Towards the QCD phase diagram from the Curci-Ferrari Model

Jan Maelger[1,2]

In collaboration with: U.Reinosa[1] and J.Serreau[2]

1. Centre de Physique Théorique, Ecole Polytechnique
2. AstroParticule et Cosmologie, Univ. Paris 7 Diderot

GDR QCD 2018, November 28

Curci-Ferrari Model
- Motivation
- Perturbation theory in CF
- Correlation functions

Phase diagram & Columbia plot
- Yang-Mills
- Heavy Quarks
- Light Quarks

Conclusion & Outlook

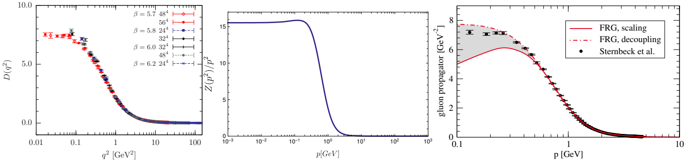
Motivation

- One of the most celebrated properties of QCD is Asymptotic Freedom, which means that $g_s(E) << 1$ for $E >> 1$ GeV
- Allows for a weak coupling expansion in the high energy regime
- Perturbation theory is a very useful tool in the UV
- Lowering the energy scale, the coupling constant eventually diverges in the Landau pole
- This is usually regarded as the onset of non-perturbative QCD and one refers to non-pert. methods such as
  - Lattice QCD
  - Dyson-Schwinger Equations
  - Functional Renormalization Group
- However, perturbation theory is based on the Fadeev-Popov Lagrangian because of the necessity to gauge fix
- While the FP Lagrangian is aligned with QCD in the UV, it is well-known that this association breaks down in the IR

Motivation

1. In all covariant gauge fixings, the FP procedure is non-complete in the IR and leaves a residual ambiguity due to the presence of Gribov copies [Singer (1978)]
2. Landau gauge gluon propagator - decoupling behavior

[Sternbeck et al. (2006)] [Huber (2018)] [Cyrol et al. (2016)]



So clearly, in order to describe IR QCD, the FP Lagrangian is not enough and needs to be modified!

CF-Model as an effective theory

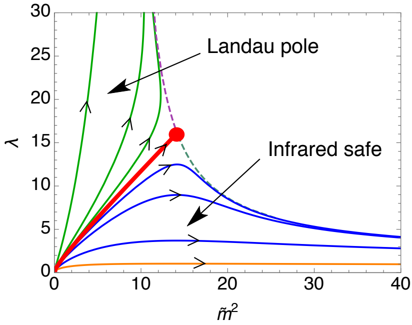
1. On a lattice, one picks one copy by hand for each gauge configuration $\longrightarrow$ Minimal Landau gauge
2. restrict the space of gauge transformations to the first Gribov region $\longrightarrow$ (refined) Gribov-Zwanziger action
3. modify the theory by the addition of an operator to obtain an effective model $\longrightarrow$ **Curci-Ferrari Model**

$$S = \int_x \left\{ \frac{1}{4}(F^a_{\mu\nu})^2 + \bar{\psi}(\not{D} + M + \mu\gamma_0)\psi \right\} + \underbrace{S_{FP}}_{\text{Landau}} + \int_x \left\{ \frac{1}{2} m^2 (A^a_\mu)^2 \right\}$$

- minimal effective theory in the IR while keeping the UV fixed
- gluon mass term softly breaks BRST symmetry
- CF is still perturbatively renormalizable

RG-flow in CF and Pert. Theory

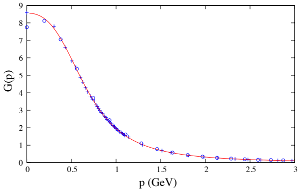
- allows for infrared safe RG flows without a Landau pole permitting a perturbative treatment at all momentum scales down to the deep IR [Tissier, Wschebor (2011)]



from [Reinosa, Serreau, Tissier, Wschebor (2017)]
lattice data correspond to [Dudal, Oliveira, Vandersickel (2010)]
and [Bogolubsky, Ilgenfritz, M.-Preussker, Sternbeck (2009)]

Curci-Ferrari Model

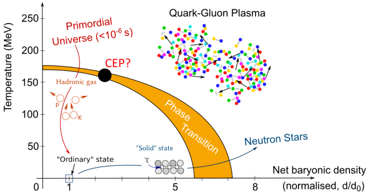
- Superficially, there is one extra parameter in the CF Model
- In principle, it should be fixed intrinsically from the theory itself (Gribov copies, Λ_{QCD} ...)
- In practice, fix the gluon mass by fitting the calculated gluon propagator against corresponding Lattice data and then keep it fixed in any further calculation.

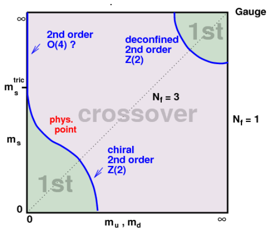


one-loop gluon propagator [Tissier, Wschebor (2011)] against lattice data [Bogolubsky et al. (2009), Dudal, Oliveira, Vandersickel (2010)]

- The optimal value is around 500 MeV
- Many more correlation functions have been computed in reasonable qualitative and quantitative agreement with lattice findings [Pelàez, Reinosa, Serreau, Tissier, Tresmontant, Wschebor]

Phase diagram & Columbia plot

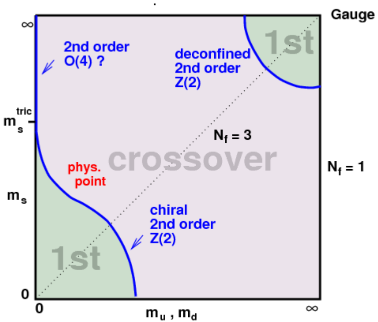




Several other approaches on the market:

- Lattice QCD [de Forcrand, Philipsen, Rodriguez-Quintero, Mendes, ...]
- Dyson Schwinger Equations [Alkofer, Fischer, Huber, ...]
- Functional Renormalization Group [Pawlowski, Mitter, Schaefer...]
- Variational Approach [Reinhardt, Quandt, ...]
- Gribov-Zwanziger Action [Dudal, Oliveira, Zwanziger...]
- Matrix-, QM-, NJL-Model,... [Pisarski, Dumitru, Schaffner-B., Stiele, ...]

At the Yang-Mills point



Phys. Lett. B **742** (2015) 61

Phys. Rev. D **91** (2015) 045035

Phys. Rev. D **93** (2016) 105002 all [Reinosa, Serreau, Tissier, Wschebor]

Polyakov loops as order parameters

At the YM point, a relevant order parameter for the deconfinement transition is the (anti-)Polyakov loop. It is related to the free energy F_q necessary to bring a quark into a "bath" of gluons.

$$\ell \equiv \frac{1}{3}\mathrm{tr}\left\langle P\exp\left(ig\int_0^\beta d\tau A_0^a t^a\right)\right\rangle \sim e^{-\beta F_q} \qquad \bar{\ell} \sim e^{-\beta F_{\bar{q}}}$$

Hence

$$\underbrace{\ell = 0 \leftrightarrow F_q = \infty \leftrightarrow \text{confinement}}_{\text{imposed by center symmetry}} \qquad \ell \neq 0 \leftrightarrow F_q < \infty \leftrightarrow \text{deconfinement}$$

→ It is thus very important to work in a choice of gauge which does **not** explicitly break center symmetry!

Landau-DeWitt gauge [Braun, Pawlowski, Gies (2010)]

$$A_\mu^a = \bar{A}_\mu^a + a_\mu^a$$

In practice, at each temperature, the background field $\bar{A}_\mu^a$ is chosen such that the expectation value $\langle a_\mu^a \rangle$ vanishes in the limit of vanishing sources.

This corresponds to finding the absolute minimum of $\tilde{\Gamma}[\bar{A}] \equiv \Gamma[\bar{A}, \langle a \rangle = 0]$, where $\Gamma[\bar{A}, \langle a \rangle]$ is the effective action for $\langle a \rangle$ in the presence of $\bar{A}$.

Seek the minima in the subspace of configurations $\bar{A}$ that respect the symmetries of the system at finite temperature.
$\longrightarrow$ One restricts to temporal and homogenous backgrounds:

$$\bar{A}_\mu(\tau, \mathbf{x}) = \bar{A}_0 \delta_{\mu 0}$$

$\longrightarrow$ functional $\tilde{\Gamma}[\bar{A}]$ reduces to an effective potential $V(\bar{A}_0)$ for the constant matrix field $\bar{A}_0$.

One can always rotate this matrix $\bar{A}_0$ into the Cartan subalgebra:

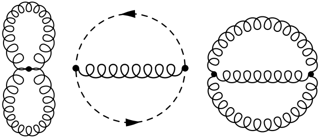
$$\beta g \bar{A}_0 = r_3 \frac{\lambda_3}{2} + r_8 \frac{\lambda_8}{2}$$

Then $V(\bar{A}_0)$ reduces to a function of 2 components $V(r_3, r_8)$.

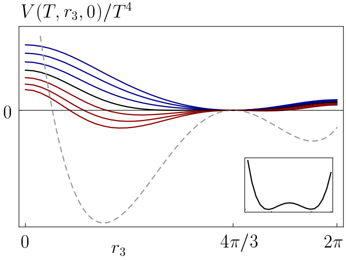
	r_3	r_8
Yang-Mills	$\mathbb{R}$	0
$\mu = 0$	$\mathbb{R}$	0
$\mu \in i\,\mathbb{R}$	$\mathbb{R}$	$\mathbb{R}$
$\mu \in \mathbb{R}$	$\mathbb{R}$	$i\,\mathbb{R}$

Yang-Mills Two-loop Expansion

$$V(r_3, r_8) = \frac{3}{2}\mathrm{Tr}\,\mathrm{Ln}\left(\bar{D}^2 + m^2\right) - \frac{1}{2}\mathrm{Tr}\,\mathrm{Ln}\left(\bar{D}^2\right) +$$



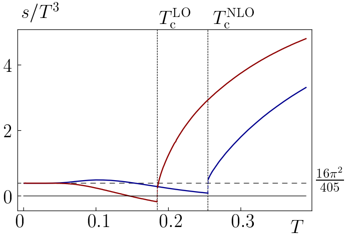
Yang-Mills Results

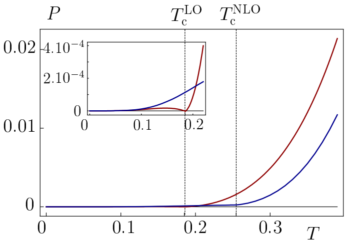


transition temperatures in MeV:

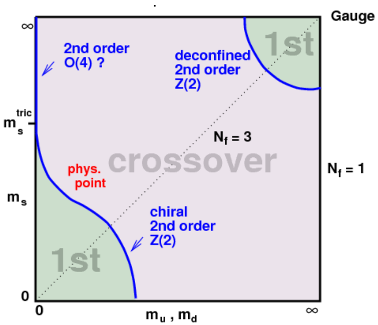
	SU(3)	SU(2)
One-loop	185	237
Two-loop	254	284
Lattice	270	295

Moreover some worrisome thermodynamic curiosities present at one-loop order disappear upon taking into account the two-loop corrections, eg. negative entropy and pressure.





Heavy Quarks



Phys. Rev. D **92** (2015) 025021 [Reinosa, Serreau, Tissier]

Phys. Rev. D **97** (2018) 074027 [Maelger, Reinosa, Serreau]

Polyakov loops as order parameters

Yang-Mills:

At the YM point, a relevant order parameter for the deconfinement transition is the (anti-)Polyakov loop. It is related to the free energy F_q necessary to bring a quark into a "bath" of gluons.

$$\ell \equiv \frac{1}{3}\mathrm{tr}\left\langle P\exp\left(ig\int_0^\beta d\tau A_0^a t^a\right)\right\rangle \sim e^{-\beta F_q} \qquad \bar{\ell} \sim e^{-\beta F_{\bar{q}}}$$

Hence

$$\ell = 0 \leftrightarrow F_q = \infty \leftrightarrow \text{confinement} \qquad \ell \neq 0 \leftrightarrow F_q < \infty \leftrightarrow \text{deconfinement}$$

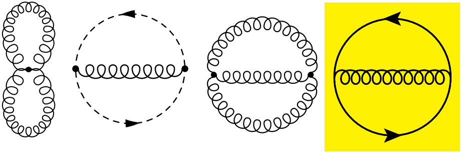
Unquenched:

Introducing quarks, center symmetry is explicitly broken. For heavy quarks, this breaking is "soft", thus:

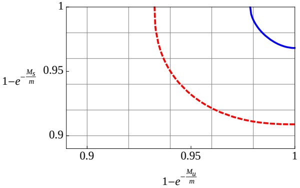
$$\ell \approx 0 \leftrightarrow F_q \approx \infty \leftrightarrow \text{confinement} \qquad \ell \not\approx 0 \leftrightarrow F_q < \infty \leftrightarrow \text{deconfinement}$$

Heavy Quark Two-loop Expansion

$$V(r_3, r_8) = -\mathrm{Tr}\,\mathrm{Ln}\left(\not{\partial} + M + \mu\gamma_0 - ig\gamma_0 \bar{A}^k t^k\right)$$
$$+\frac{3}{2}\mathrm{Tr}\,\mathrm{Ln}\left(\bar{D}^2 + m^2\right) - \frac{1}{2}\mathrm{Tr}\,\mathrm{Ln}\left(\bar{D}^2\right) +$$



Vanishing chemical potential



$$R_{N_f} \equiv \frac{M_c(N_f)}{T_c(N_f)}$$

$\mathcal{O}(1)$: $M_{\text{bare}} = M_{\text{ren.}}$
$\mathcal{O}(g^2)$: $M_{\text{bare}} = Z_M\, M_{\text{ren.}} + C_M$

$\longrightarrow$ hard to compare between different approaches!

However, Z_M, C_M are independent of N_f at $\mathcal{O}(g^2)$, and observing

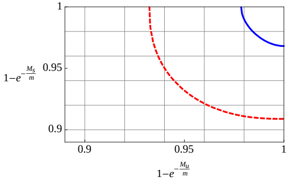
$$\frac{T_c(N_f = 3) - T_c(N_f = 1)}{T_c(N_f = 1)} \approx 0.2\%$$

allows for:

$$\overbrace{R_{N_f'}/R_{N_f} \approx M_c(N_f')/M_c(N_f)}^{\text{if } C_M=0} \qquad \overbrace{Y_{N_f} \equiv \frac{R_{N_f} - R_1}{R_2 - R_1}}^{\text{if } C_M\neq 0}$$

is scheme indep. & comparable to other approaches up to higher order corrections.

Vanishing chemical potential



$$R_{N_f} \equiv \frac{M_c(N_f)}{T_c(N_f)}$$

$$Y_{N_f} \equiv \frac{R_{N_f} - R_1}{R_2 - R_1}$$

R_{N_f}	$N_f = 1$	$N_f = 2$	$N_f = 3$	R_2/R_1	R_3/R_1	Y_3
1-loop	6.74	7.59	8.07	1.13	1.20	1.58
2-loop	7.53	8.40	8.90	1.12	1.18	1.57
Lattice [1]	7.23	7.92	8.33	1.10	1.15	1.59
DSE [2]	1.42	1.83	2.04	1.29	1.43	1.51
Matrix [3]	8.04	8.85	9.33	1.10	1.16	1.59

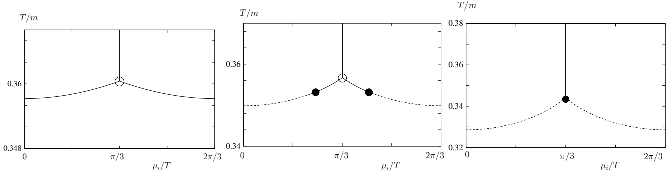
⟶ The overall good agreement seems to suggest that the underlying dynamics is well-described within perturbation theory.

[1] M. Fromm, J. Langelage, S. Lottini and O. Philipsen (2012)

[2] C. S. Fischer, J. Luecker and J. M. Pawlowski (2015)

[3] K. Kashiwa, R. D. Pisarski and V. V. Skokov (2012)

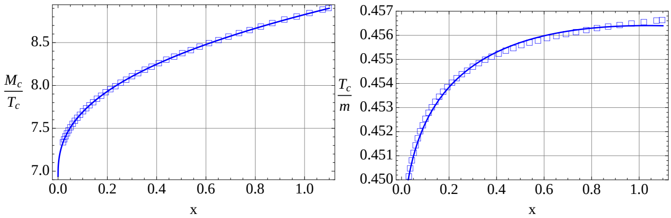
Imaginary chemical potential $\mu = i\mu_i$



The vicinity of the tricritical point is approximately described by the mean field scaling behavior

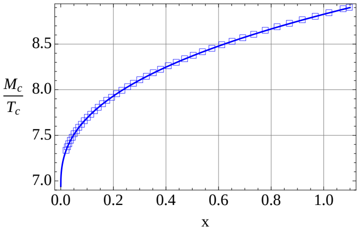
$$\frac{M_c(\mu_i)}{T_c(\mu_i)} = \frac{M_{\text{tric.}}}{T_{\text{tric.}}} + K\left[\left(\frac{\pi}{3}\right)^2 - \left(\frac{\mu_i}{T_c}\right)^2\right]^{\frac{2}{5}}$$

[de Forcrand, Philipsen (2010); Fischer, Luecker, Pawlowski (2015)]



$$x \equiv (\pi/3)^2 + (\mu/T_c)^2 = (\pi/3)^2 - (\mu_i/T_c)^2$$

Imaginary chemical potential $\mu = i\mu_i$



$$\frac{M_c}{T_c}(x) \approx 6.939 + 1.888\, x^{2/5}$$

$$\frac{M_c(N_f, \mu_i)}{M_c(N_f = 1, \mu_i)} \approx \frac{R_{N_f}(\mu_i)}{R_1(\mu_i)}$$

at $\mu = \mu_i i = i\pi/3$

$R_{N_f}(\pi/3)$	$N_f = 1$	$N_f = 2$	$N_f = 3$	R_2/R_1	R_3/R_1	Y_3
1-loop	4.74	5.63	6.15	1.19	1.30	1.57
2-loop	5.47	6.41	6.94	1.17	1.27	1.57
Lattice [1]	5.56	6.25	6.66	1.12	1.20	1.59
DSE [2]	0.41	0.85	1.11	2.07	2.70	1.59
Matrix [3]	5.00	5.90	6.40	1.18	1.28	1.56

[1] Fromm et al. (2012), [2] Fischer et al. (2015), [3] Kashiwa et al.(2012)

Real chemical potential

- $V(r_3, r_8) \in \mathbb{C}$
- $V(\ell, \bar{\ell}) \in \mathbb{C}$ $\longrightarrow$ physical point $\hat{\neq}$ absolute minimum

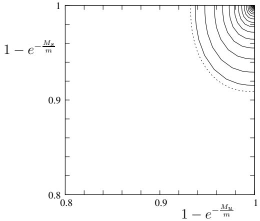
Common fix: $V = \mathrm{Re}\,V +$ ~~$i\,\mathrm{Im}\,V$~~ $\rightarrow$ No explicit breaking of charge conjugation, ie $r_8 \equiv 0$ or $q \hat{=} \bar{q}$!

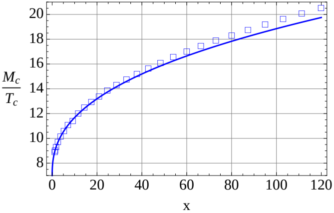
Instead, we can continue the r_8-component via $r_8 \rightarrow i r_8$

$\hat{\equiv}$ $\ell \& \bar{\ell} \in \mathbb{R}$ and indep. [Dumitru, Pisarski, Zschiesche (2005)]

Then

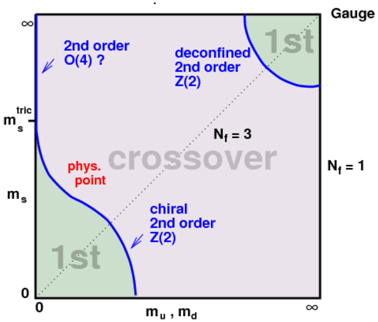
- $V(r_3, r_8) \in \mathbb{C} \longrightarrow V(r_3, i r_8) \in \mathbb{R}$
- min $V(r_3, r_8) \longrightarrow$ saddle point in $\mathbb{R} \times i\mathbb{R}$
- residual ambiguity: Wich saddle $\hat{=}$ physical point?
  $\longrightarrow$ Choose convention to pick the lowest saddle! (well-motivated around $\mu \approx 0$)





$$\frac{M_c}{T_c}(x) \approx 6.939 + 1.888\,x^{2/5} \qquad x \equiv (\pi/3)^2 + (\mu/T_c)^2 = (\pi/3)^2 - (\mu_i/T_c)^2$$

Light Quarks



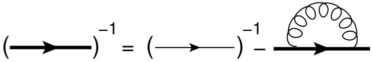
Preliminary/ In preparation [Maelger, Reinosa, Serreau]

Quark Propagator as Order Parameter

The light quark regime is governed by chiral symmetry breaking and restoration. An appropriate order parameter in the chiral limit, $M_{\text{bare}} = 0$, is the quark condensate or the mass function $B(Q)$ of the quark propagator $S(Q) = \langle q(Q)\bar{q}(0)\rangle$, where

$$B(Q) \neq 0 \leftrightarrow \text{broken}\,\chi \qquad B(Q) = 0 \leftrightarrow \text{restored}\,\chi$$

An integral equation for the full quark propagator is given by the resummed rainbow-laddder equation:



$$S^{-1}(P) \quad = \quad -i\not{P} + M_{\text{bare}} + g^2_{\text{bare}} \int_{\hat{Q}} \gamma_\mu G_{\mu\nu}(P-Q)S(Q)\gamma_\nu$$

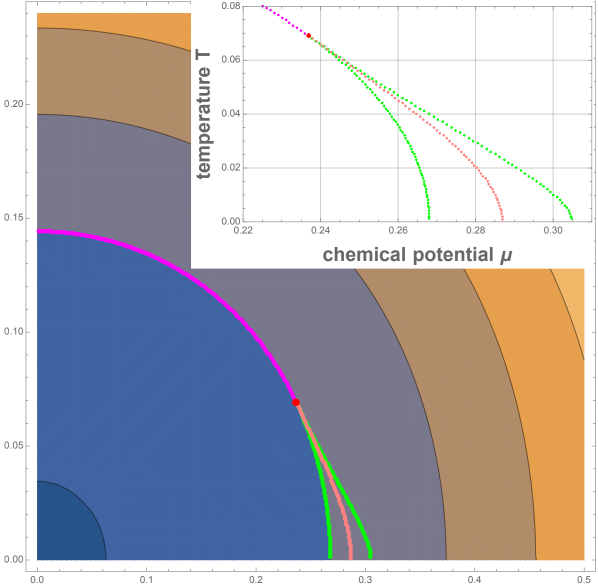
Localisation and other Approximations

- After invoking parity, charge and complex conjugation, the most general form of the propagator is

$$S^{-1}(P) = B(P)\mathbf{1} \underbrace{-i\gamma_0 A_s(P) - i\underline{\gamma}\cdot\underline{\hat{p}}A_v(P) - i\gamma_0\underline{\gamma}\cdot\underline{\hat{p}}A_t(P)}_{\text{consider trivial: } -i\not{P}}$$

- Localise the equation and consider $B(P) = B(p_0, 0)$
- analytically continue in the frequency q_0
- Finally, we fix the gluon mass to 500 MeV and phenomenologically choose the coupling such we have a chiral symmetry breaking solution of mass 300 MeV at zero temperature.

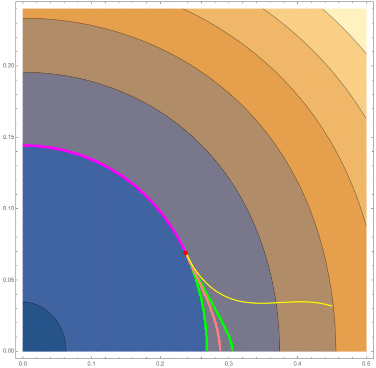
Results in the chiral limit - preliminary

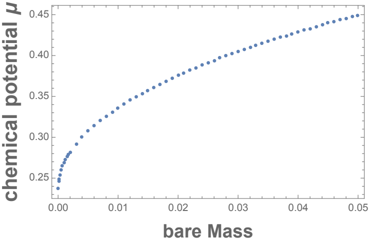


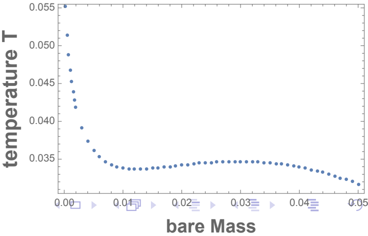
IR QCD from Curci-Ferrari

Jan Maelger

Results for non-zero bare mass - preliminary







Conclusion & Outlook

CONCLUSION:

- The heavy quark phase diagram (+YM) is qualitatively well-described by a simple CF one-loop calculation
- Two-loop corrections lead to quantitative improvements
- A localised rainbow-ladder analysis within CF is able to capture the physics underlying the light quark regime
- suggests that the description of the phase diagram within the CF model is robust

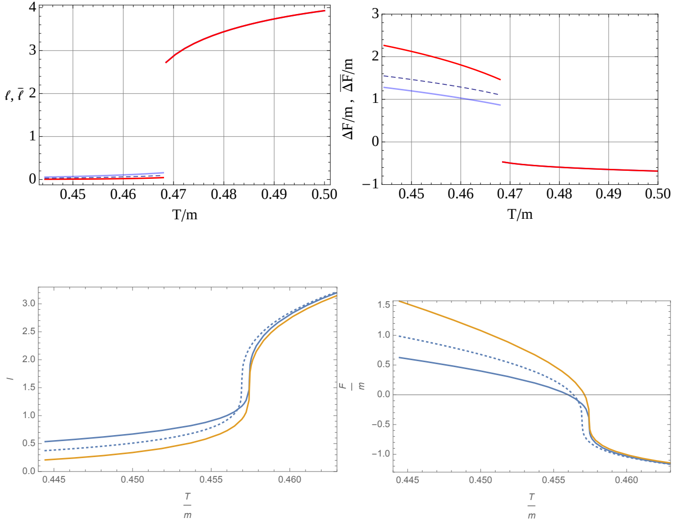
OUTLOOK:

- More refined description of the chiral sector (flavor blindness, ...)
- Off equilibrium thermodynamics?
- Real time observables?
- ...

Backup slides

Call in the reinforcements!!

Explicit breaking of charge-conjugation in Polyakov loops

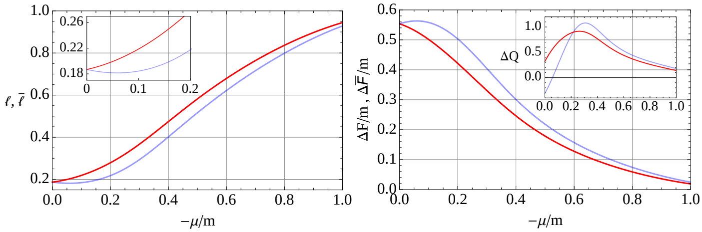


Curci-Ferrari: Motivation

To doubt everything, or, to believe everything, are two equally convenient solutions; both dispense with the necessity of reflection.

Henri Poincaré

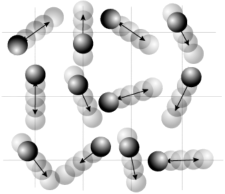
$\ell_{q,\bar{q}}(\hat{\mu})$ and $F_{q,\bar{q}}(\hat{\mu})$



- Trace $\ell_{q,\bar{q}}$ and $F_{q,\bar{q}}$ as functions of $\hat{\mu} = -\mu$

  $\longrightarrow$ ℓ and F_q change monotony, but $\bar{\ell}$ and $F_{\bar{q}}$ don't! Then $\ell, \bar{\ell}$ increase together towards 1 [Dumitru, Hatta, Lenaghan, Orginos, Pisarski (2004)]
- "Free energy must be strictly monotonically decreasing as a function of chemical potential" $\longrightarrow$ contradicts $\ell = e^{-\beta F_q}$?
- Interpretation $\ell \sim e^{-\beta F_q}$ is saved by a simple thermodynamic argument if the charge of the bath at $\hat{\mu} = 0$ is not zero

Pure Thermal bath



free energy of the bath:

$$F = -T \ln \mathrm{tr} \exp\{-\beta(H - \hat{\mu} Q)\}$$

Q is the baryonic charge and $\hat{\mu} = -\mu$

One easily obtains that

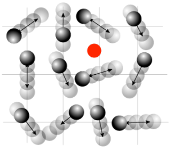
$$\frac{\partial F}{\partial \hat{\mu}} = -\langle Q \rangle \quad \text{and} \quad \frac{\partial \langle Q \rangle}{\partial \hat{\mu}} = \beta \left\langle (Q - \langle Q \rangle)^2 \right\rangle > 0 \,.$$

Now, in absence of any external sources, the thermal bath is charge-conjugation invariant for $\hat{\mu} = 0$:

$$\langle Q \rangle_{\hat{\mu}=0} = 0$$

$\longrightarrow$ for any $\hat{\mu} > 0$: $\langle Q \rangle > 0$ and thus $\frac{\partial F}{\partial \hat{\mu}} < 0$, i.e. the free energy of the bath is a decreasing function of $\hat{\mu}$

Thermal bath with charged test source



from before:

$$\frac{\partial F}{\partial \hat{\mu}} = -\langle Q \rangle \qquad \frac{\partial \langle Q \rangle}{\partial \hat{\mu}} = \beta \left\langle (Q - \langle Q \rangle)^2 \right\rangle > 0$$

In the presence of a static quark (q) or antiquark ($\bar{q}$), charge-conjugation invariance is broken s.t.:

$$\langle Q \rangle_{q,\hat{\mu}=0} < 0 \qquad \langle Q \rangle_{\bar{q},\hat{\mu}=0} > 0$$

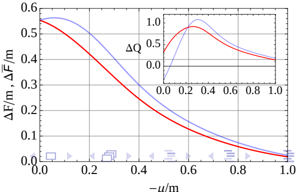
The equations above then imply that

$$\forall \hat{\mu} > 0\,, \quad \langle Q \rangle_{\bar{q}} > 0\,,$$

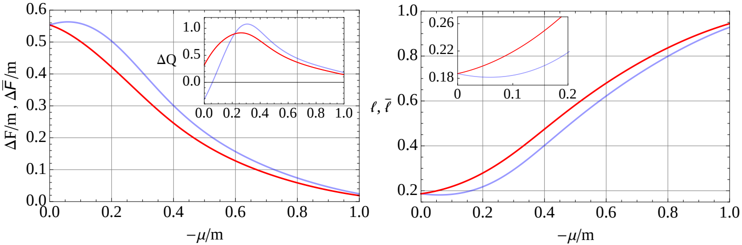
while there exists a certain $\hat{\mu}_0 > 0$ such that,

$$\forall \hat{\mu} \in [0, \hat{\mu}_0]\,,\ \langle Q \rangle_q < 0 \quad \text{and} \quad \forall \hat{\mu} > \hat{\mu}_0\,,\ \langle Q \rangle_q > 0\,.$$

Therefore
$F_{\bar{q}}$ is monotonously decreasing
for $\hat{\mu} > 0$, while
F_q first increases and then decreases



Thermal bath with charged test source



Then

$$\ell \sim e^{-\beta(F_q - F)} \qquad \bar{\ell} \sim e^{-\beta(F_{\bar{q}} - F)}$$

are found by the free energy differences wrt to the bath without any external source.

Since $\frac{\partial F}{\partial \hat{\mu}} = 0\big|_{\hat{\mu}=0}$, both are dominated for small $\hat{\mu}$ by either F_q or $F_{\bar{q}}$, which explains the different monotony.

$\Delta\langle Q_q\rangle$ and $\Delta\langle Q_{\bar{q}}\rangle$ should approach 0 at large $\hat{\mu}$, which we also observe.