Charge fluctuations in SQED in power-law inflation

Dražen Glavan

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CP3, UCLouvain, Louvain-la-Neuve, BE

Introduction

recent work with Gerasimos Rigopoulos (Newcastle)

Interacting QFT effects in primordial universe:

Macroscopic effects of vacuum polarization in SQED

Contents:

(I) Intuitive picture: interacting SQED in primordial universe

(II) Quantitative methods and results

Part I: Intuitive picture

Intuitive picture: linear (free) fields

Linear fields in cosmological patch of de Sitter:

exponentially expanding space, $a(t) = e^{Ht}$

Linear quantum fields in de Sitter:

Heavy fields $(m \geq H)$. Not so interesting: expansion effects subleading correction to flat space behaviour.
Massless/light fields fields $(0 \leq m \ll H)$.
Interesting: large IR phase space absent in flat space.

Massless linear fields in de Sitter:

Conformally coupled fields (photon)
Non-conformally coupled fields (massless minimally coupled scalar MMCS)

Intuitive picture: conformal fields

FLRW space-time conformally flat: $ds^2 = a^2(\eta)(-d\eta^2 + d\vec{x}^2)$
(conformal time $dt = a d\eta$)

Photon (vector field) is **conformally** coupled to gravity:

$$S = \int d^4x \sqrt{-g} \left[-\frac{1}{4} g^{\mu\rho} g^{\nu\sigma} F_{\mu\nu} F_{\rho\sigma} \right] = \int d^4x \left[-\frac{1}{4} \eta^{\mu\rho} \eta^{\nu\sigma} F_{\mu\nu} F_{\rho\sigma} \right]$$

Photon does not experience expansion of the universe!

Intuitive picture: non-conformal fields

$$ds^2 = a^2(\eta)(-d\eta^2 + d\vec{x}^2)$$

MMCS is **non-conformally** coupled to gravity:

$$S = \int d^4x \sqrt{-g} \left[-\frac{1}{2} g^{\mu\nu} (\partial_\mu \phi)(\partial_\nu \phi) \right]$$

Rescaling of the MMCS in FLRW: $\phi \to \phi/a$:

$$S = \int d^4x \left[-\frac{1}{2} \eta^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - \frac{1}{2} m_{\text{eff}}^2 \phi^2 \right]$$

$\Rightarrow$ scalar in flat space with an effective time-dependent mass:

$$m_{\text{eff}}^2 = (2-\epsilon)\mathcal{H}^2 \,, \qquad \mathcal{H} = \frac{a'}{a} \,, \qquad \epsilon = 1 - \frac{\mathcal{H}'}{\mathcal{H}^2}$$

Intuitive picture: conformal vs non-conformal fields

de Sitter space: $a = \dfrac{-1}{H_0\eta}$

Two-point functions of *natural* energy-minimizing states very different:

$$\left\langle \hat{A}_\mu(x)\hat{A}_\nu(x')\right\rangle = \frac{1}{4\pi^2}\frac{\eta_{\mu\nu}}{(\Delta x^2)}\,,$$

$$\left\langle \hat{\phi}(x)\hat{\phi}(x')\right\rangle = \frac{1}{4\pi^2}\left\{\frac{1}{\Delta x^2} - \frac{H_0^2}{2}\ln\!\left(\frac{y}{4}\right) + \frac{H_0^2}{2}\ln(aa') + H_0^2\left[\ln\!\left(\frac{H_0}{2k_0}\right) - \gamma_{\rm E}\right]\right\}$$

$$(\Delta x^2) = \|\Delta\vec{x}^2\| - (\eta - \eta')^2\,, \qquad y = aa'H_0^2(\Delta x^2)$$

Large superhorizon correlators for scalar!

Intuitive picture: interacting fields 1

What if scalar is charged? (complex scalar):

- Linear level: two real scalars

- Global $U(1)$ invariance $\Rightarrow$ conserved charge current $\hat{J}_\mu$

- Large superhorizon scalar field correlators

  $\Rightarrow$ Large superhorizon charge current correlators

- Rapid expansion rips pairs of charged scalars out of vacuum!

What if we couple massless charged scalar to photon?

Intuitive picture: interacting fields 2

Rapid expansion

$\Downarrow$

Superhorizon scalar fluctuations

$\Downarrow$

Charge fluctuations

$\Downarrow$

Enhanced vector fluctuations

$\Downarrow$

Charges move around

$\Downarrow$

...

ad infinitum: perturbation theory

PART II: Quantitative results

Power-law inflation

FLRW space-time:

$$g_{\mu\nu} = a^2(\eta)\eta_{\mu\nu}\,, \qquad \eta_{\mu\nu} = \mathrm{diag}(-1,1,1,1)$$

Conformal Hubble rate: $\mathcal{H} = a'/a$

Slow-roll parameter: $\epsilon = 1 - \frac{\mathcal{H}'}{\mathcal{H}^2}$

Power-law inflation: $\epsilon = \mathrm{const.}$, $0 \leq \epsilon < 1$,

$$\mathcal{H}(\eta) = H_0\Big[1 - (1-\epsilon)H_0(\eta-\eta_0)\Big]^{-1}, \qquad a(\eta) = \left(\frac{\mathcal{H}}{H_0}\right)^{\frac{1}{1-\epsilon}}$$

(de Sitter limit: $\epsilon = 0$)

Why power-law inflation? Closer to slow-roll inflation than de Sitter, captures effects of non-vanishing slow-roll parameter

Scalar electrodynamics (SQED)

Action of SQED in curved space:

$$S[A_\mu, \phi, \phi^*] = \int d^4x \sqrt{-g} \left[-\frac{1}{4} g^{\mu\rho} g^{\nu\sigma} F_{\mu\nu} F_{\rho\sigma} - g^{\mu\nu} \left(\partial_\mu \phi^* + iqA_\mu \phi^* \right) \left(\partial_\nu \phi - iqA_\nu \phi \right) \right] \quad (1)$$

Complex scalar:

$$\phi = \frac{1}{\sqrt{2}} \left(\phi_1 + i\phi_2 \right) \quad (2)$$

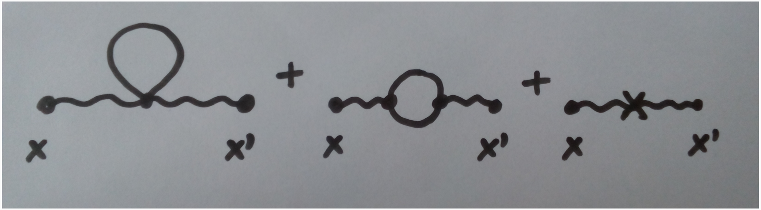
Vector field strength:

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \,. \quad (3)$$

Field strength correlator: diagram

Compute electric and magnetic field correlators,

$$\left\langle \hat{F}_{\mu\nu}(x)\hat{F}_{\rho\sigma}(x')\right\rangle = 4\Big(\partial_{[\mu}\delta^{\alpha}_{\nu]}\Big)\Big(\partial_{[\rho}\delta^{\beta}_{\sigma]}\Big) \times \left\langle \hat{A}_{\alpha}(x)\hat{A}_{\beta}(x')\right\rangle \quad (4)$$



Photon propagator in power-law inflation
in general covariant gauge   Domazet, DG, Prokopec *in preparation*
in general covariant gauge   DG *in preparation*

Not needed for just superhorizon correlator

Digression: de Sitter results

SQED in de Sitter studied by
Prokopec, Tornkvist, Woodard (2002, 2003)
Prokopec, Tsamis, Woodard (2007, 2008)

What is the effect on a single photon propagating through vacuum-polarized de Sitter?

Photon develops an effective mass term:
- perturbative result: $m_\gamma = \frac{H_0^2}{4\pi^2}\ln(a)$
- nonperturbative result: $m_\gamma \approx 3.3 H_0^2$

Mechanism for creating cosmic magnetic fields:
Prokopec, Woodard (2004)

WHAT ARE PROPERTIES OF FLUCTUATIONS (CORR.)?

Field strength correlator: superhorizon computation

Electric and magnetic field: $\hat{E}_i = \hat{F}_{0i},\ \hat{B}_i = \frac{1}{2}\varepsilon_{ijk}\hat{F}_{jk}$

Take a correlator of Maxwell's equations:

$$
\begin{aligned}
(aa')^2\Big\langle \hat{J}_0(x)\hat{J}_0(x')\Big\rangle &= \partial_i\partial'_j\Big\langle \hat{E}_i(x)\hat{E}_j(x')\Big\rangle\,, \\
(aa')^2\Big\langle \hat{J}_0(x)\hat{J}_i(x')\Big\rangle &= \partial_j\partial'_0\Big\langle \hat{E}_j(x)\hat{E}_i(x')\Big\rangle \\
&\quad + \varepsilon_{ikl}\partial_j\partial'_k\Big\langle \hat{E}_j(x)\hat{B}_l(x')\Big\rangle\,, \\
(aa')^2\Big\langle \hat{J}_i(x)\hat{J}_j(x')\Big\rangle &= \partial_0\partial'_0\Big\langle \hat{E}_i(x)\hat{E}_j(x')\Big\rangle \\
&\quad + \varepsilon_{ikl}\partial_k\partial'_0\Big\langle \hat{B}_l(x)\hat{E}_j(x')\Big\rangle \\
&\quad + \varepsilon_{jab}\partial_0\partial'_a\Big\langle \hat{E}_i(x)\hat{B}_b(x')\Big\rangle \\
&\quad + \varepsilon_{ikl}\varepsilon_{jab}\partial_k\partial'_a\Big\langle \hat{B}_l(x)\hat{B}_b(x')\Big\rangle\,,
\end{aligned}
$$

Solve perturbatively in charge for superhorizon separations

Charge currents

Source of Maxwell equations are conserved $U(1)$ currents, $\nabla^\mu J_\mu = 0$,

$$J_\mu = \frac{q}{2}\Big(\phi_1\partial_\mu\phi_2 - \phi_2\partial_\mu\phi_1\Big) - q^2\Big(\phi_1^2 + \phi_2^2\Big)A_\mu \,. \qquad (5)$$

Leading contribution – linear theory:

$$\Big\langle \hat{J}_\mu(x)\hat{J}_\nu(x')\Big\rangle = i\Delta(x;x') \times \partial_\mu\partial'_\nu\, i\Delta(x;x') - \partial_\mu\, i\Delta(x;x') \times \partial'_\nu\, i\Delta(x;x') \qquad (6)$$

Scalar propagator in power-law inflation determined by linear physics:

Janssen, Miao, Prokopec, Woodard   arXiv:0808.2449 [gr-qc]

Scalar propagator

Neutral state:

$$\left\langle \hat{\phi}_1(x)\hat{\phi}_1(x) \right\rangle = \left\langle \hat{\phi}_2(x)\hat{\phi}_2(x) \right\rangle = i\Delta(x;x')$$

Interested in superhorizon correlators:

$$\|\vec{x}-\vec{x}\,'\| \ll (\eta-\eta'), \mathcal{H}$$

Expand propagator for superhorizon separations ($\nu = \frac{3-\epsilon}{2(1-\epsilon)}$),

$$i\Delta(x;x') \sim \left[\frac{(1-\epsilon)H_0}{4\pi}\right]^2 \frac{\Gamma(2\nu)}{\Gamma(\frac{1}{2}+\nu)}$$
$$\times \left\{ \frac{-\Gamma(\nu)}{(\frac{3}{2}-\nu)\Gamma(\frac{3}{2})}\left(\frac{H_0}{k_0}\right)^{2\nu-3} + \Gamma\Big(\tfrac{3}{2}-\nu\Big)\Big[\tfrac{1}{2}(1-\epsilon)H_0\|\Delta\vec{x}\|\Big]^{2\nu-3} \right.$$
$$\left. + \frac{1}{4}\Gamma(\tfrac{5}{2}-\nu)\left(\frac{\mathcal{H}}{\mathcal{H}'}+\frac{\mathcal{H}'}{\mathcal{H}}\right)\Big[\tfrac{1}{2}(1-\epsilon)H_0\|\Delta\vec{x}\|\Big]^{2\nu-5} + \dots \right\}$$

Charge correlators

Superhorizon spatial current correlator:

$$\left\langle \hat{J}_i(\eta,\vec{x})\hat{J}_j(\eta,\vec{x}\,')\right\rangle \sim \frac{q^2[(1-\epsilon)H_0]^4}{|\Delta\vec{x}|^2}\Big[(1-\epsilon)H_0\|\Delta\vec{x}\|\Big]^{4\nu-6}\times$$
$$\times\left\{\delta_{ij}\left[\#_1+\#_2\Big[(1-\epsilon)k_0\|\Delta\vec{x}\|\Big]^{3-2\nu}\right]\right.$$
$$\left.+\frac{\Delta x_i\Delta x_j}{\|\Delta\vec{x}\|^2}\left[\#_3+\#_4\Big[(1-\epsilon)k_0\|\Delta\vec{x}\|\Big]^{3-2\nu}\right]\right\}$$

Charge correlators ⇔ vacuum polarization

Superhorizon charge correlators ⇒ macroscopic effects of vacuum polarization

Electric and magnetic field correlators

$$\left\langle \hat{E}_i(\eta,\vec{x})\hat{E}_i(\eta,\vec{x}\,')\right\rangle = \frac{1}{\pi^2\|\Delta\vec{x}\|^4}\times$$

$$\Bigg[\delta_{ij}\bigg(1+q^2a^{2+2\epsilon}\Big[(1-\epsilon)H_0\|\Delta\vec{x}\|\Big]^{4\nu-4}\Big[\#_1+\#_2\Big[(1-\epsilon)k_0\|\Delta\vec{x}\|\Big]^{3-2\nu}\Big]\bigg)$$

$$+\frac{\Delta x_i\Delta x_j}{\|\Delta\vec{x}\|^2}\bigg(2+q^2a^{2+2\epsilon}\Big[(1-\epsilon)H_0\|\Delta\vec{x}\|\Big]^{4\nu-4}\Big[\#_3+\#_4\Big[(1-\epsilon)k_0\|\Delta\vec{x}\|\Big]^{3-2\nu}\Big]\bigg)\Bigg]$$

E-E: secular $a^{2+2\epsilon}$, spatial $\|\Delta\vec{x}\|^{\frac{2+2\epsilon}{(1-\epsilon)}}\left[1+(k_0\|\Delta\vec{x}\|)^{\frac{-\epsilon}{1-\epsilon}}\right]$

E-B: secular $a^{1+2\epsilon}$, spatial $\|\Delta\vec{x}\|^{\frac{1+3\epsilon}{(1-\epsilon)}}\left[1+(k_0\|\Delta\vec{x}\|)^{\frac{-\epsilon}{1-\epsilon}}\right]$

B-B: secular $a^{2\epsilon}$, spatial $\|\Delta\vec{x}\|^{\frac{4\epsilon}{(1-\epsilon)}}\left[1+(k_0\|\Delta\vec{x}\|)^{\frac{-\epsilon}{1-\epsilon}}\right]$

Conclusions

- Interacting QFT computation in power-law inflation
- Power-law inflation long enough: correlators grow faster than $\ln(a)$ (Weinberg's result)
- Large superhorizon one-loop electric and magnetic fields compared to tree-level
- Secular growth: correlators grow in time
- Spatial growth: correlators grow with distance
- IR sensitivity: real or perturbation theory artifact?
  $\Rightarrow$ RG methods as resummation method
- Relation to primordial magnetogenesis and observable imprints?

Extra expressions

$$\left\langle \hat{E}_i(\eta, \vec{x}) \hat{E}_i(\eta, \vec{x}\,') \right\rangle \sim (\# q^2) a^{2+2\epsilon} \frac{[(1-\epsilon)H_0]^{4\nu-2}}{\pi^2 \|\Delta\vec{x}\|^{8-4\nu}} \left[\frac{\delta_{ij}}{4} - \frac{\Delta x_i \Delta x_j}{\|\Delta\vec{x}\|^2} \right]$$

$$\left\langle \hat{E}_i(\eta, \vec{x}) \hat{E}_i(\eta, \vec{x}\,') \right\rangle = \frac{1}{\pi^2 \|\Delta\vec{x}\|^4} \left[\delta_{ij} + 2 \frac{\Delta x_i \Delta x_j}{\|\Delta\vec{x}\|^2} \right]$$