Description of exclusive diffractive processes in the shock wave approach

Lech Szymanowski

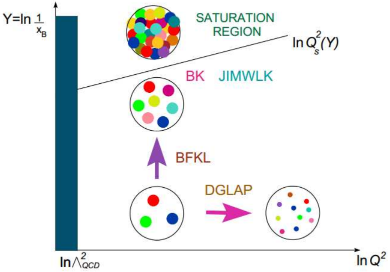
National Centre for Nuclear Research, Warsaw

in collaboration with:

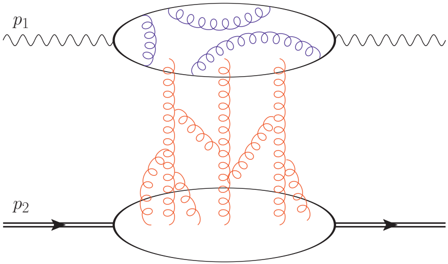
Renaud Boussarie, Andrey Grabovsky & Samuel Wallon

GDR, November 2018

The shockwave description of the Color Glass Condensate



Kinematics



$$p_1 = p^+ n_1 - \frac{Q^2}{2s} n_2$$

$$p_2 = \frac{m_t^2}{2p_2^-} n_1 + p_2^- n_2$$

$$p^+ \sim p_2^- \sim \sqrt{\frac{s}{2}}$$

Lightcone (Sudakov) vectors

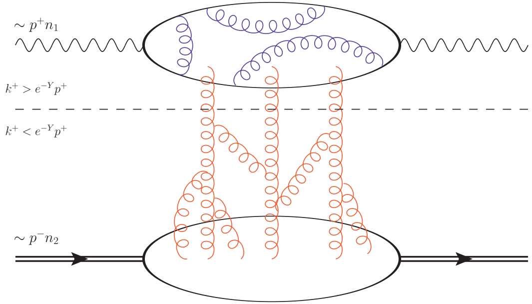
$$n_1 = \sqrt{\frac{1}{2}}(1, 0_\perp, 1), \quad n_2 = \sqrt{\frac{1}{2}}(1, 0_\perp, -1), \quad (n_1 \cdot n_2) = 1$$

Lightcone coordinates:

$$x = (x^0, x^1, x^2, x^3) \to (x^+, x^-, \vec{x})$$

$$x^+ = x_- = (x \cdot n_2) \quad x^- = x_+ = (x \cdot n_1)$$

Rapidity separation

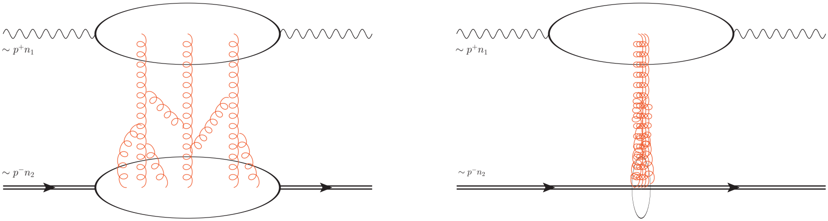


Let us split the gluonic field between "fast" and "slow" gluons

$$\mathcal{A}^{\mu a}(k^+, k^-, \vec{k}) = A_\eta^{\mu a}(|k^+| > e^\eta p^+, k^-, \vec{k}) + b_\eta^{\mu a}(|k^+| < e^\eta p^+, k^-, \vec{k})$$

$e^\eta = e^{-Y} \ll 1$

Large longitudinal boost to the projectile frame



$$b^+(x^+, x^-, \vec{x}) \longrightarrow \frac{1}{\Lambda} b^+(\Lambda x^+, \frac{x^-}{\Lambda}, \vec{x}) \to 0$$

$$b^-(x^+, x^-, \vec{x}) \longrightarrow \Lambda b^-(\Lambda x^+, \frac{x^-}{\Lambda}, \vec{x})$$

$$b^k(x^+, x^-, \vec{x}) \qquad \Lambda \sim \sqrt{\frac{s}{m_t^2}} \qquad b^k(\Lambda x^+, \frac{x^-}{\Lambda}, \vec{x}) \to 0$$

$$b^\mu(x) \to b^-(x)\, n_2^\mu = \delta(x^+)\, \mathbf{B}(\vec{x})\, n_2^\mu + O(\sqrt{\frac{m_t^2}{s}})$$

Shockwave approximation

no x^- dependence

Effective Feynman rules in the external shockwave field

Wilson lines :

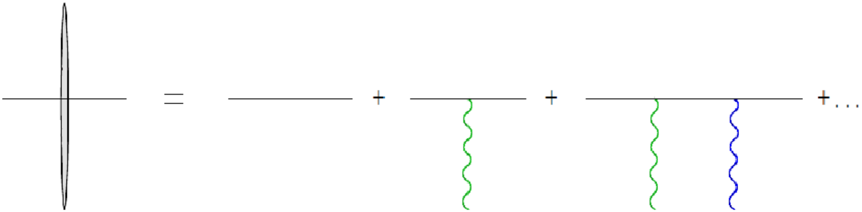
$$U_i^\eta = U_{\vec{z}_i}^\eta = P\exp\left[ig\int_{-\infty}^{+\infty} b_\eta^-(z_i^+,\vec{z}_i)\, dz_i^+\right]$$

Fourier transform of a Wilson line

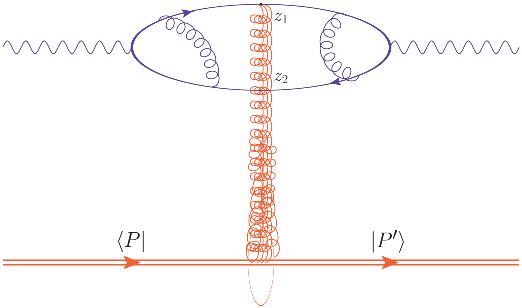
$$\tilde{U}^\eta(\vec{p}) = \int d^{D-2}\vec{z}\; e^{-i(\vec{p}\cdot\vec{z})}\, U_{\vec{z}}^\eta$$

$$U_i^\eta = 1 + ig\int_{-\infty}^{+\infty} b_\eta^-(z_i^+,\vec{z}_i)dz_i^+ + (ig)^2\int_{-\infty}^{+\infty} b_\eta^-(z_i^+,\vec{z}_i)b_\eta^-(z_j^+,\vec{z}_j)\theta(z_{ji}^+)dz_i^+dz_j^+$$

...



Factorized picture



Factorized amplitude

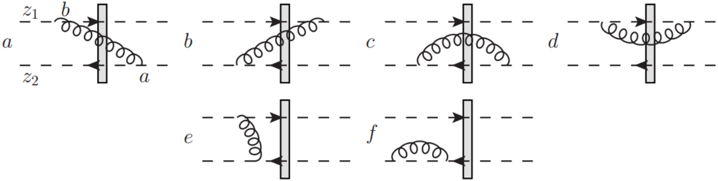
$$\mathcal{A}^\eta = \int d^{D-2}\vec{z}_1 d^{D-2}\vec{z}_2\, \Phi^\eta(\vec{z}_1, \vec{z}_2)\, \langle P'|[\mathrm{Tr}(U^\eta_{\vec{z}_1} U^{\eta\dagger}_{\vec{z}_2}) - N_c]|P\rangle$$

Dipole operator $\mathcal{U}^\eta_{ij} = \frac{1}{N_c}\mathrm{Tr}(U^\eta_{\vec{z}_i} U^{\eta\dagger}_{\vec{z}_j}) - 1$

Written similarly for any number of Wilson lines in any color representation!

Evolution for the dipole operator

$$\mathcal{U}_{12}^{\eta+\delta\eta} - \mathcal{U}_{12}^{\eta}$$



B-JIMWLK hierarchy of equations
[Balitsky, Jalilian-Marian, Iancu, McLerran, Weigert, Leonidov, Kovner]

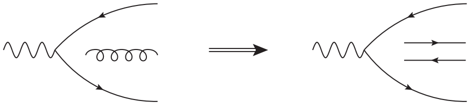
$$\frac{\partial \mathcal{U}_{12}^{\eta}}{\partial \eta} = \frac{\alpha_s N_c}{2\pi^2} \int d\vec{z}_3 \frac{\vec{z}_{12}^{\,2}}{\vec{z}_{13}^{\,2} \vec{z}_{23}^{\,2}} \left[\mathcal{U}_{13}^{\eta} + \mathcal{U}_{32}^{\eta} - \mathcal{U}_{12}^{\eta} + \mathcal{U}_{13}^{\eta} \mathcal{U}_{32}^{\eta} \right]$$

$$\frac{\partial \mathcal{U}_{13}^{\eta} \mathcal{U}_{32}^{\eta}}{\partial \eta} = \ldots$$

Evolves a dipole into a double dipole

The BK equation

Mean field approximation, or 't Hooft planar limit $N_c \to \infty$ in the dipole B-JIMWLK equation



[Mueller]

$\Rightarrow$ BK equation [Balitsky, 1995] [Kovchegov, 1999]

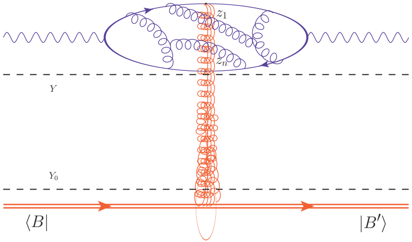
$$\frac{\partial \langle \mathcal{U}_{12}^{\eta} \rangle}{\partial \eta} = \frac{\alpha_s N_c}{2\pi^2} \int d\vec{z}_3 \, \frac{\vec{z}_{12}^{\,2}}{\vec{z}_{13}^{\,2}\vec{z}_{23}^{\,2}} \left[\langle \mathcal{U}_{13}^{\eta} \rangle + \langle \mathcal{U}_{32}^{\eta} \rangle - \langle \mathcal{U}_{12}^{\eta} \rangle + \langle \mathcal{U}_{13}^{\eta} \rangle \, \langle \mathcal{U}_{32}^{\eta} \rangle \right]$$

BFKL/BKP part Triple pomeron vertex

Non-linear term : saturation

Practical use of the formalism

- Compute the upper impact factor using the effective Feynman rules
- Build non-perturbative models for the matrix elements of the Wilson line operators acting on the target states
- Solve the B-JIMWLK evolution for these matrix elements with such non-perturbative initial conditions at a typical target rapidity $\eta = Y_0$
- Evaluate the solution at a typical projectile rapidity $\eta = Y$, or at the rapidity of the slowest gluon
- Convolute the solution and the impact factor



$$\mathcal{A} = \int d\vec{z}_1 ... d\vec{z}_n \; \Phi(\vec{z}_1, ..., \vec{z}_n) \times \langle P' | U_{\vec{z}_1} ... U_{\vec{z}_n} | P \rangle$$

Exclusive diffraction allows one to probe the $b_\perp$-dependence of the non-perturbative scattering amplitude

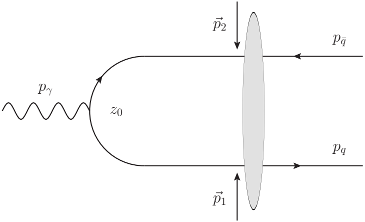
Open parton production at NLO

First step: open parton production

- Regge-Gribov limit : $s \gg Q^2 \gg \Lambda_{QCD}$
- Otherwise completely general kinematics
- Shockwave (CGC) Wilson line approach
- Transverse dimensional regularization $d = 2 + 2\varepsilon$, longitudinal cutoff

$$|p_g^+| > \alpha p_\gamma^+$$

LO diagram



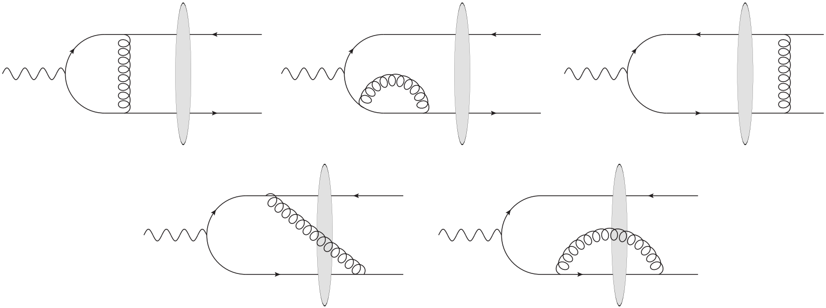
$$\mathcal{A} = \frac{\delta^{ik}}{\sqrt{N_c}} \int d^D z_0 [\bar{u}(p_q, z_0)]_{ij} (-ie_q) \hat{\varepsilon}_\gamma e^{-i(p_\gamma \cdot z_0)} [v(p_{\bar{q}}, z_0)]_{jk} \theta(-z_0^+)$$

$$= \delta(p_q^+ + p_{\bar{q}} - p_\gamma^+) \int d^d \vec{p}_1 d^d \vec{p}_2 \delta(\vec{p}_q + \vec{p}_{\bar{q}} - \vec{p}_\gamma - \vec{p}_1 - \vec{p}_2) \Phi_0(\vec{p}_1, \vec{p}_2)$$

$$\times \left\langle P' \middle| \tilde{\mathcal{U}}^\alpha(\vec{p}_1, \vec{p}_2) \middle| P \right\rangle$$

$$\tilde{\mathcal{U}}^\alpha(\vec{p}_1, \vec{p}_2) = \int d^d \vec{z}_1 d^d \vec{z}_2 \, e^{-i(\vec{p}_1 \cdot \vec{z}_1) - i(\vec{p}_2 \cdot \vec{z}_2)} [\tfrac{1}{N_c} \mathrm{Tr}(U^\alpha_{\vec{z}_1} U^{\alpha\dagger}_{\vec{z}_2}) - 1]$$

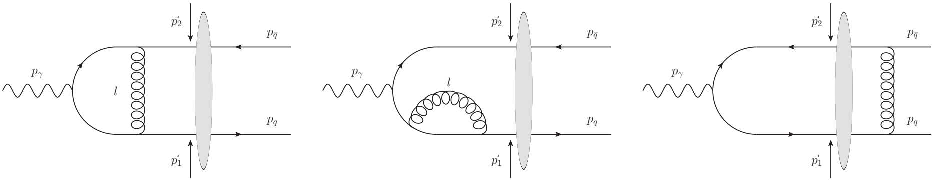
NLO open $q\bar{q}$ production



Diagrams contributing to the NLO correction

First kind of virtual corrections

no crossing of the shock wave



Color factor

$$\frac{C_F}{\sqrt{N_c}}\tilde{\mathcal{U}}^{\alpha}(\vec{p}_1,\vec{p}_2)$$

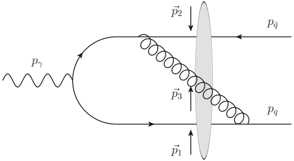
Impact factor

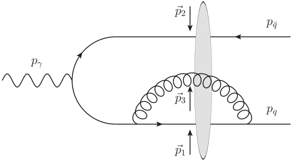
$$\mathcal{A}^{(1)}_{NLO} \propto \delta(p_q^+ + p_{\bar{q}} - p_\gamma^+)\int d^d\vec{p}_1 d^d\vec{p}_2 \delta(\vec{p}_q + \vec{p}_{\bar{q}} - \vec{p}_\gamma - \vec{p}_1 - \vec{p}_2)\,\Phi_{V1}(\vec{p}_1,\vec{p}_2)$$

$$\times C_F \left\langle P' \middle| \tilde{\mathcal{U}}^{\alpha}(\vec{p}_1,\vec{p}_2) \middle| P \right\rangle$$

Second kind of virtual corrections

crossing of the shock wave





Color factor

$$\frac{\delta^{ik}}{\sqrt{N_c}}(t^a U_1 t^b U_2^\dagger)_{ik}(U_3)^{ab}$$

Action of the Wilson line in the adjoint representation

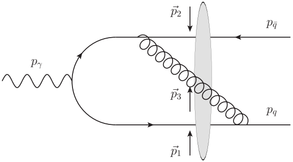
$$(U_3)^{ab} t^b = U_3 t^a U_3^\dagger \quad \Rightarrow \quad (U_3)^{ab} = 2\mathrm{Tr}(t^a U_3 t^b U_3^\dagger)$$

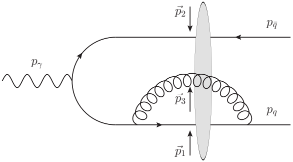
+ Fierz identity

$$C_F \mathcal{U}_{12} + \frac{1}{2}[\mathcal{U}_{13} + \mathcal{U}_{32} - \mathcal{U}_{12} + \mathcal{U}_{13}\mathcal{U}_{32}] \quad = C_F \mathcal{U}_{12} + \mathcal{W}_{123}$$

Second kind of virtual corrections

crossing of the shock wave



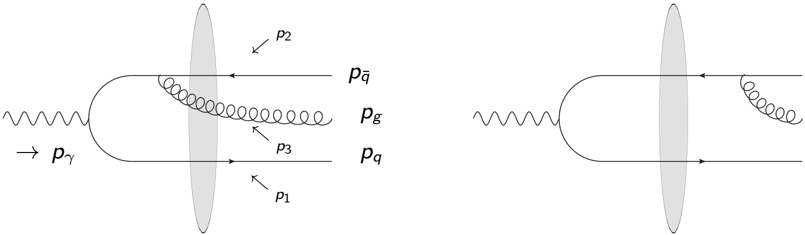


$$\mathcal{A}_{NLO}^{(2)} \propto \delta(p_q^+ + p_{\bar{q}} - p_\gamma^+) \int d^d\vec{p}_1 d^d\vec{p}_2 d^d\vec{p}_3\, \delta(\vec{p}_q + \vec{p}_{\bar{q}} - \vec{p}_\gamma - \vec{p}_1 - \vec{p}_2 - \vec{p}_3)$$

$$\times [\Phi'_{V1}(\vec{p}_1, \vec{p}_2) C_F \left\langle P' \right| \tilde{\mathcal{U}}^\alpha(\vec{p}_1, \vec{p}_2) \left| P \right\rangle (2\pi)^d \delta(\vec{p}_3)$$

$$+ \Phi_{V2}(\vec{p}_1, \vec{p}_2, \vec{p}_3) \left\langle P' \right| \tilde{\mathcal{W}}(\vec{p}_1, \vec{p}_2, \vec{p}_3) \left| P \right\rangle]$$

LO open $q\bar{q}g$ production



$$\mathcal{A}_R^{(2)} \propto \delta(p_q^+ + p_{\bar{q}} + p_g^+ - p_\gamma^+) \int d^d\vec{p}_1 d^d\vec{p}_2 d^d\vec{p}_3 \delta(\vec{p}_q + \vec{p}_{\bar{q}} + \vec{p}_g - \vec{p}_\gamma - \vec{p}_1 - \vec{p}_2 - \vec{p}_3)$$

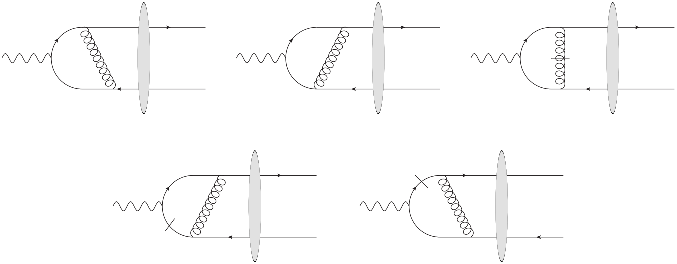
$$\times[\Phi'_{R1}(\vec{p}_1, \vec{p}_2) C_F \langle P' | \tilde{\mathcal{U}}^\alpha(\vec{p}_1, \vec{p}_2) | P \rangle (2\pi)^d \delta(\vec{p}_3)$$

$$+\Phi_{R2}(\vec{p}_1, \vec{p}_2, \vec{p}_3) \langle P' | \tilde{\mathcal{W}}(\vec{p}_1, \vec{p}_2, \vec{p}_3) | P \rangle]$$

$$\mathcal{A}_R^{(1)} \propto \delta(p_q^+ + p_{\bar{q}} + p_g^+ - p_\gamma^+) \int d^d\vec{p}_1 d^d\vec{p}_2 \delta(\vec{p}_q + \vec{p}_{\bar{q}} + \vec{p}_g - \vec{p}_\gamma - \vec{p}_1 - \vec{p}_2)$$

$$\times \Phi_{R1}(\vec{p}_1, \vec{p}_2) C_F \langle P' | \tilde{\mathcal{U}}^\alpha(\vec{p}_1, \vec{p}_2) | P \rangle$$

Generic computation method



- Perform the $k_\perp$ integration with the usual d-dimensional regularization methods
- Perform the k^+ integration with the longitudinal cutoff αp_γ^+ when possible, or isolate the divergent term by + prescription

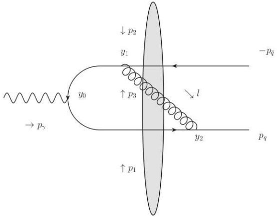
$$\int_{\alpha p_\gamma^+}^{p^+} dk^+ \frac{F(k^+)}{k^+} = \int_{\alpha p_\gamma^+}^{p^+} dk^+ \frac{F(0)}{k^+} + \int_0^{p^+} dk^+ \left[\frac{F(k^+)}{k^+}\right]_+$$

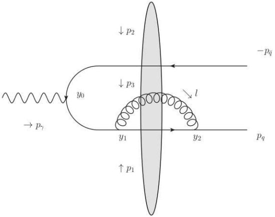
Divergences

Divergences

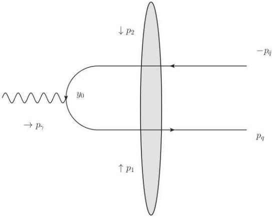
- Rapidity divergence $p_g^+ \to 0$ — $\Phi_{V2}\Phi_0^* + \Phi_0\Phi_{V2}^*$
- UV divergence $\vec{p}_g^{\,2} \to +\infty$ — $\Phi_{V1}\Phi_0^* + \Phi_0\Phi_{V1}^*$
- Soft divergence $p_g \to 0$ — $\Phi_{V1}\Phi_0^* + \Phi_0\Phi_{V1}^*, \Phi_{R1}\Phi_{R1}^*$
- Collinear divergence $p_g \propto p_q$ or $p_{\bar{q}}$ — $\Phi_{R1}\Phi_{R1}^*$
- Soft and collinear divergence $p_g = \frac{p_g^+}{p_q^+} p_q$ or $\frac{p_g^+}{p_{\bar{q}}^+} p_{\bar{q}}$, $p_g^+ \to 0$ — $\Phi_{R1}\Phi_{R1}^*$

Rapidity divergence





Double dipole virtual correction Φ_{V2}



B-JIMWLK evolution of the LO term : $\Phi_0 \otimes \mathcal{K}_{BK}$

Rapidity divergence

$$\mathcal{U}_{\vec{x}}^{e^\eta} - \mathcal{U}_{\vec{x}}^{\alpha} = \int_{\alpha}^{e^\eta} d\rho \frac{\partial \mathcal{U}_{\vec{x}}^{\rho}}{\partial \rho}$$

B-JIMWLK equation for the dipole operator

$$\frac{\partial \tilde{\mathcal{U}}_{12}^{\alpha}}{\partial \log \alpha} = 2\alpha_s N_c \mu^{2-d} \int \frac{d^d \vec{k}_1 d^d \vec{k}_2 d^d \vec{k}_3}{(2\pi)^{2d}} \delta(\vec{k}_1 + \vec{k}_2 + \vec{k}_3 - \vec{p}_1 - \vec{p}_2) \left(\tilde{\mathcal{U}}_{13}^{\alpha} \tilde{\mathcal{U}}_{32}^{\alpha} + \tilde{\mathcal{U}}_{13}^{\alpha} + \tilde{\mathcal{U}}_{32}^{\alpha} - \tilde{\mathcal{U}}_{12}^{\alpha} \right)$$
$$\times \left[2 \frac{(\vec{k}_1 - \vec{p}_1) \cdot (\vec{k}_2 - \vec{p}_2)}{(\vec{k}_1 - \vec{p}_1)^2 (\vec{k}_2 - \vec{p}_2)^2} + \frac{\pi^{\frac{d}{2}} \Gamma(1 - \frac{d}{2}) \Gamma^2(\frac{d}{2})}{\Gamma(d-1)} \left(\frac{\delta(\vec{k}_2 - \vec{p}_2)}{\left[(\vec{k}_1 - \vec{p}_1)^2 \right]^{1-\frac{d}{2}}} + \frac{\delta(\vec{k}_1 - \vec{p}_1)}{\left[(\vec{k}_2 - \vec{p}_2)^2 \right]^{1-\frac{d}{2}}} \right) \right]$$

η rapidity divide, which separates the upper and the lower impact factors

$$\Phi_0 \tilde{\mathcal{U}}_{12}^{\alpha} \to \Phi_0 \tilde{\mathcal{U}}_{12}^{\eta} + 2 \log \left(\frac{e^\eta}{\alpha} \right) \mathcal{K}_{BK} \Phi_0 \tilde{\mathcal{W}}_{123}$$

Rapidity divergence

Virtual contribution

$$(\Phi^{\mu}_{V2})_{div} \propto \Phi^{\mu}_{0} \left\{ 4 \ln\left(\frac{x\bar{x}}{\alpha^2}\right) \left[\frac{1}{\varepsilon} + \ln\left(\frac{\vec{p}_3^{\,2}}{\mu^2}\right)\right] - \frac{6}{\varepsilon} \right\}$$

BK contribution

$$(\Phi^{\mu}_{BK})_{div} \propto \Phi^{\mu}_{0} \left\{ 4 \ln\left(\frac{\alpha^2}{e^{2\eta}}\right) \left[\frac{1}{\varepsilon} + \ln\left(\frac{\vec{p}_3^{\,2}}{\mu^2}\right)\right] \right\}$$

Sum : the α dependence cancels

$$(\Phi'^{\mu}_{V2})_{div} \propto \Phi^{\mu}_{0} \left\{ 4 \ln\left(\frac{x\bar{x}}{e^{2\eta}}\right) \left[\frac{1}{\varepsilon} + \ln\left(\frac{\vec{p}_3^{\,2}}{\mu^2}\right)\right] - \frac{6}{\varepsilon} \right\}$$

Rapidity divergence

artificial UV pole

Fadin,Fiore,Grabovsky,Papa Nucl. Phys. 856

Cancellation of the remaining $1/\epsilon$ divergence

Convolution

$$
\begin{aligned}
(\Phi'^{\mu}_{V2} \otimes \mathcal{W}) &= 2\int d^d\vec{p}_1 d^d\vec{p}_2 d^d\vec{p}_3 \left\{ 4\ln\left(\frac{x\bar{x}}{e^{2\eta}}\right)\left[\frac{1}{\varepsilon} + \ln\left(\frac{\vec{p}_3^{\,2}}{\mu^2}\right)\right] - \frac{6}{\varepsilon} \right\} \\
&\times \quad \delta(\vec{p}_{q1} + \vec{p}_{\bar{q}2} - \vec{p}_3)\left[\tilde{\mathcal{U}}_{13} + \tilde{\mathcal{U}}_{32} - \tilde{\mathcal{U}}_{12} - \tilde{\mathcal{U}}_{13}\tilde{\mathcal{U}}_{32}\right]\Phi_0^{\mu}(\vec{p}_1, \vec{p}_2)
\end{aligned}
$$

- $\Phi_0(\vec{p}_1, \vec{p}_2)$ only depends on one of the t-channel momenta.
- The double-dipole operators cancels when $\vec{z}_3 = \vec{z}_1$ or $\vec{z}_3 = \vec{z}_2$.

The convolution cancels the remaining $\frac{1}{\varepsilon}$ divergence.

Then $\tilde{\mathcal{U}}^{\alpha}_{12}\Phi_0 + \Phi_{V2}$ is finite

Divergences

- Rapidity divergence
- UV divergence $\vec{p}_g^{\,2} \to +\infty$ $\qquad \Phi_{V1}\Phi_0^* + \Phi_0\Phi_{V1}^*$
- Soft divergence $p_g \to 0$ $\qquad \Phi_{V1}\Phi_0^* + \Phi_0\Phi_{V1}^*,\ \Phi_{R1}\Phi_{R1}^*$
- Collinear divergence $p_g \propto p_q$ or $p_{\bar{q}}$ $\qquad \Phi_{R1}\Phi_{R1}^*$
- Soft and collinear divergence $p_g = \frac{p_g^+}{p_q^+} p_q$ or $\frac{p_g^+}{p_{\bar{q}}^+} p_{\bar{q}},\ p_g^+ \to 0$ $\qquad \Phi_{R1}\Phi_{R1}^*$

Constructing a finite cross section

Exclusive diffractive production of a forward dijet

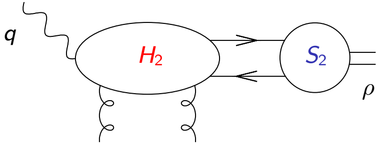
- Use a jet cone algorithm
- Combine real corrections and virtual corrections
- Possibility for precision study of the Dipole Wigner distribution at small x [Hatta, Xiao, Yuan]

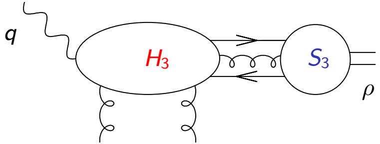
[R. Boussarie, A. Grabovsky, LS, S. Wallon]
JHEP 1611 (2016) ; arXiv:1606.00419

Collinear factorization for light vector meson production

Collinear factorization: basic principle

The impact factor is the convolution of a hard part and the vacuum-to-meson matrix element of an operator



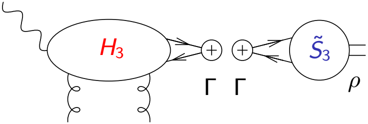


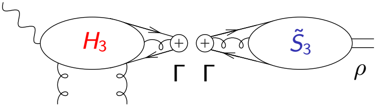
$$\int_x (H_2(x))_{ij}^{\alpha\beta} \left\langle \rho \left| \bar{\psi}_i^{\alpha}(x)\psi_j^{\beta}(0) \right| 0 \right\rangle \qquad \int_{x_1,x_2} (H_3^{\mu}(x_1,x_2))_{ij,c}^{\alpha\beta} \left\langle \rho \left| \bar{\psi}_i^{\alpha}(x_1)A_{\mu}^{c}(x_2)\psi_j^{\beta}(0) \right| 0 \right\rangle$$

H and S are by convolution and by summation over spinor and color indices

Spinorial and color factorization

Applying a simple Fierz decomposition in color space and in spinor space





$$\frac{1}{4N_c}\mathrm{Tr}_{c,D}\left[H_2\,\Gamma^\lambda\right]\left\langle\rho\left|\bar\psi\,\Gamma_\lambda\,\psi\right|0\right\rangle \qquad \frac{1}{4}\mathrm{Tr}_{c,D}\left[H_3^{\mu,c}\,t^c\Gamma^\lambda\right]\left\langle\rho\left|\bar\psi\,A_\mu\Gamma_\lambda\,\psi\right|0\right\rangle$$

We thus need to study the following matrix elements:

$$\left\langle\rho\left|\bar\psi\,\Gamma_\lambda\,\psi\right|0\right\rangle \quad \text{and} \quad \left\langle\rho\left|\bar\psi\,A_\mu\Gamma_\lambda\,\psi\right|0\right\rangle$$

Twist 2

Collinear factorization at twist 2

- Leading twist DA for a longitudinally polarized light vector meson

$$\left\langle \rho \left| \bar{\psi}(z)\gamma^{\mu}\psi(0) \right| 0 \right\rangle \to p^{\mu} f_{\rho} \int_0^1 dx e^{ix(p\cdot z)} \varphi_1(x)$$

- Leading twist DA for a transversely polarized light vector meson

$$\left\langle \rho \left| \bar{\psi}(z)\sigma^{\mu\nu}\psi(0) \right| 0 \right\rangle \to i(p^{\mu}\varepsilon_{\rho}^{\nu} - p^{\nu}\varepsilon_{\rho}^{\mu}) f_{\rho}^{T} \int_0^1 dx e^{ix(p\cdot z)} \varphi_{\perp}(x)$$

The twist 2 DA for a transverse meson is chiral odd, thus $\gamma^* A \to \rho_T A$ starts at twist 3

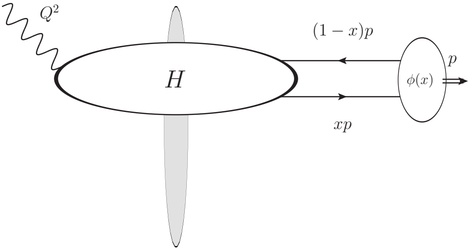
Constructing a finite amplitude

Exclusive diffractive ρ_L production

[R. Boussarie, A. Grabovsky, D.Yu. Ivanov, LS, S. Wallon]
Phys.Rev.Lett. 119 (2017) ; arXiv:1612.08026
Non-forward and non-dilute extension of [Ivanov, Kotsky, Papa]

s-channel collinear factorization

Twist 2: ρ_L production

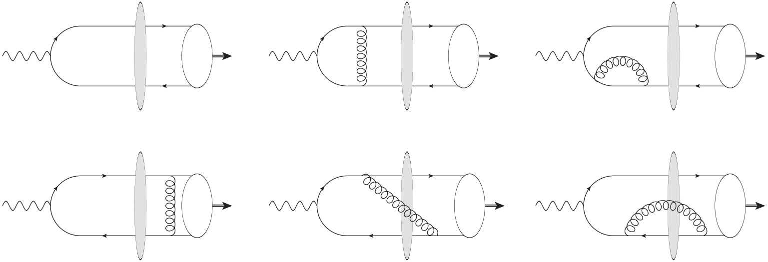


Singlet transition $\Rightarrow$ only virtual diagrams contribute.
Leading twist matrix element:

$$\left\langle \rho_L(p) \left| \bar{\psi}(z) \gamma^\lambda \psi(0) \right| 0 \right\rangle \quad \rightarrow \quad f_\rho m_\rho p^\lambda \int_0^1 dx\ e^{-ixp\cdot z} \varphi_\parallel(x)$$

Take the NLO open parton production result with collinear kinematics $(p_q, p_{\bar{q}}) = (xp, \bar{x}p)$, project on the leading twist Fierz matrix γ^- and convolute with the twist 2 DA φ_1

Exclusive diffractive production of a light neutral vector meson



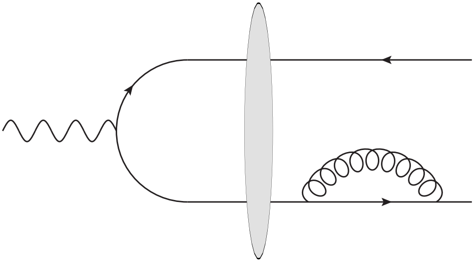
$$\mathcal{A} = -\frac{e_V\, f_V\, \varepsilon_\beta}{N_c} \int_0^1 dx \varphi_{\parallel}(x) \int \frac{d^d \vec{p}_1}{(2\pi)^d} \frac{d^d \vec{p}_2}{(2\pi)^d} \frac{d^d \vec{p}_3}{(2\pi)^d}$$
$$\times (2\pi)^{d+1} \delta\left(p_V^+ - p_\gamma^+\right) \delta\left(\vec{p}_V - \vec{p}_\gamma - \vec{p}_1 - \vec{p}_2 - \vec{p}_3\right)$$
$$\times \left[\left(\Phi_0^\beta(x,\, \vec{p}_1,\, \vec{p}_2) + C_F \Phi_{V1}^\beta(x,\, \vec{p}_1,\, \vec{p}_2) \right) \tilde{\mathcal{U}}_{12}^\eta (2\pi)^d \delta(\vec{p}_3) \right.$$
$$\left. + \Phi_{V2}^\beta(x, \vec{p}_1, \vec{p}_2, \vec{p}_3)\, \tilde{\mathcal{W}}_{123}^\eta \right]$$

Divergences

- Rapidity divergence
- UV divergence $\vec{p}_g^{\,2} \to +\infty$ — Φ_{V1}
- Soft divergence $p_g \to 0$ — $\Phi_{V1}, \Phi_{R1}\Phi_{R1}^*$
- Collinear divergence $p_g \propto p_q$ or $p_{\bar{q}}$ — $\Phi_{V1}, \Phi_{R1}\Phi_{R1}^*$
- Soft and collinear divergence $p_g = \frac{p_g^+}{p_q^+} p_q$ or $\frac{p_g^+}{p_{\bar{q}}^+} p_{\bar{q}}$, $p_g^+ \to 0$ — $\Phi_{R1}\Phi_{R1}^*$

UV divergence

Null yet crucial diagrams



In dimensional regularization, some null diagrams just contribute to turning UV divergences into IR divergences

$$\Phi = 0 \propto \left(\frac{1}{\epsilon_{IR}} - \frac{1}{\epsilon_{UV}} \right)$$

ERBL evolution equation

Efremov, Radyushkin, Brodsky, Lepage evolution equation for a DA

Renormalization of the bilocal operator

$$\bar{\psi}(z)\gamma^{\mu}\psi(0)$$

$\Rightarrow$ Evolution equation for the distribution amplitude in the $\overline{MS}$ scheme

$$\frac{\partial\varphi(x,\mu_F^2)}{\partial \ln \mu_F^2} = \frac{\alpha_s C_F}{2\pi}\frac{\Gamma(1-\epsilon)}{(4\pi)^{\epsilon}}\left(\frac{\mu_F^2}{\mu^2}\right)^{\epsilon}\int_0^1 dz\varphi(z,\mu_F^2)\mathcal{K}(x,z),$$

$\mathcal{K}$ = ERBL kernel

ERBL evolution equation

Evolution equation for the distribution amplitude in the $\overline{MS}$ scheme

$$\frac{\partial\varphi(x,\mu_F^2)}{\partial\ln\mu_F^2} = \frac{\alpha_s C_F}{2\pi}\frac{\Gamma(1-\epsilon)}{(4\pi)^\epsilon}\left(\frac{\mu_F^2}{\mu^2}\right)^\epsilon\int_0^1 dz\varphi(z,\mu_F^2)\mathcal{K}(x,z),$$

where we parameterize the ERBL kernel for consistency as

$$\begin{aligned}\mathcal{K}(x,\,z) &= \frac{x}{z}\left[1+\frac{1}{z-x}\right]\theta(z-x-\alpha) \\ &+ \frac{1-x}{1-z}\left[1+\frac{1}{x-z}\right]\theta(x-z-\alpha) \\ &+ \left[\frac{3}{2}-\ln\left(\frac{x(1-x)}{\alpha^2}\right)\right]\delta(z-x).\end{aligned}$$

It is equivalent to the usual ERBL kernel
Provides a counterterm to the divergences in the dipole term:

$$\Phi_0^\beta\otimes\varphi\to-\int_0^1 dz\,\mathcal{K}(x,z)\,\varphi\,(z,\mu_F^2)\left[\frac{1}{\epsilon}+\ln\left(\frac{\mu_F^2}{\mu^2}\right)\right]\Phi_0^\beta(z)$$

Final result

Finite result for the amplitude

$$\mathcal{A} = -\frac{e_V\, f_V\, \varepsilon_\beta}{N_c} \int_0^1 dx \varphi_{\parallel}(x, \mu_F^2) \int \frac{d^d \vec{p}_1}{(2\pi)^d} \frac{d^d \vec{p}_2}{(2\pi)^d} \frac{d^d \vec{p}_3}{(2\pi)^d}$$
$$\times (2\pi)^{d+1} \delta\left(p_V^+ - p_\gamma^+\right) \delta\left(\vec{p}_V - \vec{p}_\gamma - \vec{p}_1 - \vec{p}_2 - \vec{p}_3\right)$$
$$\times \left[\left(\Phi_0^\beta\left(x,\, \vec{p}_1,\, \vec{p}_2\right) + C_F \Phi_{V1}^\beta\left(x,\, \vec{p}_1,\, \vec{p}_2\right)\right) \tilde{\mathcal{U}}_{12}^\eta \left(2\pi\right)^d \delta(\vec{p}_3)\right.$$
$$\left. + \Phi_{V2}^\beta(x, \vec{p}_1, \vec{p}_2, \vec{p}_3)\, \tilde{\mathcal{W}}_{123}^\eta\right]$$

Residual parameter dependence

Required parameters

- Renormalization scale μ_R
- Factorization scale μ_F if assumed that $\mu_F \neq \mu_R$
- Typical target rapidity Y_0
- Typical projectile rapidity Y

In the linear BFKL limit, the cross section only depends on $Y - Y_0$, so one only needs one arbirary parameter s_0 defined by

$$Y - Y_0 = \ln\left(\frac{s}{s_0}\right).$$

Modifying any of these parameter results in a higher order (NNLO) contribution

Prospects:

- comparison of predictions based on the shock wave approach for dijets with HERA data

  ZEUS Collaboration, H. Abramowicz et. al., Production of exclusive dijets in diffractive DIS at HERA, Eur. Phys. J. C76 (2016)

- comparison of predictions based on the shock wave approach for meson production

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THANK YOU FOR YOUR ATTENTION