Directed flow from C-odd gluon correlations at small

Elena Petreska

Nikhef/VU Amsterdam

with Daniël Boer, Tom van Daal and Piet J. Mulders

JHEP 1807, 140 (2018)

CDR QCD, Ecole Polytechnique

November 27, 2018

Outline

- **Introduction:** azimuthal correlations in two-particle production in pA collisions.

Outline

- **Introduction:** azimuthal correlations in two-particle production in pA collisions.

- **This work:** Odd harmonic azimuthal correlations in forward two-particle production in peripheral pA collisions.

Outline

- **Introduction:** azimuthal correlations in two-particle production in pA collisions.

- **This work:** Odd harmonic azimuthal correlations in forward two-particle production in peripheral pA collisions.

- **Source for the odd harmonics:** the radial nuclear profile.

  ⊖ Manifested through the C-odd dipole GTMD correlator of nucleons in the nucleus (or the odderon Wigner distribution).

Outline

- **Introduction:** azimuthal correlations in two-particle production in pA collisions.

- **This work:** Odd harmonic azimuthal correlations in forward two-particle production in peripheral pA collisions.

- **Source for the odd harmonics:** the radial nuclear profile.
    - Manifested through the C-odd dipole GTMD correlator of nucleons in the nucleus (or the odderon Wigner distribution).

- **GTMDs:** Parametrization of the GTMD correlator in terms of GTMDs.
    - At small-x there is only one independent dipole-type GTMD.

Outline

- **Introduction:** azimuthal correlations in two-particle production in pA collisions.

- **This work:** Odd harmonic azimuthal correlations in forward two-particle production in peripheral pA collisions.

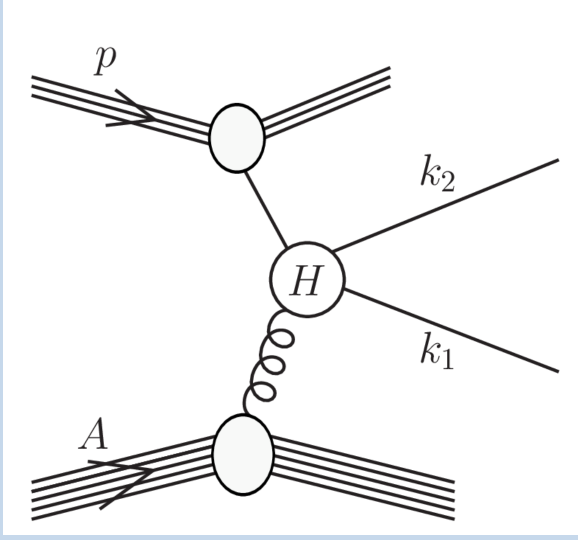
- **Source for the odd harmonics:** the radial nuclear profile.
  - Manifested through the C-odd dipole GTMD correlator of nucleons in the nucleus (or the odderon Wigner distribution).

- **GTMDs:** Parametrization of the GTMD correlator in terms of GTMDs.
  - At small-*x* there is only one independent dipole-type GTMD.

- **Model calculation:** First odd harmonic v_1 in the Color Glass Condensate (CGC) framework.
  - Percent level v_1 from initial state effects in the large N_c limit.

Introduction

Azimuthal correlations in two-particle production in pA collisions



- Decomposition of the cross section into Fourier modes in the relative azimuthal angle between the produced particles:

$$\frac{d\sigma}{d^2\boldsymbol{k_1}d^2\boldsymbol{k_2}} \propto 1 + 2V_1(\boldsymbol{k_1},\boldsymbol{k_2})\cos\left(\phi_{k_1}-\phi_{k_2}\right)$$

$$+2V_2(\boldsymbol{k_1},\boldsymbol{k_2})\cos 2\left(\phi_{k_1}-\phi_{k_2}\right)$$

$$+2V_3(\boldsymbol{k_1},\boldsymbol{k_2})\cos 3\left(\phi_{k_1}-\phi_{k_2}\right)+\cdots$$

- Azimuthal correlations are quantified through the flow coefficients:

$$v_n(\boldsymbol{k},\boldsymbol{k}^{\mathrm{ref}}) \equiv \frac{V_n(\boldsymbol{k},\boldsymbol{k}^{\mathrm{ref}})}{\sqrt{V_n(\boldsymbol{k}^{\mathrm{ref}},\boldsymbol{k}^{\mathrm{ref}})}}$$

Introduction

Azimuthal correlations in two-particle production in pA collisions

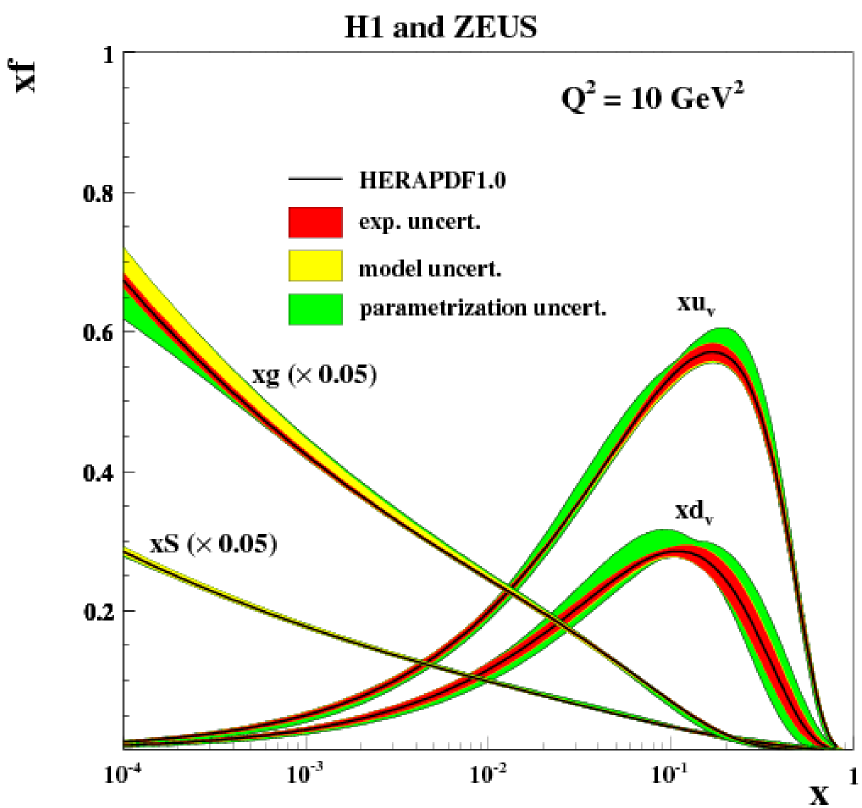
- Source of the azimuthal asymmetries: Initial vs Final state effects

- **Final state effects:** Collective behavior from strong interactions in the final state (“hydrodynamic flow”).
  - In analogy to hydrodynamic flow in AA collisions.
  - Agreement with data, but sensitive to the initial conditions for the hydrodynamic flow.
  - Can hydrodynamic flow develop in small systems, or initial state correlations alone can provide an explanation?

- **Initial state effects:** Correlation effects already present in the wave function of the incoming hadrons or nuclei.
  - Most calculations performed in the CGC theory.
  - Even harmonics are easier to obtain; Non-vanishing odd harmonics are more challenging.

Odd harmonics from initial state correlations in the CGC theory

- Color glass condensate (CGC): effective field theory for the high-gluon density regime of ultra-relativistic protons and nuclei.
  - Momentum transfer squares Q^2 is fixed.
  - Center of mass squared $s \to \infty$.
  - Longitudinal momentum fraction $x \to 0$.

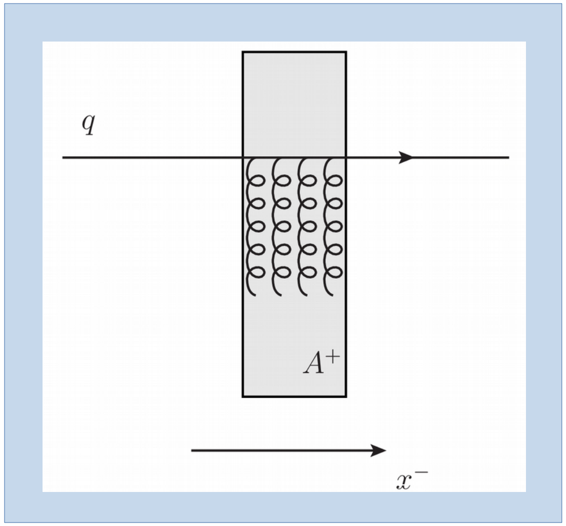


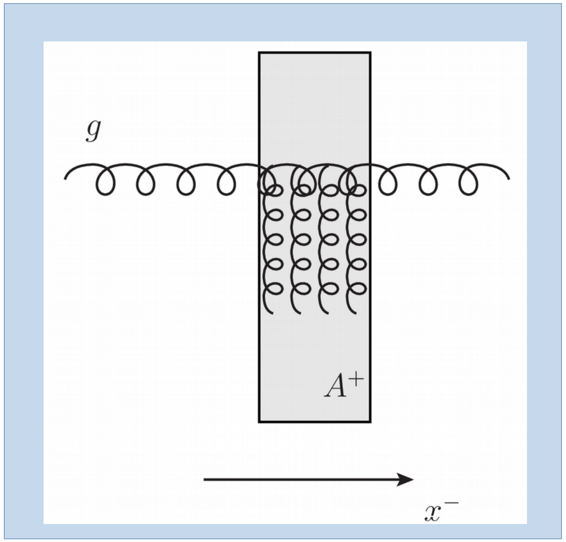
From electron-proton deep inelastic scattering
(H1 and ZEUS collaborations 2010)

Odd harmonics from initial state correlations in the CGC theory

- Scattering off the classical CGC field of a large nucleus (scattering off the shock-wave) in the eikonal approximation.
  - Resummation of multiple gluon scatterings into Wilson lines.

$$\alpha_s^2 A^{1/3} \sim 1$$





$$U(\mathbf{x}) = \mathcal{P}\exp\left[ig\int_{-\infty}^{+\infty} dx^- A_a^+(x^-,\mathbf{x})t^a\right] \qquad V(\mathbf{x}) = \mathcal{P}\exp\left[ig\int_{-\infty}^{+\infty} dx^- A_a^+(x^-,\mathbf{x})T^a\right]$$

Fundamental/Adjoint eikonal Wilson line

Odd harmonics from initial state correlations in the CGC theory

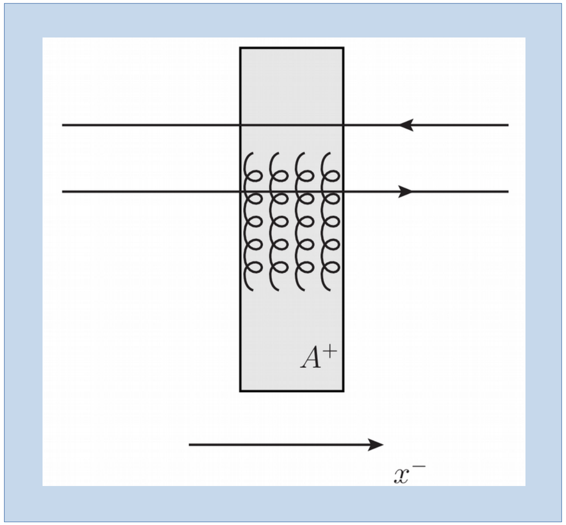
- Scattering off the classical CGC field of a large nucleus (scattering off the shock-wave) in the eikonal approximation.
  - Resummation of multiple gluon scatterings into Wilson lines.

$$\alpha_s^2 A^{1/3} \sim 1$$

- Quark production at forward rapidity in pA in the hybrid formalism:
  - Dipole scattering amplitude

$$\int \frac{d^2\boldsymbol{r}}{(2\pi)^2}\, e^{-i\boldsymbol{k}\cdot\boldsymbol{r}} \left\langle \frac{1}{N_c} \mathrm{tr}\, U\left(\boldsymbol{b}+\frac{\boldsymbol{r}}{2}\right) U^\dagger\left(\boldsymbol{b}-\frac{\boldsymbol{r}}{2}\right) \right\rangle$$

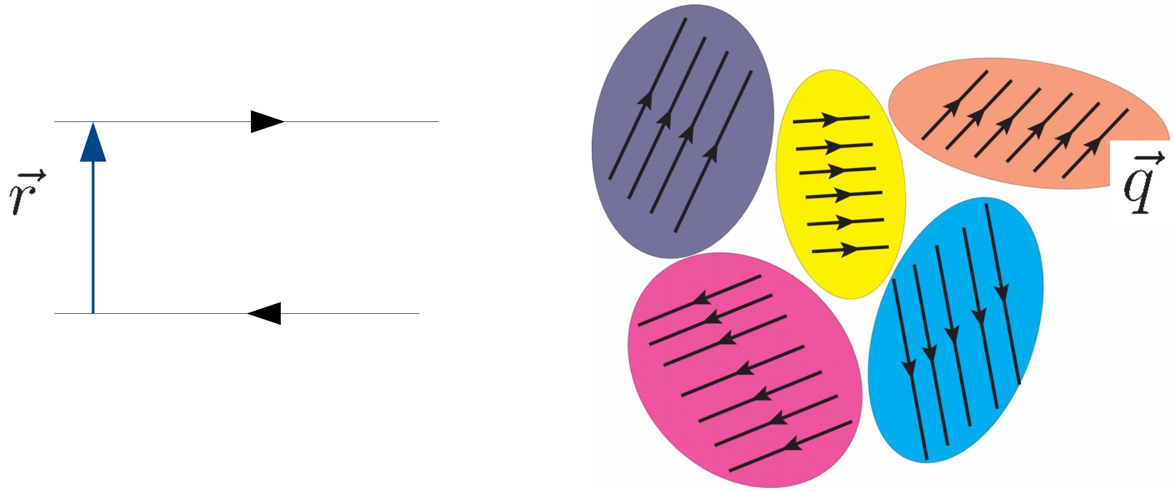
$$U(\mathbf{x}) = \mathcal{P}\exp\left[ig\int_{-\infty}^{+\infty} dx^- A_a^+(x^-,\mathbf{x})t^a\right]$$



Odd harmonics from initial state correlations in the CGC theory

- Odd harmonics from the imaginary part of the quark dipole scattering amplitude (the odderon) + broken rotational symmetry of the nucleus due to fluctuations.

⊖ Correlations between the orientation of the dipole and the direction of the fluctuations in the target $\vec{r} \leftrightarrow \vec{q}$

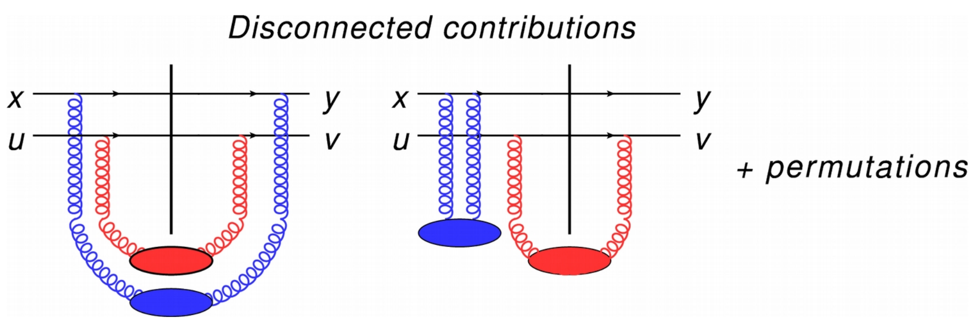


A. Dumitru, A. V. Giannini, 2015

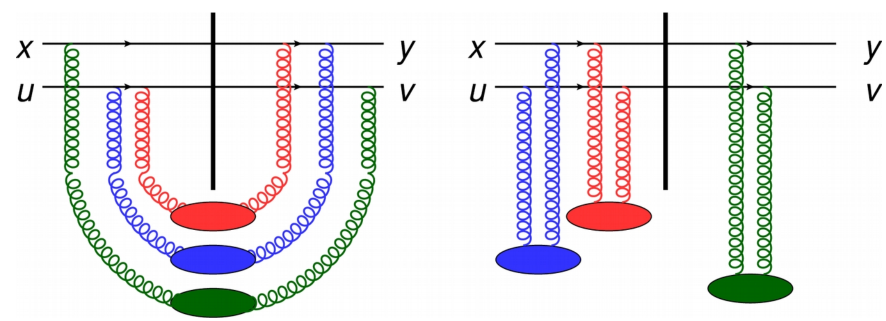
A. Dumitru, V. Skokov, 2015

Odd harmonics from initial state correlations in the CGC theory

- Odd harmonics in two-quark production in pA from connected diagrams (independent of the impact parameter).
  - Correlations arise from diagrams subleading in $1/N_c$.



Connected diagrams subleading in $1/N_c$



A. Dumitru, F. Gelis, L. McLerran, R. Venugopalan, 2008

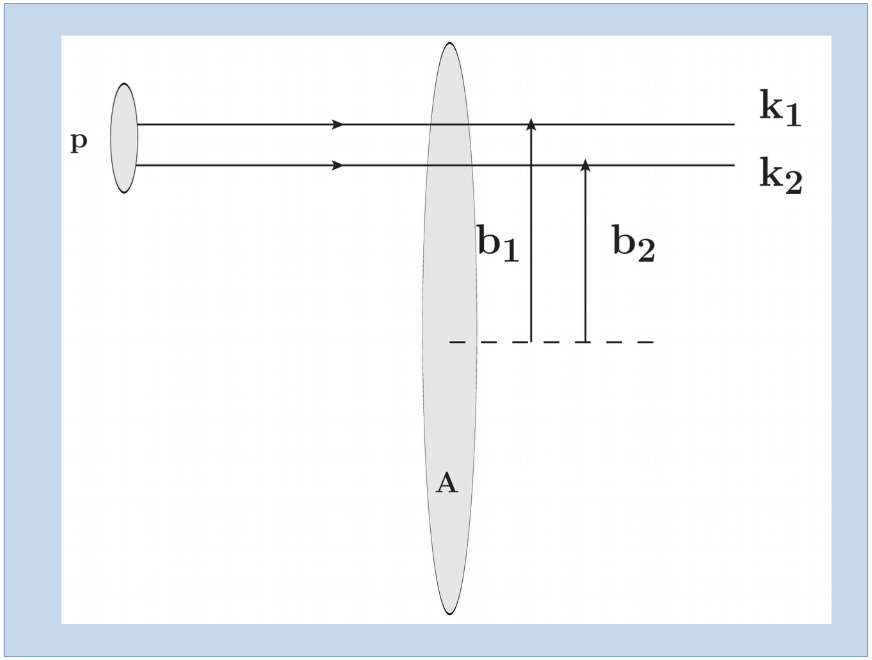
T. Lappi, 2015;

T. Lappi, B. Schenke, S. Schlichting and R. Venugopalan, 2016

K. Dusling, M. Mace, R. Venugopalan, 2018

Odd harmonics from initial state correlations in the CGC theory

- Odd harmonics in two-quark production in pheripheral pA collisions that:
  - Arise from the inhomogeneity in the nucleus in the radial direction (no breaking of the rotational symmetry).
  - Are leading in $1/N_c$.



Di-hadron production from double parton scattering

D. Boer, T. van Daal, P. J. Mulders, EP; JHEP 2018

Odd harmonics from initial state correlations in the CGC theory

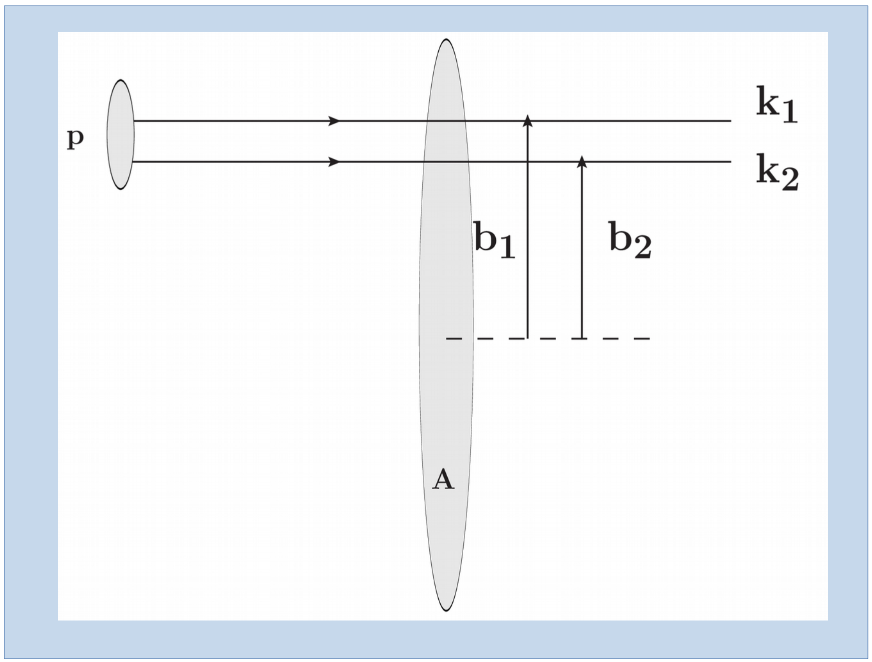
- Odd harmonics in two-quark production in pheripheral pA collisions that:
    - Arise from the inhomogeneity in the nucleus in the radial direction (no breaking of the rotational symmetry).
    - Are leading in $1/N_c$.

- Even harmonics from the orientation of the dipole with respect to its impact parameter:

B. Z. Kopeliovich, H. J. Pirner, A. H. Rezaeian, I. Schmidt, 2008
B. Z. Kopeliovich, A. H. Rezaeian, I. Schmidt, 2008
E. Levin, A. H. Rezaeian, 2011
J. Zhou, 2016
E. Iancu, A. H. Rezaeian 2017

Two-quark production in pA from double parton scattering

- Hybrid formalism *(Dumitru and Jalilian-Marian (2002))* for forward particle production.



The proton is probed at large values of x (dilute):

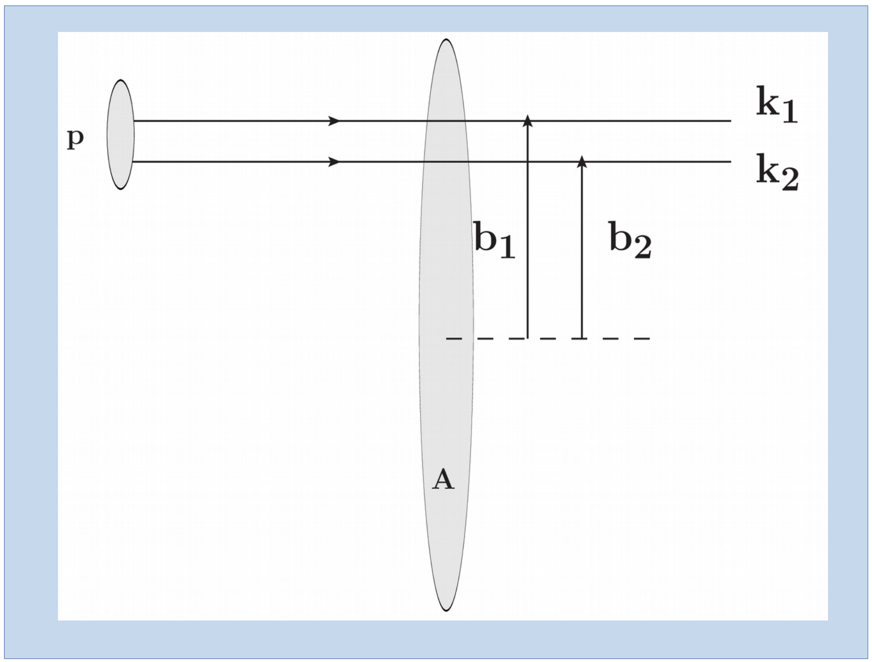
$$x_1 = \frac{1}{\sqrt{s}} \left(|p_{1t}|e^{y_1} + |p_{2t}|e^{y_2}\right) \sim 1$$

The nucleus is probed at small values of x (dense):

$$x_2 = \frac{1}{\sqrt{s}} \left(|p_{1t}|e^{-y_1} + |p_{2t}|e^{-y_2}\right) \ll 1$$

Two-quark production in pA from double parton scattering

- Hybrid formalism *(Dumitru and Jalilian-Marian (2002))* for forward particle production.

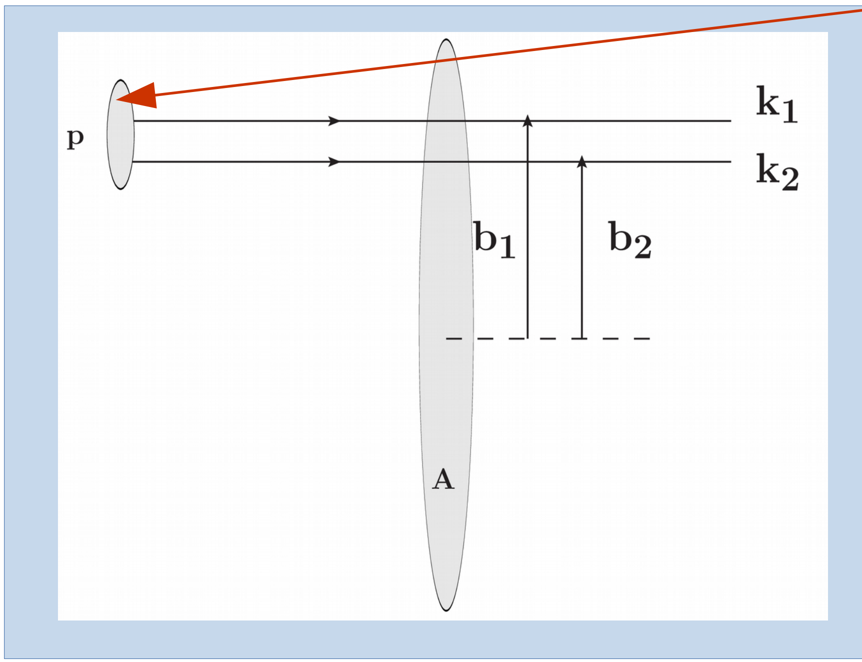


$$\frac{d\sigma^{pA}}{dy_1 dy_2\, d^2\boldsymbol{k}_1 d^2\boldsymbol{k}_2} \propto \int d^2\boldsymbol{b}_1\, d^2\boldsymbol{b}_2\, F_p(x_1, x_2, \boldsymbol{b}_1 - \boldsymbol{b}_2) \int \frac{d^2\boldsymbol{r}_1 d^2\boldsymbol{r}_2}{(2\pi)^4}\, e^{-i\boldsymbol{k}_1\cdot\boldsymbol{r}_1 - i\boldsymbol{k}_2\cdot\boldsymbol{r}_2}$$

$$\times \left\langle S\left(\boldsymbol{b}_1 + \tfrac{\boldsymbol{r}_1}{2}, \boldsymbol{b}_1 - \tfrac{\boldsymbol{r}_1}{2}\right)\, S\left(\boldsymbol{b}_2 + \tfrac{\boldsymbol{r}_2}{2}, \boldsymbol{b}_2 - \tfrac{\boldsymbol{r}_2}{2}\right)\right\rangle_{x,A}$$

Two-quark production in pA from double parton scattering

- Hybrid formalism *(Dumitru and Jalilian-Marian (2002))* for forward particle production.



Double parton distribution for the proton:

- Gaussian ansatz

$$F_p(x_1, x_2, \boldsymbol{b}_1 - \boldsymbol{b}_2) = f_p(x_1, x_2)\, \frac{1}{4\pi R_N^2}\, e^{-\frac{(\boldsymbol{b}_1 - \boldsymbol{b}_2)^2}{4R_N^2}}$$

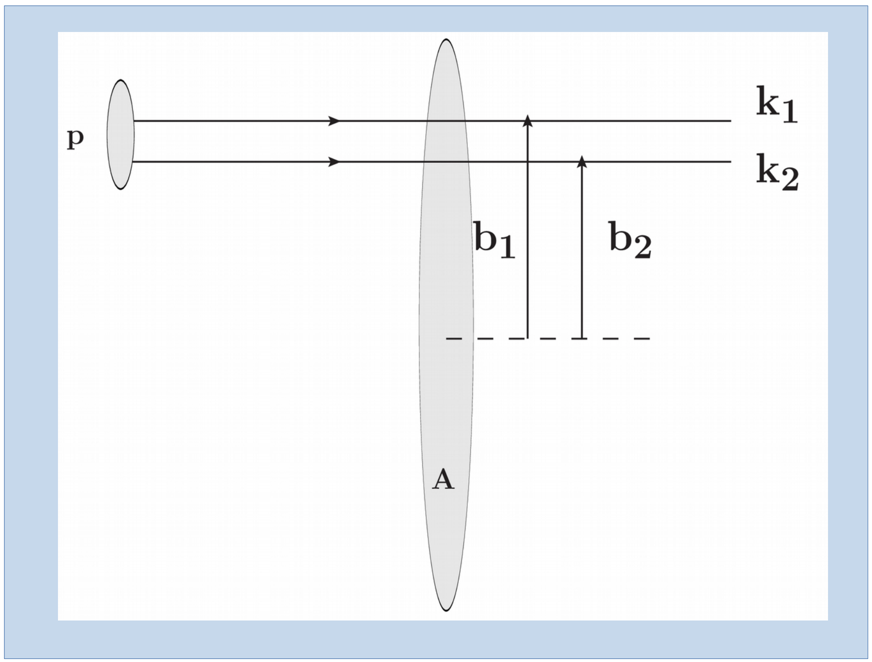
R_N is the radius of the proton

$$\frac{d\sigma^{pA}}{dy_1 dy_2\, d^2\boldsymbol{k}_1 d^2\boldsymbol{k}_2} \propto \int d^2\boldsymbol{b}_1\, d^2\boldsymbol{b}_2\, F_p(x_1, x_2, \boldsymbol{b}_1 - \boldsymbol{b}_2) \int \frac{d^2\boldsymbol{r}_1 d^2\boldsymbol{r}_2}{(2\pi)^4}\, e^{-i\boldsymbol{k}_1\cdot\boldsymbol{r}_1 - i\boldsymbol{k}_2\cdot\boldsymbol{r}_2}$$

$$\times \left\langle S\left(\boldsymbol{b}_1 + \tfrac{\boldsymbol{r}_1}{2}, \boldsymbol{b}_1 - \tfrac{\boldsymbol{r}_1}{2}\right)\, S\left(\boldsymbol{b}_2 + \tfrac{\boldsymbol{r}_2}{2}, \boldsymbol{b}_2 - \tfrac{\boldsymbol{r}_2}{2}\right)\right\rangle_{x,A}$$

Two-quark production in pA from double parton scattering

- Hybrid formalism *(Dumitru and Jalilian-Marian (2002))* for forward particle production.



- Scattering of two quarks off the CGC field in the light-cone gauge of the proton $A^- = 0$ (the proton is moving in the x^- direction):

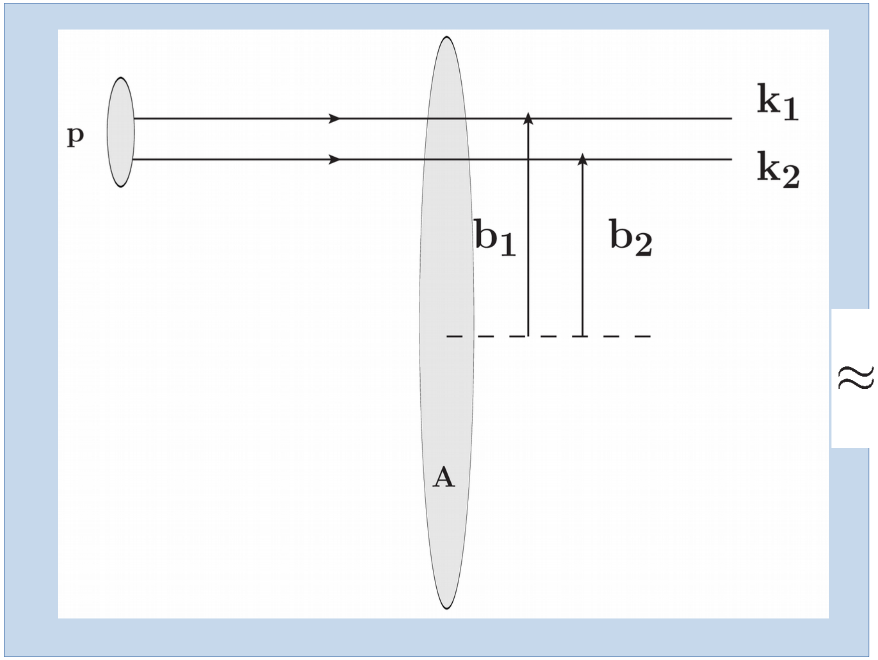
$$\left\langle \frac{1}{N_c} \mathrm{tr}\left[U(\boldsymbol{x}_1)\, U^\dagger(\boldsymbol{y}_1)\right] \frac{1}{N_c} \mathrm{tr}\left[U(\boldsymbol{x}_2)\, U^\dagger(\boldsymbol{y}_2)\right] \right\rangle_{x,A}$$

$$\frac{d\sigma^{pA}}{dy_1 dy_2\, d^2\boldsymbol{k}_1 d^2\boldsymbol{k}_2} \propto \int d^2\boldsymbol{b}_1\, d^2\boldsymbol{b}_2\, F_p(x_1, x_2, \boldsymbol{b}_1 - \boldsymbol{b}_2) \int \frac{d^2\boldsymbol{r}_1 d^2\boldsymbol{r}_2}{(2\pi)^4}\, e^{-i\boldsymbol{k}_1\cdot\boldsymbol{r}_1 - i\boldsymbol{k}_2\cdot\boldsymbol{r}_2}$$

$$\times \left\langle S\left(\boldsymbol{b}_1 + \tfrac{\boldsymbol{r}_1}{2}, \boldsymbol{b}_1 - \tfrac{\boldsymbol{r}_1}{2}\right) S\left(\boldsymbol{b}_2 + \tfrac{\boldsymbol{r}_2}{2}, \boldsymbol{b}_2 - \tfrac{\boldsymbol{r}_2}{2}\right) \right\rangle_{x,A}$$

Two-quark production in pA from double parton scattering

- Hybrid formalism *(Dumitru and Jalilian-Marian (2002))* for forward particle production.



In the limit $N_c \to \infty$:

$$\left\langle \frac{1}{N_c} \mathrm{tr}\left[U(\boldsymbol{x}_1)\, U^\dagger(\boldsymbol{y}_1)\right] \frac{1}{N_c} \mathrm{tr}\left[U(\boldsymbol{x}_2)\, U^\dagger(\boldsymbol{y}_2)\right] \right\rangle_{x,A}$$

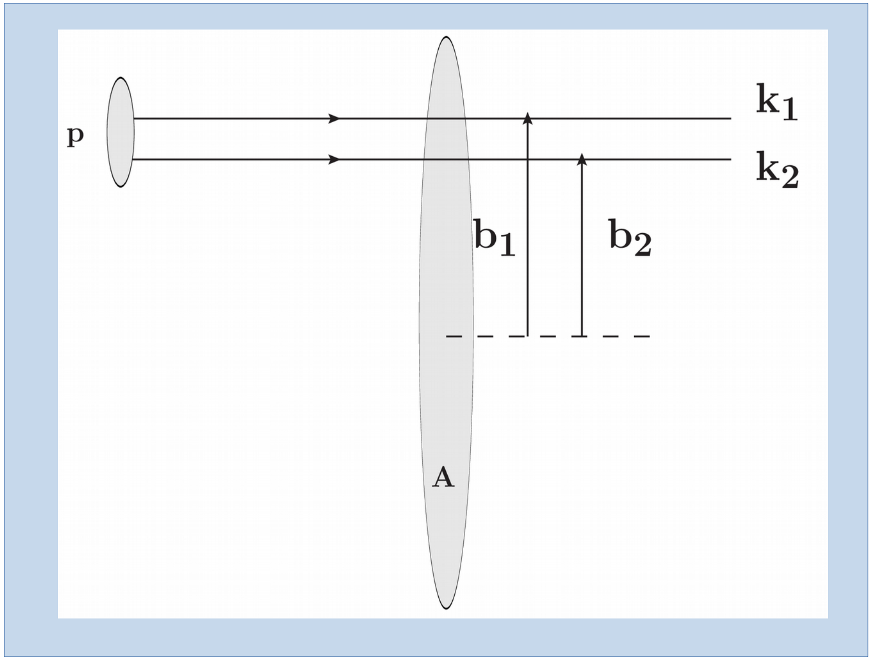
$$\approx \left\langle \frac{1}{N_c} \mathrm{tr}\left[U(\boldsymbol{x}_1)\, U^\dagger(\boldsymbol{y}_1)\right] \right\rangle \left\langle \frac{1}{N_c} \mathrm{tr}\left[U(\boldsymbol{x}_2)\, U^\dagger(\boldsymbol{y}_2)\right] \right\rangle$$

$$\frac{d\sigma^{pA}}{dy_1 dy_2\, d^2\boldsymbol{k}_1 d^2\boldsymbol{k}_2} \propto \int d^2\boldsymbol{b}_1\, d^2\boldsymbol{b}_2\, F_p(x_1, x_2, \boldsymbol{b}_1 - \boldsymbol{b}_2) \int \frac{d^2\boldsymbol{r}_1 d^2\boldsymbol{r}_2}{(2\pi)^4}\, e^{-i\boldsymbol{k}_1\cdot\boldsymbol{r}_1 - i\boldsymbol{k}_2\cdot\boldsymbol{r}_2}$$

$$\times \left\langle S\left(\boldsymbol{b}_1 + \tfrac{\boldsymbol{r}_1}{2}, \boldsymbol{b}_1 - \tfrac{\boldsymbol{r}_1}{2}\right)\, S\left(\boldsymbol{b}_2 + \tfrac{\boldsymbol{r}_2}{2}, \boldsymbol{b}_2 - \tfrac{\boldsymbol{r}_2}{2}\right)\right\rangle_{x,A}$$

Odd harmonics in two-particle production in pA

- Hybrid formalism *(Dumitru and Jalilian-Marian (2002))* for forward particle production.



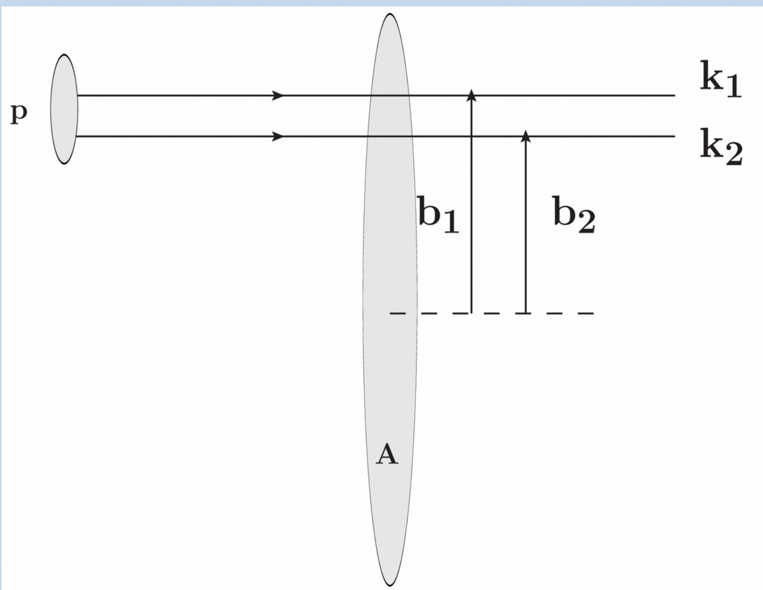
Dipole Wigner distribution:

$$W(\boldsymbol{b},\boldsymbol{k}) = \int \frac{d^2\boldsymbol{r}}{(2\pi)^2}\, e^{-i\boldsymbol{k}\cdot\boldsymbol{r}} \left\langle S\left(\boldsymbol{b}+\frac{\boldsymbol{r}}{2}, \boldsymbol{b}-\frac{\boldsymbol{r}}{2}\right)\right\rangle$$

$$\frac{d\sigma^{pA}}{dy_1 dy_2\, d^2\boldsymbol{k}_1 d^2\boldsymbol{k}_2} \propto \int d^2\boldsymbol{b}_1\, d^2\boldsymbol{b}_2\, F_p(x_1, x_2, \boldsymbol{b}_1 - \boldsymbol{b}_2)\, xW(x, \boldsymbol{b}_1, \boldsymbol{k}_1)\, xW(x, \boldsymbol{b}_2, \boldsymbol{k}_2)$$

Odd harmonics in two-particle production in pA

- Hybrid formalism *(Dumitru and Jalilian-Marian (2002))* for forward particle production.



Dipole Wigner distribution:

$$W(\boldsymbol{b},\boldsymbol{k}) = \int \frac{d^2\boldsymbol{r}}{(2\pi)^2}\, e^{-i\boldsymbol{k}\cdot\boldsymbol{r}} \left\langle S\left(\boldsymbol{b}+\tfrac{\boldsymbol{r}}{2}, \boldsymbol{b}-\tfrac{\boldsymbol{r}}{2}\right)\right\rangle$$

$$\frac{d\sigma^{pA}}{dy_1 dy_2\, d^2\boldsymbol{k}_1 d^2\boldsymbol{k}_2} \propto \int d^2\boldsymbol{b}_1\, d^2\boldsymbol{b}_2\, F_p(x_1, x_2, \boldsymbol{b}_1 - \boldsymbol{b}_2)\, xW(x, \boldsymbol{b}_1, \boldsymbol{k}_1)\, xW(x, \boldsymbol{b}_2, \boldsymbol{k}_2)$$

GTMDs and Wigner distributions at small *x*

Gluon GTMD correlator and gluon Wigner distribution

- Gluon-gluon generalized transverse momentum dependent (GTMD) correlator for unpolarized hadron (combined information on TMDs and GPDs):

$$G(x,\boldsymbol{k},\xi,\boldsymbol{\Delta}) \equiv \frac{2}{P^+}\int \frac{dz^-\, d^2\boldsymbol{z}}{(2\pi)^3}\, e^{ik\cdot z}\left\langle p' \left| \mathrm{Tr}\left(F^{\mu\nu}\left(-\tfrac{z}{2}\right) U_{[-\frac{z}{2},\frac{z}{2}]}\, F^{\rho\sigma}\left(\tfrac{z}{2}\right) U'_{[\frac{z}{2},-\frac{z}{2}]}\right)\right| p\right\rangle\Bigg|_{z^+=0}$$

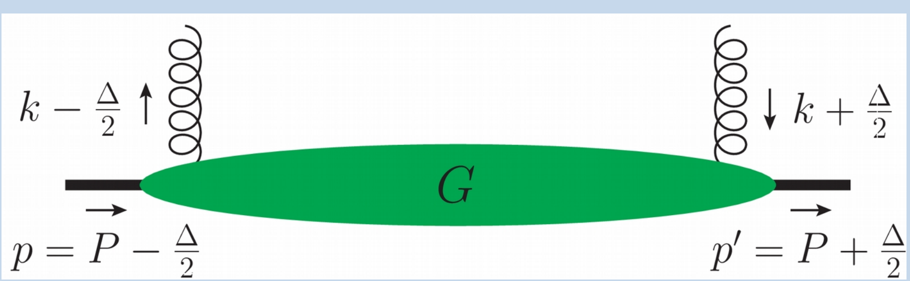
x → longitudinal momentum fraction

$\boldsymbol{k}$ → transverse momentum

ξ → skewness parameter

$\boldsymbol{\Delta}$ → transverse momentum transfer

C. Lorcé and B. Pasquini, 2013



Off-forwardness: $\Delta \equiv p' - p$

$P = (P^-, P^+, \mathbf{0})$

$\Delta = (2\xi P^-, -2\xi P^+, \boldsymbol{\Delta})$

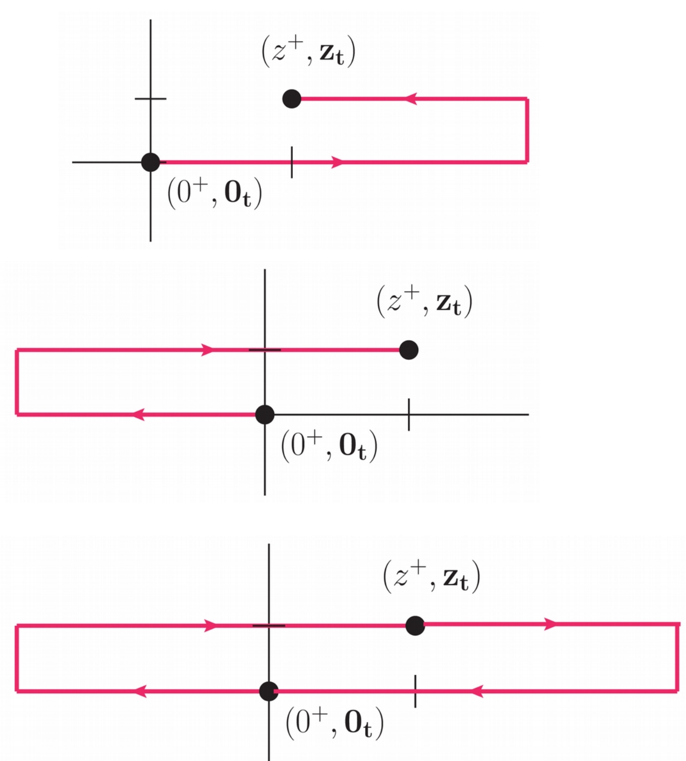
$k = (k^-, xP^+, \boldsymbol{k})$

Gluon GTMD correlator and gluon Wigner distribution

- Gluon-gluon generalized transverse momentum dependent (GTMD) correlator for unpolarized hadron (combined information on TMDs and GPDs):

$$G(x, \boldsymbol{k}, \xi, \boldsymbol{\Delta}) \equiv \frac{2}{P^+} \int \frac{dz^- \, d^2\boldsymbol{z}}{(2\pi)^3} \, e^{ik\cdot z} \left\langle p' \left| \mathrm{Tr}\left(F^{\mu\nu}\left(-\tfrac{z}{2}\right) U_{\left[-\frac{z}{2},\frac{z}{2}\right]} F^{\rho\sigma}\left(\tfrac{z}{2}\right) U'_{\left[\frac{z}{2},-\frac{z}{2}\right]} \right) \right| p \right\rangle \Big|_{z^+=0}$$

Process dependent gauge links.



$$\mathcal{U}^{[\pm]} = U(0, \pm\infty; \mathbf{0}) U^T(\pm\infty; \mathbf{0}, \boldsymbol{\xi}) U(\pm\infty, \xi^+; \boldsymbol{\xi})$$

$$\mathcal{U}^{[\square]} = \mathcal{U}^{[+]}\mathcal{U}^{[-]\dagger} = \mathcal{U}^{[-]}\mathcal{U}^{[+]\dagger}$$

$$U(a, b; \mathbf{x}) = \mathcal{P} \exp\left[ig \int_a^b dx^+ A_a^-(x^+, \mathbf{x}) t^a \right]$$

Gluon GTMD correlator and gluon Wigner distribution

- Gluon-gluon generalized transverse momentum dependent (GTMD) correlator for unpolarized hadron (combined information on TMDs and GPDs):

$$G(x,\boldsymbol{k},\xi,\boldsymbol{\Delta}) \equiv \frac{2}{P^+}\int \frac{dz^-\,d^2\boldsymbol{z}}{(2\pi)^3}\, e^{ik\cdot z}\left\langle p'\left|\mathrm{Tr}\left(F^{\mu\nu}\left(-\tfrac{z}{2}\right)U_{\left[-\frac{z}{2},\frac{z}{2}\right]}F^{\rho\sigma}\left(\tfrac{z}{2}\right)U'_{\left[\frac{z}{2},-\frac{z}{2}\right]}\right)\right|p\right\rangle\Big|_{z^+=0}$$

Fourier transform $\boldsymbol{\Delta} \to \boldsymbol{b}$, $\boldsymbol{b}$ is the impact parameter.

$W(x,\mathbf{k},\xi,\mathbf{b})$ Gluon Wigner distribution.

Gluon GTMD correlator and gluon Wigner distribution

- Gluon-gluon generalized transverse momentum dependent (GTMD) correlator for unpolarized hadron (combined information on TMDs and GPDs):

$$G(x,\boldsymbol{k},\xi,\boldsymbol{\Delta}) \equiv \frac{2}{P^+}\int \frac{dz^-\,d^2\boldsymbol{z}}{(2\pi)^3}\, e^{ik\cdot z}\left\langle p' \middle| \mathrm{Tr}\left(F^{\mu\nu}\left(-\tfrac{z}{2}\right) U_{[-\frac{z}{2},\frac{z}{2}]}\, F^{\rho\sigma}\left(\tfrac{z}{2}\right) U'_{[\frac{z}{2},-\frac{z}{2}]}\right) \middle| p \right\rangle\Big|_{z^+=0}$$

- Parametrization:

$$G^{[U,U']\,ij}(x,\mathbf{k},\xi,\Delta) = x\left(\delta_T^{ij}\,F_1 + \frac{k_T^{ij}}{M^2}\,F_2 + \frac{\Delta_T^{ij}}{M^2}F_3 + \frac{k_T^{[i}\Delta_T^{j]}}{M^2}\,F_4\right)$$

GTMDs: $F_i = F_i^{[U,U']}(x,\xi,\boldsymbol{k}^2,\boldsymbol{\Delta}^2,\boldsymbol{k}\cdot\boldsymbol{\Delta})$

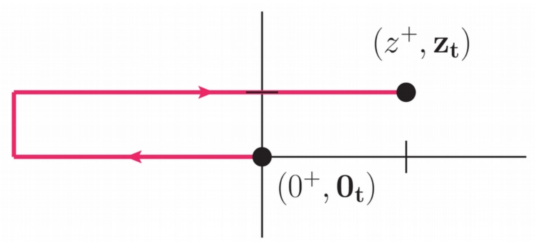
C. Lorcé and B. Pasquini, 2013

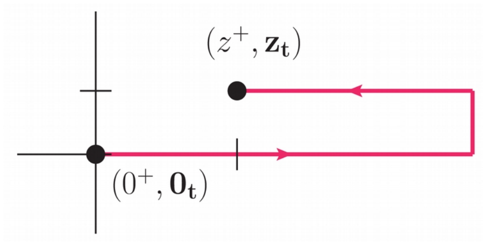
D. Boer, T. van Daal, P. J. Mulders, EP; JHEP 2018

Dipole type GTMDs at small *x*

- Dipole GTMD gluon distribution:

$$G^{[+,-]ij}(x,\boldsymbol{k},\xi,\boldsymbol{\Delta}) \equiv \frac{2}{P^+}\int \frac{dz^-\,d^2\boldsymbol{z}}{(2\pi)^3}\, e^{ik\cdot z}\left\langle p'\left|\mathrm{Tr}\left(F^{\mu\nu}\left(-\tfrac{z}{2}\right)U^{[-]\dagger}_{[-\frac{z}{2},\frac{z}{2}]}F^{\rho\sigma}\left(\tfrac{z}{2}\right)U^{[+]}_{[\frac{z}{2},-\frac{z}{2}]}\right)\right|p\right\rangle\Big|_{z^+=0}$$

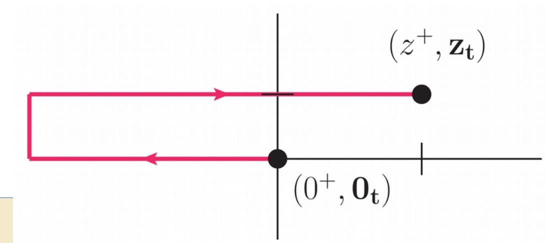


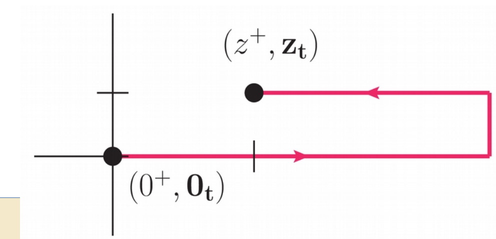


Dipole type GTMDs at small *x*

- Dipole GTMD gluon distribution:

$$G^{[+,-]ij}(x,\boldsymbol{k},\xi,\boldsymbol{\Delta}) \equiv \frac{2}{P^+}\int \frac{dz^-\,d^2\boldsymbol{z}}{(2\pi)^3}\, e^{ik\cdot z}\left\langle p'\left|\mathrm{Tr}\left(F^{\mu\nu}\left(-\tfrac{z}{2}\right)U^{[-]\dagger}_{[-\frac{z}{2},\frac{z}{2}]}\,F^{\rho\sigma}\left(\tfrac{z}{2}\right)U^{[+]}_{[\frac{z}{2},-\frac{z}{2}]}\right)\right|p\right\rangle\Big|_{z^+=0}$$

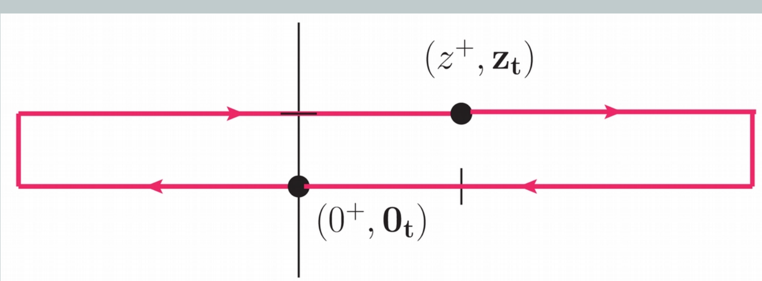




- In the small-*x* limit:

$$G^{[+,-]\,ij}(\boldsymbol{k},\boldsymbol{\Delta}) = \frac{2N_c}{\alpha_s}\left[\frac{1}{2}\left(\boldsymbol{k}^2-\frac{\boldsymbol{\Delta}^2}{4}\right)\delta_T^{ij}+k_T^{ij}-\frac{\Delta_T^{ij}}{4}-\frac{k_T^{[i}\Delta_T^{j]}}{2}\right]G^{[\Box]}(\boldsymbol{k},\boldsymbol{\Delta})$$

$$G^{[\Box]}(\boldsymbol{k},\boldsymbol{\Delta}) \equiv \int \frac{d^2\boldsymbol{x}\,d^2\boldsymbol{y}}{(2\pi)^4}\, e^{-i\boldsymbol{k}\cdot(\boldsymbol{x}-\boldsymbol{y})+i\boldsymbol{\Delta}\cdot\frac{\boldsymbol{x}+\boldsymbol{y}}{2}}\,\frac{\langle p'|S^{[\Box]}(\boldsymbol{x},\boldsymbol{y})|p\rangle\big|_{\mathrm{LF}}}{\langle P|P\rangle}$$



$$S^{[\Box]}(\boldsymbol{x},\boldsymbol{y}) \equiv \frac{1}{N_c}tr\left(U^{[\Box]}\right)$$

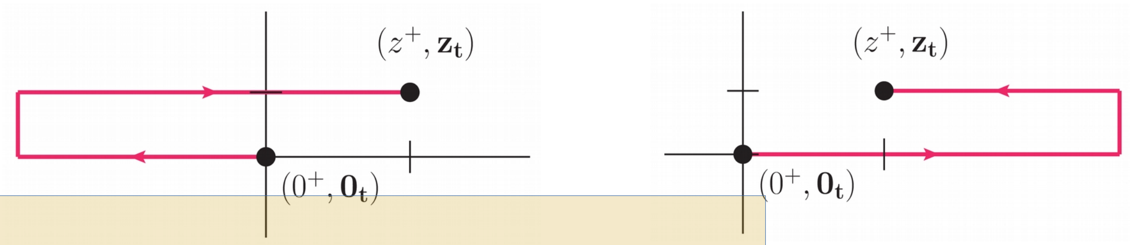
D. Boer, T. van Daal, P. J. Mulders, EP, 2018

Y. Hatta, B.-W. Xiao and F. Yuan, 2016

Dipole type GTMDs at small *x*

- Dipole GTMD gluon distribution:

$$G^{[+,-]ij}(x,\boldsymbol{k},\xi,\boldsymbol{\Delta}) \equiv \frac{2}{P^+}\int \frac{dz^-\,d^2\boldsymbol{z}}{(2\pi)^3}\, e^{ik\cdot z}\left\langle p'\left|\mathrm{Tr}\left(F^{\mu\nu}\left(-\tfrac{z}{2}\right)U^{[-]\dagger}_{[-\frac{z}{2},\frac{z}{2}]}F^{\rho\sigma}\left(\tfrac{z}{2}\right)U^{[+]}_{[\frac{z}{2},-\frac{z}{2}]}\right)\right|p\right\rangle\Bigg|_{z^+=0}$$



- In the small-*x* limit:

$$G^{[+\,-]\,ij}(x,\mathbf{k},\xi,\Delta) = x\left(\delta_T^{ij}\,F_1 + \frac{k_T^{ij}}{M^2}\,F_2 + \frac{\Delta_T^{ij}}{M^2}F_3 + \frac{k_T^{[i}\Delta_T^{j]}}{M^2}\,F_4\right)$$

$$\lim_{x,\xi\to 0} xF_1 = \lim_{x,\xi\to 0} xF_2^{(1)} = -4\lim_{x,\xi\to 0} xF_3^{(1)} = -2\lim_{x,\xi\to 0} xF_4^{(1)} = \mathcal{E}^{(1)}$$

  - In the small-x limit there is only one independent GTMD.

D. Boer, T. van Daal, P. J. Mulders, EP, 2018

C-odd and T-odd dipole GTMDs at small *x*

- Odd harmonics arise from the C-odd and T-odd dipole GTMD gluon distribution:

$$G_{(d)}^{(\text{T-odd})\,ij}(\boldsymbol{k},\boldsymbol{\Delta}) \propto G^{[\Box]}(\boldsymbol{k},\boldsymbol{\Delta}) - G^{[\Box^\dagger]}(\boldsymbol{k},\boldsymbol{\Delta}) \propto \int \frac{d^2\boldsymbol{x}\, d^2\boldsymbol{y}}{(2\pi)^4}\; e^{-i\boldsymbol{k}\cdot(\boldsymbol{x}-\boldsymbol{y})+i\boldsymbol{\Delta}\cdot\frac{\boldsymbol{x}+\boldsymbol{y}}{2}}\; \langle \mathcal{O}(\boldsymbol{x},\boldsymbol{y})\rangle$$

where $\mathcal{O}(\boldsymbol{x},\boldsymbol{y})$ is the odderon operator, the imaginary part of the Wilson loop operator:

$$S^{[\Box]}(\boldsymbol{x},\boldsymbol{y}) = \mathcal{P}(\boldsymbol{x},\boldsymbol{y}) + i\mathcal{O}(\boldsymbol{x},\boldsymbol{y})$$

C-odd and T-odd dipole GTMDs at small *x*

- Odd harmonics arise from the C-odd and T-odd dipole GTMD gluon distribution:

$$G_{(d)}^{(\text{T-odd})\,ij}(\boldsymbol{k},\boldsymbol{\Delta}) \propto G^{[\Box]}(\boldsymbol{k},\boldsymbol{\Delta}) - G^{[\Box^\dagger]}(\boldsymbol{k},\boldsymbol{\Delta}) \propto \int \frac{d^2\boldsymbol{x}\,d^2\boldsymbol{y}}{(2\pi)^4}\; e^{-i\boldsymbol{k}\cdot(\boldsymbol{x}-\boldsymbol{y})+i\boldsymbol{\Delta}\cdot\frac{\boldsymbol{x}+\boldsymbol{y}}{2}}\,\langle\mathcal{O}(\boldsymbol{x},\boldsymbol{y})\rangle$$

where $\mathcal{O}(\boldsymbol{x},\boldsymbol{y})$ is the odderon operator, the imaginary part of the Wilson loop operator:

$$S^{[\Box]}(\boldsymbol{x},\boldsymbol{y}) = \mathcal{P}(\boldsymbol{x},\boldsymbol{y}) + i\mathcal{O}(\boldsymbol{x},\boldsymbol{y})$$

- The C-odd and T-odd dipole GTMD contains only odd powers of $\boldsymbol{k}\cdot\boldsymbol{\Delta}$
  - The odderon contains only odd harmonics:

$$\cos\left[(2n-1)(\phi_k-\phi_\Delta)\right]\;,\quad n\geq 1$$

Odderon Wigner distribution at small *x*

- The Fourier transform of the GTMD correlator is the Wigner distribution:

$$W^{[\Box]}(\boldsymbol{b}, \boldsymbol{k}) = \int d^2\boldsymbol{\Delta}\; e^{-i\boldsymbol{\Delta}\cdot\boldsymbol{b}}\, G^{[\Box]}(\boldsymbol{k}, \boldsymbol{\Delta})$$

and the correlations $k \leftrightarrow \Delta$ translate into correlations $k \leftrightarrow b$.

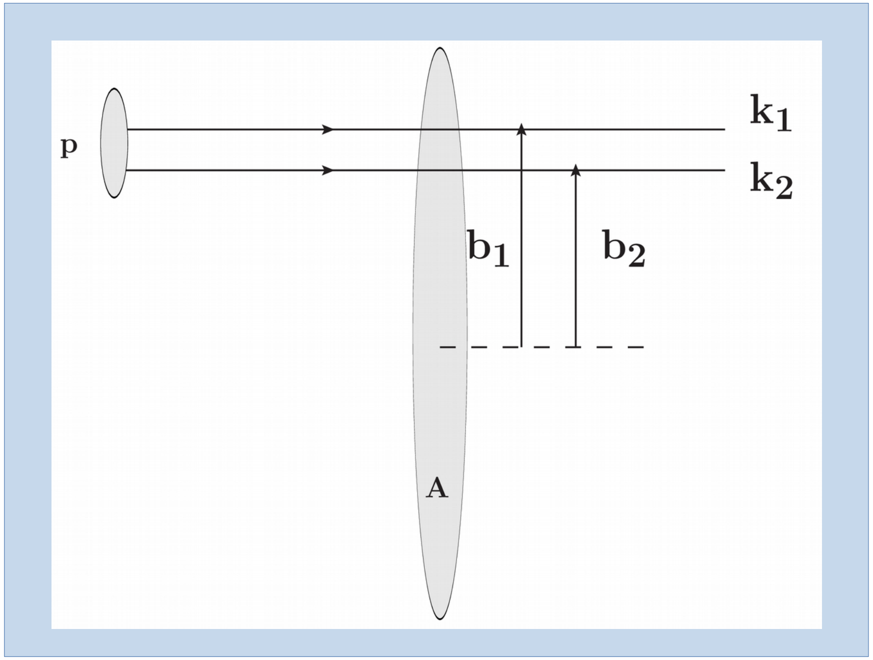
- To extract the angular correlations we parametrize the Wigner distribution:

$$xW(x, \boldsymbol{b}, \boldsymbol{k}) = x\mathcal{W}_0(x, \boldsymbol{b}^2, \boldsymbol{k}^2) + 2\cos(\phi_b - \phi_k)\, x\mathcal{W}_1(x, \boldsymbol{b}^2, \boldsymbol{k}^2)$$

$$+2\cos 2(\phi_b - \phi_k)\, x\mathcal{W}_2(x, \boldsymbol{b}^2, \boldsymbol{k}^2) + \ \ldots$$

Odderon Wigner distribution

Odd harmonics in two-particle production in pA



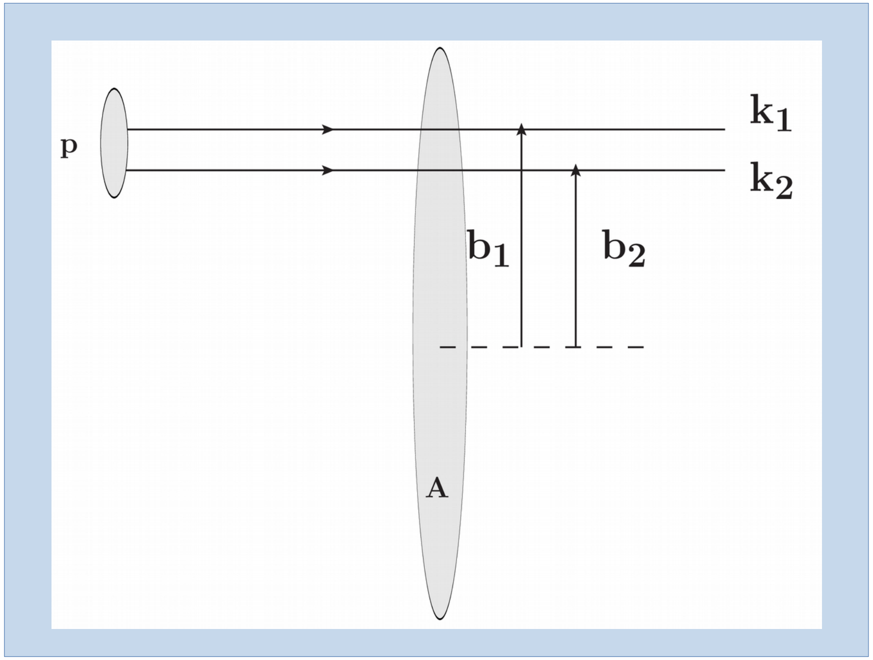
$$xW(x,\boldsymbol{b},\boldsymbol{k}) = x\mathcal{W}_0(x,\boldsymbol{b}^2,\boldsymbol{k}^2)$$

$$+2\cos(\phi_b-\phi_k)\,x\mathcal{W}_1(x,\boldsymbol{b}^2,\boldsymbol{k}^2)+\cdots$$

$$\frac{d\sigma^{pA}}{dy_1 dy_2\, d^2\boldsymbol{k}_1 d^2\boldsymbol{k}_2} \propto \int d^2\boldsymbol{b}_1\, d^2\boldsymbol{b}_2\, F_p(x_1,x_2,\boldsymbol{b}_1-\boldsymbol{b}_2)\, xW(x,\boldsymbol{b}_1,\boldsymbol{k}_1)\, xW(x,\boldsymbol{b}_2,\boldsymbol{k}_2)$$

$$\sim 1+2V_1\cos\left(\phi_{k_1}-\phi_{k_2}\right)+2V_2\cos 2\left(\phi_{k_1}-\phi_{k_2}\right)+\cdots$$

Odd harmonics in two-particle production in pA



$$xW(x,\boldsymbol{b},\boldsymbol{k}) = x\mathcal{W}_0(x,\boldsymbol{b}^2,\boldsymbol{k}^2)$$

$$+2\cos(\phi_b-\phi_k)\,x\mathcal{W}_1(x,\boldsymbol{b}^2,\boldsymbol{k}^2)$$

$$+2\cos 2(\phi_b-\phi_k)\,x\mathcal{W}_2(x,\boldsymbol{b}^2,\boldsymbol{k}^2)+\cdots$$

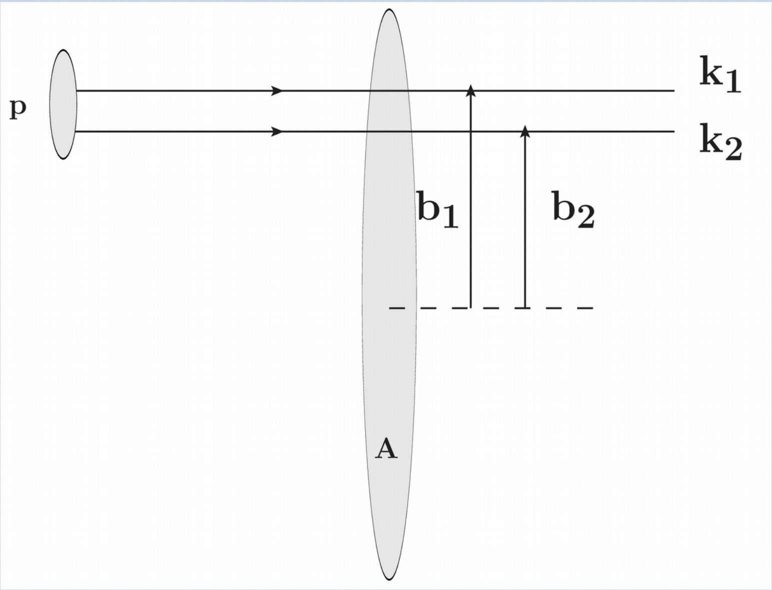
Eliptic Wigner distribution

$$\frac{d\sigma^{pA}}{dy_1 dy_2\, d^2\boldsymbol{k}_1 d^2\boldsymbol{k}_2} \propto \int d^2\boldsymbol{b}_1\, d^2\boldsymbol{b}_2\, F_p(x_1,x_2,\boldsymbol{b}_1-\boldsymbol{b}_2)\, xW(x,\boldsymbol{b}_1,\boldsymbol{k}_1)\, xW(x,\boldsymbol{b}_2,\boldsymbol{k}_2)$$

$$\sim 1+2V_1\cos\left(\phi_{k_1}-\phi_{k_2}\right)+2V_2\cos 2\left(\phi_{k_1}-\phi_{k_2}\right)+\cdots$$

Y. Hagiwara, Y. Hatta, B.-W. Xiao, F. Yuan, 2017

Odd harmonics in two-particle production in pA



$$xW(x,\boldsymbol{b},\boldsymbol{k}) = x\mathcal{W}_0(x,\boldsymbol{b}^2,\boldsymbol{k}^2)$$

$$+2\cos(\phi_b-\phi_k)\, x\mathcal{W}_1(x,\boldsymbol{b}^2,\boldsymbol{k}^2)+\cdots$$

$$V_1(\boldsymbol{k}_1^2,\boldsymbol{k}_2^2) \equiv \frac{\int db_1^2\, db_2^2\, e^{-\frac{b_1^2+b_2^2}{4R_N^2}}\, I_1\left(\frac{b_1 b_2}{2R_N^2}\right) x\mathcal{W}_1(x,\boldsymbol{b}_1^2,\boldsymbol{k}_1^2)\, x\mathcal{W}_1(x,\boldsymbol{b}_2^2,\boldsymbol{k}_2^2)}{\int db_1^2\, db_2^2\, e^{-\frac{b_1^2+b_2^2}{4R_N^2}}\, I_0\left(\frac{b_1 b_2}{2R_N^2}\right) x\mathcal{W}_0(x,\boldsymbol{b}_1^2,\boldsymbol{k}_1^2)\, x\mathcal{W}_0(x,\boldsymbol{b}_2^2,\boldsymbol{k}_2^2)}$$

Models for W_0 and W_1

$$V_1(\boldsymbol{k}_1^2, \boldsymbol{k}_2^2) \equiv \frac{\int db_1^2\, db_2^2\, e^{-\frac{b_1^2+b_2^2}{4R_N^2}}\, I_1\left(\frac{b_1 b_2}{2R_N^2}\right) x\mathcal{W}_1(x, \boldsymbol{b}_1^2, \boldsymbol{k}_1^2)\, x\mathcal{W}_1(x, \boldsymbol{b}_2^2, \boldsymbol{k}_2^2)}{\int db_1^2\, db_2^2\, e^{-\frac{b_1^2+b_2^2}{4R_N^2}}\, I_0\left(\frac{b_1 b_2}{2R_N^2}\right) x\mathcal{W}_0(x, \boldsymbol{b}_1^2, \boldsymbol{k}_1^2)\, x\mathcal{W}_0(x, \boldsymbol{b}_2^2, \boldsymbol{k}_2^2)}$$

The angular independent distribution W_0 can be obtained from the real part of the dipole operator in the McLerran-Venugopalan (MV) model:

$$\left\langle \mathcal{P}\left(\boldsymbol{b} + \tfrac{\boldsymbol{r}}{2}, \boldsymbol{b} - \tfrac{\boldsymbol{r}}{2}\right)\right\rangle_{x,A} = \exp\left[-\frac{1}{4}\, r^2 Q_s^2(b) \ln \frac{1}{r\Lambda}\right]$$

- The saturation scale can be defined in terms of the nuclear profile function $T(b)$

$$Q_s^2(b) \equiv \frac{4\pi\alpha_s^2 C_F}{N_c}\, T(b)$$

Models for W_0 and W_1

- Nuclear profile function:

$$T(\boldsymbol{b}) = \int_{-\infty}^{\infty} dz\, \rho_A\left(\sqrt{\boldsymbol{b}^2 + z^2}\right)$$

- ρ_A is the density of nucleons in the nucleus. We use Woods-Saxon distribution:

$$\rho_A(r) = \frac{N_A}{1 + \exp\left(\frac{r - R_A}{\delta}\right)}$$

$R_A = (1.12\,\mathrm{fm})A^{1/3}$ is the nuclear radius.

$\delta = 0.54\,\mathrm{fm}$ is the width of the 'nuclear edge'.

Similarly to E. Iancu and A. H. Rezaeian (2017)

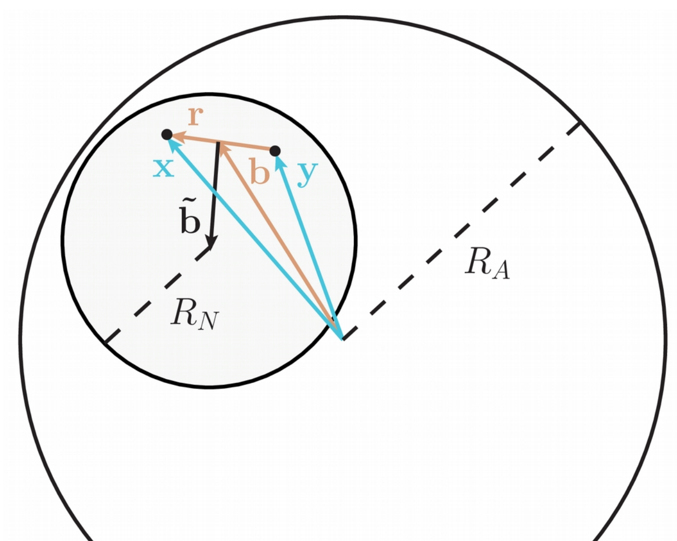
Models for W_0 and W_1

- The first odd-harmonic W_1 can be obtained from the odderon operator *(Kovchegov, Szymanowski and Wallon (2004), Hatta, Iancu, Itakura and McLerran (2005), Jeon and Venugopalan (2005), Kovchegov and Sievert (2012))*.

$$\left\langle \mathcal{O}\left(\boldsymbol{b}+\tfrac{\boldsymbol{r}}{2},\boldsymbol{b}-\tfrac{\boldsymbol{r}}{2}\right)\right\rangle = -\frac{3}{4R_N^4}\, c_0\alpha_s^3 r^2 \exp\left[-\frac{1}{4}\, r^2 Q_s^2(b)\ln\frac{1}{r\Lambda}\right]$$

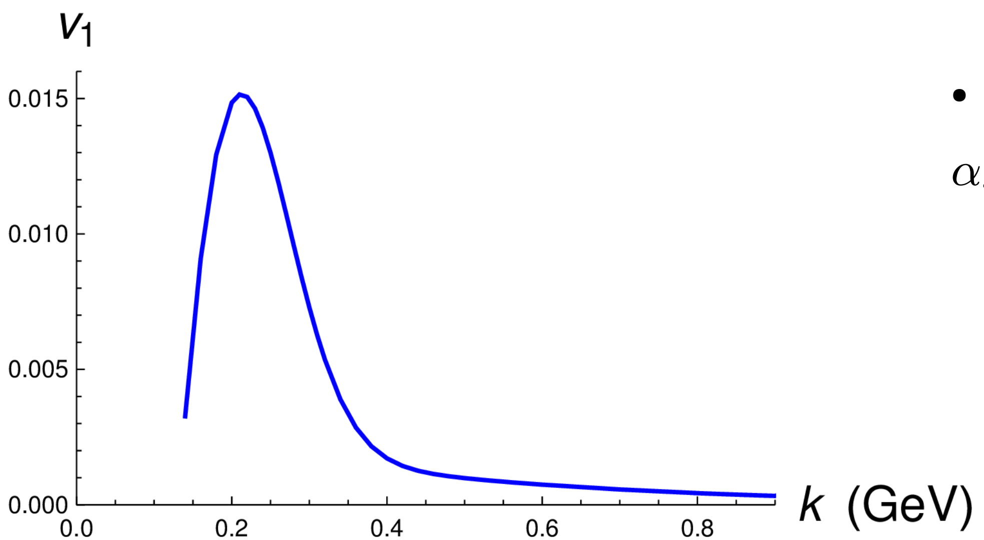
$$\times\, \boldsymbol{r}\cdot\int d^2\tilde{\boldsymbol{b}}\;\tilde{\boldsymbol{b}}\; T\left(|\boldsymbol{b}+\tilde{\boldsymbol{b}}|\right)\theta(R_N-\tilde{b})$$

$$T\left(|\boldsymbol{b}+\tilde{\boldsymbol{b}}|\right) = \left[1+\tilde{b}^i\frac{\partial}{\partial b^i}+\frac{1}{2}\,\tilde{b}^i\tilde{b}^j\frac{\partial^2}{\partial b^i\partial b^j}+\frac{1}{3!}\,\tilde{b}^i\tilde{b}^j\tilde{b}^k\frac{\partial^3}{\partial b^i\partial b^j\partial b^k}+\ldots\right]T(b)$$



First odd harmonic in two-quark production in pA

$$V_1(\boldsymbol{k}_1^2, \boldsymbol{k}_2^2) \equiv \frac{\int db_1^2\, db_2^2\, e^{-\frac{b_1^2+b_2^2}{4R_N^2}}\, I_1\left(\frac{b_1 b_2}{2R_N^2}\right) x\mathcal{W}_1(x, \boldsymbol{b}_1^2, \boldsymbol{k}_1^2)\, x\mathcal{W}_1(x, \boldsymbol{b}_2^2, \boldsymbol{k}_2^2)}{\int db_1^2\, db_2^2\, e^{-\frac{b_1^2+b_2^2}{4R_N^2}}\, I_0\left(\frac{b_1 b_2}{2R_N^2}\right) x\mathcal{W}_0(x, \boldsymbol{b}_1^2, \boldsymbol{k}_1^2)\, x\mathcal{W}_0(x, \boldsymbol{b}_2^2, \boldsymbol{k}_2^2)}$$



- For a lead nucleus with A=208, α_s=0.3 , $\Lambda = 0.24$ GeV, k_{ref}=$0.8 GeV$.

- Magnitude of v_1 of the same order as the maximal value observed in the data from ATLAS, Phys. Rev. C90 (2014) 044906.

D. Boer, T. van Daal, P. J. Mulders, EP; JHEP 2018

Summary

Generalized TMDs at small x :
source of azimuthal asymmetries
in small systems from initial state effects
(the radial nuclear profile).

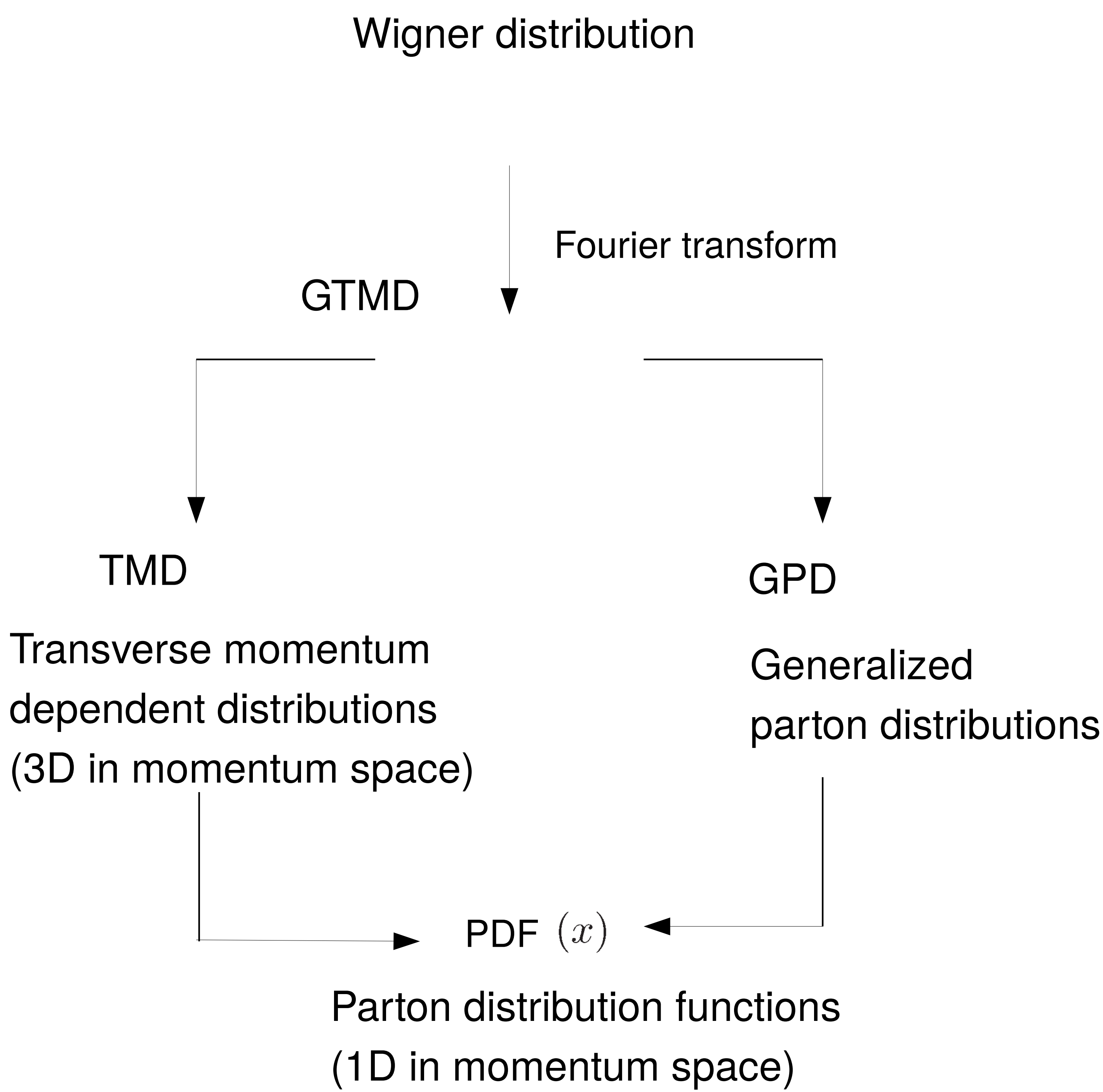
- In the small-x limit there is only one independent GTMD.

Outlook

Generalized TMDs at small x :
Weizsacker-Williams
gluon distribution at EIC through
diffractive dijet production

Third odd harmonic v_3

Thank you


Wigner distribution
Fourier transform
GTMD
TMD
Transverse momentum
dependent distributions
(3D in momentum space)
GPD
Generalized
parton distributions
PDF (x)
Parton distribution functions
(1D in momentum space)

Parametrizatization: Hadronic GTMD and TMD correlators cannot be calculated perturbatively, but they can be parametrized in terms of GTMDs and TMDs that give more explicit information and whose interpretation, properties and evolution can be separately studied.

Parametrization of the GTMD gluon-gluon correlator:

$$G^{[U,U']\,ij}(x,\mathbf{k},\xi,\Delta) = x\left(\delta_T^{ij}\,F_1 + \frac{k_T^{ij}}{M^2}\,F_2 + \frac{\Delta_T^{ij}}{M^2}F_3 + \frac{k_T^{[i}\Delta_T^{j]}}{M^2}\,F_4\right)$$

Kronecker delta in transverse space

GTMDs

[ ] → antisymmetrization of the indices

$k_T^{ij} \quad \Delta_T^{ij}$

rank-2 symmetric traceless tensors

$$a_T^{ij} \equiv a_T^i a_T^j + \frac{1}{2}\boldsymbol{a}^2 g_T^{ij}$$

GTMDs: $F_i = F_i^{[U,U']}(x,\xi,\boldsymbol{k}^2,\boldsymbol{\Delta}^2,\boldsymbol{k}\cdot\boldsymbol{\Delta})$

C. Lorcé and B. Pasquini, 2013
D. Boer, T. van Daal, P. J. Mulders, EP; JHEP 2018

Parametrizatization: Hadronic GTMD and TMD correlators cannot be calculated perturbatively, but they can be parametrized in terms of GTMDs and TMDs that give more explicit information and whose interpretation, properties and evolution can be separately studied.

Parametrization of the GTMD gluon-gluon correlator:

$$G^{[U,U']\,ij}(x,\mathbf{k},\xi,\Delta) = x\left(\delta_T^{ij}\,F_1 + \frac{k_T^{ij}}{M^2}\,F_2 + \frac{\Delta_T^{ij}}{M^2}F_3 + \frac{k_T^{[i}\Delta_T^{j]}}{M^2}\,F_4\right)$$

- In the forward limit the GTMDs are related to the TMDs:

$\lim_{\Delta\to 0} F_1 = f_1$ → Unpolarized gluons in unpolarized hadrons.

$\lim_{\Delta\to 0} F_2 = h_1^{\perp}$ → Linearly polarized gluons in unpolarized hadrons.

$\lim_{\Delta\to 0} F_3 = \lim_{\Delta\to 0} F_4 = 0$

- Upon integration over ***k***, the GTMDs reduce to the GPDs.

C. Lorcé and B. Pasquini, 2013
D. Boer, T. van Daal, P. J. Mulders, EP; JHEP 2018

Azimuthal correlations in di-hadron production in pA from initial-state effects

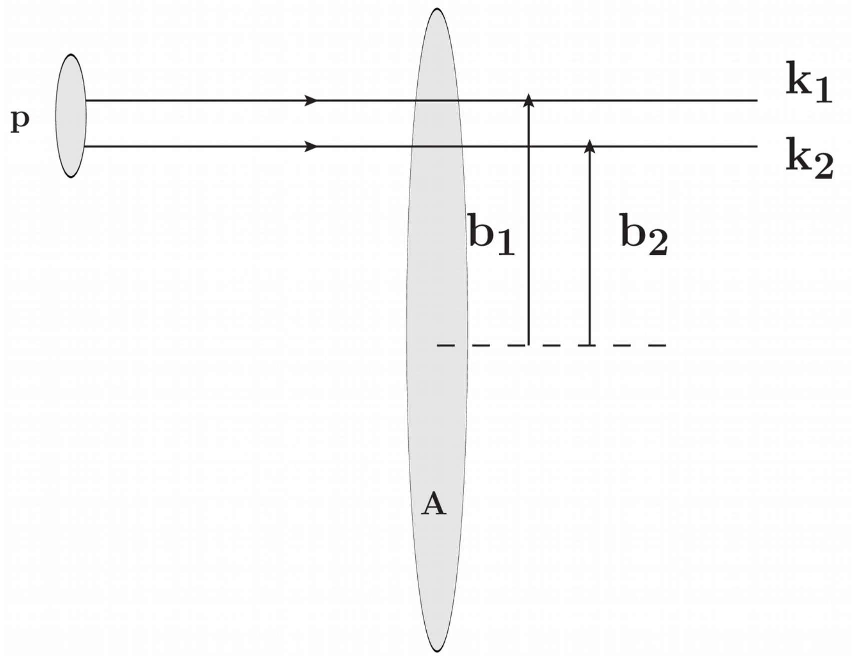
Odd harmonics for gluon scattering:

- Evolution of the fields in the final state:

  B. Schenke, S. Schlichting, R. Venugopalan; 2015

- Initial state effects:

  A. Kovner, M. Lublinsky, V. Skokov, 2017
  Y. V. Kovchegov, V. V. Skokov, 2018



Odd harmonics in forward two-particle production in peripheral pA collisions from the radial nuclear profile, leading in $1/N_c$.

↓

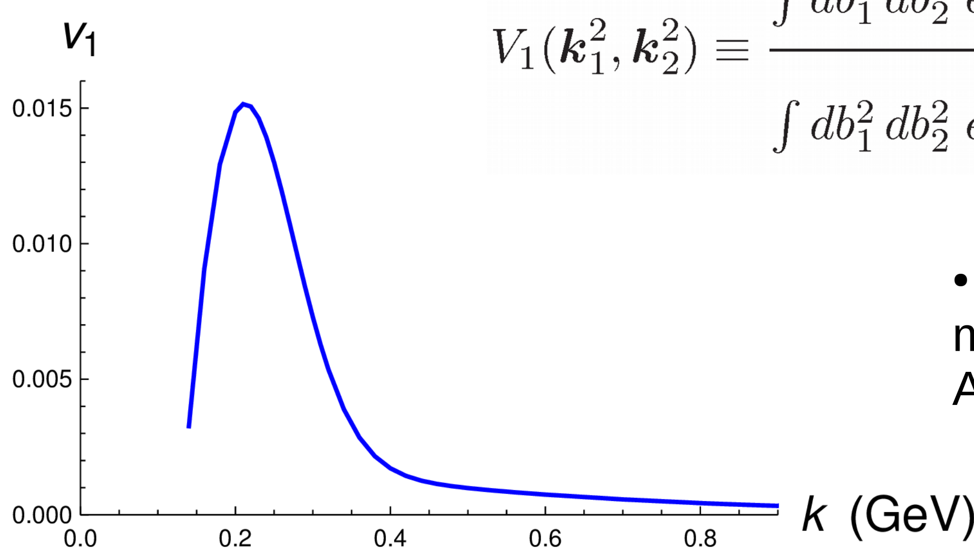
Woods-Saxon distribution

↓

$V_1, V_3 \ldots$

Odd harmonics survive when antiquarks are included as well

$$V_1(\boldsymbol{k}_1^2, \boldsymbol{k}_2^2) \equiv \frac{\int db_1^2\, db_2^2\, e^{-\frac{b_1^2+b_2^2}{4R_N^2}}\, I_1\left(\frac{b_1 b_2}{2R_N^2}\right) x\mathcal{W}_1(x, \boldsymbol{b}_1^2, \boldsymbol{k}_1^2)\, x\mathcal{W}_1(x, \boldsymbol{b}_2^2, \boldsymbol{k}_2^2)}{\int db_1^2\, db_2^2\, e^{-\frac{b_1^2+b_2^2}{4R_N^2}}\, I_0\left(\frac{b_1 b_2}{2R_N^2}\right) x\mathcal{W}_0(x, \boldsymbol{b}_1^2, \boldsymbol{k}_1^2)\, x\mathcal{W}_0(x, \boldsymbol{b}_2^2, \boldsymbol{k}_2^2)}$$



- Magnitude of v_1 of the same order as the maximal value observed in the data from ATLAS, Phys. Rev. C90 (2014) 044906

D. Boer, T. van Daal, P. J. Mulders, EP; JHEP 2018