The complex gluon, ghost and Higgs

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Non-perturbative QFT in Euclidean and Minkowski, Coimbra, 10-12 September 2019

Collaborators on this research

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Overview

(QCD) Propagators in the complex plane

- Spectral forms of correlation functions are widely studied using lattice simulations, in particular in relation to meson spectra, charmonia (at finite T), dissociation temperatures, what happens at deconfinement, transport in the quark-gluon plasma, ...
- They can also enter the QCD bound state equations (Bethe-Salpeter, Dyson-Schwinger), in terms of their constituents.
- They are indispensable to connect fictitious Euclidean results to physical Minkowski observables.
- Unfortunately, getting clear-cut information on the full spectral properties of propagators is a very hard job (see this meeting!).
- Sometimes leads to typical one sentence editor/referee reports: "the application to gauge dependent local operators raises too much doubt about the physical relevance".

Anyhow, to us:

gauge (in)variant spectral properties are important!

Overview

Playing with the Källén-Lehmann representation

Let us assume $D(p^2)$ describes a physical observable degree of freedom: Euclidean propagator can be written as

$$G(p^2) = \int_0^\infty \frac{\tilde{\rho}(\mu)}{\mu + p^2} d\mu = \text{Stieltjes integral transform}$$

with $\tilde{\rho}(t) \geq 0$.

- $\tilde{\rho}(t) \propto \text{Disc}_{\text{cut=negative real axis}} G(t)$ via Cauchy's theorem.
- In principle, no need to know $G(p^2)$ for $p^2 \in \mathbb{C}$. Via (inverse) Stieltjes transform (1941,Widder)

$$\tilde{\rho}(t) = \lim_{n\to+\infty} (-1)^{n+1} \frac{1}{(n!)^2} \partial_t^n \left[t^{2n+1} \partial_t^{n+1} G(t) \right], \qquad t \geq 0$$

Playing with the Källén-Lehmann representation

$$\tilde{\rho}(t) = \lim_{n\to+\infty}(-1)^{n+1}\frac{1}{(n!)^2}\partial_t^n\left[t^{2n+1}\partial_t^{n+1}G(t)\right]\,, \qquad t\geq 0$$

- "Small" obstacle: ∞th derivative is a numerical beast. In some cases stability can be reached for large n using appropriate numerical derivation. Some analytical cases can be worked out exactly.
- As, in principle, $D(p^2\geq 0)$ is sufficient, perhaps nonperturbative lattice data can be used to construct an approximation for $\tilde{\rho}(\mu)$? $\rightarrow$ we can try to get at least some information on the analytical structure of Green functions?

Why is it so cumbersome to invert the Stieltjes transform?

Consider the Källén-Lehmann spectral form

$$G(p^2) \quad = \quad \int_0^\infty \frac{\tilde{\rho}(\mu)}{\mu + p^2} \mathrm{d}\mu$$

then

$$\mathcal{G} = \mathcal{L}^2 \tilde{\rho} = \mathcal{L}\mathcal{L}^* \tilde{\rho}$$

with $\mathcal{L}f(t) = \int dse^{-st} f(s) =$ (self-adjoint) Laplace transform

Källén-Lehmann = double Laplace

- Inverse Laplace $=$ known ill-posed (hard) problem, let stand alone doing it twice:(
- Intuitively clear due to exponential dampening: tiny variation in $\mathcal{L}f$ ($\sim$ propagator) will ask for massive changes in f ($\sim$ spectral density).

A meaningful (Källén-Lehmann)$^{-1}$

Consider a generic ill-posed problem

$$y = \mathcal{K}x\,, \quad ||y - y^{\delta}|| \leq \delta$$

y^{δ} = data we have for the quantity y, polluted with “noise” δ. In human language: numerical or experimental stuff always comes with errors.

Regularization

We want a controllable solution for x: if we get better and better data, we come closer and closer to the exact solution. A direct inversion is useless, thus we need to regularize the system.

A meaningful (Källén-Lehmann)$^{-1}$: Tikhonov regularization

- We search for a solution x^δ such that

$$\mathcal{J}_\alpha = ||\mathcal{K}x - y||^2 + \alpha||x||^2$$

  is minimal; $\alpha > 0$ is the regularization parameter.
- Intuitively clear: we search for a solution with "sufficiently small L_2 norm".
- In case of relevant preknowledge, $\alpha||x||^2 \to \alpha||x - x^*||^2$
- One shows that x^α = (unique) solution of the normal equation:

$$\alpha x^\alpha + \mathcal{K}^*\mathcal{K}x^\alpha = \mathcal{K}^*y$$

- Intuitively clear that the latter equation is well-posed: $\mathcal{K}^*\mathcal{K} + \alpha$ is a strictly positive —and thus invertible—operator.
- $\alpha > 0$ acts as a screening filter on the very small singular values.

A meaningful (Källén-Lehmann)$^{-1}$: Tikhonov regularization

- Concerning the choice of α: an a posteriori fixing, by making use of x^α. A controllable way is the Morozov discrepancy principle: choose that α with

$$||\mathcal{K}x^\alpha - y^\delta|| = \delta$$

  A unique solution $x^{\alpha,\delta}$ exists (can be proven).
- Looks quite reasonable: if the noise on the input data vanishes, $\delta \to 0$, the "noise" on the approximate equation will also vanish. We search for "output" of similar quality as the "input".
- The discrepancy principle avoids selecting a too small α, which would drive us back dangerously close to the ill-posed case.

Tikhonov regularization

A few observations

- The assumption made is that the integral equation has a solution (not trivial!)
- Everything is linear, so computational effort is well under control.
- No a priori assumptions are made about sign of solution.
- Other methods, as (standard) Maximum Entropy Method, assume positive $\tilde{\rho}$ and are nonlinear in nature. (extensions to non-positive case exist, see f.i. LANGFELD ET AL, NUCL.PHYS. B621 (2002) 131.

Overview

Generalities ($T = 0$, scalar)

(More technical details can be found in ROELFS ET AL, ARXIV:1901.05348.)

- We depart from ("p^2" version)

$$G(p_4) = \int_{\omega_0}^{\infty} \frac{2\omega\rho(\omega)}{p_4^2+\omega^2}\,\mathrm{d}\omega \equiv \int_{\omega_0^2}^{\infty} \frac{\tilde{\rho}(\mu)}{p_4^2+\mu}\,\mathrm{d}\mu$$

- Equivalent, but numerically different ("ip" version):

$$G(p_4) = \int_{-\infty}^{\infty} \Omega(\omega,\omega_0)\frac{\rho(\omega)}{\omega - ip_4}\,\mathrm{d}\omega\,, \qquad \Omega(\omega,\omega_0) = \begin{cases} 0 & |\omega| < \omega_0 \\ 1 & \text{otherwise} \end{cases}.$$

- We minimize

$$J_\alpha = (\boldsymbol{K}\boldsymbol{\rho} - \boldsymbol{G})^T\boldsymbol{\Sigma}^{-1}(\boldsymbol{K}\boldsymbol{\rho} - \boldsymbol{G}) + \alpha^2\boldsymbol{\rho}^T\boldsymbol{\rho},$$

with $\boldsymbol{\Sigma}$ the covariance matrix.

Generalities ($T = 0$, scalar)

- In general,

$$\Sigma(p_i,p_j) = \frac{1}{N_{\text{Conf}}} \sum_{k=1}^{N_{\text{Conf}}} \Big(G_k(p_i) - \langle G(p_i)\rangle\Big) \Big(G_k(p_j) - \langle G(p_j)\rangle\Big) \, ,$$

but in practice, Σ almost diagonal, so $\Sigma_{ij} = \sigma_i^2 \delta_{ij}$ (no sum); σ_i^2 the variance of $G(p_i)$:

$$\sigma^2(p_i) = \frac{1}{N_{\text{Conf}}} \sum_{k=1}^{N_{\text{Conf}}} \Big(G_k(p_i) - \langle G(p_i)\rangle\Big) \Big(G_k(p_i) - \langle G(p_i)\rangle\Big) \, .$$

- With $\boldsymbol{c} = \boldsymbol{K}\boldsymbol{\rho} - \boldsymbol{G}$, we have

$$\boldsymbol{\rho} = -\frac{1}{\alpha^2} \, \boldsymbol{K}^T \, \boldsymbol{\Sigma}^{-1} \, \boldsymbol{c} \, .$$

Generalities ($T = 0$, scalar)

- We solve the linear system

$$\boldsymbol{c} + \frac{1}{\alpha^2} \boldsymbol{M} \boldsymbol{\Sigma}^{-1} \boldsymbol{c} = -\boldsymbol{G} \qquad \text{where} \qquad \boldsymbol{M} = \boldsymbol{K}\boldsymbol{K}^T \; .$$

  The propagator can be reconstructed from

$$\boldsymbol{G} = -\frac{1}{\alpha^2} \boldsymbol{M} \boldsymbol{\Sigma}^{-1} \boldsymbol{c} \; .$$

- The discrepancy principle is written as

$$\|\boldsymbol{K}\boldsymbol{\rho} - \boldsymbol{G}\|_2^2 = \sum_i \sigma_i^2 ,$$

  and can be solved for the regularization parameter α.

Generalities ($T = 0$, scalar)

- Similar relations can be derived for the p^2-version of the KL representation.
- The quality of reconstruction will be measured from the coefficient of determination,

$$R^2 = 1 - \frac{\Delta_{\rm res}^2}{\Delta_{\rm tot}^2} ,$$

where

$$\Delta_{\rm res}^2 = \sum_i (\rho_{\rm orig}(\omega_i) - \rho_{\rm re}(\omega_i))^2, \qquad \Delta_{\rm tot} = \sum_i (\rho_{\rm orig}(\omega_i) - \bar{\rho}_{\rm orig})^2 ,$$

$\rho_{\rm orig}$ are the input data points used to build the propagator and $\bar{\rho}_{\rm orig}$ is the mean value of $\rho_{\rm orig}$.

$R \leq 1$, with $R = 1$ optimal fitting.

Generalities ($T = 0$, scalar)

- In order to mimic lattice data, for a given "test spectral function", we computed the "propagator" G_{orig}.
- From this G_{orig}, $N_{\text{bootstrap}}$ data sets G_ε are generated satisfying a Gaussian distribution with mean value G_{orig} and variance $(\varepsilon G_{\text{orig}})^2$, i.e. the G_ε are distributed according to a probability distribution $G_\varepsilon \sim \mathcal{N}(G_{\text{orig}}, (\varepsilon G_{\text{orig}})^2)$, where ε is the noise level (in percentage) of the samples.

Test model 1: Breit-Wigner

- $$\rho(\omega) = \frac{1}{\pi} \frac{2\omega\gamma}{(\omega^2 - \gamma^2 - M^2)^2 + 4\omega^2\gamma^2},$$
  with $M = 3$ and $\gamma = 1$ (dimensionless). This toy model was also investigated in TRIPOLT ET AL, COMPUT.PHYS.COMMUN. 237 (2019) 129.
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Test model 1: Breit-Wigner

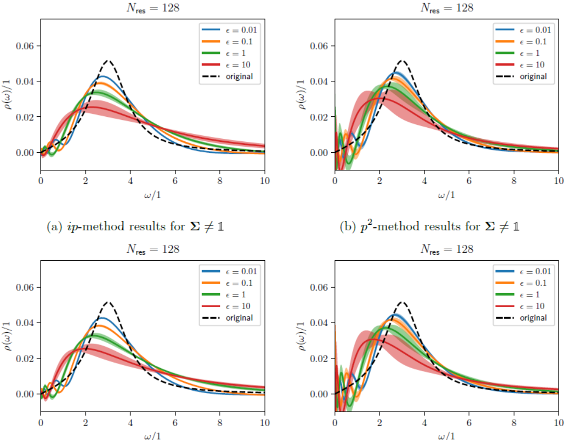


Figure: Breit-Wigner spectral function and its reconstructions.

Test model 1: Breit-Wigner

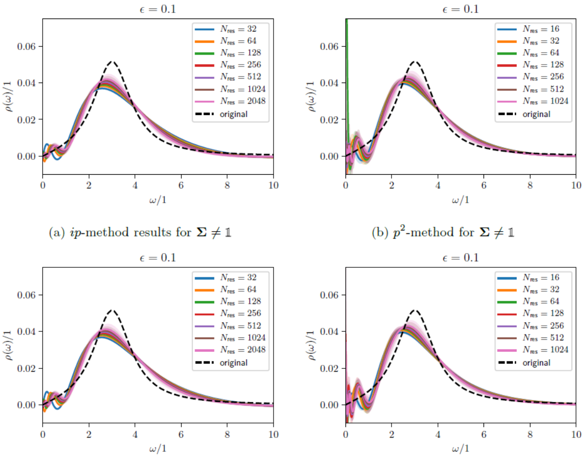
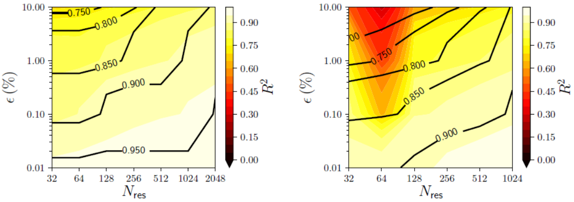


Figure: Breit-Wigner spectral function and its reconstructions.

Test model 1: Breit-Wigner



(a) *ip*-method results for $\Sigma \neq \mathbb{1}$ (b) p^2-method results for $\Sigma \neq \mathbb{1}$

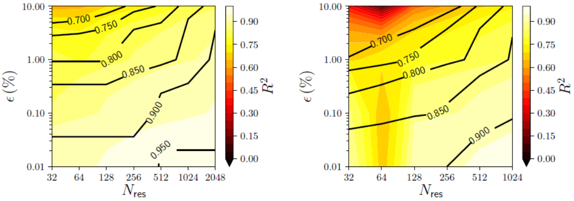


Figure: R^2. Solid black contour lines indicate the accuracy in finding the dominant peak position.

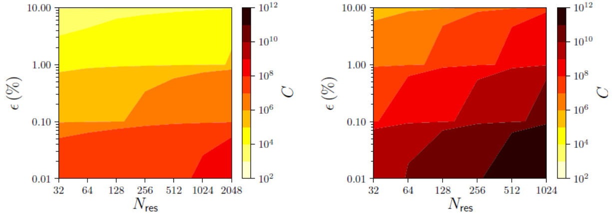
A few observations

- Taking account the statistical errors results in a reconstructed ρ with smaller errors.
- In general, reducing the noise level results in a computed spectral function that is closer to the exact $\rho(\omega)$.
- The *ip*-method provides a spectral function that is less oscillatory in the IR.
- Given R^2, the *ip*-method allows for a larger noise value than the p^2-method. For example, if one requires an $R^2 > 0.9$, a noise level of about $\varepsilon \lesssim 0.1\%$ is needed for the *ip*-method, while $\varepsilon \lesssim 0.05\%$ is required for the p^2-method.
- Looking at the effect of the parameters ε and N_{res}, it is clear that ε has a much larger effect on the quality of reconstruction than N_{res}.

ip vs. p^2

- Although formally equivalent, *ip*-version works better.
- Intuitive reason 1: KL $=$ double Laplace for the p^2-version, while KL $=$ Fourier/Laplace for the *ip*-version.

  The latter sounds already more stable from inversion viewpoint.
- Also visible from the condition number (ratio of largest/smallest singular value) of $\left(\mathbb{1} + \frac{1}{\alpha^2}\boldsymbol{M\Sigma}^{-1}\right)$. p^2-version is 2-3 order of magnitude worse than *ip*.



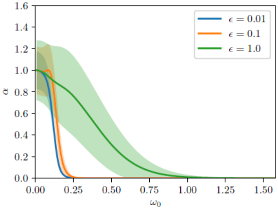
What if there is a cut-off in the KL?

- (Physical) KL integrals can have a cut-off (threshold) ω_0,

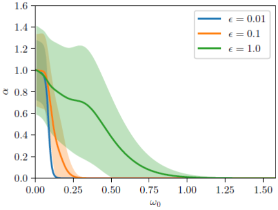
$$G(p_4) = \int_{\omega_0}^{\infty} \frac{2\omega\rho(\omega)}{p_4^2+\omega^2}\,\mathrm{d}\omega \equiv \int_{\omega_0^2}^{\infty} \frac{\tilde{\rho}(\mu)}{p_4^2+\mu}\,\mathrm{d}\mu$$

  How to fix ω_0 from the data?
- ω_0 should be independent of α, i.e. $\frac{\partial\omega_0}{\partial\alpha} \approx 0$.
- We have rather access to the inverse function, $\alpha(\omega_0)$, so we will search for that ω_0^* where $\alpha(\omega_0)$ is steepest.

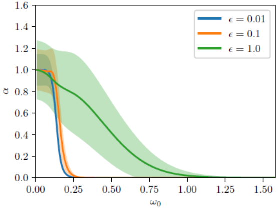
First test: the previous Breit-Wigner with no cut-off

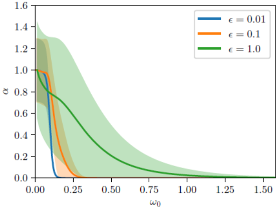


(a) *ip*-method, including $\mathbf{\Sigma}$



(b) p^2-method, including $\mathbf{\Sigma}$



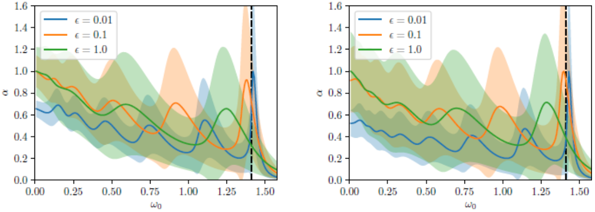


Suggests $\omega_0 = 0$ is a good choice.

Test model 2: cut-off

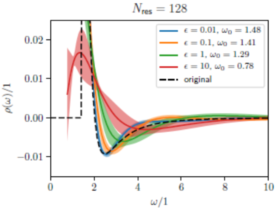
$$\rho(\omega) = -\frac{1}{\omega^4+4} + \frac{A}{\omega^6+2} \text{ for } \omega \geq \sqrt{2}\,,$$

with A such that $\int_{\omega_0}^{\infty} \rho(\omega)\omega\, d\omega = 0$ (cf. sum rule for gluons). This means that ρ is not positive-definite.

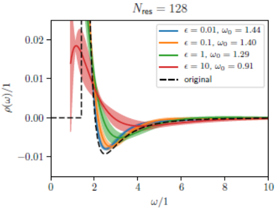


(a) ip-method with $\Sigma \neq \mathbb{1}$ (b) p^2-method with $\Sigma \neq \mathbb{1}$

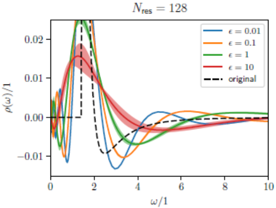
Test model 2: cut-off



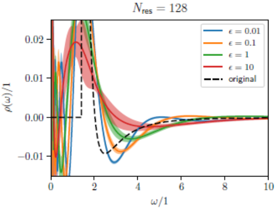
(a) ip-method with $\omega_0 > 0$.



(b) p^2-method with $\omega_0 > 0$.



(c) ip-method, $\omega_0 = 0$



(d) p^2-method, $\omega_0 = 0$

Overview

- Propagators in the complex plane
- The Källén-Lehmann representation
- Setup and tests
- Application to the complex gluon
- Application to the complex ghost
- Something about complex conjugate (*cc*) poles
- Comments about part 1
- Spectral functions for gauge-Higgs systems
- Comments about part 2

The Landau gauge gluon

- $V = 80^4$, $\beta = 6.0$, $a = 0.1016(25)$ fm, corresponding to a physical volume of $(8.1\text{ fm})^4$. The lattice data shown below refers to renormalized data within the MOM scheme at the scale $\mu = 4$ GeV, i.e.

$$G(p^2)\big|_{p^2=\mu^2} = \frac{1}{\mu^2}\ .$$

  We used 550 configs. Noise level is of the order $\varepsilon \sim 0.5\%$. $N_{\text{res}} = 219$ momentum values.
- Methodology as explained before.

The Landau gauge gluon

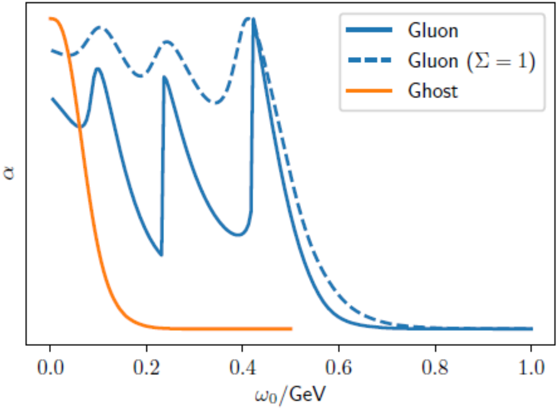
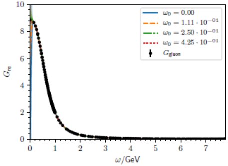
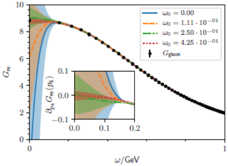


Figure: To determine the cut-off.

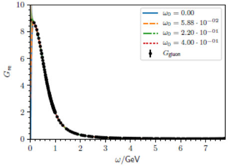
The Landau gauge gluon



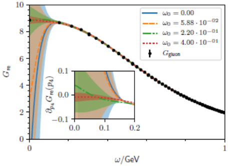
(a) Reconstructed gluon propagator for the full range of lattice momenta for the ω_0 associated with the maxima of $\alpha(\omega_0)$.



(b) IR reconstructed propagator and its derivatives for the reconstructions associated with the maxima of $\alpha(\omega_0)$.



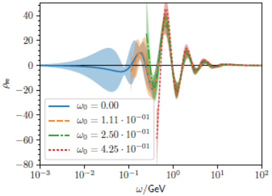
(c) Reconstructed gluon propagator for the full range of lattice momenta for the ω_0 associated with the minima of $\alpha(\omega_0)$.



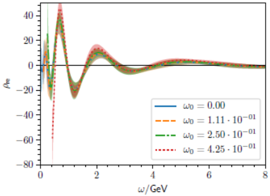
(d) IR reconstructed propagator and its derivatives for the reconstructions associated with the minima of $\alpha(\omega_0)$.

Figure: Gluon data and reconstruction

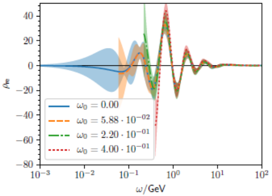
The Landau gauge gluon



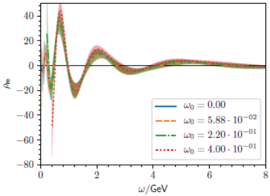
(a) Reconstructions at the maxima of $\alpha(\omega_0)$, logarithmic scale.



(b) Reconstructions at the maxima of $\alpha(\omega_0)$, linear scale.



(c) Reconstructions at the minima of $\alpha(\omega_0)$, logarithmic scale.



(d) Reconstructions at the minima of $\alpha(\omega_0)$, linear scale.

Figure: Corresponding spectral function

The Landau gauge gluon

- Main peak $\omega = 0.64 - 0.70$ GeV and a main minimum around 1 GeV where $\rho(\omega)$ is negative. Similar structures found by other inversion methods.
- We do not attempt to check the sum rule as in our formulation, by construction, the correct UV asymptotic logarithmic tails of neither propagator nor spectral function are reproduced. Recall that the sum rule follows from

$$\tilde{\rho}_{\gg}(t) \overset{t\to\infty}{\sim} \frac{1}{t}\left(\ln\frac{t}{\mu^2}\right)^{-\gamma-1}$$

  with γ the anomalous dimension.

  Could in principle be built into a prior choice ρ^*, but will not learn so much. UV spectral density is rather small/not so interesting an sich.
- IR ringing phenomenon is annoying. Could be overcome by building in IR prior information, see e.g. CYROL ET AL, SCIPOST PHYS. 5 (2018) NO.6, 065. Unlike UV asymptotics (perturbative RG), IR is not a universal modelling, so we refrained from doing so. Also here, if ringing amplitude is small, not so relevant.
- Error estimation was done via a bootstrap analysis.

A curiosity by Orlando

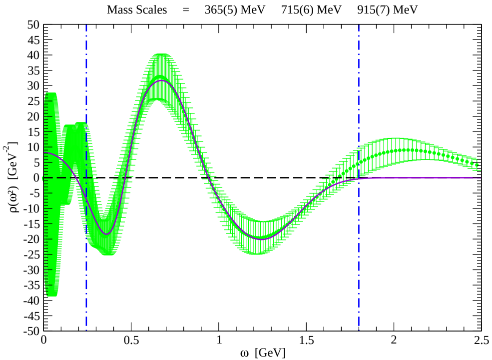


Figure: A “window fit” to the 3-peaked spectral function with 2 Gaussians + derivative of a Gaussian. Compatible with P. Lowdon et al, arXiv:1907.10073 [hep-th], see yesterday’s talk.

Overview

Propagators in the complex plane

The Källén-Lehmann representation

Setup and tests

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Application to the complex ghost

Something about complex conjugate (*cc*) poles

Comments about part 1

Spectral functions for gauge-Higgs systems

Comments about part 2

The Landau gauge ghost

- We expect a massless ghost $\to$ spectral $\delta(\omega)$-peak. Of course impossible to reconstruct numerically.
- Better approach: first isolate the δ.
- Via the form factor

$$g(p) = p^2 G(p) = \int_{-\infty}^{\infty} \frac{\sigma(\omega)}{\omega - ip} \mathrm{d}\omega ,$$

From

$$\hat{\rho}(\omega) = -\frac{\sigma(\omega)}{\omega^2}$$

one proves

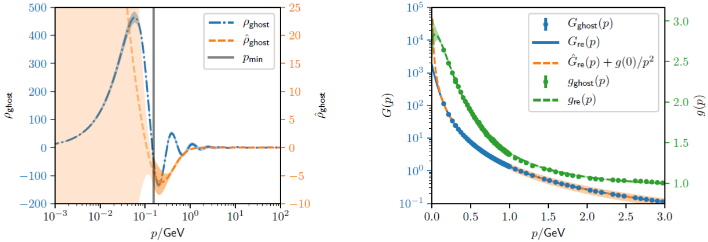
$$\hat{G}(p) = \int_{-\infty}^{\infty} \frac{\hat{\rho}(\omega)}{\omega - ip} \mathrm{d}\omega = -\frac{g(0)}{p^2} + G(p)$$

and therefore

$$\rho(\omega) = -g(0)\delta'(\omega) + \hat{\rho}(\omega),$$

The interesting information is hiding in $\hat{\rho}$.

The Landau gauge ghost



(a) Spectral density before (ρ_{ghost}) and after $\delta(0)$ removal ($\hat{\rho}_{\text{ghost}}$). The vertical black line indicates $p_{\text{min}} = 0.15$ GeV, the smallest momentum value measured.

(b) Reconstruction of the ghost propagator and dressing function. Error bars are not significant compared to the resolution of the plot.

Figure: Ghost data and reconstruction and corresponding spectral function

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Quarkyonic *cc* poles

- An option is to work e.g. with quark models, for example using quark propagators with complex conjugate (*cc*) poles.
- Inspired by results in 3D QED, see P. Maris, Phys.Rev. D52 (1995) 6087, found by solving the DSE with a contour deformation method.

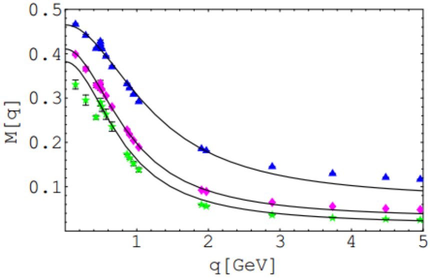
  See also model work of P. Tandy et al, Phys.Rev. D67 (2003) 054019 or Alkofer et al, Phys.Rev. D70 (2004) 014014 or S. Strauss et al, Phys.Rev.Lett. 109 (2012) 252001.

Quarkyonic *cc* poles

- Such *cc* masses were also used in the quark sector in S. BENIC, D. BLASCHKE AND M. BUBALLA, HYS.REV. D86 (2012) 074002, singnalling certain thermodynamic instability properties.

  (oscillating pressure for example)

Quarkyonic *cc* poles



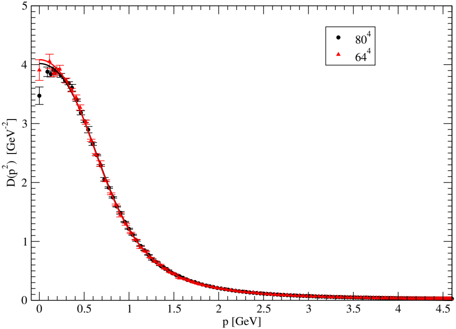
- Fit performed with (Landau gauge)

$$\langle\overline{\psi}\psi\rangle_p = \frac{i\not{p} + \mathcal{M}(p^2)}{p^2 + \mathcal{M}^2(p^2)}\,, \quad \mathcal{M}(p^2) = \frac{a}{p^2+b} + m_{\text{bare}}$$

S. Furui, H. Nakajima, hep-lat/0511045, see also Dudal et al, Annals Phys. 365 (2016) 155-179

- Similar form (+log corrections) in Coulomb gauge, G. Burgio et al, Phys.Rev. D86 (2012) 014506

Gluonic *cc* poles



Fit performed with

$$D(p^2) = Z \frac{p^2 + M^2}{p^4 + (M^2 + m^2)p^2 + \lambda^4}$$

OLIVEIRA ET AL, PHYS.REV. D86 (2012) 105005; SEE ALSO A. CUCCHIERI, D. DUDAL, T. MENDES, N. VANDERSICKEL, PHYS.REV. D85 (2012) 094513

Gluonic *cc* poles

- $D(p^2)$ corresponds to tree-level "Refined Gribov-Zwanziger" propagator, see D. Dudal, J.A. Gracey, S.P. Sorella, N. Vandersickel, H. Verschelde, Phys.Rev. D78 (2008) 065047 and C.Felix et al, Eur.Phys.J. C79 (2019) no.9, 731 : dynamical improvement of Gribov/Zwanziger's treatment of gauge copies.
- We encounter 2 *cc* poles at $\approx 0.352 \pm i0.513$ (GeV units)
- An inversion of lattice and/or DSE gluon propagator data in D. Binosi, R.-A. Tripolt, arXiv:1904.08172 predicted also (sets of) *cc* poles, using a Padé approximation method. (aka. Schlessinger point method tested before in Tripolt et al, Comput.Phys.Commun. 237 (2019) 129.)

  (Not such a surprise of *cc* poles given the rational function).
- Big question: how to "translate" unphysical constituent *cc* poles into physical bound state correlation functions with only desirable spectral properties? Or how to improve thermodynamics? (also in gluonic case, similar thermodynamics instabilities, see Dudal, Pais, et al, Eur.Phys.J. C75 (2015) no.7, 326.

  Despite the problems, *cc* poles are interesting. Very simple way to model in confinement (no "free" quarks/gluons).

Overview

- Propagators in the complex plane
- The Källén-Lehmann representation
- Setup and tests
- Application to the complex gluon
- Application to the complex ghost
- Something about complex conjugate (*cc*) poles
- Comments about part 1
- Spectral functions for gauge-Higgs systems
- Comments about part 2

How to further improve our Tikhonov regularization scheme

- can be used to construct estimates for spectral densities of physical and unphysical two-point functions.
- The numerical burden of the Tikhonov spectroscopy is well-behaved (linear vs nonlinear for MEM).
- Under assumption of location of the cut(s)! In principle, also cuts in the complex plane can be considered, but we need to “guess” where. In any case, all analytical suggestions can be tested against *data*.
- Sets of complex conjugate poles can be added in principle, i.e. to investigate

$$D(p^2) = \frac{Z}{p^2+m^2} + \frac{\bar{Z}}{p^2+\bar{m}^2} + \int_0^\infty \frac{\tilde{\rho}(\mu)}{\mu+p^2} d\mu$$

How to further improve our Tikhonov regularization scheme

This is under investigation, but not yet clear what best strategy is. Regularization for Z, $\bar{Z}$, m^2, $\bar{m}^2$ necessary? Notice that m^2 and $\bar{m}^2$ make the problem partially non-linear.

Notice however that choosing the integral representation to invert is a choice. The inversion itself will never discriminate between the several possibilities. Notice indeed that without *cc* poles, we can already "perfectly" invert the gluon data.

- Inclusion of temperature $\rightarrow$ quasi-gluon, consistent with Ilgenfritz et al, Eur.Phys.J. C78 (2018) no.2, 127?
- Even more interesting: extension to (scalar) glueball, including "molten spectral function" at higher T (deconfinement signal). Problem to be faced there: the KL integral needs subtraction to make sense, e.g.

$$D(p^2) \;=\; a_0 - (p^2 - M^2)\int_0^\infty \frac{\hat{\rho}(\mu)}{\mu + p^2}\mathrm{d}\mu\,, \quad \hat{\rho}(\mu) = \frac{\tilde{\rho}(\mu)}{(\mu + M^2)}$$

How to further improve our Tikhonov regularization scheme

- The subtraction polynomial in p^2 corresponds to $\delta^{(n)}(x-y)$-contact terms in renormalization procedure of correlation function.
- Not so evident how to numerically control the subtraction (we tried:)). We are now developing inversion for the Schwinger function, obeying

$$\begin{aligned} f(t) &= \frac{1}{2\pi}\int_{-\infty}^{+\infty} e^{-ipt} D(p^2) \\ &= \int_0^{+\infty} \tilde{\rho}(y^2) e^{-ty} \end{aligned}$$

For $t > 0$, the possible subtraction constants are irrelevant, as then

$$f(t) = b_1\delta(t) + \ldots + b_n\delta^{(n)}(t) + \int_0^{+\infty} \tilde{\rho}(y^2) e^{-ty}$$

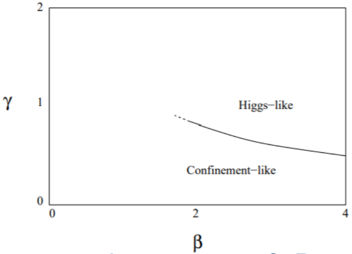
Overview

Gauge field propagators are (usually) not gauge invariant

- So what physical information can be drawn from them?
- Powerful ally: BRST invariance $\rightarrow$ Slavnov-Taylor identity $\rightarrow$ Nielsen identities $\rightarrow$ (perturbative) mass poles are gauge parameter independent. So is vacuum energy $V(\phi)$ when considering gauge-Higgs systems, albeit that $\langle\phi\rangle$ is gauge variant.
- Beyond perturbation theory? What about spectral functions? Gauge parameter dependent? Positive? What about the Higgs scalar?

Why our interest in non-Abelian Higgs systems?

- Assume a fundamental Higgs, in the Higgs coupling $\to \infty$ limit to freeze the VEV (for simplicity). Then no clear order parameter between confinement-Higgs behaviour (FRADKIN-SHENKER LATTICE STUDY, PHYS.REV. D19 (1979) 3682)
- Phase diagram sketch of W. CAUDY, J. GREENSITE, PHYS.REV. D78 (2008) 025018



- Wait, is $\langle \phi \rangle \neq 0$ not an order parameter? Perturbatively perhaps yes, but non-perturbatively, topological DOFs might destroy the condensate according to FMS (J. FROHLICH, G. MORCHIO, F. STROCCHI, PHYS. LETT. 97B (1980) 249; NUCL. PHYS. B190 (1981) 553).

Why our interest in Higgs systems?

- Should we able to see this behaviour in (gauge invariant) spectrum related quantities?
- For example: *cc* poles (or other "objects" mentioned during this meeting) emerging in certain corners of the (gauge coupling, Higgs mass)-diagram, representing "confinement"? Standard mass poles in other corners, representing "massive Higgs physics"?
- Let us be modest, and first learn a few (new) things for Abelian Higgs systems.

  Based on D. Dudal, D.M. van Egmond et al, Phys. Rev. D/arXiv:1905.10422 and work in progress.

Abelian Higgs model

$$S=\int d^4x\left\{\frac{1}{4}F_{\mu\nu}F_{\mu\nu}+(D_\mu\varphi)^\dagger D_\mu\varphi+\frac{\lambda}{2}\left(\varphi^\dagger\varphi-\frac{v^2}{2}\right)^2\right\},$$

The spontaneous symmetry breaking is implemented by expressing the scalar field as an expansion around its vev, namely

$$\varphi=\frac{1}{\sqrt{2}}((v+h)+i\rho),$$

h is identified as the Higgs field and ρ is the (unphysical) Goldstone boson, with $\langle\rho\rangle=0$. Here we choose to expand around the classical value of the vev, so that $\langle h\rangle$ is zero at the classical level, but receives loop corrections. That is, tadpole graphs are to be kept!

Abelian Higgs model

In the condensed vacuum, the gauge field and the Higgs field acquire the following masses

$$m^2 = e^2 v^2,\ m_h^2 = \lambda v^2.$$

Quantization in the 't Hooft or R_ξ-gauge (to remove mixing between Goldstone/photon)

$$\begin{aligned} \mathcal{S}_{gf} &= s\int d^dx \left\{ -i\frac{\xi}{2}\bar{c}b + \bar{c}(\partial_\mu A_\mu + \xi m\rho) \right\}, \\ &= \int d^dx \left\{ \frac{\xi}{2}b^2 + ib\partial_\mu A_\mu + ib\xi m\rho + \bar{c}\partial^2 c - \xi m^2 \bar{c}c - \xi m e \bar{c}hc \right\}. \end{aligned}$$

For the BRST transformation we have

$$\begin{aligned} sA_\mu &= -\partial_\mu c, sc = 0, s\varphi = iec\varphi, s\varphi^\dagger = -iec\varphi^\dagger, \\ sh &= -ec\rho, s\rho = ec(v+h), s\bar{c} = ib, sb = 0. \end{aligned}$$

Pole mass, residue and spectral functions

If

$$G(p^2) = \frac{1}{p^2 + m^2 - \Pi(p^2)},$$

then

$$m^2_{pole} = m^2 - \Pi^{1-loop}(-m^2) + \mathcal{O}(\hbar^2),$$

is the consistent way to derive the pole mass. Formally, the residue is given by

$$Z = \lim_{p^2 \to -m^2_{pole}} (p^2 + m^2_{pole}) G(p^2).$$

leading to

$$Z = \frac{1}{1 - \partial_{p^2} \Pi(p^2)|_{p^2=-m^2}} = 1 + \partial_{p^2} \Pi(p^2)|_{p^2=-m^2} + \mathcal{O}(\hbar^2).$$

Pole mass: wrong and right

We could also solve exactly

$$p^2 + m^2 - \Pi^{1-loop}(p^2) = 0.$$

The pole mass will be gauge dependent (at odds with Nielsen identity). For very small values of ξ, the approximated pole mass even gets complex (conjugate) values. This is due to the fact that the branch point, is ξ-dependent, and we can end up "on the cut", splitting the pole mass in 2 *cc* values.

This has been done in Y. Hayashi, K.I. Kondo, Phys. Rev. D99 (2019) no.7, 074001 using the "massive Landau gauge" (Curci-Ferrari) to model nonperturbative physics (Tissier-Serreau-Wschebor-Reinosa-et al model). Correct identification of pole masses in perturbation theory requires care, in all cases!

Pole mass: wrong and right

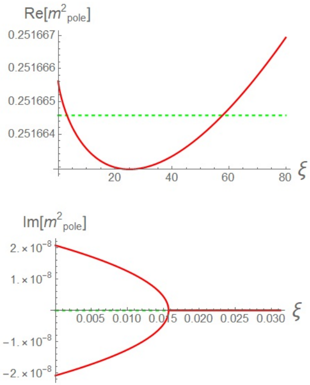


Figure: Gauge dependence of the Higgs pole mass obtained iteratively to first order (Green) and the approximated pole mass (Red), for the parameter values $m = 2$ GeV, $m_h = \frac{1}{2}$ GeV, $\mu = 10$ GeV, $e = \frac{1}{10}$.

Residue

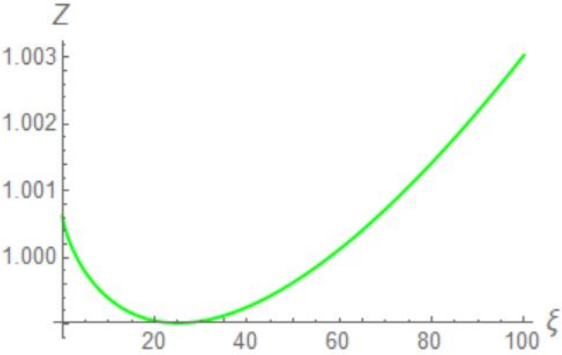


Figure: Gauge dependence of the residue of the pole for the Higgs field, for the parameter values $m = 2$ GeV, $m_h = \frac{1}{2}$ GeV, $\mu = 10$ GeV, $e = \frac{1}{10}$.

Elementary spectral functions

$$G(p^2) = \int_0^\infty dt \frac{\rho(t)}{t+p^2},$$

is rewritten as

$$G(p^2) = \frac{Z}{p^2+m_{pole}^2} + \int_0^\infty dt \frac{\widetilde{\rho}(t)}{t+p^2}$$

with

$$\widetilde{G}(p^2) = \int_0^\infty dt \frac{\widetilde{\rho}(t)}{t+p^2} = Z\left(\frac{\widetilde{\Pi}(p^2) - (p^2+m_{pole}^2)\frac{\partial \widetilde{\Pi}(p^2)}{\partial p^2}\big|_{p^2=-m^2}}{(p^2+m_{pole}^2)^2}\right).$$

while, using Cauchy's integral theorem,

$$\widetilde{\rho}(t) = \frac{1}{2\pi i}\lim_{\varepsilon\to 0^+}\left(\widetilde{G}(-t-i\varepsilon) - \widetilde{G}(-t+i\varepsilon)\right).$$

Photon spectral function

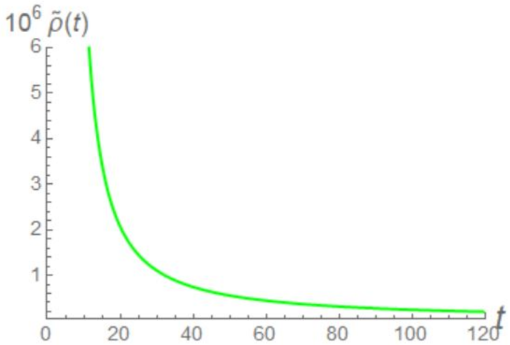


Figure: Spectral function of the photon, with t given in GeV^2, for the parameter values $m = 2$ GeV, $m_h = \frac{1}{2}$ GeV, $\mu = 10$ GeV, $e = \frac{1}{10}$. It is gauge independent, as consistent with Nielsen identity. Or even simpler/stronger: the transverse part of the photon is gauge invariant.

Higgs spectral function

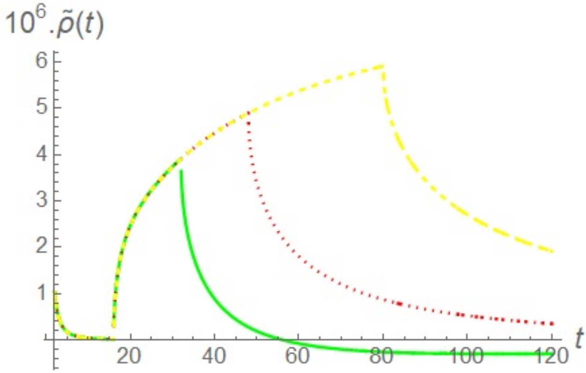


Figure: Spectral function of the Higgs boson, with t given in GeV^2, for $\xi = 2$ (Green, solid), $\xi = 3$ (Red, dotted), $\xi = 5$ (Yellow, dashed) and the parameter values $m = 2$ GeV, $m_h = \frac{1}{2}$ GeV, $\mu = 10$ GeV, $e = \frac{1}{10}$. Clearly, it is gauge dependent/non-positive. Interesting limit: the larger ξ gets, the longer positive the spectral functions stays. Visual interpretation of the unitary gauge, $\xi \to \infty$ being a "physical" gauge. But also non-renormalizable, visible from the growth at larger t.

On the branch point

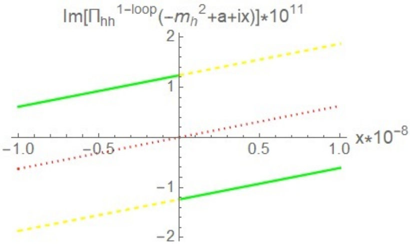


Figure: Behaviour of the one-loop correction of the Higgs propagator $\Pi_{hh}(p^2)$ around the pole mass, for the values $a = -10^{-6}$ (Yellow, dashed), $a = 0$ (Red, dotted), $a = -10^{-6}$ (Green, solid). The value x is a small imaginary variation of the argument in $\Pi_{hh}(p^2)$. Only for $a = 0$ we find a continuous function at $x = 0$, meaning that for any other value, we are on the branch cut. $\Pi_{hh}(p^2)$ is non-differentiable at $p^2 = -m_h^2$ and we cannot extract a residue for this pole. In order to avoid such a problem, we should move away from the Landau gauge and take a larger value for ξ, so that the threshold for the branch cut will be smaller than $-m_h^2$. For this we need that $4\xi m^2 > m_h^2$, which in the case of our parameters set means to require that $\xi > \frac{1}{64}$.

Intermezzo: the massive Abelian Landau gauge

- Remember: massive Landau gauge frequently used to (quite successfully) describe non-perturbative QCD propagators.
- DOFs are confined, so let us not worry about non-unitarity of the elementary gluons.
- But what if we were to worry, how to see the non-unitarity via the spectral functions?

Intermezzo: the massive Abelian Landau gauge

Consider a Higgs-Curci-Ferrari model

$$S_{CF} = \int d^d x \left\{ \frac{1}{4} F_{\mu\nu} F_{\mu\nu} + \frac{m^2}{2} A_\mu A_\mu + (D_\mu \varphi)^\dagger D_\mu \varphi + m_\varphi^2 \varphi^\dagger \varphi + \lambda (\varphi \varphi^\dagger)^2 \right.$$
$$\left. - \alpha \frac{b^2}{2} + b \partial_\mu A_\mu + \bar{c} \partial^2 c - \alpha m^2 \bar{c} c \right\},$$

There is a non-nilpotent BRST invariance:

$$s_m A_\mu = -\partial_\mu c, s_m c = 0, s_m \varphi = iec\varphi,$$
$$s_m \varphi^\dagger = -iec\varphi^\dagger, s_m \bar{c} = b, s_m b = -m^2 c.$$

Bad property 1

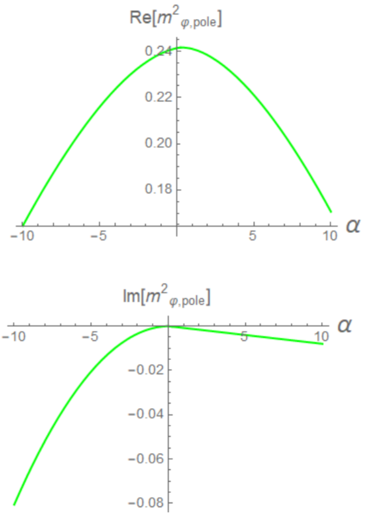


Figure: Gauge dependence of the first order pole mass for the scalar field.

Bad property 2

We could ignore the elementary excitations, and focus on the s_m-invariant operators ("physical" subspace). We notice that $s_m\left(\frac{b^2}{2}+m^2\bar{c}c\right)=0$.

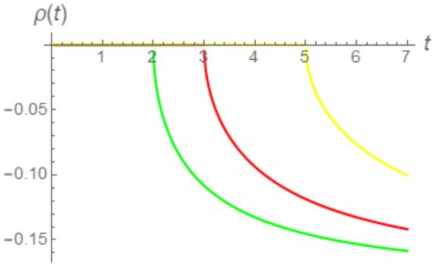


Figure: Spectral function of the composite operator $\frac{b^2}{2}+m^2\bar{c}c$, for $\alpha=2$ (Green, dotted), $\alpha=3$ (Red, solid), $\alpha=5$ (Yellow, dashed). The chosen parameter values are $m=\frac{1}{2}$ GeV, $\mu=10$ GeV. This is a ghost! Functional version of the asymptotic Fock space ghost constructed by I. Ojima, Z. Phys. C13 (1982) 173.

Some comments

Even in one-loop perturbation theory in the Abelian Higgs model, the typical problems should have become clear already:

- unphysical (gauge variant) thresholds
- non-positive spectral functions for would-be observables
- gauge variant spectral functions for would-be observables
- imagine the non-Abelian case, where the transverse gauge bosons are also not gauge invariant anymore

Gauge invariant observables

Who else than 't Hooft (1979 Cargèse lectures)

However, this model is *not* fundamentally different from a model with "permanent confinement". One could interpret the same physical particles as being all gauge singlets, bound states of

TOPOLOGICAL FEATURES OF A GAUGE THEORY 119

the fundamental fields with extremely strong confining forces, due to the gauge fields A^a_μ of the group SU(2). We have scalar quarks (the Higgs field ϕ) and fermionic quarks (the ψ_L field) both as fundamental doublets. Let us call them q. Then there are "mesons" ($q\bar{q}$) and "baryons" (qq). The neutrino is a "meson". Its field is the composite, SU(2)-invariant

$$\phi^* \psi_L = F\nu_L + \text{negligible higher order terms.}$$

The e_L field is a "baryon", created by the SU(2)-invariant

$$\varepsilon_{ij}\phi_i\psi_j = Fe_L + \ldots \qquad \text{(II4)}$$

the e_R field remains an SU(2) singlet.
Also bound states with angular momentum occur: The neutral intermediate vector boson is the "meson"

$$\phi^* D_\mu \phi = \frac{i}{2} g F^2 A^{(3)}_\mu + \text{total derivative + higher orders,} \qquad \text{(II5)}$$

if we split off the total derivative term (which corresponds to a spin-zero Higgs particle).
The $W^\pm_\mu$ are obtained from the "baryons" $\varepsilon_{ij}\phi_i D_\mu \phi_j$, and the Higgs particle can also be ontained from $\phi^*\phi$.
Apparently some mesonic and baryonic bound states survive perturbation expansion, most do not (only those containing a Higgs "quark" may survive).

Gauge invariant observables

- Spectrum should be described by gauge invariant operators. Could be used to "interpolate" between bound states ($\sim$ strong coupling) and elementary excitations ($\sim$ weak coupling).
- Later on formalized by FMS in J. Frohlich, G. Morchio, F. Strocchi, Phys. Lett. 97B (1980) 249; Nucl. Phys. B190 (1981) 553.
- Recent review and new (lattice) results in A. Maas,Prog. Part. Nucl. Phys. **106** (2019) 132.
- Let us work out the details in the $U(1)$ case, already quite instructive (and new as far as we are aware).

Gauge invariant observables

Consider the two local gauge invariant composite operators $O(x)$ and $O_\mu(x)$

$$
\begin{aligned}
O(x) &= \varphi^\dagger(x)\varphi(x) \,, \\
O_\mu(x) &= -i\varphi^\dagger(x)(D_\mu\varphi)(x) \,.
\end{aligned}
$$

In the Higgs vacuum, one gets

$$
\begin{aligned}
\langle O(x)O(y)\rangle &\sim \frac{v^4}{4} + \langle h(x)h(y)\rangle_{(tree\ level)} + \text{higher orders} \,, \\
O_\mu(x) &\sim \frac{ev^2}{2} A_\mu(x) + \text{total derivative} + \text{higher orders} \,.
\end{aligned}
$$

$O(x)$ related to the Higgs excitation, $O_\mu(x)$ to the photon.
Let me spare you the technical details.

Spectral function for the gauge invariant scalar

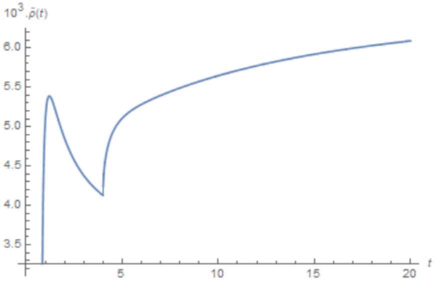


Figure: Spectral function for the propagator $\langle O(p)O(-p)\rangle$, with t given in units of μ^2, for the parameter values $e = 1$, $v = 1\,\mu$, $\lambda = \frac{1}{5}$. The spectral function is now positive, gauge invariant and no more plagued by unphysical threshold effects. We see the close similarity with $\langle hh\rangle$ in the unitary gauge, making clear the physicalness of the latter gauge. Moreover, we can also show the (now genuinely gauge invariant) mass pole coincides with that of $\langle hh\rangle$.

Spectral function for the transverse vector propagator

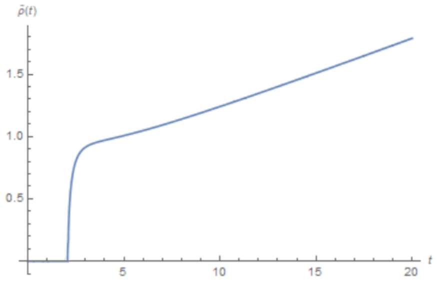


Figure: Spectral function for the propagator $\langle O(p)O(-p)\rangle^T$, with t given in unity of μ^2, for the parameter values $e = 1$, $v = 1\,\mu$, $\lambda = \frac{1}{5}$. Also here, everything perfectly physical; the pole is again coincident with the elementary one. (not shown, but the longitudinal propagator is nicely re-describing the scalar mode.)

Overview

What to do next?

- Generalize to $SU(2)$ case, including FMS operators.
- Add Gribov to the game to allow for some semiclassical non-perturbative physics. Interplay of gap equations with couplings/Higgs VEV.
- Any access to Fradkin-Shenker like predictions?

The End!

Thanks!