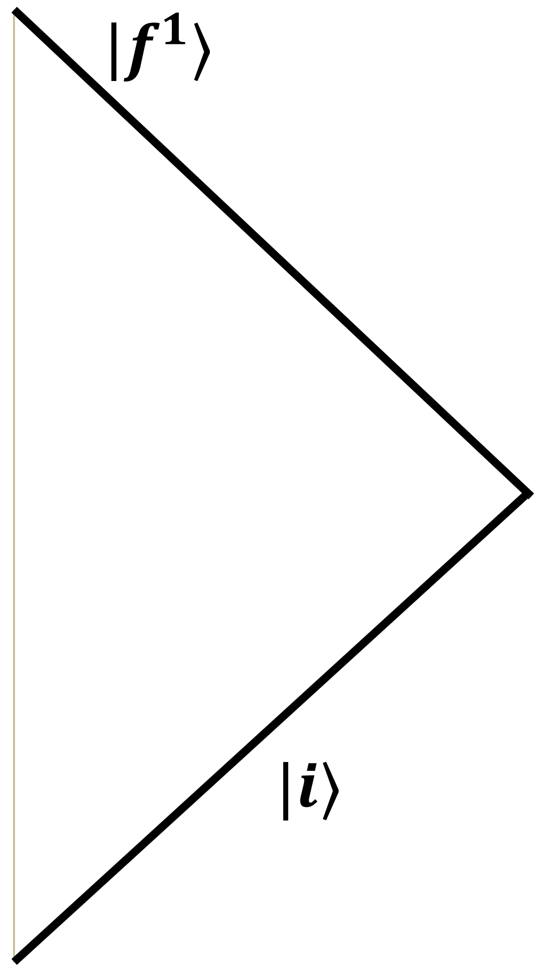
Understanding information loss problem

$$\langle f|i\rangle = \int_{i\to f^j} DgD\phi\, e^{-S_E}$$



Understanding information loss problem

$$\langle f|i\rangle = \int_{i\to f^j} DgD\phi\, e^{-S_E}$$



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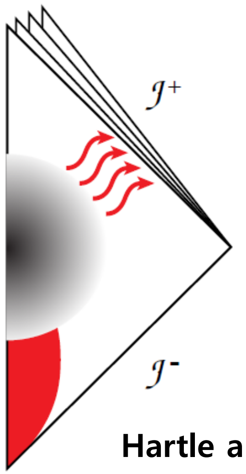
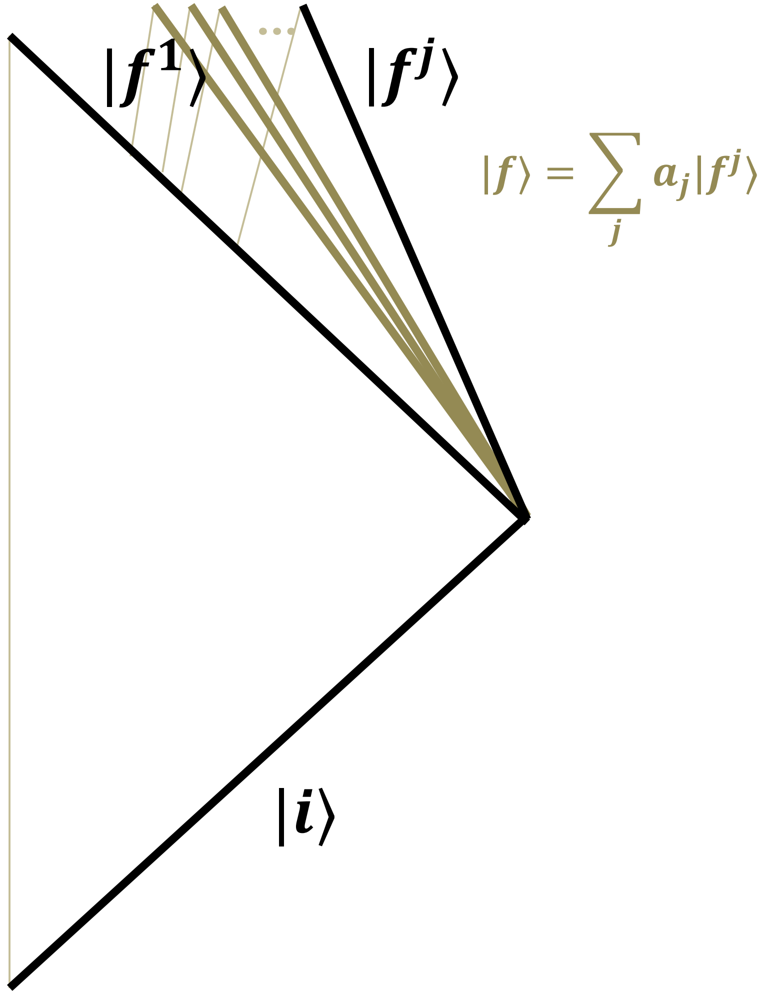
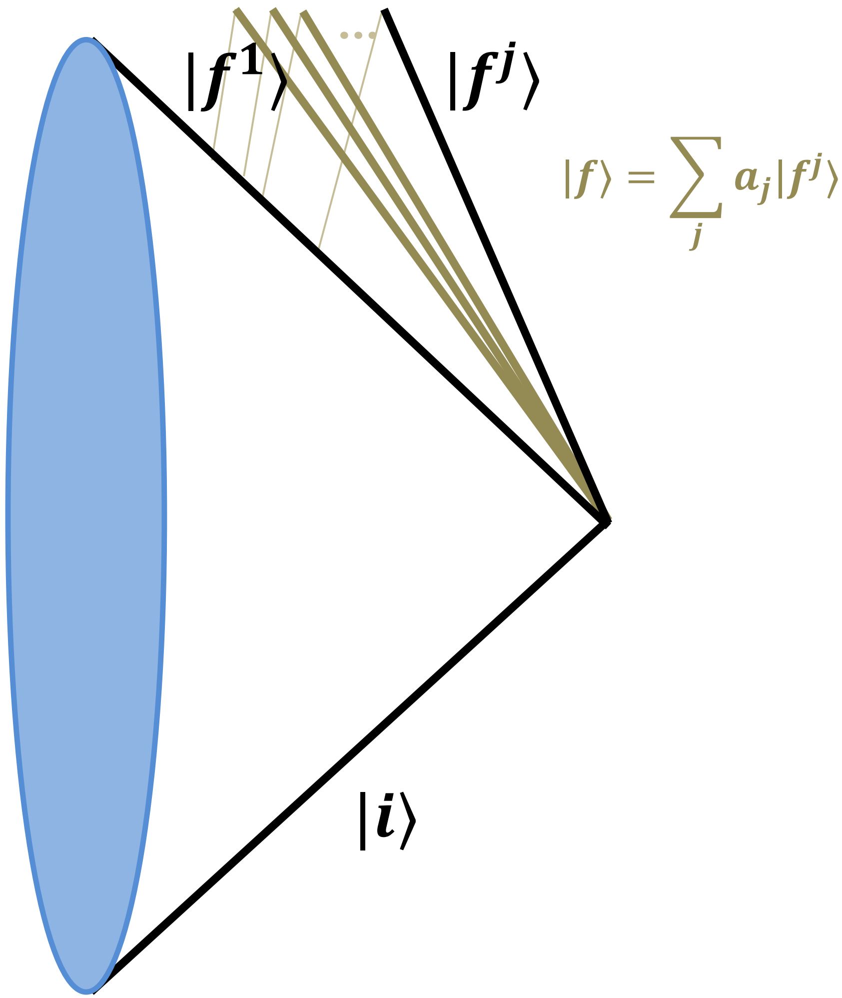


Hartle and Hertog, 2015



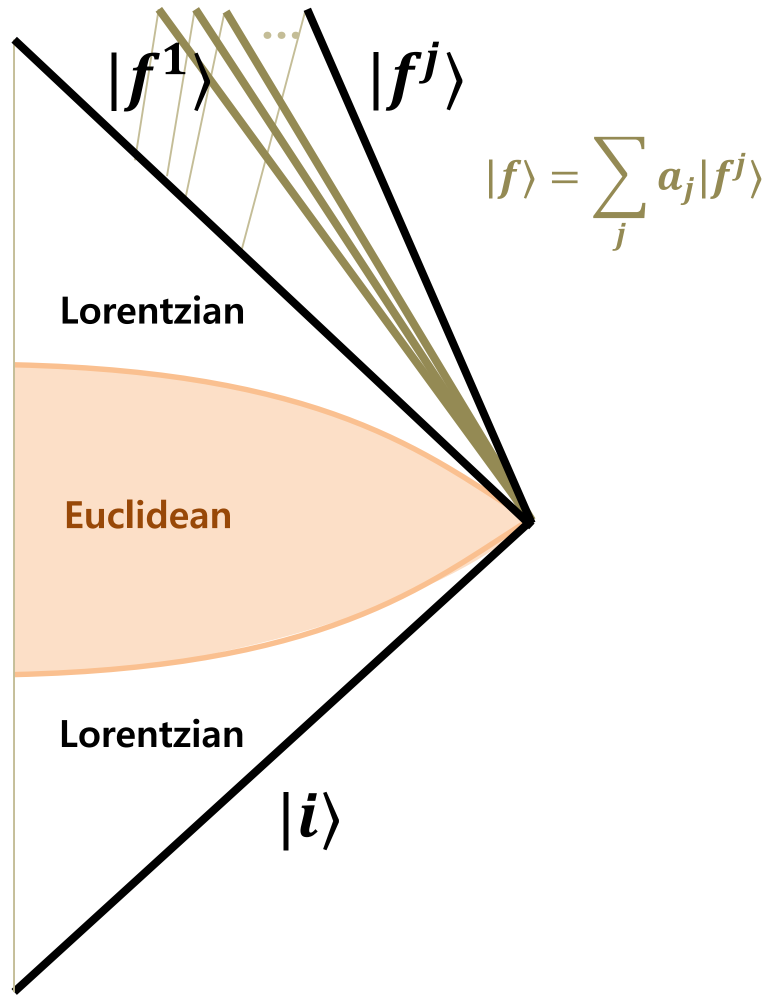
Understanding information loss problem

$$\langle f|i\rangle = \int_{i\to f^j} DgD\phi\, e^{-S_E}$$



Understanding information loss problem

$$\langle f|i\rangle \cong \sum_{i\to f^j} e^{-S_E^{\text{on-shell}}}$$



Understanding information loss problem

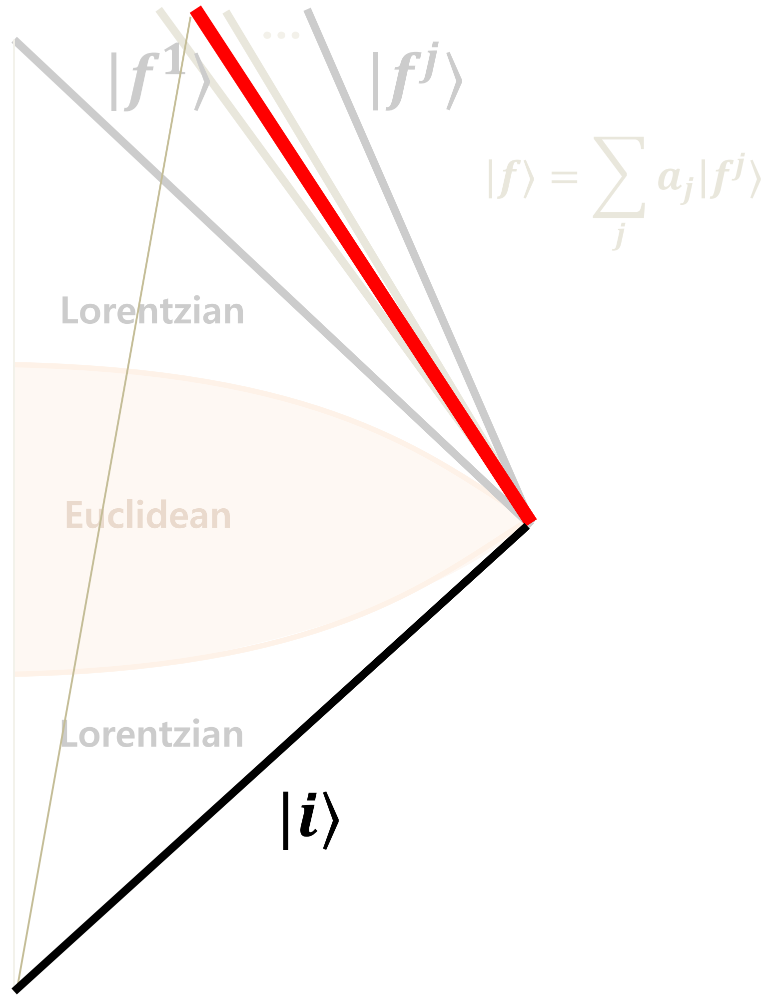
$$\langle f|i\rangle \cong \sum_{i\to f^j} e^{-S_E^{\text{on-shell}}}$$

If there is a trivial geometry, then every correlations will be recovered by the geometry.

Information exists in the wave function.

$$\langle g_{\mu\nu}\rangle \simeq p_1 g_{\mu\nu}^{(1)} + p_2 g_{\mu\nu}^{(2)} + \ldots$$

$$\langle\phi\phi\rangle \simeq p_1\langle\phi\phi\rangle_1 + p_2\langle\phi\phi\rangle_2 \simeq \text{const}\times e^{-A}$$



Understanding information loss problem

$$\langle f|i\rangle \cong \sum_{i\to f^j} e^{-S_E^{\text{on-shell}}}$$

If there is a trivial geometry,

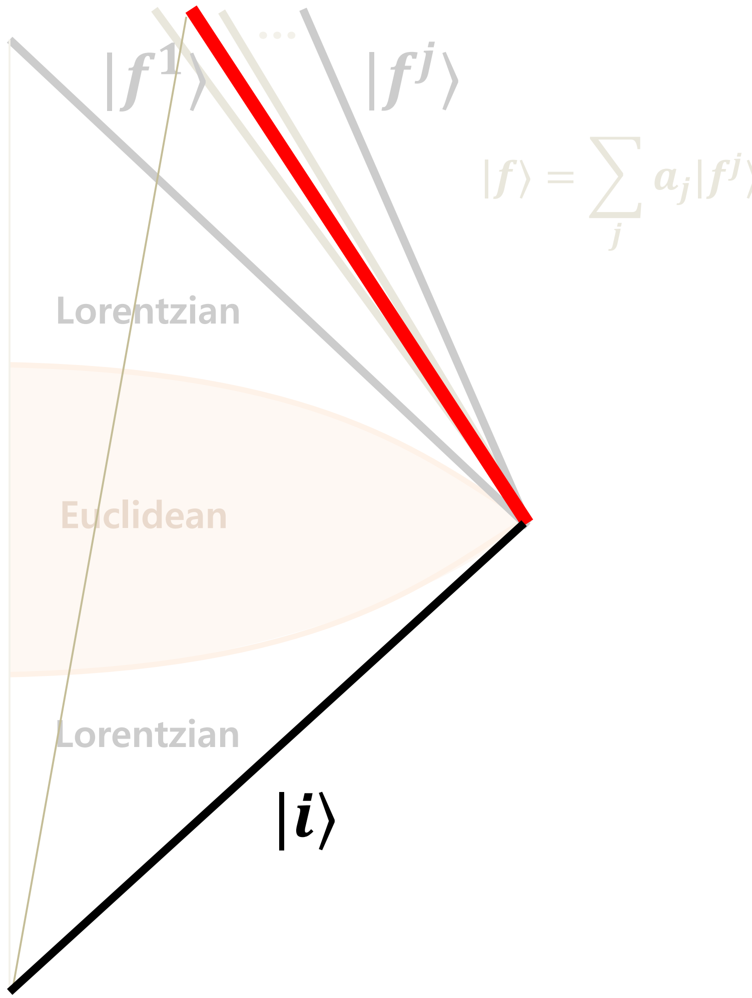
then every correlations will be recovered by the geometry.

Information exists in the wave function.

The asymptotic observer should superpose geometries: GR may not be satisfied.

$$\langle g_{\mu\nu}\rangle \simeq p_1 g_{\mu\nu}^{(1)} + p_2 g_{\mu\nu}^{(2)} + \ldots$$

$$\langle\phi\phi\rangle \simeq p_1\langle\phi\phi\rangle_1 + p_2\langle\phi\phi\rangle_2 \simeq \text{const}\times e^{-A}$$



Understanding information loss problem

$$\langle f|i\rangle \cong \sum_{i\to f^j} e^{-S_E^{\text{on-shell}}}$$

If one follows only one dominant history,

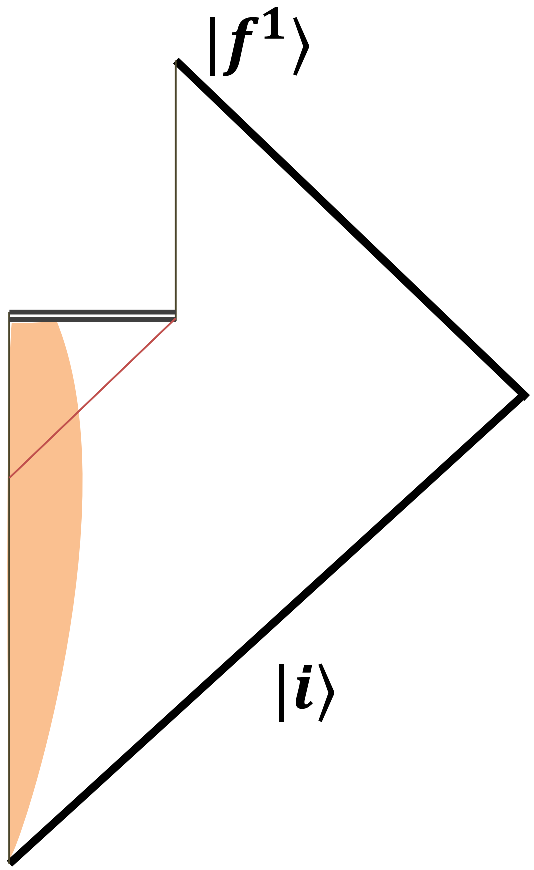
then information cannot be recovered,

though GR and local QFT can be satisfied.

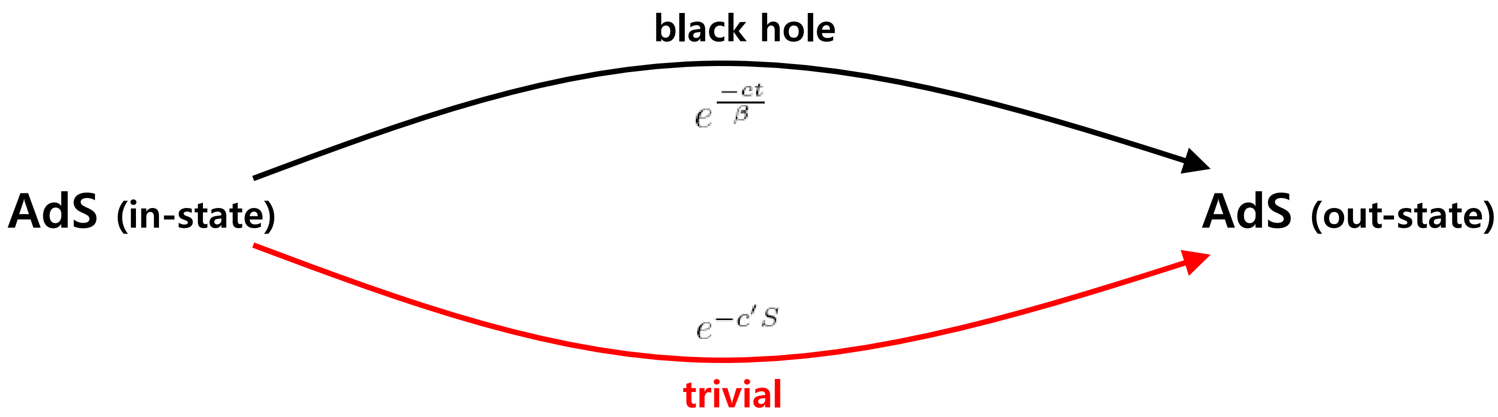
"Effective loss of information"

$$\langle g_{\mu\nu}\rangle \simeq p_1 g_{\mu\nu}^{(1)} + p_2 g_{\mu\nu}^{(2)} + \ldots$$

$$\langle\phi\phi\rangle \simeq p_1\langle\phi\phi\rangle_1 + p_2\langle\phi\phi\rangle_2 \simeq \text{const}\times e^{-A}$$



Maldacena argument, revisited (Maldacena, 2001)



So in the end everyone was right in a way. Information is lost in topologically non-trivial metrics like black holes. This corresponds to dissipation in which one loses sight of the exact state. On the other hand, information about the exact state is preserved in topologically trivial metrics. The confusion and paradox arose because people thought classically in terms of a single topology for spacetime. It was either R^4 or a black hole. But the Feynman sum over histories allows it to be both at once. One can not tell which topology contributed to the observation, any more than one can tell which slit the electron went through in the two slits experiment. All that observation at infinity can determine is that there is a unitary mapping from initial states to final and that information is not lost.

Hawking, 2005

Example: True vacuum bubble in AdS

Let us consider the true vacuum bubble in AdS.

$$ds^2 = -f(R)dT^2 + \frac{1}{f(R)}dR^2 + R^2 d\Omega^2,$$

outside: $$f_+(R) = 1 - \frac{2M_+}{R} + \frac{R^2}{l_+^2},$$

inside: $$f_-(R) = 1 - \frac{2M_-}{R} + \frac{R^2}{l_-^2}.$$

junction equation: $$\epsilon_- \sqrt{\dot{r}^2 + f_-(r)} - \epsilon_+ \sqrt{\dot{r}^2 + f_+(r)} = 4\pi r \sigma$$

$$\dot{r}^2 + V(r) = 0,$$

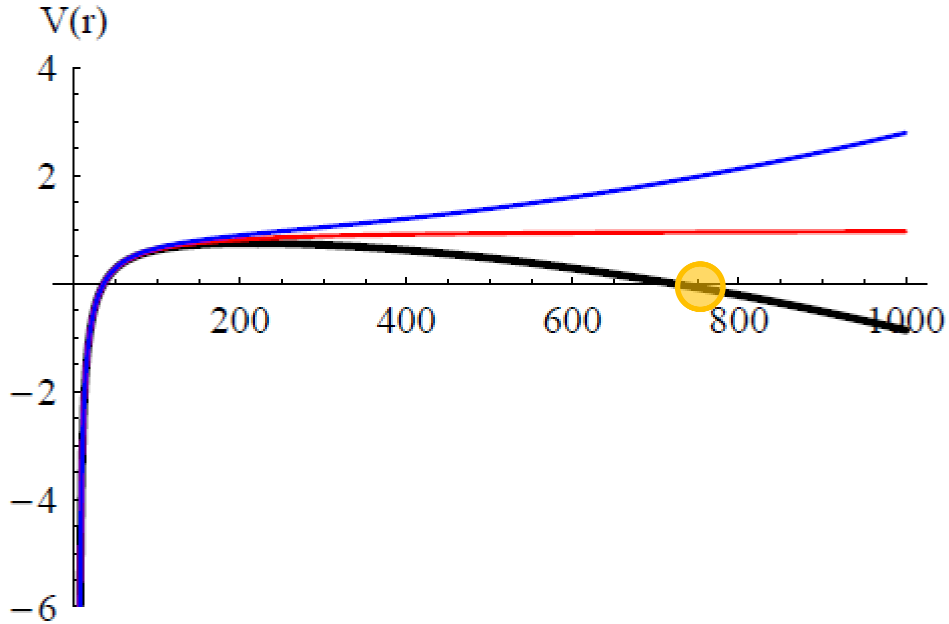
effective potential: $$V(r) = f_+(r) - \frac{\left(f_-(r) - f_+(r) - 16\pi^2\sigma^2 r^2\right)^2}{64\pi^2\sigma^2 r^2}$$

Example: True vacuum bubble in AdS

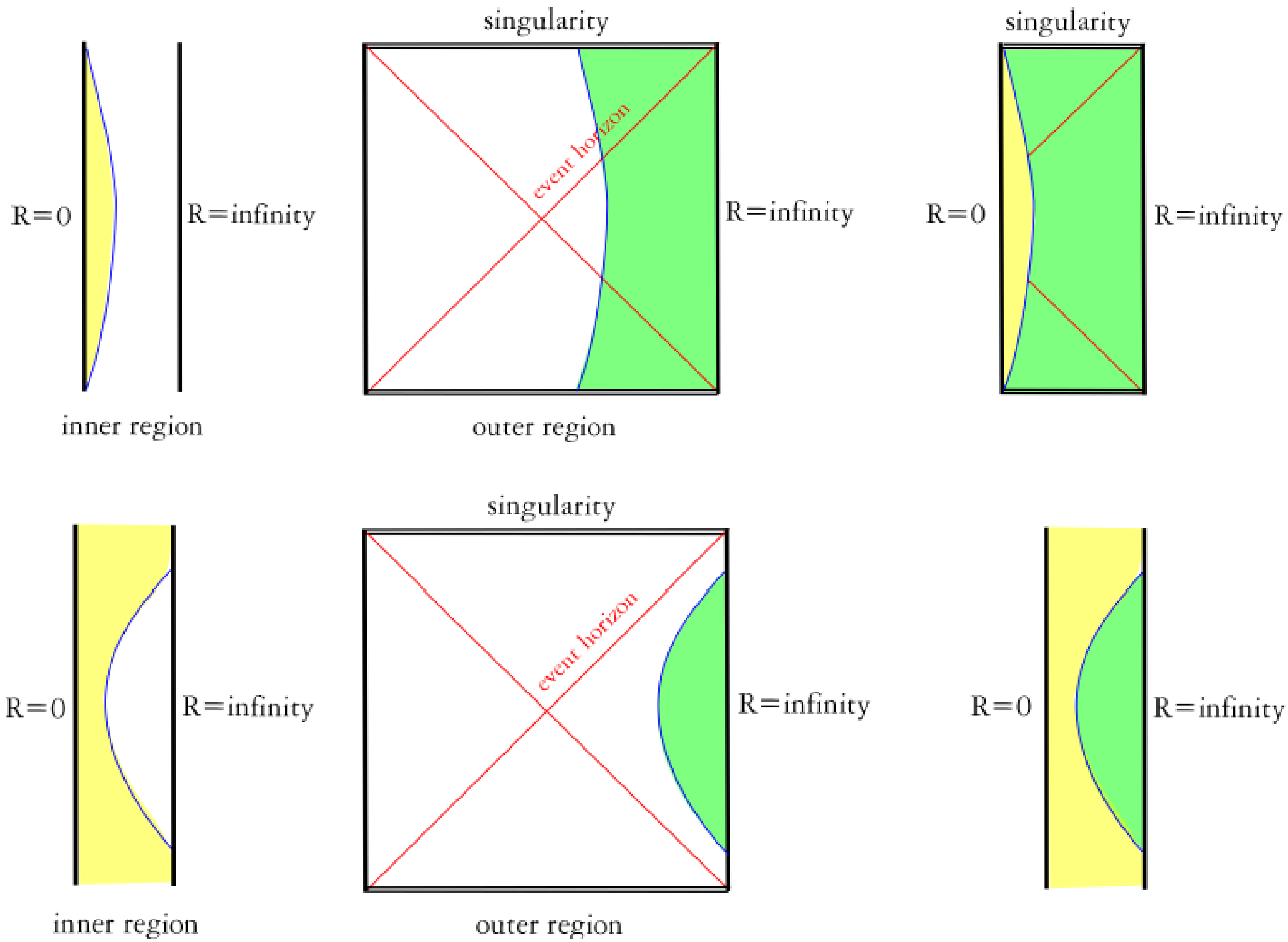
For given M_+, l_+ and $M_- = 0$,

there exists l_- such that $V(r)$ allows a bouncing solution,

even for a large (hence, eternal) black hole in AdS.

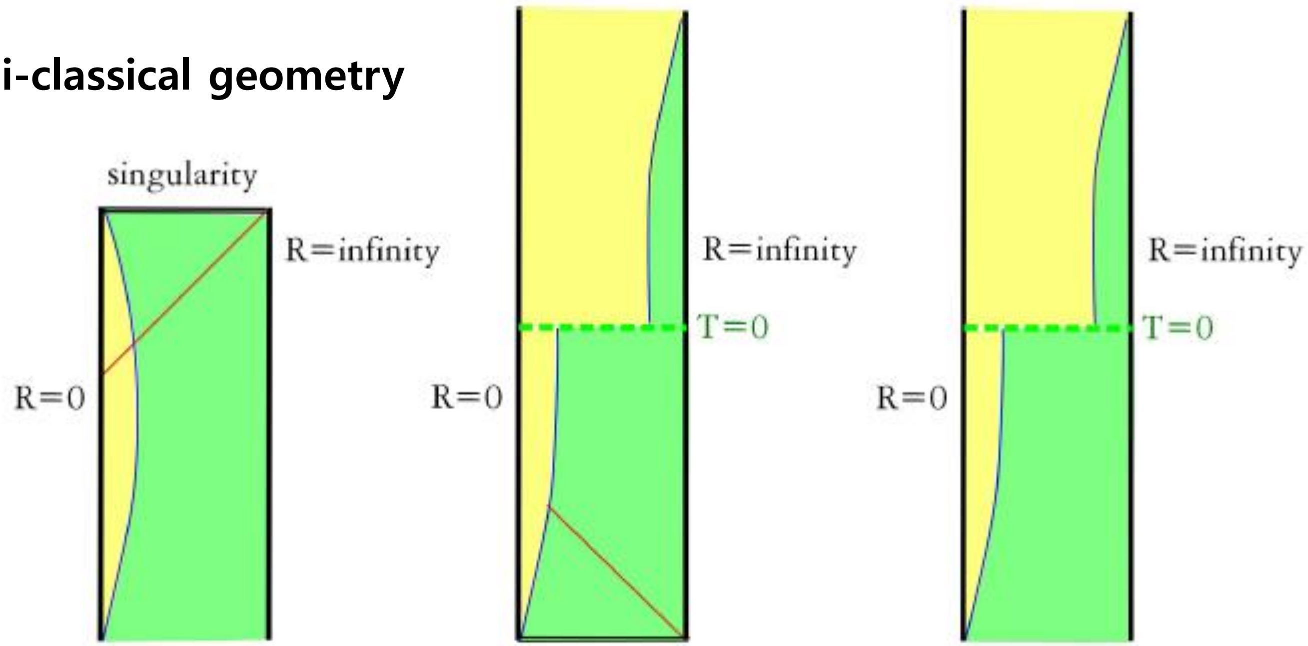


Classical trajectories



Power of non-perturbative effects

Semi-classical geometry



Fischler-Morgan-Polchinski tunneling

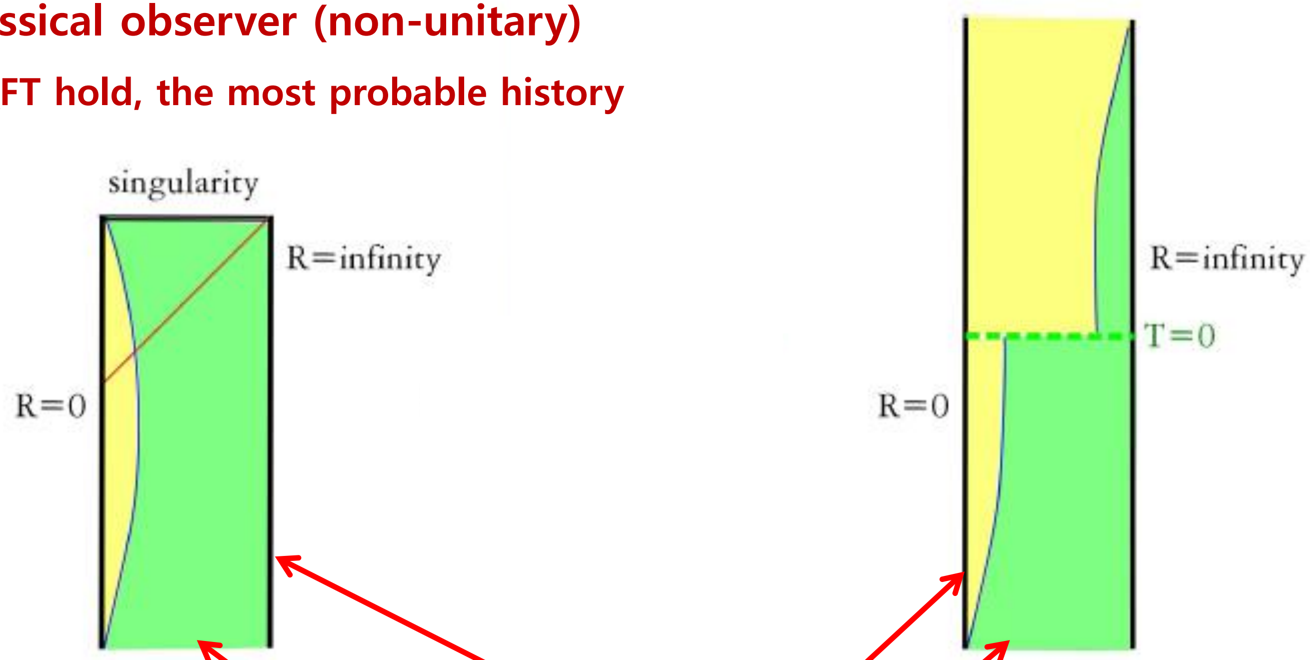
(non-perturbative process)

toward trivial topology

Power of non-perturbative effects

Semi-classical observer (non-unitary)

GR and QFT hold, the most probable history



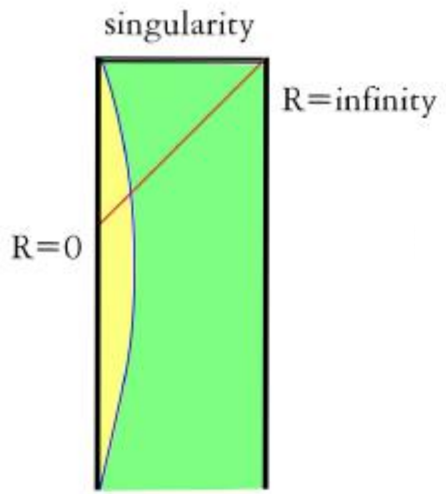
$$\langle g_{\mu\nu}\rangle \simeq p_1 g_{\mu\nu}^{(1)} + p_2 g_{\mu\nu}^{(2)} + \cdots$$

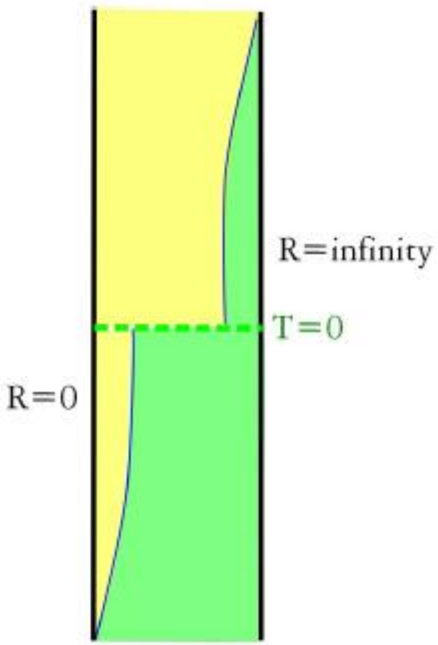
$$\langle\phi\phi\rangle \simeq p_1\langle\phi\phi\rangle_1 + p_2\langle\phi\phi\rangle_2 \simeq \text{const} \times e^{-A}$$

Power of non-perturbative effects

Unitary observer

Doesn't have to satisfy GR: firewall phenomena





$$\langle g_{\mu\nu} \rangle \simeq p_1 g^{(1)}_{\mu\nu} + p_2 g^{(2)}_{\mu\nu} + \ldots$$

$$\langle \phi\phi \rangle \simeq p_1 \langle \phi\phi \rangle_1 + p_2 \langle \phi\phi \rangle_2 \simeq \mathrm{const} \times e^{-A}$$

Comments and future aspects

- ✓ One can show that very **generically** there exists an instanton toward the trivial geometry (Chen, Sasaki and DY, 2018).
- ✓ One needs to check whether this is **enough** to explain the information loss problem or not.
- ✓ The nature of **dynamical spherical symmetric instantons** needs to be investigated further.