Finite-size scaling, intermittency and the QCD critical point

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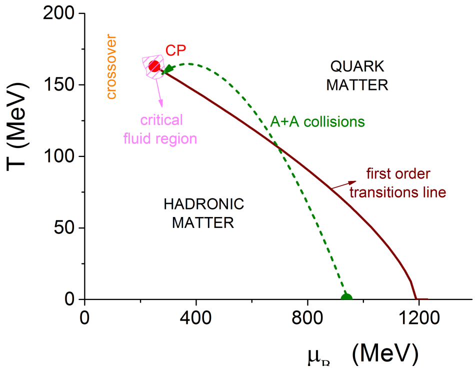
1. Critical region ; scaling properties
2. Ising-QCD thermodynamics in the critical region
3. Size of the critical region
4. Locating the CEP
5. Conclusions

A sketch for finite system(s)



Objective: Detection (existence?) of the QCD Critical Point (CP) (remnant of chiral transition)

$\Downarrow$

Systems: Final states in A+A collisions

Relevant properties:

Equilibrium? seems to be **yes**, (experiment)

Theoretical input:

3d-Ising universality class, scaling properties with known critical exponents

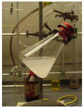
How to observe a critical point?

Lessons from conventional (QED) matter: **Critical opalescence**

A simple example: binary liquids

$T > T_c$

$T < T_c$



$T = T_c$

Where to look for scaling properties?

Order parameter fluctuations! (here density fluctuations)

How to observe them?

Scatter light through the system!

Order parameter fluctuations in **space**	$\Rightarrow$	**power-law correlations** of photon **momenta!**

How to transfer this to the QCD critical point?

- **Order parameter?**

  The condensate $\langle \bar{q}q \rangle$ (σ-field) and the net-baryon density n_B

$$\Downarrow$$

σ-field

Pro: Statistics

Contra: not directly measurable $\Rightarrow$ form (π^+, π^-)-pairs (combinatorial background),

Contra: fast component $\Rightarrow$ fluctuations wash-out quickly

net baryon density

Pro: direct measurable

Pro: Slow component $\Rightarrow$ fluctuations sustain (baryon number conservation)

Contra: Statistics

How to transfer this to the QCD critical point?

- **Fluctuations of the order parameter?**

At the **critical point** $\Rightarrow$ **self-similar fluctuations**

(exact for an infinite system)

$\Downarrow$

fractal geometry

density-density correlation is of power-law form:

$\langle n_B(\mathbf{r_1}) n_B(\mathbf{r_2}) \rangle \sim |\mathbf{r_1} - \mathbf{r_2}|^{-(d-d_F)}$ (d_F = fractal dimension)

Correlation length ξ_∞ becomes **infinite**!

Critical fluid: **thermally** and **chemically** excited **QCD vacuum**

$\Downarrow$

The **momenta** of protons and (π^+, π^-)-pairs created from the excited vacuum possess **power-law correlations**

Dealing with finite systems

Finite system $\Rightarrow$ **linear size** L is an **additional length scale**.

When $\xi_\infty \gtrsim L$ we enter into the **finite size scaling (FSS)** regime!

Thermodynamic quantity Q (infinite system) $\Rightarrow$ Q_L (finite system):

$$Q_L(t_\pm) = L^p f_\pm(L/\xi_\infty(t_\pm)) \quad ; \quad t = \pm\frac{T - T_c}{T}$$

Correlations: $\langle n_B(\mathbf{r_1}) n_B(\mathbf{r_2})\rangle \sim |\mathbf{r_1} - \mathbf{r_2}|^{-(d-d_F)}$ $\Rightarrow$ Valid at distances $|\mathbf{r_1} - \mathbf{r_2}| \approx \xi_\infty \gtrsim L$

Not realizable in real space but in **momentum space** for **small momentum differences** (Fourier transform, **fractal** in momentum space)!

$$\lim_{\mathbf{k_1}\to\mathbf{k_2}} \langle n_B(\mathbf{k_1}) n_B(\mathbf{k_2})\rangle \sim |\mathbf{k_1} - \mathbf{k_2}|^{-d_F}$$

$$\Downarrow$$

Intermittency = Critical opalescence in ion collisions

Application to the QCD critical point

- **Order parameter:** net baryon density $n_B \Rightarrow$ proton density
- **FSS:** $n_B \sim L^{-\beta/\nu} \Rightarrow d_F = d - \frac{\beta}{\nu}$.
- **Universality class:** **3d Ising** $\Rightarrow$ critical exponents $\beta \approx \frac{1}{3}$, $\nu \approx \frac{2}{3}$.
- **Fractal dimension:** $\boxed{d_F \approx \frac{5}{2}}$
- Rapidity mixes space and time $\Rightarrow$ need for stationary variables!

$\swarrow$

Project into transverse space $\Rightarrow$ possible in central region

$\Downarrow$

3d geom. $\approx$ long. $\bigotimes$ trans. space

Application to the QCD critical point

- **Fractal dimension in transverse space:** $d_{F,\perp} = \frac{2}{3} d_F \approx \frac{5}{3}$
- **Critical opalescence $\Rightarrow$ fractal in transverse momentum space!**
- **Fractal dimension in transverse momentum space:**

$$\tilde{d}_{F,\perp} = 2 - d_{F,\perp} \approx \frac{1}{3}$$ (Fourier rule)

$\Downarrow$

Observable through **intermittency** in transverse momentum space:

$$F_2(M) \sim M^{2\phi_2} = M^{2-\tilde{d}_{F,\perp}}$$

(intermittency rule)

$\Downarrow$

$$\phi_2 = 1 - \frac{\tilde{d}_{F,\perp}}{2} \approx \frac{5}{6}$$ (model independent result!)

Measuring critical exponents through intermittency

Experimental observation of local, power-law distributed fluctuations

$\Downarrow$

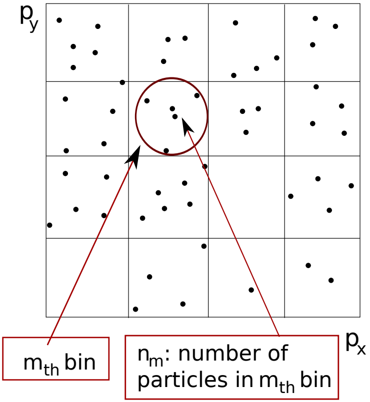
Intermittency in transverse momentum space (net protons at mid-rapidity)

(Critical opalescence in ion collisions)

- Transverse momentum space is partitioned into M^2 cells
- Calculate second factorial moments $F_2(M)$ as a function of cell size $\Leftrightarrow$ number of cells M:

$$F_2(M) \equiv \frac{\sum_m \langle n_m(n_m-1)\rangle}{\sum_m \langle n_m\rangle^2} \sim M^{2\phi_2}$$

where $\langle \ldots \rangle$ denotes averaging over events.



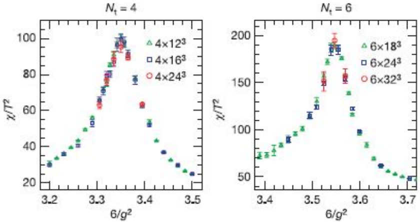
Summarizing....

To detect the QCD critical point:

1. Search for self-similar fluctuations of the **order parameter(s)** ⇒ **scaling** is the **benchmark** of the critical point!
2. **Non-monotonic** behaviour of **response functions** (global quantities) is **insufficient** to declare criticality (valid also for cross-over).

⇓

FSS of the extremal value with size L must hold! ⇒ **unattainable** task, needs **many systems of different size** freezing within FFS region!



Summarizing....

To detect the QCD critical point:

1. Critical opalescence transfers **FSS** to **fractal geometry in momentum space**!

$\Downarrow$

Detection of **scaling** through **power-law** behaviour of factorial moments (local quantity) in a **single system!**

$\Downarrow$

Intermittency phenomenon!

$F_2(M) \sim M^{2\phi_2}$ for a wide range of M-values with $M \gg 1$

Additional tasks ....

- Estimate the **size** of the **critical region**!
- Find **the equation of state** in the neighbourhood of the CP

$$\Downarrow$$

A **theoretical description** of the **QCD-thermodynamics** in the **critical region** is needed!

$$\Downarrow$$

Universality can help here!

3d-Ising effective action

3d-Ising effective action (dimensionless form) for the order parameter ϕ in the critical region:

$$S_{eff} = \int_V d^3\hat{\mathbf{x}} \left[\frac{1}{2} |\hat{\nabla}\phi|^2 + U(\phi) - \hat{h}\phi \right] \qquad \text{with}$$

$$U(\phi) = \frac{1}{2}\hat{m}^2\phi^2 + \hat{m}g_4\phi^4 + g_6\phi^6 \quad ; \quad \phi = \beta_c^3 \lim_{\delta V \to 0} \frac{n_\uparrow - n_\downarrow}{\delta V}$$

$\hat{x} = x\beta_c^{-1}$, $\hat{m} = \beta_c m$, $m = \xi^{-1}$, $\hat{h} = h\beta_c^{-1}$ — ξ= correlation length

universal constants $g_4 \approx 0.97$, $g_6 \approx 2.1$ — h=ordering field

Partition function $\mathcal{Z} = \sum_{\{\phi\}} \exp(-S_{eff}[\phi])$

M. M. Tsypin, Phys. Rev. Lett. ***73****, 2015 (1994)*

Ising-QCD partition function

Constructing the **Ising-QCD** partition function in the critical region:

$$(n_\uparrow, n_\downarrow) \Longrightarrow (n_B, n_{\bar{B}})$$

Scaling properties describable restricting to **protons**!

Y. Hatta and M. A. Stephanov, PRL 91, 102003 (2003)

Use **constant configurations** for the field $\phi = \frac{N}{V}$ with $N = N_p$:

$$\mathcal{Z}_{IQCD} = \sum_{N=0}^{\Lambda} \zeta^N \exp\left[-\frac{1}{2}\hat{m}^2 \frac{N^2}{\Lambda} - g_4 \hat{m} \frac{N^4}{\Lambda^3} - g_6 \frac{N^6}{\Lambda^5}\right]$$

with $\zeta = \exp[(h - h_c)\beta_c]$ ($h_c = 0$ for 3d-Ising) and $\Lambda = \dfrac{V}{V_0}$

(V_0 = proton volume)

Ising-QCD partition function (continued)

Thermodynamic quantities in $\mathcal{Z}_{IQCD}$:

$$\mathcal{Z}_{IQCD} = \sum_{N=0}^{\Lambda} \zeta^N \exp\left[-\frac{1}{2}\hat{m}^2\frac{N^2}{\Lambda} - g_4\hat{m}\frac{N^4}{\Lambda^3} - g_6\frac{N^6}{\Lambda^5}\right]$$

Direct mapping

$h - h_c$ is mapped to $\mu_B - \mu_c$ in ζ

T in $\hat{m} = \xi^{-1}\beta_c$

$\xi = \xi_{0,\pm}|1 - \frac{T}{T_c}|^{-\nu}$,

$\xi_{0,\pm}$ non-universal with $\frac{\xi_{0,+}}{\xi_{0,-}} = 2$

More general

$h \to (\mu_B - \mu_c) - \tan\alpha(T - T_c)$

$\xi \to \xi_{0,\pm}|\frac{T}{T_c} - 1 + \tan\alpha\frac{(\mu_B - \mu_c)}{T_c}|^{-\nu}$

J.J. Rehr and N.D. Mermin, PRA 8, 472 (1973)

Robustness for small α!

Volume in Λ $\quad$ ($\nu = \frac{2}{3}$ for 3d-Ising)

Proton multiplicity moments and FSS

For $\mu_B = \mu_c$, $T = T_c$ we find:

$$\langle N^k \rangle \sim \Lambda^{kq}, \quad q = d_F/d, \quad k = 1, 2, ..$$

$$\Downarrow$$

Finite size scaling (FSS) law with $d_F = \frac{5}{2}$ (and $d = 3$)

FSS exponent q is related to the isothermal critical exponent δ

$$q = \frac{d_F}{d} = \frac{\delta}{\delta + 1} \quad ; \quad \delta = 5 \ (\mathrm{3d - Ising})$$

Measurement of $q \Rightarrow$ measurement of δ

Possible through **proton Intermittency in $p_\perp$!**

N.G. Antoniou, N. Davis, F.K. D., PRC 93, 014908 (2016)

Other multiplicity moments

The non-Gaussian kurtosis:

$$\kappa_{nG} = \frac{C_4 - 3C_2^2}{C_2^2} \quad ; \quad C_k = \langle (N - \langle N \rangle)^k \rangle, \quad k = 2, 3, ..$$

becomes **negative** approaching the critical point

M.A. Stephanov, PRL 107, 052301 (2011)

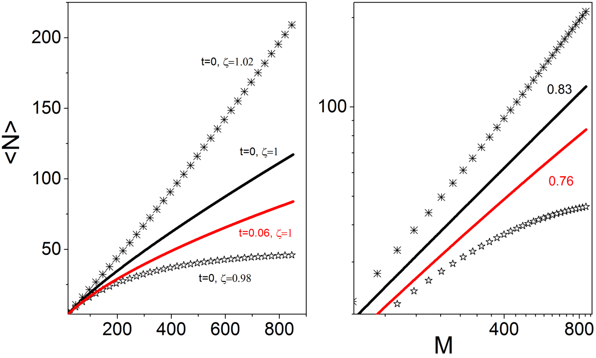
Calculate cumulants C_k and κ_{nG} through $\mathcal{Z}_{IQCD}$:

$$\frac{\partial^2}{\partial(\ln\zeta)^2} \ln \mathcal{Z}_{IQCD} = C_2 \quad ; \quad \frac{\partial^4}{\partial(\ln\zeta)^4} \ln \mathcal{Z}_{IQCD} = C_4 - 3C_2^2$$

and explore their behaviour close to the critical point!

Size of the critical region

Departing from the critical point $\Rightarrow$ Gradual destruction of the FSS law:
$(\zeta = 1, t = 0)$ $\langle N \rangle \sim \Lambda^{\frac{5}{6}}$



Notation: $\zeta = \exp[(\mu_B - \mu_c)\beta_c]$, $t = \frac{T - T_c}{T_c}$ $(\alpha = 0)$

Size of the critical region (continued)

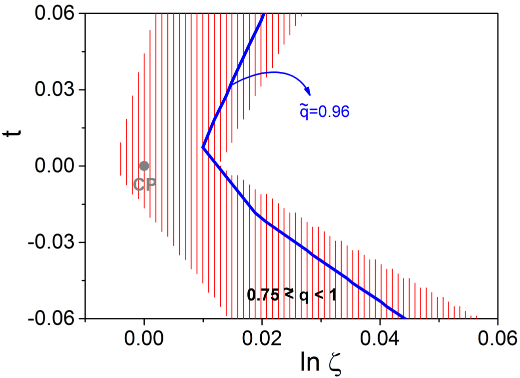
In a region around $\zeta = 1$, $t = 0$ (CP) it holds:

$$\langle N \rangle \sim \Lambda^{\tilde{q}}$$

- $\tilde{q} = \frac{3}{4} \Rightarrow$ scaling (q) in mean field theory
- $\tilde{q} = 1 \Rightarrow$ trivial scaling

Critical region: region in $(\ln \zeta, t)$-plane for which $\frac{3}{4} < \tilde{q} < 1$

Size of the critical region (first result)



Critical region $\Delta\mu_B \approx 5\ MeV$

(for $T_c \approx 160\ MeV$)

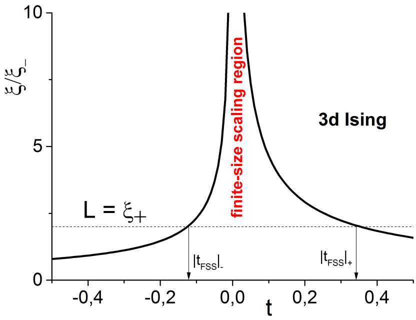
$\Downarrow$

Very **narrow** along μ_B-axis

N.G. Antoniou, F.K. D., X.N. Maintas, C.E. Tsagkarakis, PRD 97, 034015 (2018)

Finite size scaling region

FSS condition: $\xi_\infty > V^{1/3}$



Bounds along the t axis!

System dependent!

For **medium** $(20 < A < 50)$ size nucleii,

FSS region:
$3\ MeV < \Delta T < 5\ MeV$

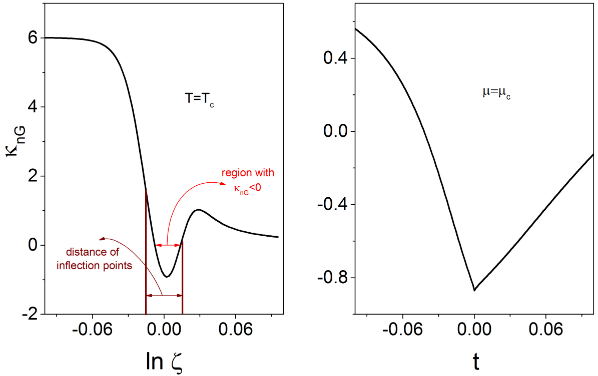
(for $T_c \approx 160\ MeV$)

$\Downarrow$

Narrowness along T-axis too

N.G. Antoniou, F.K. D., arXiv:1802.05857 [hep-ph]

Kurtosis within the critical region



Negative sharp minimum

of κ_{nG} at the CP

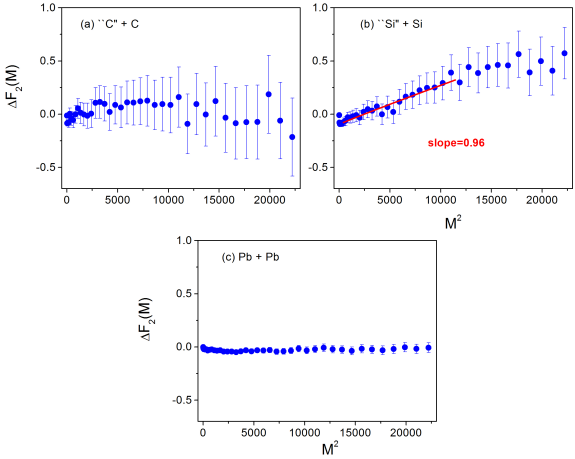
N.G. Antoniou, F.K. D., N. Kalntis, A. Kanargias

arXiv:1711.10315 [nucl-th]

Alternative(s) for the critical region size

N.G. Antoniou, F.K. D., arXiv:1802.05857 [hep-ph]

Intermittency in Si + Si collisions (NA49, SPS, CERN)



Si + Si central collisions at $\sqrt{s} = 17.2$ GeV create a final state **within the critical region:** $\tilde{q} \approx 0.96$!

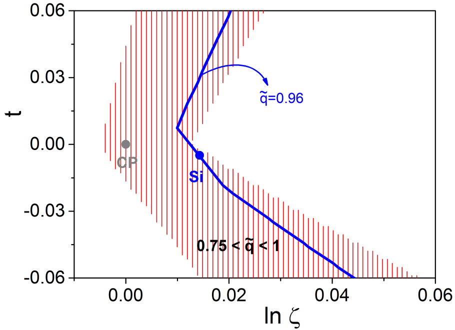
$\Downarrow$

Use this result to locate CEP!

Ignore **large** statistical errors...

Typical range for spatial correlations $[1, 8]$ *fm*
$\Rightarrow$ tr. mom. space $M^2 \in [400, 11000]$

Locating Si + Si final state within the critical region



Line $\tilde{q} = 0.96$ determines μ_c for known T_c (Lattice QCD)

recent result: $T_c = 163\ MeV$

S. Datta et al, PRD 95, 054512 (2017)

<u>Freeze-out of Si*:</u>

$$(\mu_{Si}, T_{Si}) = (260, 162.2)\ MeV$$

$$T_c = 163\ MeV$$
$$\Downarrow$$

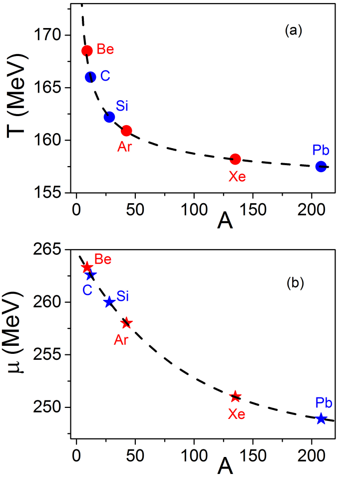
$$\ln \zeta_{Si} = 0.0143 \Rightarrow$$

$$\mu_c = 257.7\ MeV$$

*: *F. Becattini, J. Manninen and M. Gazdzicki, PRC 73, 044905 (2006)*

Predictions for NA61/SHINE freeze-out states

Freeze-out conditions for Ar+Sc and Xe+La $\Rightarrow$ use NA49 results



$$\sqrt{s} = 17.2\ GeV$$

<u>Freeze-out of central Ar+Sc:</u>

$$(\mu_{ArSc}, T_{ArSc}) = (258, 160.9)\ MeV$$

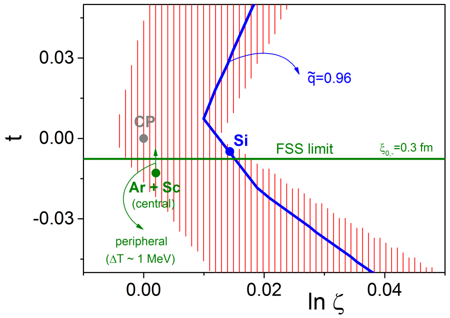
<u>Freeze-out of central Xe+La:</u>

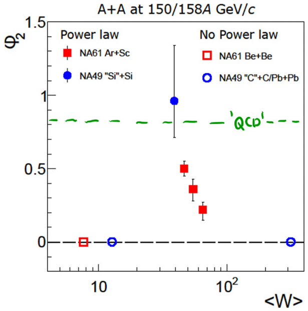
$$(\mu_{XeLa}, T_{XeLa}) = (251, 158.2)\ MeV$$

N.G. Antoniou, F.K. D., arXiv:1802.05857 [hep-ph]

M. Gazdzicki, indico.cern.ch;

N. Davis, CPOD 2018 talk





F. Becattini, et al, PRC 90, 054907 (2014);

N.G. Antoniou, F.K. D., arXiv:1802.05857 [hep-ph]

Conclusions

- **Critical (FSS) region** is **very narrow** $O(5\ MeV)$ along the μ_B and the T axis.
- Beam energy scan program at RHIC with $\Delta\mu_B \approx 50\ MeV$ is very unlike to approach the critical region.
- **Important NA49 result: freeze-out state of central Si+Si collisions at $\sqrt{s} = 17.2\ GeV$ lies within the critical (FSS) region!** (needs accurate measurements to reduce statistical errors)

$$\Downarrow$$

Can be used as a **guide** for detecting the QCD CEP.

- **Basic strategy: Accurate measurements of FSS exponent $\tilde{q}$ (intermittency analysis) and corresponding freeze-out parameters (μ_B, T) in A+A collisions with $25 < \mathrm{A} < 50$.**

Conclusions (continued)

- $\sqrt{s} \approx 17\ GeV$ seems to be the **appropriate beam energy** for approaching μ_c. **Peripheral collisions** can be used for **fine changes in** T allowing the entrance into the FSS region.

  For A+A collisions at $\sqrt{s} = 17.2\ GeV$ we propose:

  Accurate measurements of $(\tilde{q}, \mu_B, T)$ in central collisions for $25 < \mathrm{A} < 32$.

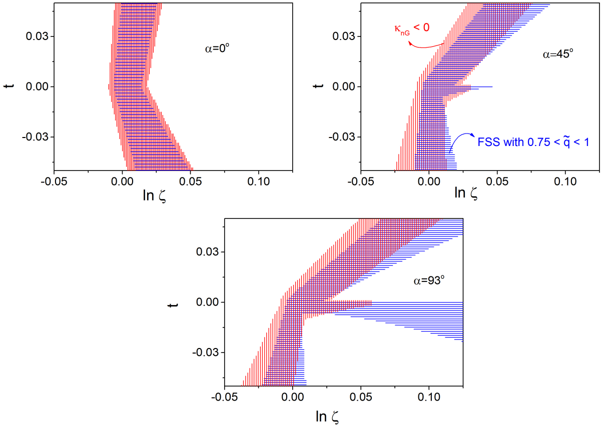
  Accurate measurements of $(\tilde{q}, \mu_B, T)$ in peripheral collisions for $32 < \mathrm{A} < 50$.

  Prediction: Strong intermittency effect in peripheral Ar+Sc collisions at $\sqrt{s} \approx 17\ GeV$ (NA61/SHINE experiment).

  (See *N.G. Antoniou, F.K. D., arXiv:1802.05857 [hep-ph]*)

Thank you!

Critical region size: κ_{nG} vs. FSS varying α



Critical region size along μ_B is $3\ MeV \leq \Delta\mu_B \leq 11 MeV$ for all α!

N.G. Antoniou, F.K. D., arXiv:1802.05857 [hep-ph]

Enriched sketch of the critical region for $\alpha \neq 0$