From QCD kinetics to hydrodynamics

Derek Teaney
Stony Brook University

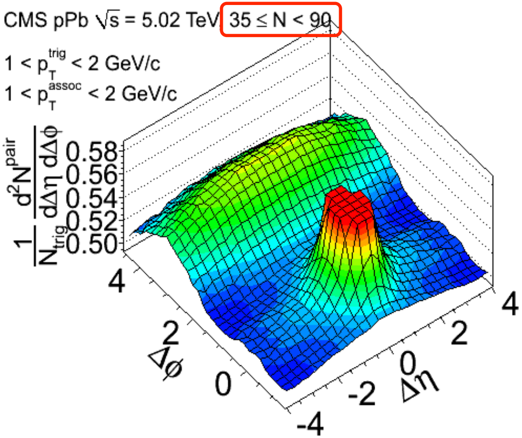
- G. Basar, DT, Phys. Rev. C (2013)
- L. Keegan, A. Kurkela, A. Mazeliauskas, DT, JHEP (2016)
- A. Kurkela, A. Mazeliauskas, J.F. Paquet, S. Schlichting, DT: 1805.01604
- A. Kurkela, A. Mazeliauskas, J.F. Paquet, S. Schlichting, DT: 1805.00961

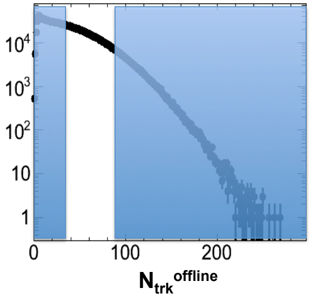
Two Questions:

1. How does v_2 depends on multiplicity?
2. At what multiplicity does hydro set in?

A typical pPb event

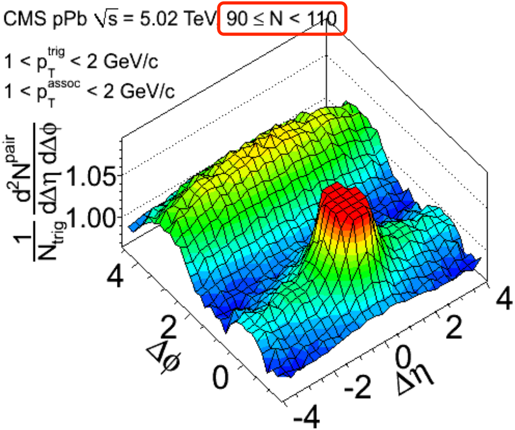
(figure G. Roland)

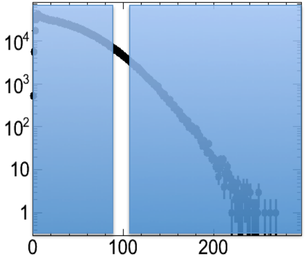




Somewhat rare event 3%

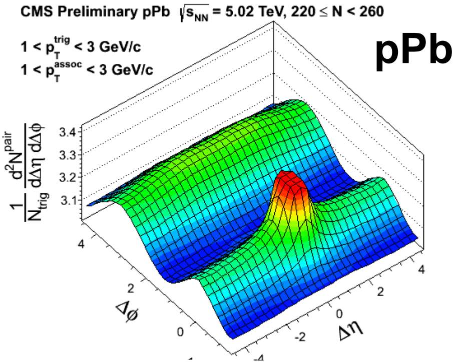
(figure G. Roland)

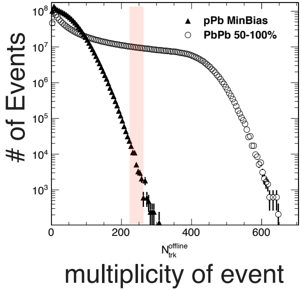




Rare event – one in 20 thousand

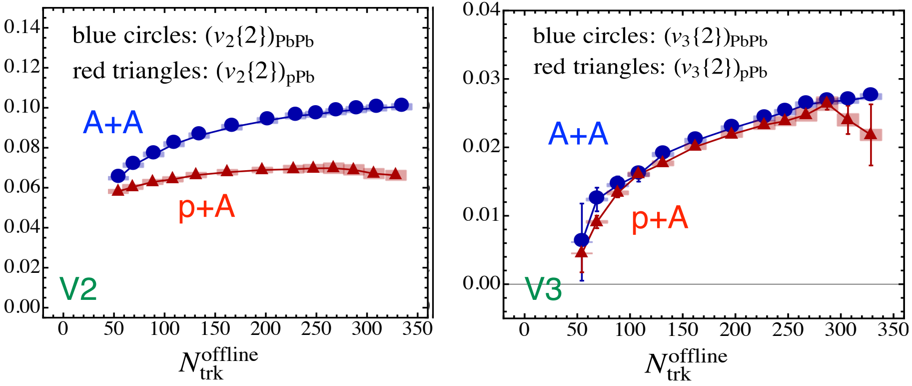
(figure G. Roland)





Want to understand this *transition*!

Can extract a v_2 and a v_3 from these data ... (CMS Data)



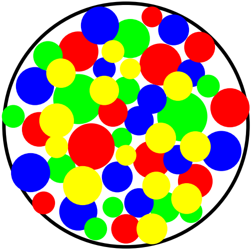
The v_3 are *really the same*!
Is the mechanism for the correlation the same in pPb and PbPb?
Is hydrodynamics/kinetics the origin of pPb correlations?

Flow in pA and AA from hydro and kinetics:

$$\underbrace{v_2}_{\text{Measured E by E flow}} = \underbrace{k_2}_{\text{Response coefficient}} \times \underbrace{\epsilon_2}_{\text{geometry}}$$

1. Are the response coefficients the same?
2. Is the geometry or ϵ_2 the same? ϵ_3?

What is the ℓ_{mfp}/R in high multiplicity p+A and A+A?



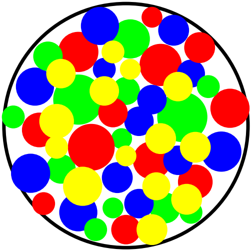
1. Throw N_{clust} clusters in in the transverse plane:
   - This defines a momentum scale (the saturation momentum)

$$Q_s^2 \propto \frac{N_{\mathrm{clust}}}{\pi R^2} \qquad\qquad (R = \text{transverse size})$$

2. Assume Q_s is the only scale for the 1D expansion, $1/Q_s \ll \tau \ll R$

$$\ell_{\mathrm{mfp}} \propto \frac{1}{Q_s} \propto \frac{1}{T_{\mathrm{o}}}$$

What is the $\ell_{\rm mfp}/R$ in high multiplicity p+A and A+A?



3. Assume the multiplicity is proportional to the number of clusters

$$\frac{dN}{dy} \propto N_{\rm clust}$$

4. Find that $\ell_{\rm mfp}/R$ (at early time) is fixed by dN/dy

$$\frac{\ell_{\rm mfp}}{R} \propto \frac{1}{T_o R} \propto \frac{1}{Q_s R} \propto \frac{1}{\sqrt{dN/dy}}$$

$\frac{\ell_{\rm mfp}}{R}$ is the same in pA and AA at fixed multiplicity!

What is the ℓ_{mfp}/R when the flow develops?

1. Need to estimate ℓ_{mfp}/R when the flow develops, $1/Q_s \ll \tau \sim R$

$$\tau \sim \frac{R}{c_s} \quad \Leftarrow \qquad \text{elliptic flow develops at this time}$$

2. Using the Bjorken estimate:

$$\ell_{\mathrm{mfp}} \propto \frac{1}{T(\tau)} \propto \frac{1}{T_o} \left(\frac{\tau}{\tau_o}\right)^{1/3}$$

Find:

$$\frac{1}{w(R)} \equiv \left.\frac{\ell_{\mathrm{mfp}}}{R}\right|_{\tau \sim R} \propto \frac{1}{\left(\frac{dN}{dy}\right)^{1/3}}$$

Again $\frac{\ell_{\mathrm{mfp}}}{R}$ is the same in pA and AA at *fixed* multiplicity

Flow in pA and AA from hydro and kinetics:

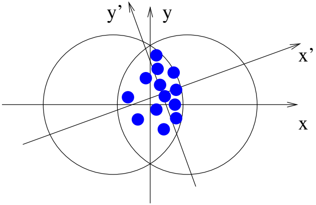
$$\underbrace{v_2}_{\text{Measured E by E flow}} = \underbrace{k_2}_{\text{Response coefficient}} \times \underbrace{\epsilon_2}_{\text{geometry}}$$

1. Are the response coefficients the same?

   Yes. The $\ell_{\rm mfp}/R$ is the same at fixed multiplicity

2. Is the geometry or ϵ_2 the same? ϵ_3?

Understanding the fluctuating geometry in AA: Bhalerao and Ollitrault '06



- Throw N_{clust} sampled from the *average/smooth* geometry
- Calculate $\epsilon_2\{2\} = \sqrt{\langle \epsilon_2^2 \rangle}$

$$\left\langle \epsilon_2^2 \right\rangle_{AA} = \underbrace{\epsilon_s^2}_{\text{ave. geometry}} + \underbrace{\left\langle \delta\epsilon_2^2 \right\rangle}_{\text{fluctuations}} \qquad \langle \delta\epsilon_2^2 \rangle_{AA} \equiv \frac{\left\langle r^4 \right\rangle_{AA}}{N_{\mathrm{clust}} \left\langle r^2 \right\rangle_{AA}^2}$$

Reproduces the results of more complex Glauber models

Understanding the fluctuating geometry in p+A:

- In pA the average geometric eccentricity is zero, $\epsilon_s = 0$

$$\underbrace{\left\langle \epsilon_2^2 \right\rangle_{pA}^2}_{\text{only flucts}} = \underbrace{\langle \delta\epsilon_2^2 \rangle_{pA}}_{\text{only flucts}} \equiv \frac{\left\langle r^4 \right\rangle_{pA}}{N_{\text{clust}} \left\langle r^2 \right\rangle_{pA}^2}$$

- So to compare to pA, we scale out the average geometry out of the AA system

$$\underbrace{(v_2\{2\})_{\text{PbPb,rscl}}}_{\text{only flucts}} \equiv \sqrt{1 - \frac{\epsilon_s^2}{\left\langle \epsilon_2^2 \right\rangle_{AA}}}\,(v_2\{2\})_{\text{PbPb}}\,.$$

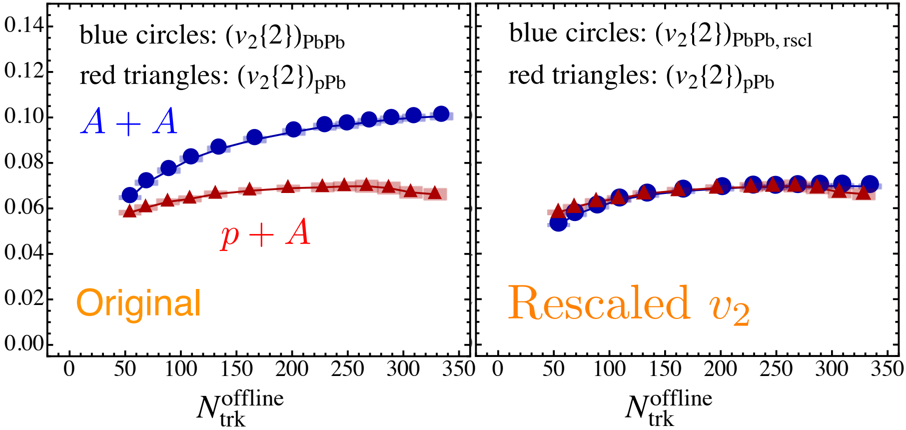
- With this rescaling we expect

$$(v_2\{2\})_{\text{PbPb,rscl}} \equiv k_2 \sqrt{\left\langle \delta\epsilon_2^2 \right\rangle_{AA}}$$

$$(v_2\{2\})_{pA} \equiv k_2 \sqrt{\left\langle \delta\epsilon_2^2 \right\rangle_{pA}}$$

Lets compare $(v_2\{2\})_{\text{PbPb,rscl}}$ to $(v_2\{2\})_{\text{pPb}}$ ...

Comparing fluctuation driven v_2 ... (CMS Data)



Fluctuation driven v_2's are equal to 5% accuracy!
Strong indication that v_2 is a response to geometry in $p+A$

Ratio of fluctuation v_2's and eccentricities

- Fluctuation driven v_2

$$\frac{(v_2\{2\})_{\rm pPb}}{(v_2\{2\})_{\rm PbPb}} = \frac{k_2}{k_2}\sqrt{\frac{\langle \delta\epsilon_2^2\rangle_{pA}}{\langle \delta\epsilon_2^2\rangle_{AA}}} = \sqrt{\frac{(\langle r^4\rangle \,/\, \langle r^2\rangle^2)_{pA}}{(\langle r^4\rangle \,/\, \langle r^2\rangle^2)_{AA}}}$$

- Don't know the radial profile in pA:

But, it doesn't it matter:

- Ratio of fluct-driven v_2 is determined by a root of a double ratio!
  - Very different profiles give the almost same answer

$$\sqrt{\frac{\langle \delta\epsilon_2^2\rangle_{\rm hard-sphere}}{\langle \delta\epsilon_2^2\rangle_{\rm Gaussian}}} \approx 0.85$$

Flow in pA and AA from hydro and kinetics:

$$\underbrace{v_2}_{\text{Measured E by E flow}} = \underbrace{k_2}_{\text{Response coefficient}} \times \underbrace{\epsilon_2}_{\text{geometry}}$$

1. Are the response coefficients the same? Yes.
2. Is the fluctuating geometry is the same? Yes.

v_2 scales with multiplicity as:

$$v_2 \propto \underbrace{\left(\frac{dN}{dy}\right)^{1/3}}_{\text{response}} \times \underbrace{\left(\frac{dN}{dy}\right)^{-1/2}}_{\text{geometry}} \propto \left(\frac{dN}{dy}\right)^{-1/6}$$

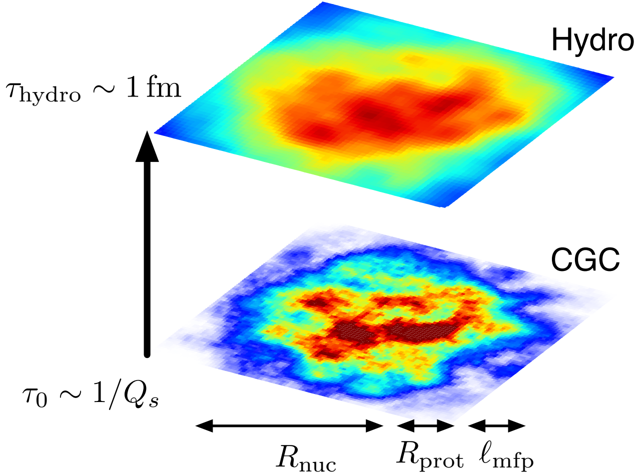
v_2 from fluctuations slowly *increases* with *decreasing* multiplicity

Two Questions:

1. How does v_2 depends on multiplicity ?
2. At what multiplicity does hydro set in?

    - Studied the approach to hydro in AA

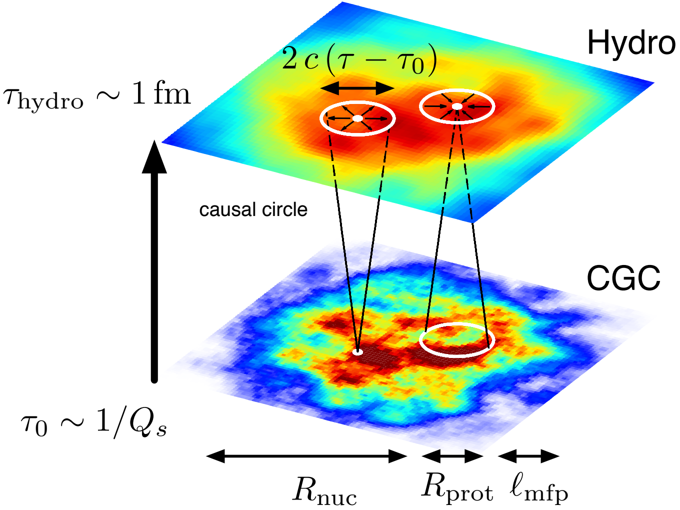
Mapping the CGC fluctuating initial conditions to hydro



Use QCD kinetic theory to map the CGC initial state to hydrodynamics

$$R_{\rm nuc} \gg R_{\rm prot} \sim \ell_{\rm mfp} \gg 1/Q_s$$

Mapping the CGC fluctuating initial conditions to hydro

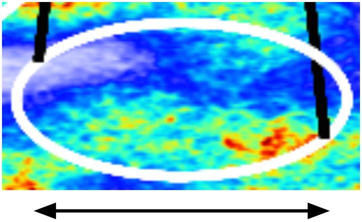


Causality limits the equilibration dynamics within a causal circle

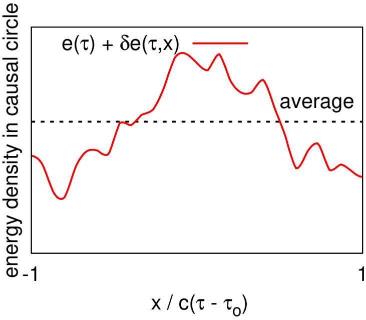
$$R_{\rm nuc} \gg R_{\rm prot} \sim \ell_{\rm mfp} \sim c\tau_{\rm hydro} \gg 1/Q_s$$

An approximation scheme for the equilibration dynamics:

look in causal circle



$$2\,c\,(\tau - \tau_0)$$



1. Determine the evolution of the average (homogeneous) background

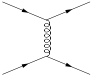
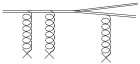
   Bottom-Up Thermalization!

2. Construct a Green function to propagate the linearized fluctuations.

$$\underbrace{\frac{\delta e(\tau, \boldsymbol{x})}{e(\tau)}}_{\text{final energy perturb}} = \int d^2\boldsymbol{x}'\, G(\boldsymbol{x} - \boldsymbol{x}'; \tau, \tau_o) \underbrace{\frac{\delta e(\tau_0, \boldsymbol{x}')}{e(\tau_0)}}_{\text{initial energy perturb}}$$

How to compute the background and perturbations:

$$\partial_\tau f + \frac{\boldsymbol{p}}{|p|}\cdot\nabla f - \underbrace{\frac{p_z}{\tau}\partial_{p_z} f}_{\text{Bjorken expansion}} = -\underbrace{\mathcal{C}_{2\leftrightarrow 2}[f]} - \underbrace{\mathcal{C}_{1\leftrightarrow 2}[f]} \,,$$

Gluon distribution function for background and perturbations

$$f = \underbrace{\bar{f}_{\boldsymbol{p}}}_{\text{uniform background}} + \underbrace{\delta f_{\boldsymbol{k}_\perp,\boldsymbol{p}} e^{i\boldsymbol{k}_\perp\cdot\boldsymbol{x}_\perp}}_{\text{transverse perturbations}} \,.$$

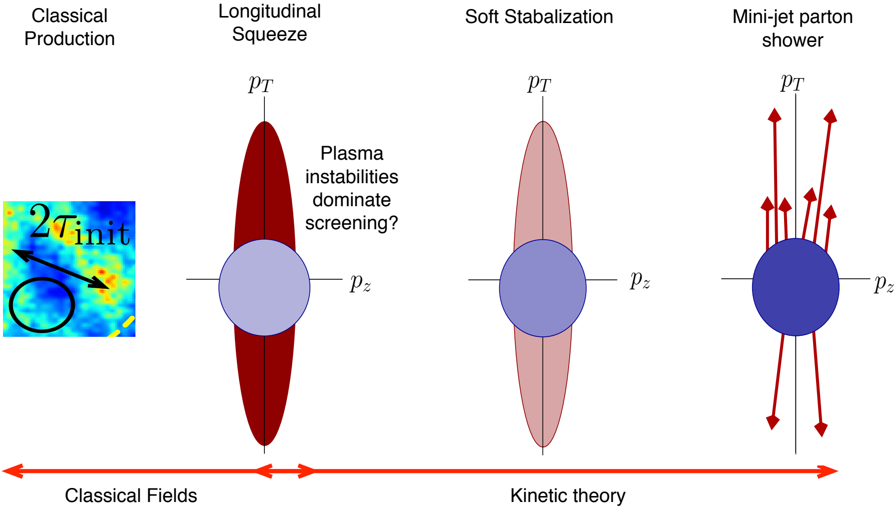
$$(\partial_\tau - \frac{p_z}{\tau}\partial_{p_z})\bar{f}_{\boldsymbol{p}} = -\mathcal{C}[\bar{f}] \qquad \text{background}$$

$$(\partial_\tau - \frac{p_z}{\tau}\partial_{p_z} + \frac{i\boldsymbol{p}_\perp\cdot\boldsymbol{k}_\perp}{p})\delta f_{\boldsymbol{k}_\perp,\boldsymbol{p}} = -\delta\mathcal{C}[\bar{f},\delta f] \qquad \text{perturbation}$$

We will discuss the background and perturbations separately

The background and "bottom-up" thermalization

Baier, Mueller, Schiff, Son

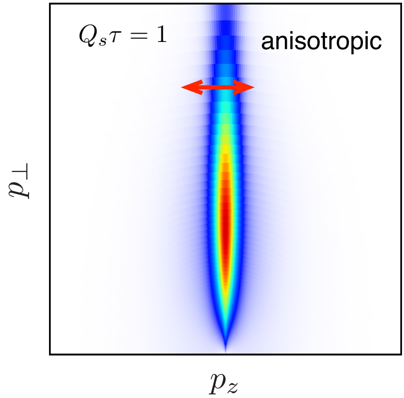


Reach a thermal state in $\tau_{\rm hydro} \sim 1/(\alpha_s^{13/5} Q_s)$

A numerical realization of bottom-up

- Builds upon the first numerical realization — Kurkela, Zhu PRL (2015)

$p^2 f(p_\perp, p_z)$



Initialization:

1. Partons are initialized with:

$$\langle p_\perp^2 \rangle \sim Q_s^2 \qquad \langle p_z^2 \rangle \simeq 0$$

2. Take a coupling of $\alpha_s = 0.3$

$$\underbrace{\lambda \equiv 4\pi\alpha_s N_c = 10}_{\text{theorists version of } \alpha_s = 0.3}$$

corresponding to

$$\frac{\eta}{s} = 0.6 = \frac{7.5}{4\pi}$$

We see "Bottom-Up" in the computer code.

A numerical realization of bottom-up

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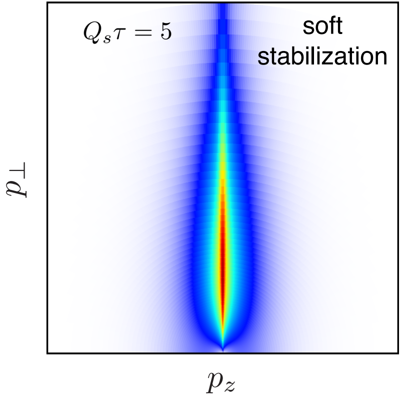

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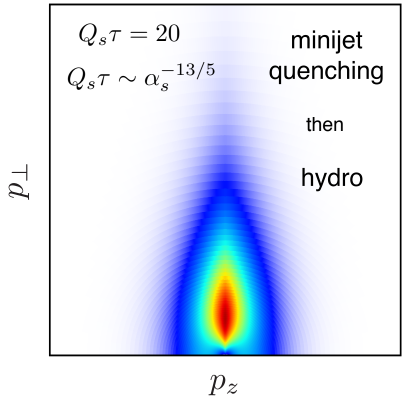

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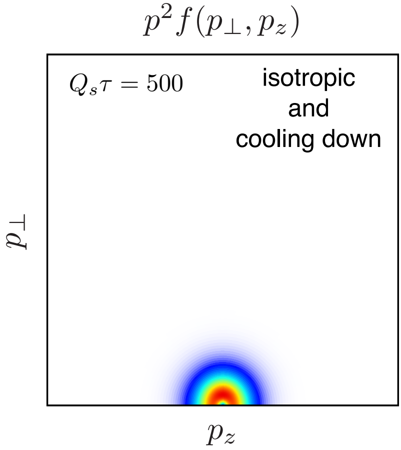

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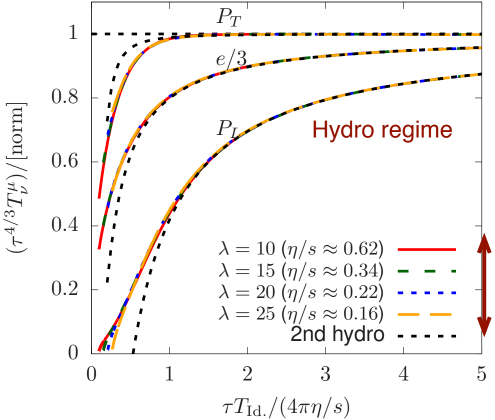
We see "Bottom-Up" in the computer code.

When does the stress tensor approach 2nd order hydrodynamics?



Different values of coupling give different η/s

In terms of eta/s, all couplings thermalize at same scaled time
Keegan, Kurkela, Romatschke,Schee, Zhu

Gives a basis for interpolating from weak coupling results to stronger coupling

Measure time in a physical relaxation time given by $\tau_R \equiv 4\pi\eta/sT_{\mathrm{eff}}(\tau)$:

$$\frac{\tau}{\tau_R} \equiv \frac{\tau T_{\mathrm{eff}}(\tau)}{4\pi\eta/s} \qquad \text{with} \qquad T_{\mathrm{Id}} \equiv T_{\mathrm{eff}}(\tau) \equiv \Lambda_T/(\Lambda_T\tau)^{1/3}$$

Can start hydro when $\tau T_{\mathrm{eff}}(\tau)/4\pi\eta/s \sim 1$

Translating earliest hydro starting time into physical units:

- At late times the dynamics is ideal hydro: $T_{\text{eff}}(\tau) = \Lambda_T/(\Lambda_T \tau)^{1/3}$
- Hydro fits to multiplicity give:

$$\langle \tau s \rangle\big|_{\tau=1.2\,\text{fm}} = \underbrace{4.1\,\text{GeV}^2 \propto \Lambda_T^2}_{\text{highly constrained by } \frac{dN}{dy}!}$$

- The estimate for τ_{hydro}:

$$\frac{\tau_{\text{hydro}} T_{\text{eff}}(\tau_{\text{hydro}})}{4\pi(\eta/s)} = 1$$

Find that hydrodynamics is applicable for times later than:

$$\tau_{\mathsf{hydro}} \approx 1.1\,\text{fm} \left(\frac{4\pi(\eta/s)}{2}\right)^{\frac{3}{2}} \left(\frac{4.1\,\text{GeV}}{\langle \tau s \rangle}\right)^{-1/2} \left(\frac{\nu_{\text{eff}}}{40}\right)^{1/2}$$

Estimate for the applicability of hydro for system of radius R:

- For a hydrodynamic regime the system should equilibrate by $\tau \sim R$:

$R/\ell_{\rm mfp}$ at time R: $$w(R) \equiv \left.\frac{\tau T_{\rm id}(\tau)}{4\pi(\eta/s)}\right|_{\tau=R} > 1$$

- Since $T \sim (dN/dy)/R^{1/3}$, then $w(R) \propto RT$ is *independent of* R.

$$w(R) = \left(\frac{4\pi(\eta/s)}{2}\right)^{-1}\left(\frac{dN_{\rm ch}/d\eta}{63}\right)^{1/3} > 1$$

  - We used the EOS to relate dS/dy to measured $dN_{\rm ch}/dy$

- Corrections to ideal hydro response coefficients come in powers of $w(R)^{-1}$.

$$k_2(w) = k_2^{\rm ideal}\Big(1 - \underbrace{\frac{C_1}{w(R)}}_{\text{first order hydro}} - \underbrace{\frac{C_2}{w(R)^2}}_{\text{second order}} + \ldots\Big)$$

Two Questions with Answers:

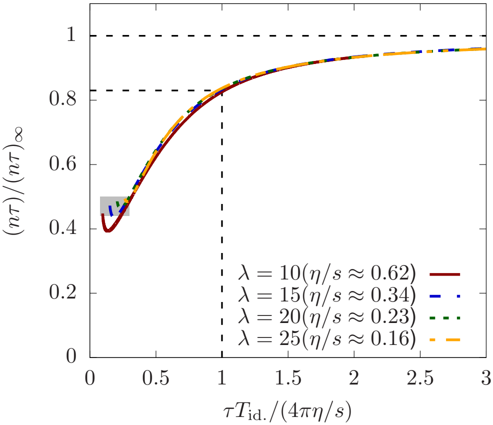
1. How does v_2 in small systems depends on multiplicity ?

$$v_2 \propto \left(\frac{dN}{dy}\right)^{-1/6}$$

2. At what multiplicity does hydro set in?

$$\left(\frac{dN}{dy}\right)_{\rm hydro} = 63\left(\frac{4\pi(\eta/s)}{2}\right)^3$$

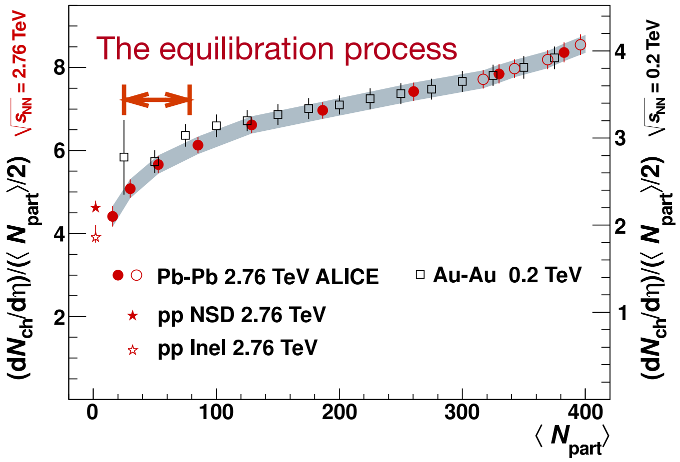
Changes during the equilibration process: increase in multiplicity



$$n\tau = \frac{1}{A}\frac{dN}{dy}$$

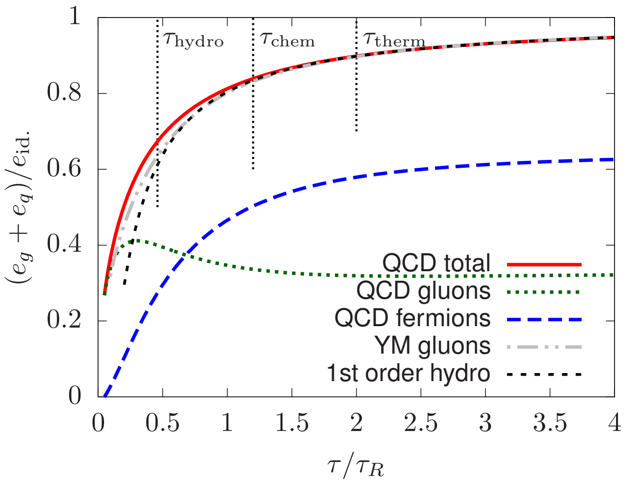
The final gluon multiplicity is 2.5 times the initial gluon multiplicity independent of the coupling or η/s!

Multiplicity increase in peripheral collisions:



Changes during the equilibration process: change in chemistry

- Kurkela and Mazeliauskas:1811.03040

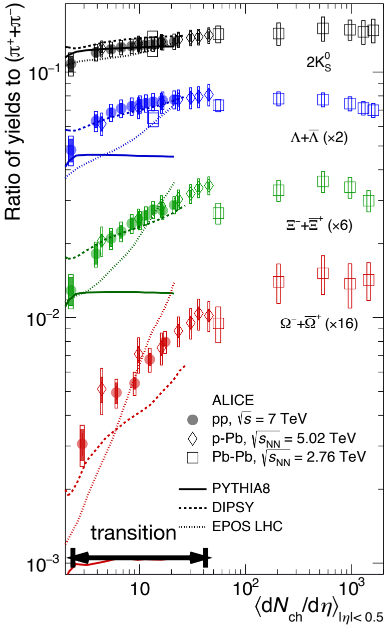


During the first one or two relaxation times
the energy density becomes more than half quarks

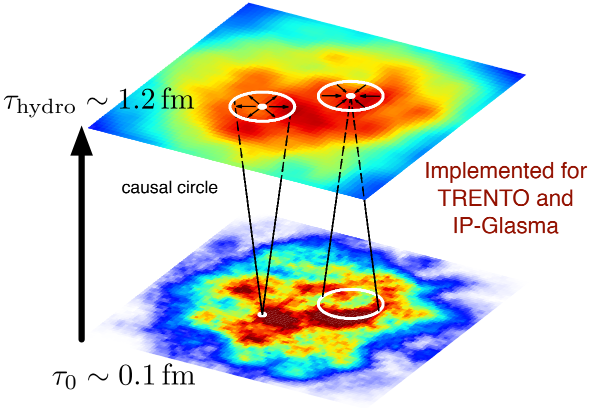
Chemical changes:

Strangeness enhancement

Alice: 1606.07424



A practical algorithm for pre-hydro evolution in A+A:

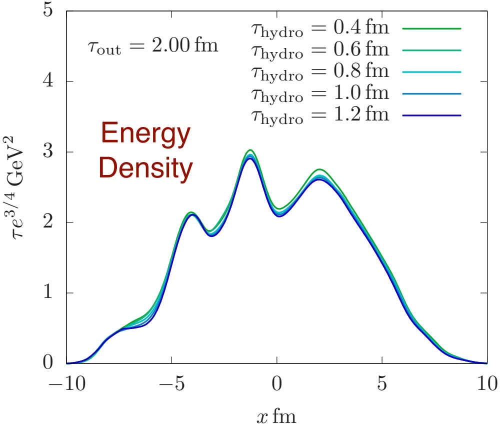


(i) For each point, average the energy in causal circle:
  - ⋆ Determine $\tau_{\rm hydro}$ in units of the scaled time, $\tau_{\rm hydro}/\tau_R$
  - ⋆ Evolve the background in scaled time.

(ii) Propagate background and perturbations in scaled time
  - ⋆ Sometimes need to regulate the response

Do hydro results depend on τ_{hydro} ?

1. Implementation in `TRENTO`. $\eta/s = 2/4\pi$. Central LHC.
2. Kinetics runs from $\tau_0 = 0.1$ up to τ_{hydro}, then hydro up to τ_{out}.

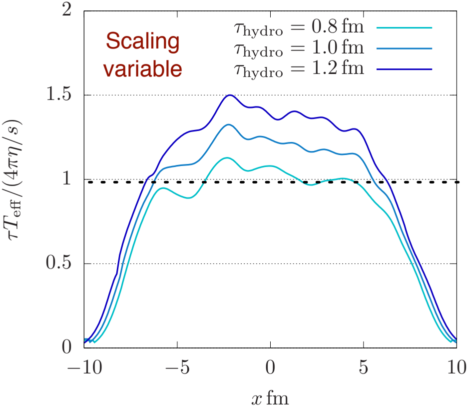


Remarkably insensitive to τ_{hydro} as we want !

Are the constitutive relations are satisfied at late times?

- For times sufficiently late times Navier-Stokes should be valid:

$$\pi^{\mu\nu} = \underbrace{-\eta\sigma^{\mu\nu}}_{\text{navier stokes}} \qquad \text{for} \qquad \frac{\tau T_{\rm eff}}{4\pi\eta/s} > 1$$



Expect the cells near the line to be equilibrated and obey constitutive equations

Are the constitutive relations are satisfied at late times?

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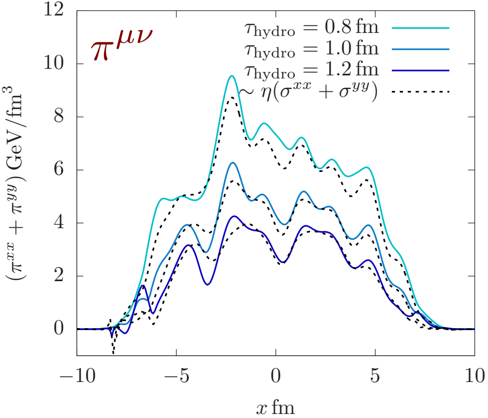

Black lines are navier stokes.

Color lines are kinetics

Constitutive relations are satisfied!

Summary:

- Want to see a transition to flow:
  - Counter productive to only study flow, and to ignore non-flow
  - Can become impossible to define them separately
- At what multiplicity does hydro set in?

$$\left(\frac{dN}{dy}\right)_{\text{hydro}} = 63\left(\frac{4\pi(\eta/s)}{2}\right)^3$$

- Equilibration leaves fingerprints on multiplicities and strangeness
- For the AA system we have constructed a useful tool for pre-flow.