INSTITUT DENIS POISSON
ORLÉANS · TOURS
Mathématiques & Physique Théorique

Casimir Effect in Gauge Theories in 2+1

Ha NGUYEN

Institut Denis Poisson, Université de Tours, France

In collaboration with: M. N. Chernodub[1], V. A. Goy[2], A. V. Molochkov[2]

1. Institut Denis Poisson, Université de Tours, France
2. Far Eastern Federal University, Vladivostok, Russia

Based on: Phys. Rev. Lett. **121**, 191601 (2018)

GDR QCD 2018, 26-28 novembre 2018, École Polytechnique

The Casimir Effect

Named after Dutch physicist Hendrik Casimir
[H.B.G. Casimir, Proc. K. Ned. Acad. Wet. 51, 793 (1948)]
(2.5 page–long article).

[Source: Wikipedia]

Mathematics. — *On the attraction between two perfectly conducting plates.* By H. B. G. CASIMIR.

(Communicated at the meeting of May 29, 1948.)

Outline:

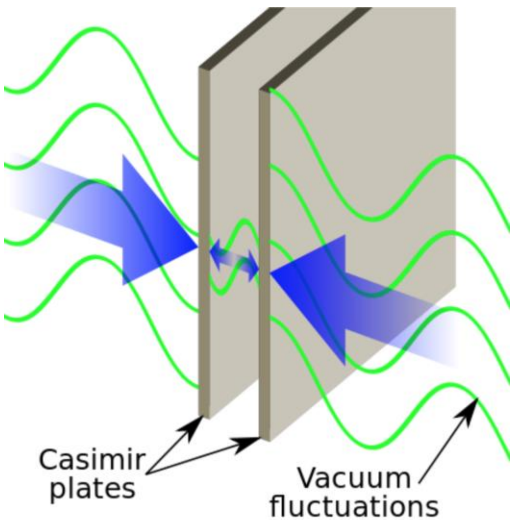
– Short description of Casimir Effect
– Casimir effect in Compact QED

– Casimir effect in Yang-Mills fields

.

Casimir effect

Simplest setup: two parallel perfectly conducting plates at finite distance *R*.



From Wikipedia

- The plates modify the energy spectrum of the electromagnetic field, and lead to a finite contribution to the vacuum energy.

$$\langle E \rangle = \frac{1}{2} \sum_n E_n$$

- The energy depends on the inter-plate distance *R*,

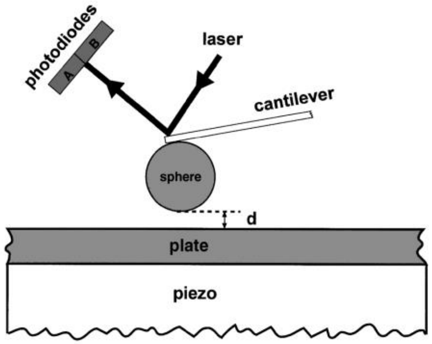
$$\frac{\langle E \rangle}{\text{Area}} = -\frac{\pi^2}{720} \frac{1}{R^3} \hbar c$$

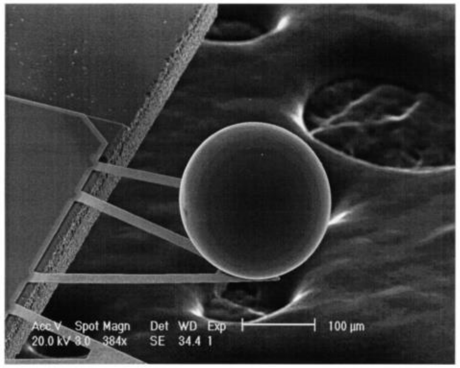
Proves existence of the zero-point energy? Questioned in [R. L. Jaffe, Phys. Rev. D72, 021301 (2005)]

leading to an attraction between the neutral plates.

Experimentally confirmed
(in plate-sphere geometries)
1% agreement with the theory

A very small force at human scales. However, at *R* =10 nm the pressure is about 1 atmosphere.





[S. K. Lamoreaux, Phys. Rev. Lett. 78, 5 (1997)]

From U. Mohideen and A. Roy, Phys. Rev. Lett. 81, 4549 (1998), down to 100 nm scale.

Types of boundary conditions for a gauge field

- Ideal electric conductor: at the boundary
  normal magnetic field and tangential electric field are vanishing:

$$B_\perp \Big|_{x\in S} = 0\,, \qquad \boldsymbol{E}_{||}\Big|_{x\in S} = 0$$

$\rightarrow$ These conditions were originally considered by H. Casimir.

- Ideal magnetic conductor:
  normal electric field and tangential magnetic field are vanishing:

$$E_\perp \Big|_{x\in S} = 0\,, \qquad \boldsymbol{B}_{||}\Big|_{x\in S} = 0$$

$\rightarrow$ A non-Abelian version of these conditions is suitable for the MIT bag model (normal component of the classical gluon current vanishes at the boundary).

Toy model: (2+1)d compact U(1) gauge theory (cQED)

The compact QED (cQED) is a toy model for QCD:
- exhibits both confinement and mass gap generation at $T = 0$
- possesses a deconfinement phase transition at $T > 0$
- can be treated analytically and, of course, numerically.

[A. M. Polyakov, Nucl.Phys. B120, 429 (1977)]

Simple Lagrangian: $$\mathcal{L} = \frac{1}{4} F_{\mu\nu}^2$$

Field strength: $$\underbrace{F_{\mu\nu} = F_{\mu\nu}^{\mathrm{ph}}}_{\text{photons}} + \underbrace{F_{\mu\nu}^{\mathrm{mon}}}_{\text{monopoles}}$$

$$g_{\mathrm{mon}} = \frac{2\pi}{g}$$

photons

monopoles

Photon field strength:

$$F_{\mu\nu}^{\mathrm{ph}}[A] = \partial_\mu A_\nu - \partial_\nu A_\mu$$

monopole density and charge

$$\rho(\boldsymbol{x}) = \sum_a q_a \delta^{(3)}(\boldsymbol{x} - \boldsymbol{x}_a)$$

Field strength of the monopoles:

$$F_{\mu\nu}^{\mathrm{mon}}(\boldsymbol{x}) = -g_{\mathrm{mon}} \epsilon_{\mu\nu\alpha} \partial_\alpha \int d^3 y D(\boldsymbol{x} - \boldsymbol{y}) \rho(\boldsymbol{y})$$

Basic non-perturbative properties of cQED (at $T = 0$)

– The cQED may be mapped into the sine-Gordon model:

$$\mathcal{L}_s = \frac{1}{2g^2_{\mathrm{mon}}} \left(\partial_\mu \chi\right)^2 - 2\zeta \cos\chi$$

χ is the scalar real-valued field. The mean density of monopoles

$$\varrho_{\mathrm{mon}} = 2\zeta$$

is controlled by the fugacity parameter ζ. The monopoles lead to

– mass gap generation (photon mass / Debye screening):

$$m_{\mathrm{ph}} = g_{\mathrm{mon}} \sqrt{2\zeta} \equiv \frac{2\pi}{g} \sqrt{\varrho_{\mathrm{mon}}}$$

– confinement of charges:

$$V(R) = \sigma R \quad \text{at} \quad R \to \infty$$

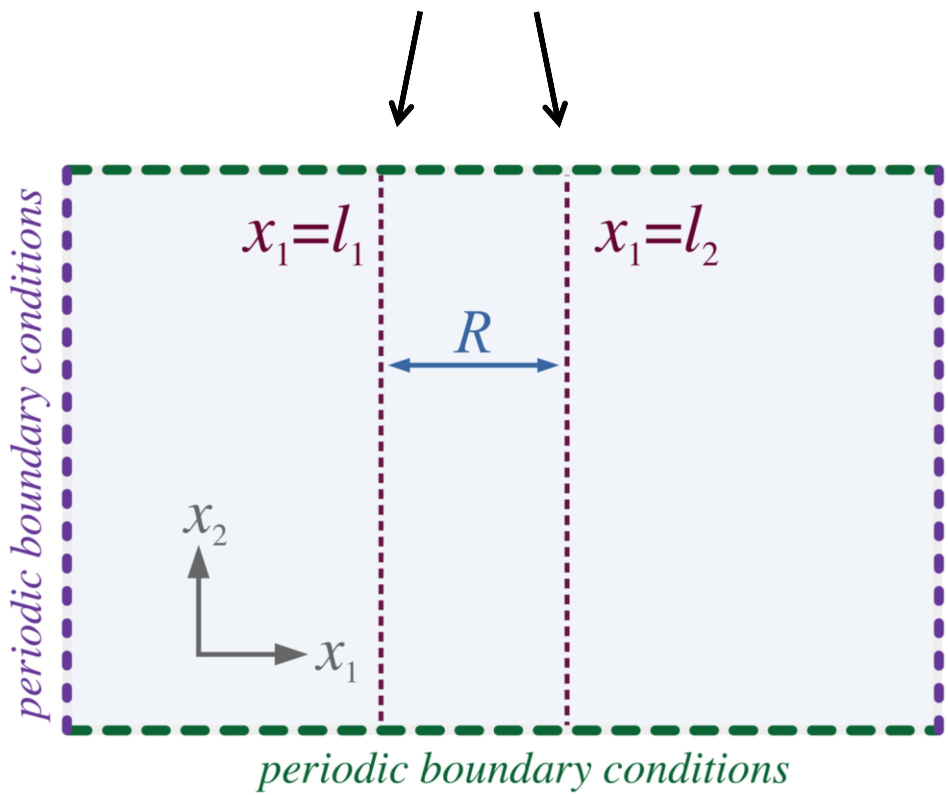
String tension:

$$\sigma = \frac{8\sqrt{2\zeta}}{g_{\mathrm{mon}}} \equiv \frac{4g\sqrt{\varrho_{\mathrm{mon}}}}{\pi}$$

(all formulae are written in a dilute gas approximation)

Casimir effect in cQED: formulation

Two parallel metallic wires in two spatial dimensions



Boundary conditions:

– tangential electric field vanishes at any point of each wire

$$E_{||}(\boldsymbol{x}) = 0$$

– there is no true magnetic field (a pseudo-scalar *B* in 2+1d), so one has no condition on *B*.

Relativistically invariant formulation: $F^{\mu\nu}(\boldsymbol{x}) s_{\mu\nu}(\boldsymbol{x}) = 0$

The world-surface *S* of the wires is parameterized by a vector $\bar{\boldsymbol{x}} = \bar{\boldsymbol{x}}(\tau, \xi)$

Characteristic function of *S* : $$s_{\mu\nu}(\boldsymbol{x}) = \int d\tau \int d\xi \frac{\partial \bar{x}_{[\mu,}}{\partial \tau} \frac{\partial \bar{x}_{\nu]}}{\partial \xi} \delta^{(3)}\left(\boldsymbol{x} - \bar{\boldsymbol{x}}(\tau, \xi)\right)$$

Casimir effect in cQED

Path integral formulation:

$$Z = \int \mathcal{D}A \sum\!\!\!\!\!\!\!\int_{\mathrm{mon}} e^{-S[A,\rho]}$$

Integral over monopole configurations

$$\sum\!\!\!\!\!\!\!\int_{\mathrm{mon}} = \sum_{N=0}^{\infty} \frac{1}{N!} \prod_{a=1}^{N} \left(\sum_{q_a=\pm 1} \zeta \int d^3 x_a \right)$$

The ideal-metal boundary condition $F^{\mu\nu}(\boldsymbol{x}) s_{\mu\nu}(\boldsymbol{x}) = 0$

corresponds to a δ function $\delta_S[F] = \prod_{\boldsymbol{x}} \delta\Big(F^{\mu\nu}(\boldsymbol{x}) s_{\mu\nu}(\boldsymbol{x}) \Big)$

which restricts the fields at the (hyper)surfaces *S*:

$$Z_S = \int DA \sum\!\!\!\!\!\!\!\int_{\mathrm{mon}} e^{-S[A,\rho]} \delta_S[F]$$

It may be realized with the help of Lagrange multiplier field λ:

$$\delta_S[F] = \int \mathcal{D}\lambda \exp\left[\frac{i}{2} \int d^3x \, \lambda(\boldsymbol{x}) F^{\mu\nu}(\boldsymbol{x}) s_{\mu\nu}(\boldsymbol{x}) \right] \equiv \int \mathcal{D}\lambda \exp\left[\frac{i}{2} \int d^3x \, F^{\mu\nu}(\boldsymbol{x}) J_{\mu\nu}(\boldsymbol{x};\lambda) \right]$$

with the "surface tensor field" $J_{\mu\nu}(\boldsymbol{x};\lambda) = \lambda(\boldsymbol{x}) s_{\mu\nu}(\boldsymbol{x})$

Analytical calculation of Casimir energy in cQED

The Casimir energy (potential) is then calculated as a properly normalized 00-component of the energy-momentum tensor in the presence of the boundaries. In an Abelian gauge theory

$$T^{\mu\nu} = -\frac{1}{g^2} F^{\mu\alpha} F^{\nu}_{\alpha} + \frac{1}{4g^2} \eta^{\mu\nu} F_{\alpha\beta} F^{\alpha\beta}$$

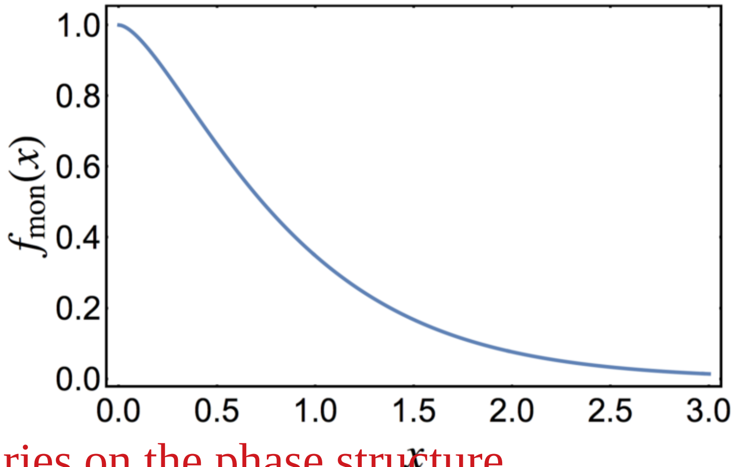
For (2+1)D cQED with two parallel wires at the distance *R*

$$V_{\mathrm{Cas}}(R) = -\underbrace{\frac{\zeta(3)}{16\pi}\frac{1}{R^2}}_{\text{Usual tree-level term}}\ \underbrace{f_{\mathrm{mon}}\left(m_{\mathrm{ph}} R\right)}_{\text{Nonperturbative term due to monopoles}}$$

Non-perturbative photon mass $m_{\mathrm{ph}} = \frac{2\pi}{g}\sqrt{\varrho_{\mathrm{mon}}}$

Usual tree-level term

Nonperturbative term due to monopoles
(appears basically due to the Debye screening)

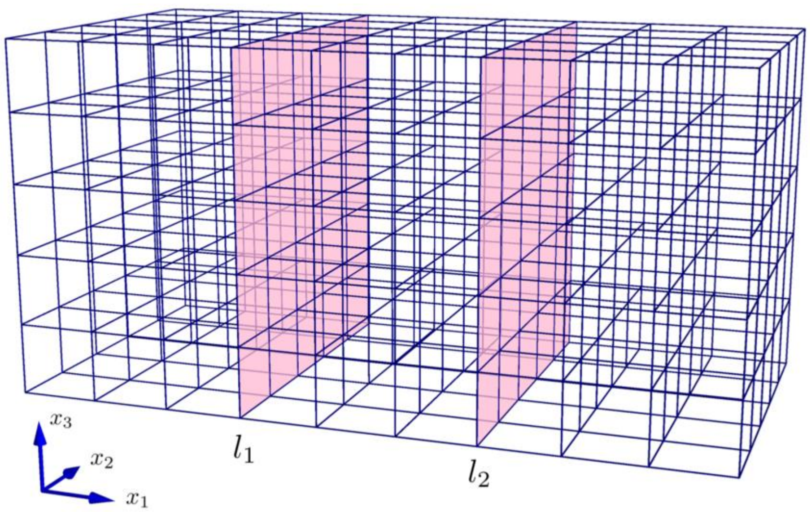


*) Dilute gas approximation, assumes no effect of boundaries on the phase structure.
**) Translationally-invariant anzats for the monopole density.

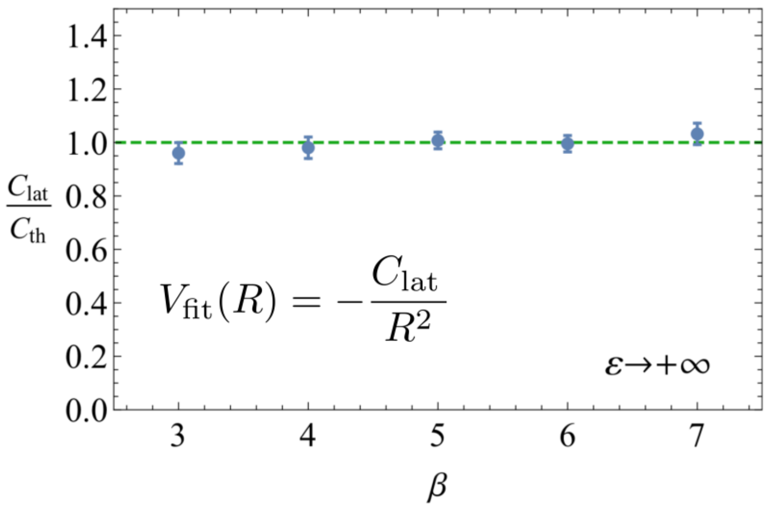
[M. Chernodub,V.A. Goy, A.V. Molochkov, Phys.Rev. D94, 094504 (2016)]

A good setup for first-principle lattice simulations

Take a lattice formulation of the theory and impose the appropriate conditions via the Lagrange multipliers at the boundaries.



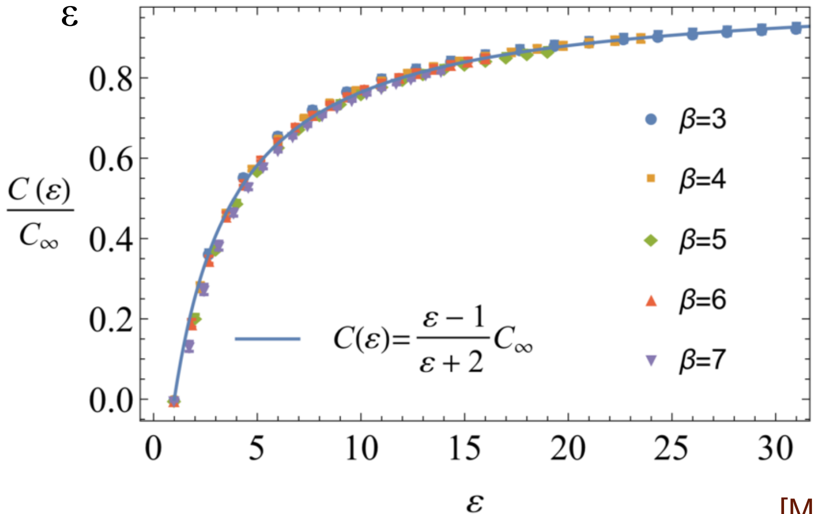
Check of the approach in a free theory (no monopoles, weak coupling regime):



Perfectly conducting wires

[= infinite static permittivity ε in (2+1)d]

Casimir energy for finite static permittivity ε



[M.N. Chernodub,V.A.Goy, A.V. Molochkov,, Phys.Rev. D94, 094504 (2016)]

Phase structure: deconfinement transition at $T = 0$

Electric charges exhibit a linear confinement in a Coulomb gas of monopoles.

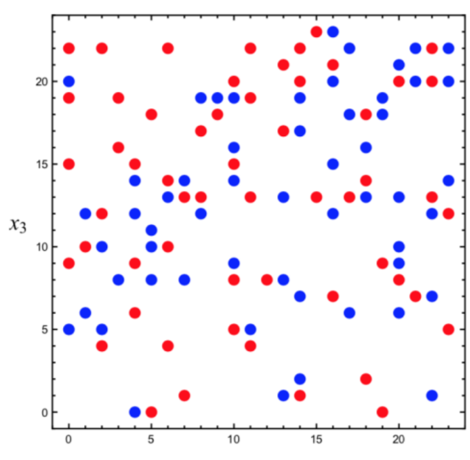
If the wires are close enough, then
→ between the wires, the dynamics of monopoles is dimensionally reduced;
→ the inter-monopole potential becomes log-confining;

$$D_{3D}(\boldsymbol{x}) = -\frac{1}{4\pi|\boldsymbol{x}|} \to D_{2D}(\boldsymbol{x}) = \frac{2}{R} \ln \frac{|\boldsymbol{x}|}{R}$$

→ the monopoles form magnetic-dipole pairs (and are suppressed);
→ the confinement of electric charges disappears (a deconfining transition).

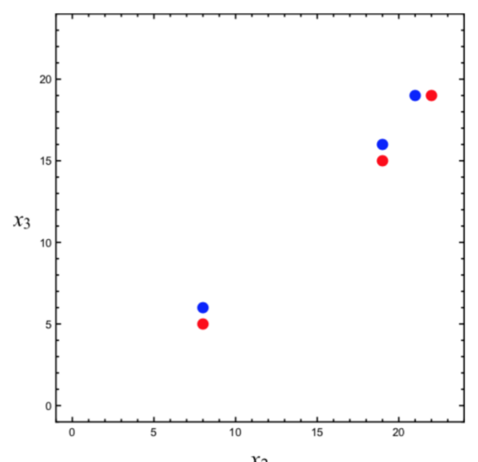
Examples of (anti-)monopole configurations

widely-spaced wires



(a Coulomb gas of monopoles)

narrowly-spaced wires



(a dilute gas of magnetic dipoles)

Berezinskii–Kosterlitz–Thouless transition appears when $R = R_C = \frac{\pi}{3g^2}$

M.N. Chernodub,V.A. Goy, A.V. Molochkov. H. N, work in progress

[M.N. Chernodub,V.A. Goy, A.V. Molochkov, Phys.Rev. D95, 074511 (2017); Phys.Rev. D96, 094507 (2017)]

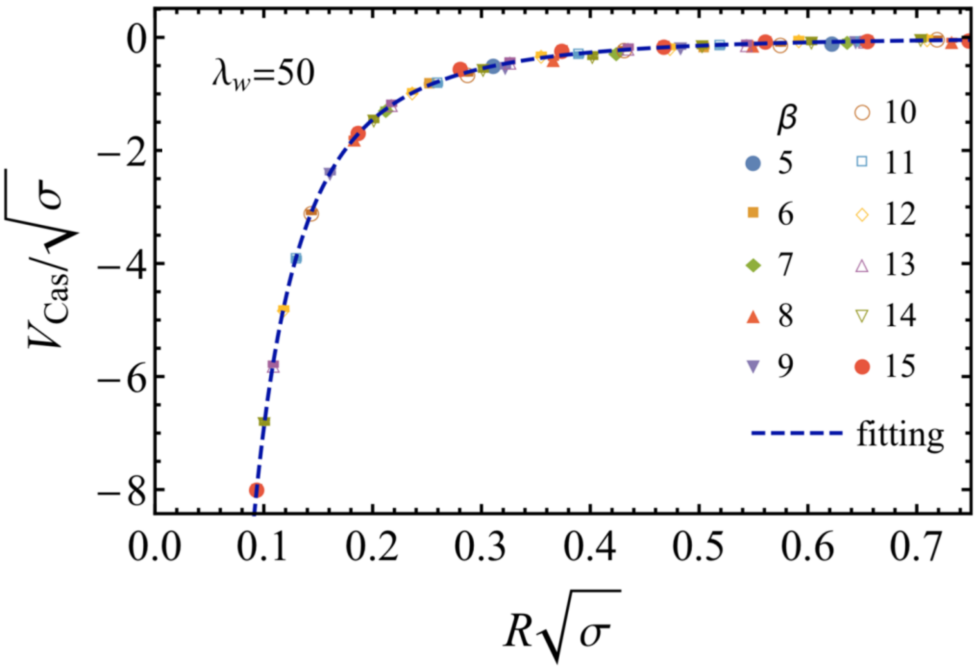
Non-Abelian Casimir effect: Casimir energy

The approach is easily generalizable to non-Abelian gauge groups.

Conditions at an ideal “chromo–metallic” boundary:

$$SU(N_c): \qquad F^{\mu\nu,a}(x) s_{\mu\nu}(x) = 0, \quad a = 1, \dots N_c^2 - 1$$

The Casimir potential in (2+1)d for SU(2) gauge theory at $T = 0$:



σ is the fundamental string tension at $T = 0$
R is the distance between the wires (plates)

Features:

- excellent scaling
- may be described by the function

$$V_{\rm Cas}(R) = 3\frac{\zeta(3)}{16\pi}\frac{1}{R^2}\frac{1}{(\sqrt{\sigma}R)^\nu}e^{-M_{\rm Cas}R}$$

- Tree-level contribution
- (Perturbative) anomalous dimension $\nu = 0.05(2)$
- Nonperturbative Casimir mass $M_{\rm Cas} = 1.38(3)\sqrt{\sigma}$

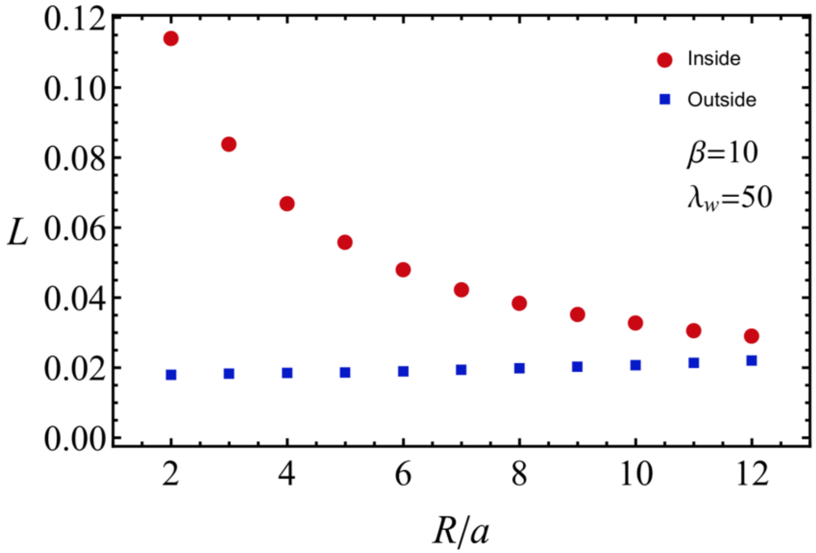
(*cf*. the glueball mass $M_{0^{++}} \approx 4.7\sqrt{\sigma}$)

[M. J. Teper, Phys.Rev. D59, 014512 (1999)]

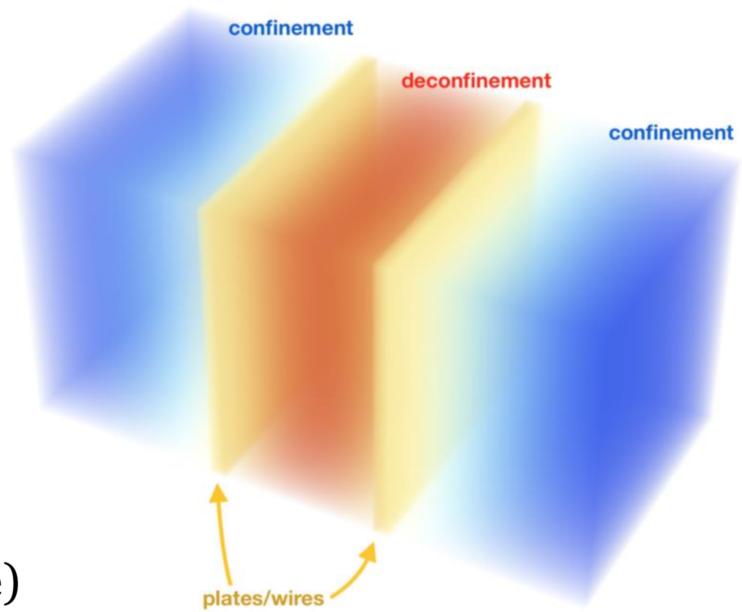
[M.N. Chernodub,V.A. Goy, A.V. Molochkov. H. N , Phys. Rev. Lett. **121**, 191601 (2018)]

Non-Abelian Casimir effect: phase structure

- The expectation value of the Polyakov line indicates deconfinement in between the plates (wires).



- No signal for a phase transition at finite R. (an infinite-order BKT–type transition or a crossover?)



The finite Casimir geometry leads to a very smooth deconfinement transition in between the plates. The absence of a thermodynamic transition marks the difference with the finite temperature case.

In a finite-temperature SU(2) gauge theory the phase transition is of the second order (Ising-type) [M. Teper, Phys.Lett. B313,417 (1993)].

[M.N. Chernodub,V.A. Goy, A.V. Molochkov. H. N , Phys. Rev. Lett. **121**, 191601 (2018)]

Conclusions

- **Casimir Effect in cQED:**
  - Screening due to mass gap generation,
  - Deconfinement of charge at small distance between the wires

- **Casimir effect in Yang-Mills field**
  - Anomalous at the small distance
  - Casimir mass at large distance
  - BKT phase transition in the middle