

# Quantum scale invariance and Weyl gravity

D. Ghilencea

National Institute of Physics IFIN Bucharest

“Warsaw Workshop on Gravity and Cosmology” - 21-23 Jan 2019

Based on: arXiv: 1812.08613, 1712.06024, 1809.09174 with Hyun Min Lee

PRD 96 (2017), with Z. Lalak, P. Olszewski

## Outline:

- I. Flat space:
  - Quantum corrections in scale-invariant regularization  $\lambda\phi^4$  & SM
  - spontaneous breaking (SB)  $\Rightarrow$  flat direction (dilaton)
  - $\rightarrow$  classical hierarchy of: higgs vev  $\ll$  dilaton vev's  $\Rightarrow$  quantum stable?
  
- II. Curved space:
  - $M_{\text{Planck}} \sim$  dilaton vev  $\Rightarrow$  Weyl gravity/gauge symmetry
  - Stueckelberg: dilaton “eaten” by Weyl gauge field  $\omega_\mu$  ( $m_\omega \sim M_{\text{Planck}}$ )
  - $\rightarrow$  SB of Weyl quadratic gravity  $\Rightarrow$  Einstein gravity + massive  $\omega_\mu$
  - Weyl geometry  $\Rightarrow$  Riemann geometry (below  $m_\omega$ )
  - Adding matter (Higgs)  $\Rightarrow$  EWSB.

- “New physics” beyond SM: new symmetry?

- SUSY @ TeV: hierarchy problem [and other problems] solved [in theory....]

- scale invariance (SI); - SM with  $m_h = 0$  has **classical** scale invariance.

[Bardeen 1995]

$$x \rightarrow \rho x, \quad \phi \rightarrow \rho^{-1} \phi, \quad [\phi] = 1, \quad \text{SI forbids} \quad \int d^4x m^2 \phi^2 + \dots$$

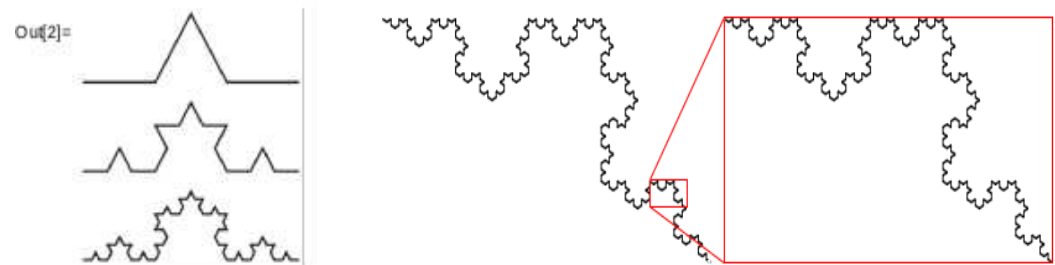
- no dimensionful couplings; **All scales from vev's! (including  $M_{\text{Planck}}$ !)**

- Classical SI: models: “Higgs portal”:  $\lambda \phi^2 \sigma^2$ . Quantum level?

- **Nature:** discrete SI: self-similarity

across different scales [fractals]

In[2]= Column[Table[Graphics[KochCurve[n]], {n, 1, 3}]] snowflake/Koch curve



- Scale-invariant regularization (SIR):

[Englert 1976, Itzykson/Zuber, Shaposhnikov 2008]

DR:  $d=4-2\epsilon$ :  $L = \frac{1}{2}(\partial_\mu\phi)^2 - \lambda_\phi \mu^{2\epsilon} \phi^4$ , explicit ~~SI~~ by UV regulators, to avoid.

replace  $\mu \rightarrow$  field  $\sigma$ , spontaneous  $\langle\sigma\rangle \neq 0 \Rightarrow$  Goldstone (dilaton):  $\sigma = \langle\sigma\rangle e^\tau$ ,  $\tau \rightarrow \tau - \ln \rho$

[nothing new, recall  $M_{\text{string}}$  moduli dep]

- Action:  $d=4$ ,  $S = \int d^4x \left[ \underbrace{(\partial_\mu\phi)^2 - V(\phi)}_{\text{visible}} \right] + \int d^4y \underbrace{L_h(\sigma, \partial\sigma)}_{\text{hidden}}$

- spectrum **extended** by  $\sigma$ ! potential:  $\lambda_\sigma \sigma^4$  but Poincaré symmetry demands  $\lambda_\sigma = 0$  [Fubini 1976]

- each sector SI (shift symmetry)  $\rightarrow$  **enhanced** shift symmetry:  $S_h \times S_v \Rightarrow \lambda_m \phi^2 \sigma^2$ :  $\lambda_m$  naturally small

[Volkas, Kobakhidze, Foot 2013]

$d=4-2\epsilon$ :  $\mu = z \sigma^{2/(d-2)}$ ,  $V \rightarrow \tilde{V} = \left[ z \sigma^{2/(d-2)} \right]^{4-d} V(\phi)$ , ( $z$ : dim-less),  $\sigma = \langle\sigma\rangle + \tilde{\sigma}$

$$\tilde{V} = (z \langle\sigma\rangle^{1/(1-\epsilon)})^{2\epsilon} \left[ 1 + 2\epsilon \left( \frac{\tilde{\sigma}}{\langle\sigma\rangle} - \frac{\tilde{\sigma}^2}{2\langle\sigma\rangle^2} + \dots \right) + \epsilon^2 \left( \frac{2\tilde{\sigma}}{\langle\sigma\rangle} + \dots \right) + \mathcal{O}(\epsilon^3) \right] V(\phi),$$

$\Rightarrow$  SIR=DR+dilaton with  $\infty$ -many  $\epsilon$ -couplings.  $\langle\sigma\rangle$ : 'new physics'. Q: if  $\langle\phi\rangle \ll \langle\sigma\rangle$ : is it quantum stable?

[ $\sigma\sigma \rightarrow \sigma\sigma$  at 3 loops!  $(\partial_\mu \ln \sigma)^4$ ]

[ 4 ]

• One-loop SI potential: 
$$L = \frac{1}{2}(\partial_\mu\phi)^2 + \frac{1}{2}(\partial_\mu\sigma)^2 - \overbrace{\frac{1}{4!}\lambda\phi^4}^{=V(\phi)}, \quad V \rightarrow \tilde{V} = z^{2\epsilon} \sigma^{2\epsilon/(1-\epsilon)} V(\phi).$$

$$V_1 = -\frac{i}{2} \int \frac{d^d p}{(2\pi)^d} \text{tr} \ln [p^2 - \tilde{V}_{\alpha\beta} + i\epsilon] = \sum_{s=\phi,\sigma} \frac{\tilde{M}_s^4}{4\kappa} \left[ \frac{-1}{\epsilon} + \ln \frac{\tilde{M}_s^2}{c_0} \right] \mu(\sigma)^{2\epsilon},$$

$$\tilde{M}_\phi^4 = M_\phi^4 + \epsilon\dots, \quad \tilde{M}_\sigma^4 \sim \epsilon^2, \quad \kappa = (4\pi)^2$$

One-loop correction: 
$$U_1 = \frac{V_{\phi\phi}^2}{4\kappa} \left[ \overline{\ln} \frac{V_{\phi\phi}}{(z\sigma)^2} - \frac{1}{2} \right] = \underbrace{\frac{V_{\phi\phi}^2}{4\kappa} \left[ \overline{\ln} \frac{V_{\phi\phi}}{(z\langle\sigma\rangle)^2} - \frac{1}{2} \right]}_{CW} + \underbrace{\frac{V_{\phi\phi}^2}{4\kappa} \left[ \frac{-\tilde{\sigma}}{\langle\sigma\rangle} + \frac{\tilde{\sigma}^2}{2\langle\sigma\rangle^2} + \dots \right]}_{\rightarrow 0; \text{ small } \tilde{\sigma} \text{ maintains SI}}$$

$$\sigma = \langle\sigma\rangle + \tilde{\sigma}, \quad \tilde{\sigma}: \text{fluctuations}$$

$$V_{\phi\phi} = \frac{\lambda}{2} \phi^2$$

$\Rightarrow U_1$  scale invariant due to dilaton ( $\ln \sigma$ ). No new poles, same beta function  $\beta_\lambda^{(1)}$  (as for no dilaton)

$\lambda^B = \lambda Z_\lambda Z_\phi^{-2}$ ,  $d\lambda^B/d(\ln z) = 0$ ,  $\beta_\lambda^{(1)} = d\lambda/d(\ln z) = 3\lambda^2/\kappa$ . Callan-Symanzik:  $d(U_1 + V)/d(\ln z) = 0$

$\Rightarrow$  Quantum SI with  $\beta_\lambda \neq 0$  [ $\beta_\lambda = 0$  is sufficient but not necessary for QSI: different spectrum/symmetry]

$\Rightarrow$  dilatation current:  $\partial_\mu D^\mu = 0$

[Englert et al 1979, Shaposhnikov et al; Tamarit 2014]

• **Two-loop SI potential:** using:  $\tilde{V}(\phi, \sigma) = z^{2\epsilon} \sigma^{2\epsilon/(1-\epsilon)} V(\phi) \sim \sigma^{2\epsilon} V(\phi)$  [background field method]

$$\Rightarrow \tilde{V}(\phi + \delta_\phi, \sigma + \delta_\sigma) = \tilde{V}(\phi, \sigma) + \tilde{V}_\alpha \delta_\alpha + \frac{1}{2} \tilde{V}_{\alpha\beta} \delta_\alpha \delta_\beta + \frac{1}{3!} \tilde{V}_{\alpha\beta\gamma} \delta_\alpha \delta_\beta \delta_\gamma + \frac{1}{4!} \tilde{V}_{\alpha\beta\gamma\rho} \delta_\alpha \delta_\beta \delta_\gamma \delta_\rho + \dots \quad \alpha, \beta = \phi, \sigma.$$

$$\tilde{V}_{\alpha\beta\dots} = \partial_\alpha \partial_\beta \dots \tilde{V}$$

$$V_2 = \frac{i}{12} \text{[bubble]} + \frac{i}{8} \text{[figure-eight]} + \frac{i}{2} \text{[triangle with cross]} = \frac{i}{12} \tilde{V}_{\alpha\beta\gamma} \tilde{V}_{\alpha'\beta'\gamma'} \int \frac{d^d p}{(2\pi)^d} \frac{d^d q}{(2\pi)^d} (\tilde{D}_p)_{\alpha\alpha'} (\tilde{D}_q)_{\beta\beta'} (\tilde{D}_{p+q})_{\gamma\gamma'} + \dots$$

$$= (z\sigma)^{2\epsilon} \frac{\lambda^3 \phi^4}{32\kappa^2} \left\{ -\frac{3}{\epsilon^2} + \frac{2}{\epsilon} + \mathcal{O}(\epsilon^0) \right\}; \quad (\tilde{D}_p)_{\alpha\beta} = (D_p)_{\alpha\beta} + \epsilon (\dots)_{\alpha\beta} + \epsilon^2 (\dots)_{\alpha\beta}$$

same poles,  $\epsilon$ -shifts to propagators, vertices:

$$\tilde{V}_{\alpha\beta\gamma\dots} = V_{\alpha\beta\gamma\dots} + \epsilon (\dots)_{\alpha\beta\gamma\dots} + \epsilon^2 (\dots)_{\alpha\beta\gamma\dots}$$

[Lalak, Olszewski, DG]

[1712.06024]

Two-loop correction:

$$U_2 = \frac{\lambda}{4!} \phi^4 \left\{ \frac{3\lambda^2}{4\kappa^2} \left( 4 + A_0 - 4 \bar{\ln} \frac{V_{\phi\phi}}{(z\sigma)^2} + 3 \bar{\ln}^2 \frac{V_{\phi\phi}}{(z\sigma)^2} \right) + \frac{5\lambda^2 \phi^2}{\kappa^2 \sigma^2} + \frac{7\lambda^2 \phi^4}{24\kappa^2 \sigma^4} \right\}, \quad V_{\phi\phi} \equiv \frac{\lambda}{2} \phi^2.$$

$V^{(2)}$ : 'old result' [ $\mu \rightarrow z\sigma$ ]

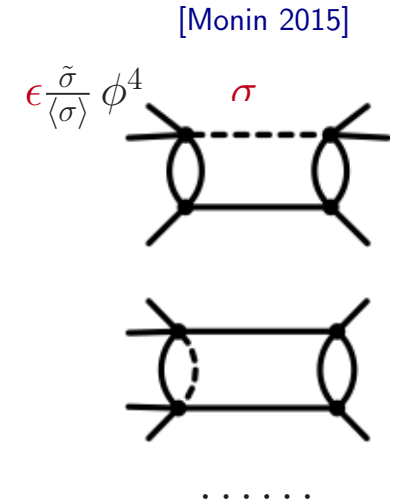
$V^{(2,n)}$ : new, finite; no  $z$ , (!) if  $\phi \sim \sigma$

[Callan-Symanzik verified]

• **Three-loop SI potential: Counterterms!**

$$\delta L_3 = \frac{1}{2} \delta_\phi^{(3)} (\partial_\mu \phi)^2 - \mu^{2\epsilon} \left\{ \frac{1}{4!} \delta_\lambda^{(3)} \lambda \phi^4 + \frac{1}{6} \delta_{\lambda_6}^{(3)} \lambda_6 \frac{\phi^6}{\sigma^2} + \frac{1}{8} \delta_{\lambda_8}^{(3)} \lambda_8 \frac{\phi^8}{\sigma^4} \right\}$$

$$\delta_{\lambda_6}^{(3)} = \frac{3}{2} \frac{\lambda^4}{\lambda_6 \kappa^3 \epsilon}, \quad \delta_{\lambda_8}^{(3)} = \frac{275}{864} \frac{\lambda^4}{\lambda_8 \kappa^3 \epsilon} \Rightarrow \gamma_\phi^{(3)}, \beta_{\lambda_6}, \beta_{\lambda_8} = \dots$$



Integrate CS: Three-loop term:  $U_3 = \Delta V + V^{(3)} + V^{(3,n)}$ .  $V^{(3)}$ ='old' [ $\mu \rightarrow z\sigma$ ]; **new:**  $V^{(3,n)}$  and  $\Delta V$

$$\Delta V = \frac{\lambda_6 \phi^6}{\sigma^2} + \frac{\lambda_8 \phi^8}{\sigma^4}, \quad V^{(3)} = \frac{\lambda^4 \phi^4}{\kappa^3} \left\{ \mathcal{Q} + \left( \frac{97}{128} + \frac{9}{64} A_0 + \frac{\zeta[3]}{4} \right) \overline{\ln} \frac{V_{\phi\phi}}{(z\sigma)^2} - \frac{31}{96} \overline{\ln}^2 \frac{V_{\phi\phi}}{(z\sigma)^2} + \frac{9}{64} \overline{\ln}^3 \frac{V_{\phi\phi}}{(z\sigma)^2} \right\}$$

$$V^{(3,n)} = \frac{\lambda^3 \phi^4}{2 \kappa^3} \left\{ \left( 27\lambda - \frac{\lambda_6}{2} \right) \frac{\phi^2}{8 \sigma^2} + \left( \frac{401\lambda}{72} - \lambda_8 \right) \frac{\phi^4}{16 \sigma^4} \right\} \overline{\ln} \frac{V_{\phi\phi}}{(z\sigma)^2}. \quad V_{\phi\phi} \equiv \frac{\lambda}{2} \phi^2.$$

[DG 1712.06024]

$\Rightarrow$  SI effective operators always **suppressed by  $\sigma$**   $\Rightarrow$ . At large  $\sigma$  enhanced symmetry  $S_v \times S_h$  restored  $\Rightarrow$

$\Rightarrow$  **No c-terms**  $\lambda^2 \phi^2 \sigma^2 = \lambda^2 \langle \sigma \rangle^2 \phi^2 + \dots$  No tuning of Higgs selfcoupling  $\lambda$  for **large  $\sigma$** .

• SM+dilaton: one-loop SI potential

[Shaposhnikov et al 2008; DG, Lalak, Olszewski]

$$V = \frac{\lambda_\phi}{3!} (H^\dagger H)^2 + \frac{\lambda_m}{2} (H^\dagger H) \sigma^2 + \frac{\lambda_\sigma}{4!} \sigma^4 + \dots$$

$$= \frac{\lambda_\phi}{4!} \phi^4 + \frac{\lambda_m}{4} \phi^2 \sigma^2 + \frac{\lambda_\sigma}{4!} \lambda_\sigma \sigma^4 + \dots; \quad (\lambda_m < 0),$$

$$\rightarrow \frac{\lambda_\phi}{4} \left( \phi^2 + \frac{3\lambda_m}{\lambda_\phi} \sigma^2 \right)^2 + \text{loops.}$$

$$\min \langle \sigma \rangle \neq 0: \quad \begin{array}{l} \mathbf{1)} \quad 9\lambda_m^2 = \lambda_\phi \lambda_\sigma + \text{loops} \\ \mathbf{2)} \quad \frac{\langle \phi \rangle^2}{\langle \sigma \rangle^2} = \frac{-3\lambda_m}{\lambda_\phi} (1 + \text{loops}) \end{array}$$

~~SI~~: tuning (i) or  $V_{\min} = 0 \Rightarrow$  EWSB;  $m_{\tilde{\phi}} \approx \lambda_\phi \langle \phi \rangle^2 = (-3)\lambda_m \langle \sigma \rangle^2 \ll \langle \sigma \rangle^2$  if  $|\lambda_m| \ll \lambda_\phi$  one classical tuning



• **SM+dilaton: one-loop SI potential**

[Shaposhnikov et al 2008; DG, Lalak, Olszewski]

$$\begin{aligned}
 V &= \frac{\lambda_\phi}{3!} (H^\dagger H)^2 + \frac{\lambda_m}{2} (H^\dagger H) \sigma^2 + \frac{\lambda_\sigma}{4!} \sigma^4 + \frac{4\lambda_6}{3} \frac{(H^\dagger H)^3}{\sigma^2} + \dots \\
 &= \frac{\lambda_\phi}{4!} \phi^4 + \frac{\lambda_m}{4} \phi^2 \sigma^2 + \frac{\lambda_\sigma}{4!} \lambda_\sigma \sigma^4 + \frac{\lambda_6}{6} \frac{\phi^6}{\sigma^2} \dots; (\lambda_m < 0), \\
 &\rightarrow \frac{\lambda_\phi}{4} \left( \phi^2 + \frac{3\lambda_m}{\lambda_\phi} \sigma^2 \right)^2 + \text{loops}.
 \end{aligned}$$

**1)**  $9\lambda_m^2 = \lambda_\phi \lambda_\sigma + \text{loops}$   
**2)**  $\frac{\langle \phi \rangle^2}{\langle \sigma \rangle^2} = \frac{-3\lambda_m}{\lambda_\phi} (1 + \text{loops})$

min  $\langle \sigma \rangle \neq 0$ :

$\cancel{SI}$ : tuning (i) or  $V_{\min} = 0 \Rightarrow$  EWSB;  $m_{\tilde{\phi}} \approx \lambda_\phi \langle \phi \rangle^2 = (-3)\lambda_m \langle \sigma \rangle^2 \ll \langle \sigma \rangle^2$  if  $|\lambda_m| \ll \lambda_\phi$  one classical tuning

$$V^{(1)} \equiv \sum_{j=\phi, \sigma; G, t, W, Z} \frac{n_j m_j^4(\phi, \sigma)}{4\kappa} \ln \frac{m_j^2(\phi, \sigma)}{c_j (z\sigma)^2}, \quad \text{'old CW' } [\mu \rightarrow z\sigma]$$

$$\begin{aligned}
 V^{(1,n)} &\equiv \frac{1}{48\kappa} \left[ (-16\lambda_m \lambda_\phi - 18\lambda_m^2 + \lambda_\phi \lambda_\sigma) \phi^4 - \lambda_m (48\lambda_m + 25\lambda_\sigma) \phi^2 \sigma^2 - 7\lambda_\sigma^2 \sigma^4 \right. \\
 &\quad \left. + (\lambda_\phi \lambda_m + 6\lambda_6 \lambda_\sigma) \frac{\phi^6}{\sigma^2} + 8(4\lambda_\phi - 2\lambda_m) \lambda_6 \frac{\phi^8}{\sigma^4} + (192\lambda_6 + 2\lambda_\phi) \lambda_6 \frac{\phi^{10}}{\sigma^6} + 40\lambda_6^2 \frac{\phi^{12}}{\sigma^8} \right], \quad \text{large } \sigma : S_v \times S_h
 \end{aligned}$$

$$\Delta V = \frac{\lambda_p}{p} \frac{\phi^p}{\sigma^{p-4}}, \quad p = 6, 8, 10, 12. \quad \rightarrow 0 \text{ if } \phi \ll \sigma \text{ and } \lambda_m \rightarrow 0. \quad \text{no } \lambda_\phi^2 \phi^2 \sigma^2 \sim \lambda_\phi \phi^2 \langle \sigma \rangle^2$$

One-loop corrected potential  $U = V + V^{(1)} + V^{(1,n)}$  gives:

$$\Delta m_{\tilde{\phi}}^2 = \frac{-\lambda_m \langle \sigma \rangle^2}{\lambda_\phi 16\kappa} \left\{ 27 \left[ g^4 \left( \ln \frac{g^2}{4} + \frac{1}{3} \right) + 2 g_2^4 \left( \ln \frac{g_2^2}{4} + \frac{1}{3} \right) - 16 h_t^4 \left( \ln \frac{h_t^2}{2} - \frac{1}{3} \right) \right] \right. \\ \left. + 4 \lambda_\phi^2 \left[ 5 \ln \frac{\lambda_\phi^2}{12} - 8 + \ln 27 \right] \right\} = \lambda_m \lambda_\phi \langle \sigma \rangle^2 + \dots \ll \langle \sigma \rangle^2, \text{ but no } \lambda_\phi^2 \langle \sigma \rangle^2.$$

- no tuning of  $\lambda_m$  beyond classical one:  $\beta_{\lambda_m} \propto \lambda_m$ , so  $\lambda_m$  stays ultraweak, natural.
- classical hierarchy of vev's:  $\langle \phi \rangle \ll \langle \sigma \rangle$  protected by quantum scale inv  $S_v \times S_h$ , (all orders, spont ~~S~~)
- All scales from fields' vev's  $\Rightarrow$  result is **not** a DR artefact! More scalars?

- **Curved spacetime** - Einstein gravity breaks  $S_v \times S_h$  by  $(\xi_1 \phi^2 + \xi_2 \sigma^2) R \Rightarrow \beta_{\lambda_m} = \xi_1(\dots) + \xi_2(\dots) + \lambda_m(\dots)$ 
  - what happens to the flat direction (dilaton)?
  - hierarchy of vev's (Higgs vs dilaton)?

• Einstein Gravity  $\rightarrow$  Weyl gauge symmetry:

$$g'_{\mu\nu}(x) = e^{2\alpha(x)} g_{\mu\nu}(x), \quad \phi'_j(x) = e^{-\alpha(x)} \phi_j(x), \quad (\text{a})$$

Wanted:  $L_E$  by spont breaking. An invariant of (a)

$$\phi_j : \phi, \sigma$$

$$L_E = -\frac{1}{2}\sqrt{g}M_p^2 R, \quad \rightarrow \quad L_1 = -\sqrt{g} \frac{\xi_j}{2} \left[ \frac{1}{6} \phi_j^2 R + g^{\mu\nu} \partial_\mu \phi_j \partial_\nu \phi_j \right] - V(\phi_j)$$

1) ghost (or  $G_N < 0$ ), 2) Fake conf symmetry?<sup>[Jackiw, Pi 2015]</sup> 3) Go to Einstein frame: # dof violated! **Avoid.**

[‘something’ is missing here....]

$$\Rightarrow \text{Add: } \omega'_\mu = \omega_\mu - \frac{2}{q} \partial_\mu \alpha(x) \quad (\text{b})$$

$$L_2 = \frac{\sqrt{g}}{2}(1 + \xi_j) g^{\mu\nu} \tilde{D}_\mu \phi_j \tilde{D}_\nu \phi_j, \quad \text{invariant,} \quad \tilde{D}_\mu \phi \equiv (\partial_\mu - \frac{q}{2} \omega_\mu) \phi$$

$$L_{12} \equiv L_1 + L_2 = \sqrt{g} \left\{ -\frac{\xi_j}{12} \phi_j^2 R + \frac{g^{\mu\nu}}{2} (\partial_\mu \phi_j) (\partial_\nu \phi_j) - \frac{q}{4} g^{\mu\nu} \omega_\mu \partial_\nu K + \frac{q^2}{8} K \omega_\mu \omega^\mu - V(\phi_j) \right\},$$

$$L_G = -\frac{\sqrt{g}}{4} F_{\mu\nu} F^{\mu\nu}, \quad F_{\mu\nu} = \partial_\mu \omega_\nu - \partial_\nu \omega_\mu, \quad K \equiv (1 + \xi_j) \phi_j^2.$$

$\Rightarrow L = L_{12} + L_G$ : Weyl gravity, gauge invariant under (a), (b) for any  $\xi_j$ , no ghost,  $G_N > 0$ . Dirac 1973

• Riemann  $\rightarrow$  Weyl geometry:

Riemann:  $D_\mu g_{\alpha\beta} = 0$ , with  $\Gamma_{\mu\nu}^\rho$

Weyl:  $\tilde{\nabla}_\mu g_{\alpha\beta} = -q \omega_\mu g_{\alpha\beta}$ ,  $\Rightarrow (\tilde{\nabla}_\mu + q \omega_\mu) g_{\alpha\beta} = D_\mu g_{\alpha\beta} = 0$ ,  $\tilde{\nabla}_\mu$ : Weyl-covariant deriv with  $\tilde{\Gamma}_{\mu\nu}^\rho$

$$\tilde{\Gamma}_{\mu\nu}^\rho = \Gamma_{\mu\nu}^\rho + \frac{q}{2} \left[ \delta_\mu^\rho \omega_\nu + \delta_\nu^\rho \omega_\mu - g_{\mu\nu} \omega^\rho \right], \text{ no torsion; invariant (a), (b)}$$

$$\Rightarrow \tilde{R} = R - 3q D_\mu \omega^\mu - \frac{3}{2} q^2 \omega^\mu \omega_\mu. \Rightarrow \tilde{R} \rightarrow \tilde{R}' = e^{-2\alpha(x)} \tilde{R}. \quad (\text{like } \phi^2!)$$

Simplest Lagrangian:  $L_{1w} = \frac{-\sqrt{g}}{12} \xi_j \phi_j^2 \tilde{R}$ : invariant!  $L_{2w} = \sqrt{g} \left[ \frac{g^{\mu\nu}}{2} \tilde{D}_\mu \phi_j \tilde{D}_\nu \phi_j - V(\phi_j) \right]$ ; invariant!

$$\Rightarrow L_{1w} + L_{2w} + L_G = L_{12} + L_G + \text{tot deriv} = L, \quad (L_G \text{ and } F_{\mu\nu} \text{ unchanged})$$

[Smolin 1979; Cheng 1988; H. M. Lee and D.G. 2018]

•  $L$  is not most general Weyl gravity: missing:  $\sqrt{g} \tilde{R}^2$  and  $\sqrt{g} \tilde{C}_{\mu\nu\rho\sigma} \tilde{C}^{\mu\nu\rho\sigma}$ , each invariant under (a),(b)

• If  $\omega_\mu \rightarrow 0$ : Weyl geometry  $\rightarrow$  Riemann geometry:  $\tilde{\Gamma}_{\mu\nu}^\rho \rightarrow \Gamma_{\mu\nu}^\rho$ ,  $\tilde{R} \rightarrow R$  and Weyl tensor  $\tilde{C}_{\mu\nu\rho\sigma} \rightarrow C_{\mu\nu\rho\sigma}$

$\Rightarrow$  Let us first turn off matter  $\phi_j$

note:  $\tilde{X}$  denotes the expression of  $X$  in Weyl-geometry.

- No matter: Weyl quadratic gravity  $\rightarrow$  Einstein gravity + massive  $\omega_\mu$

$$\mathcal{L}_1 = \sqrt{g} \frac{\xi_0}{4!} \tilde{R}^2, \quad \xi_0 > 0, \quad \mathcal{L}_1 + \mathcal{L}_2: \text{ Weyl quadratic action}$$

$$\mathcal{L}_2 = \frac{\sqrt{g}}{\zeta} \tilde{C}_{\mu\nu\rho\sigma} \tilde{C}^{\mu\nu\rho\sigma} = \frac{\sqrt{g}}{\zeta} \left[ C_{\mu\nu\rho\sigma} C^{\mu\nu\rho\sigma} + \frac{3}{2} q^2 F_{\mu\nu} F^{\mu\nu} \right]$$

+ Gauss-Bonnet.

So:

$$\begin{aligned} \mathcal{L}_1 &= \sqrt{g} \frac{\xi_0}{4!} \left[ -2 \phi_0^2 \tilde{R} - \phi_0^4 \right] & \text{eom: } \phi_0^2 &= -\tilde{R}; \quad \text{dilaton: } \ln \phi_0 \rightarrow \ln \phi_0 - \ln \Omega \\ &= \sqrt{g} \left[ -\frac{\xi_0}{12} \phi_0^2 R - \frac{q}{4} g^{\mu\nu} \omega_\mu \partial_\nu K + \frac{q^2}{8} g^{\mu\nu} K \omega_\mu \omega_\nu - \frac{\xi_0}{4!} \phi_0^4 \right] + \text{total deriv,} & K &= \xi_0 \phi_0^2 \end{aligned}$$

-  $\mathcal{L}_1$  has no k.t. for  $\omega_\mu$ : integrating  $\omega_\mu \Rightarrow \mathcal{L}_{\text{eff}} = \sqrt{g} (-\xi_0/6) \left[ 1/6 \phi_0^2 R + g^{\mu\nu} \partial_\mu \phi_0 \partial_\nu \phi_0 \right] - \xi_0/4! \phi_0^4.$

- Include  $\mathcal{L}_2$  or only  $\delta\mathcal{L}_2 = \frac{-1}{4} \sqrt{g} F_{\mu\nu} F^{\mu\nu}$ :

- Consider  $\mathcal{L} = \mathcal{L}_1 + \delta\mathcal{L}_2$ : then: 
$$\partial^\alpha (F_{\alpha\mu} \sqrt{g}) + \frac{1}{2} \sqrt{g} \xi_0 \phi_0 \left[ \partial_\mu - \frac{q}{2} \omega_\mu \right] \phi_0 = 0$$
  
conserved current

Canonical Einstein term  $\rightarrow$  Einstein gauge (“frame”)

$$\hat{g}_{\mu\nu} = \Omega g_{\mu\nu}, \quad \phi'_0 = \frac{1}{\sqrt{\Omega}} \phi_0, \quad \Omega \equiv \frac{\xi_0 \phi_0^2}{6 M^2},$$

$$M \sim \langle \phi_0 \rangle$$

Then: 
$$\mathcal{L}_1 = \sqrt{\hat{g}} \left[ \frac{-1}{2} M^2 \hat{R} + \frac{3 M^2}{4 \Omega^2} \hat{g}^{\mu\nu} (\partial_\mu \Omega)(\partial_\nu \Omega) + \frac{\hat{g}^{\mu\nu}}{\Omega} \left( \frac{-q}{4} \omega_\mu \partial_\nu K + \frac{q^2}{8} K \omega_\mu \omega_\nu - \frac{\xi_0 \phi_0^4}{4! \Omega} \right) \right].$$

$$K = \xi_0 \phi_0^2$$

Also 
$$\omega'_\mu = \omega_\mu - \frac{1}{q} \partial_\mu \ln \Omega,$$

$$\Rightarrow \mathcal{L}_1 + \delta \mathcal{L}_2 = \sqrt{\hat{g}} \left[ -\frac{1}{2} M^2 \hat{R} - \frac{3 M^4}{2 \xi_0} \right] + \sqrt{\hat{g}} \left[ -\frac{1}{4} \hat{g}^{\mu\rho} \hat{g}^{\nu\sigma} F'_{\mu\nu} F'_{\rho\sigma} + \frac{3 q^2}{4} M^2 \hat{g}^{\mu\nu} \omega'_\mu \omega'_\nu \right]$$

Weyl quadratic action

Einstein

Proca

[D.G. 1812.08613]

$$m_{\omega'}^2 = (3/2) q^2 M^2.$$

- **Stueckelberg:**  $\phi_0$  eaten by  $\omega_\mu$ : massive;  $\#$  dof conserved (=3):  $\phi_0$ +massless  $\omega_\mu \rightarrow$  no  $\phi_0$ +massive  $\omega_\mu$

- Below  $m_\omega \sim M_p$ , Weyl geometry  $\rightarrow$  Riemann geometry ( $\omega_\mu=0$ ): Avoids criticisms of Weyl gravity!

$\Rightarrow$  SB: Weyl quadratic gravity (no matter)= Einstein action (+ $\Lambda$ ) + massive Weyl  $\omega_\mu$  action.

$\Rightarrow$  The same Stueckelberg mechanism also works in the linear Weyl gravity case, with  $L$  of page 10 and 11.

• Adding matter: Weyl gravity  $\rightarrow$  EW breaking:

Add  $\phi_1$  higgs field

In Weyl-geometry language:

$$\begin{aligned}\mathcal{L} &= \sqrt{g} \left[ \frac{\xi_0}{4!} \tilde{R}^2 - \frac{\sqrt{g}}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{12} \xi_1 \phi_1^2 \tilde{R} + \frac{1}{2} g^{\mu\nu} \tilde{D}_\mu \phi_1 \tilde{D}_\nu \phi_1 - \frac{\lambda_1}{4!} \phi_1^4 \right] & \tilde{D}_\mu \phi_1 &\equiv \partial_\mu - \frac{q}{2} \omega_\mu \\ &= \sqrt{g} \left[ \frac{\xi_0}{4!} (-2\phi_0^2 \tilde{R} - \phi_0^4) - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{12} \xi_1 \phi_1^2 \tilde{R} + \frac{1}{2} g^{\mu\nu} \tilde{D}_\mu \phi_1 \tilde{D}_\nu \phi_1 - \frac{\lambda_1}{4!} \phi_1^4 \right],\end{aligned}$$

In Riemannian language:

$$\begin{aligned}\mathcal{L} &= \sqrt{g} \left[ -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{12} (\xi_0 \phi_0^2 + \xi_1 \phi_1^2) R - \frac{q}{4} \omega^\mu \partial_\mu \mathcal{K} + \frac{q^2}{8} \mathcal{K} \omega^\mu \omega_\mu + \frac{1}{2} g^{\mu\nu} \partial_\mu \phi_1 \partial_\nu \phi_1 - \mathcal{V} \right] \\ \mathcal{V} &= \frac{\xi_0}{4!} \phi_0^4 + \frac{\lambda_1}{4!} \phi_1^4, & \mathcal{K} &= \xi_0 \phi_0^2 + (1 + \xi_1) \phi_1^2,\end{aligned}$$

Einstein gauge: only one kinetic term left!

$$\hat{g}_{\mu\nu} = \Omega g_{\mu\nu}, \quad \Omega = \frac{1}{6M^2} (\xi_0 \phi_0^2 + \xi_1 \phi_1^2), \quad \mathcal{Z} = \xi_0 \frac{\phi_0^2}{\phi_1^2} + \xi_1, \quad \mathcal{L} \Big|_{k.t.} = \frac{3}{4} \frac{\sqrt{M^4 \hat{g}}}{1 + \mathcal{Z}} \partial^\mu \ln \mathcal{Z} \partial_\mu \ln \mathcal{Z}$$

Use:  $\phi_0 = \frac{\rho \sin \theta}{\sqrt{\xi_0}}, \quad \phi_1 = \frac{\rho \cos \theta}{\sqrt{1 + \xi_1}}, \quad \Rightarrow \quad \mathcal{K} = \rho^2, \quad \mathcal{L}|_{k.t.} = \frac{\sqrt{g}}{2} M^2 f(\theta) (\partial^\mu \cot \theta) (\partial_\mu \cot \theta)$

Then  $\omega'_\mu = \omega_\mu - \frac{1}{q} \partial_\mu \ln \rho^2$ , dilaton  $\rho$  “eaten” by  $\omega'_\mu$ .  $\frac{\tilde{h}}{M} = \frac{\cot \theta}{\sqrt{1 + \xi_1}} (1 + c_1 \cot^2 \theta + \dots) \ll 1$ .

$$\mathcal{L} = \sqrt{\hat{g}} \left[ -\frac{M^2}{2} \hat{R} + \frac{\hat{g}^{\mu\nu}}{2} (\partial_\mu \tilde{h}) (\partial_\nu \tilde{h}) - \frac{1}{4} F'_{\mu\nu} F'^{\mu\nu} + \frac{m^2(\tilde{h})}{2} \omega'_\mu \omega'^\mu - \hat{\mathcal{V}}(\tilde{h}) \right], \quad \text{with}$$

$$m^2(\tilde{h}) = \frac{3q^2 M^2}{2} \left[ 1 + \frac{\tilde{h}^2}{6M^2} + \mathcal{O}\left(\frac{\tilde{h}^4}{M^4}\right) \right], \quad \hat{\mathcal{V}}(\tilde{h}) = \frac{3M^4}{2\xi_0} - \frac{\xi_1 M^2}{2\xi_0} \tilde{h}^2 + \left( \frac{\lambda_1}{4!} + \frac{\xi_1 (3\xi_1 - 2)}{72\xi_0} \right) \tilde{h}^4 + \mathcal{O}\left(\frac{\tilde{h}^6}{M^6}\right)$$

$\Rightarrow$  Einstein action +  $\Lambda$  + Higgs + massive  $\omega'_\mu$ ; # dof (=4) is conserved (Jordan: 2 scalars + massless  $\omega_\mu$ ).

$\Rightarrow$  EWSB breaking;  $m_{higgs}^2 = \xi_1/\xi_0 M^2$ . Quantum stable? The limit  $\xi_1 \rightarrow 0 \Rightarrow S_v \times S_h \Rightarrow \beta_{\xi_1} \propto \xi_1!$   
not  $(\xi + 1/6)!$

$\Rightarrow$  renormalizable Weyl quadratic gravity?  $\tilde{R}^2 + \tilde{C}_{\mu\nu\rho\sigma}^2$ ? (quadratic gravity renorm [K. Stelle 1977]).



## • Conclusions

- I. Flat space:
- Quantum SI:  $V$  in  $\lambda\phi^4$  (3loop) & SM (1 loop) spontaneous breaking (SB)
  - classical hierarchy of: higgs vev  $\ll$  dilaton vev  $\Rightarrow$  quantum stable
  - SI effective operators, suppressed by dilaton
  - all scales from fields' vevs, result not a DR artefact
- II. Curved space:
- $M_{\text{Planck}} \sim$  dilaton vev  $\Rightarrow$  Weyl gravity/gauge symmetry
  - Stueckelberg: dilaton “eaten” by Weyl gauge field  $\omega_\mu$
  - $\rightarrow$  SB: Weyl quadratic gravity (no matter) = Einstein gravity + massive  $\omega_\mu$
  - Weyl geometry  $\Rightarrow$  Riemann geometry (near  $m_\omega \sim M_{\text{Planck}}$ )
  - Adding matter (higgs)  $\xi_1\phi_1^2\tilde{R}$ :  $\rightarrow$  EWSB;  $m_h$  quantum stable?
  - renormalizability of Weyl quadratic gravity  $\tilde{R}^2 + \tilde{C}_{\mu\nu\rho\sigma}^2$ ?

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 BACKUP SLIDES
 

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- **Two-loop: No new poles.** Two-loop  $\beta_\lambda^{(2)}$ , anom dims  $\gamma_\phi^{(2)}$  unchanged. Usual counterterm:

$$\delta L_2 = \frac{1}{2} (\partial_\mu \phi)^2 \delta_\phi^{(2)} - \mu(\sigma)^{2\epsilon} \frac{1}{4!} \lambda \phi^4 \delta_\lambda^{(2)}, \quad \delta_\lambda^{(2)} = \frac{\lambda^2}{\kappa^2} \left( \frac{9}{4\epsilon^2} - \frac{3}{2\epsilon} \right), \quad \delta_\phi^{(2)} = \frac{-\lambda^2}{24\kappa^2\epsilon}.$$

$$\beta_\lambda^{(2)} = -\frac{17}{3\kappa^2} \lambda^3, \quad \text{unchanged (as if no dilaton \& } \mu = \text{const)}. \quad \gamma_\sigma^{(2)} = 0.$$

Callan-Symanzik (consistency check):

$$\frac{\partial V^{(2)}}{\partial \ln z} + \left[ \beta_\lambda^{(2)} \frac{\partial}{\partial \lambda} + \gamma_\phi^{(2)} \phi \frac{\partial}{\partial \phi} + \gamma_\sigma^{(2)} \sigma \frac{\partial}{\partial \sigma} \right] V + \beta_\lambda^{(1)} \frac{\partial V^{(1)}}{\partial \lambda} = O(\lambda^4), \quad \text{"usual" CS}$$

$$\frac{\partial V^{(2,n)}}{\partial \ln z} = O(\lambda^4), \quad \beta^{(k)}, \gamma^{(k)}, V^{(k)}: \text{ k-loop correction.}$$

- if  $V \rightarrow V + \frac{\lambda_m}{2} \phi^2 \sigma^2$ ,  $V^{(2,n)} \supset \frac{\lambda \lambda_m}{96\kappa^2 \epsilon} \left\{ 7(2\lambda_m - \lambda) \frac{\phi^6}{\sigma^2} - \frac{3\lambda_m \phi^8}{2\sigma^4} \right\} \Rightarrow \beta_\lambda \rightarrow \beta_\lambda + \frac{\lambda_m}{\kappa^2} \left[ 12\lambda_m^2 - 7\lambda_m \lambda - 40\lambda^2 \right]$

$$\beta_{\lambda_m} \propto \lambda_m^2; \quad \lambda_m \rightarrow 0: S_v \times S_h.$$

- **Two-loop SI potential:** Taylor expand about  $\sigma = \langle \sigma \rangle + \tilde{\sigma}$  :

$$U = \frac{\lambda}{4!} \phi^4 \left\{ 1 + \frac{3\lambda}{2\kappa} \left( \overline{\ln} \frac{V_{\phi\phi}}{(z\langle\sigma\rangle)^2} - \frac{1}{2} \right) + \frac{3\lambda^2}{4\kappa^2} \left( 4 + A_0 - 4 \overline{\ln} \frac{V_{\phi\phi}}{(z\langle\sigma\rangle)^2} + 3 \overline{\ln}^2 \frac{V_{\phi\phi}}{(z\langle\sigma\rangle)^2} \right) + \mathcal{O}\left(\frac{1}{\langle\sigma\rangle}\right) \right\}$$

- This is the **“usual” CW result** with  $\mu = \langle \sigma \rangle$ , broken SI, **no dilaton present**. [Cheng, I. Jack, T. Jones, S. Martin]
- new terms comparable/larger than standard two-loop terms

$$\frac{\phi^n}{\sigma^n} \sim 1., \quad n = 1, 2; \quad \frac{\phi}{\sigma} = \frac{\phi}{\langle\sigma\rangle} \left( 1 - \frac{\tilde{\sigma}}{\langle\sigma\rangle} + \frac{\tilde{\sigma}^2}{\langle\sigma\rangle^2} + \dots \right)$$

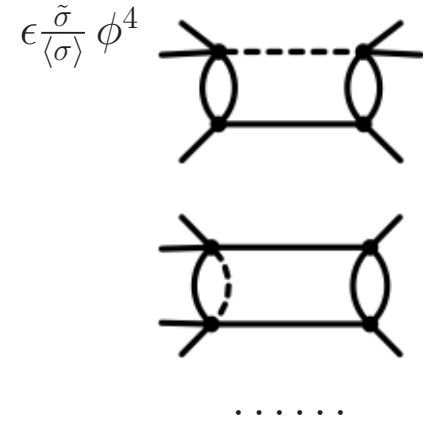
- Non-polynomial terms:
  - vanish for  $\phi \ll \sigma$ , only  $\log \sigma$  is left.
  - respect symmetries of the theory.
  - finite counterterms, cannot be “seen” in a scheme that breaks this symmetry.
  - if not forbidden (by a symmetry), operators generated at quantum level.
  - non-renormalizability.

• **Three-loop SI potential: Counterterms!**

$$\delta L_3 = \frac{1}{2} \delta_\phi^{(3)} (\partial_\mu \phi)^2 - \mu^{2\epsilon} \left\{ \frac{1}{4!} \delta_\lambda^{(3)} \lambda \phi^4 + \frac{1}{6} \delta_{\lambda_6}^{(3)} \lambda_6 \frac{\phi^6}{\sigma^2} + \frac{1}{8} \delta_{\lambda_8}^{(3)} \lambda_8 \frac{\phi^8}{\sigma^4} \right\}$$

$$\delta_\phi^{(3)} = -\frac{\lambda^3}{4\kappa^3} \left( \frac{1}{6\epsilon^2} - \frac{1}{12\epsilon} \right), \quad \delta_{\lambda_6}^{(3)} = \frac{3}{2} \frac{\lambda^4}{\lambda_6 \kappa^3 \epsilon}, \quad \delta_{\lambda_8}^{(3)} = \frac{275}{864} \frac{\lambda^4}{\lambda_8 \kappa^3 \epsilon}.$$

[Monin 2015]



So  $\gamma_\phi^{(3)} = \lambda^3/(16\kappa^3)$ . With  $Z_X = 1 + \delta_X$  and  $\lambda_6^B = \mu^{2\epsilon}(\sigma)\lambda_6 Z_{\lambda_6} Z_\phi^{-3} Z_\sigma$  and  $(d/d \ln z) \lambda_6^B = 0$ ,

$$\Rightarrow \beta_{\lambda_6} = \frac{\lambda^2 \lambda_6}{2\kappa^2} + \frac{\lambda^3}{\kappa^3} \left( 9\lambda - \frac{3}{8}\lambda_6 \right), \quad (\text{similar } \beta_{\lambda_8}).$$

Callan-Symanzik:  $V^{(3)}$  [“usual” with  $\mu \rightarrow \sigma$ ] +  $V^{(3,n)}$  [new]:

$$\frac{\partial V^{(3)}}{\partial \ln z} + \beta_{\lambda}^{(1)} \frac{\partial V^{(2)}}{\partial \lambda} + \beta_{\lambda}^{(2)} \frac{\partial V^{(1)}}{\partial \lambda} + \beta_{\lambda}^{(3)} \frac{\partial V}{\partial \lambda} + \gamma_\phi^{(2)} \frac{\partial V^{(1)}}{\partial \ln \phi} + \gamma_\phi^{(3)} \frac{\partial V}{\partial \ln \phi} = \mathcal{O}(\lambda_j^5).$$

$$\frac{\partial V^{(3,n)}}{\partial \ln z} + \beta_{\lambda_j}^{(1)} \frac{\partial V^{(2,n)}}{\partial \lambda_j} + \beta_{\lambda_j}^{(3,n)} \frac{\partial V}{\partial \lambda_j} = \mathcal{O}(\lambda_j^5), \quad \lambda_j = \lambda, \lambda_6, \lambda_8.$$

- Dilatation current  $D_\mu$ : 
$$D^\mu = \frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi_j)} (x^\nu \partial_\nu \phi_j + d_\phi) - x^\mu \mathcal{L}, \quad d_\phi = (d-2)/2 \quad (\text{scalars}),$$

- in  $d = 4 - 2\epsilon$ , potential  $\tilde{V}$ : 
$$\begin{aligned} \partial_\mu D^\mu &= (d_\phi + 1) (\partial_\mu \phi_j) \frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi_j)} + d_\phi \phi_j \frac{\partial \mathcal{L}}{\partial \phi_j} - d \mathcal{L} \\ &= d \tilde{V} - \frac{d-2}{2} \phi_j \frac{\partial \tilde{V}}{\partial \phi_j}, \quad \phi_j = \phi, \sigma; \quad (\text{onshell, canonical k.t.}) \end{aligned}$$

- in SI theory:  $\tilde{V}$  homogeneous in  $d$  dim's:  $\tilde{V}(\rho \phi_j) = \rho^{2d/(d-2)} \tilde{V}(\phi_j) \Rightarrow \partial_\mu D^\mu = 0$ .

- in “usual” reg:  $\mu = \text{const}$ , no  $\sigma$ :  $\tilde{V} = \mu^{2\epsilon} V(\phi)$ , and  $V$  is scale inv in  $d = 4$ :

$$\partial_\mu D^\mu = 2\epsilon \mu^{2\epsilon} V \sim 2\epsilon \mu^{2\epsilon} \left[ \lambda + \frac{\beta_\lambda}{\epsilon} + \dots \right] \frac{\partial V}{\partial \lambda} \propto \beta_\lambda \frac{\partial V}{\partial \lambda}, \quad [=0 \text{ only if } \beta_\lambda = 0]. \quad (\text{scale anomaly})$$

- different field content! [Shaposhnikov et al, Tamarit 2013]

“For scale invariance, though, the situation is hopeless; any cutoff procedure necessarily involves a large mass and a large mass necessarily breaks scale invariance in a large way. This argument does not show that the occurrence of anomalies is inevitable [.....]” (S. Coleman: Aspects of Symmetry, p.82, 1985).

- **SM + dilaton SI potential:** one-loop Beta functions:

$$\beta_{\lambda_\phi} = \frac{1}{\kappa} \left[ 3 \left( \frac{9}{4}g_2^4 + \frac{3}{4}g_1^4 + \frac{3}{2}g_1^2g_2^2 - 12h_t^4 \right) - 4\lambda_\phi \left( \frac{3}{4}g_1^2 + \frac{9}{4}g_2^2 - 3h_t^2 \right) + 4\lambda_\phi^2 + 3\lambda_m^2 + 96\lambda_m\lambda_6 \right]$$

$$\beta_{\lambda_m} = \frac{2\lambda_m}{\kappa} \left[ \lambda_\phi + 2\lambda_m + \frac{1}{2}\lambda_\sigma - \left( \frac{3}{4}g_1^2 + \frac{9}{4}g_2^2 - 3h_t^2 \right) \right] \propto \lambda_m, \quad \Rightarrow \lambda_m \text{ stays small (f.p.).}$$

- for new couplings

$$\beta_{\lambda_6} = \frac{3\lambda_6}{\kappa} \left[ 6\lambda_\phi - 8\lambda_m + \lambda_\sigma - 2 \left( \frac{3}{4}g_1^2 + \frac{9}{4}g_2^2 - 3h_t^2 \right) \right]$$

$$\beta_{\lambda_8} = \frac{2}{\kappa} \left[ 2\lambda_6 (28\lambda_6 + \lambda_m) - 4\lambda_8 \left( \frac{3}{4}g_1^2 + \frac{9}{4}g_2^2 - 3h_t^2 \right) \right]$$

$$\beta_{\lambda_{10}} = 10 \left[ 4\lambda_6^2 - \lambda_{10} \left( \frac{3}{4}g_1^2 + \frac{9}{4}g_2^2 - 3h_t^2 \right) \right]$$

$$\beta_{\lambda_{12}} = 2 \left[ 3\lambda_6^2 - 6\lambda_{12} \left( \frac{3}{4}g_1^2 + \frac{9}{4}g_2^2 - 3h_t^2 \right) \right]$$

$\Rightarrow$  if one sets  $\lambda_{6,8,10,\dots} = 0$  at tree level, then  $\beta_{\lambda_{6,8,10,12}} = 0$  at one-loop, but emerge at 2-loops.