POSITIVITY VIOLATION IN THE MINKOWSKI SPACE QUARK PROPAGATOR

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SPRACE

Nonperturbative QFT in Euclidean and Minkowski

Coimbra, 10 - 12 September 2019

Outline

— Motivation

— Spectral representation

— Toy-model calculation

— Positivity violation

— Complex-mass poles

— Conclusions

Work with students
- Caroline Costa, Vivian Luiz, Enzo Solis

Why Minkowski space?

1. Time-like form factors
2. Inelastic processes, particle production
3. Fragmentation functions
4. Many-body transport properties
5. Confinement, positivity violation, complex-mass poles

Work somewhat related to work of:
— Biernat et al., Binosi et al., Carbonel et al., Cornwall, Dudal et al., Frederico et al., Lowdon, Salmè et al., Sauli, Siringo, Wschebor et al, …

Fermion propagator
— model-independent features

1. Spectral representation
2. Positivity
3. One instead of two spectral functions
4. Getting rid of the Dirac structure
5. No zeros no poles or zeros off real axis
6. Renormalization

Fermion propagator
— review, notation

Renormalized propagator
(omit ren. scale **μ**)

$$\psi_\Lambda(x) = \sqrt{Z_\psi}\,\psi(x) \qquad m_\Lambda = Z_m m$$

$$iS_{\alpha\beta}(x-y) = \langle\Omega|T[\psi_\alpha(x)\overline{\psi}_\beta(y)]|\Omega\rangle$$

$$= Z_\psi^{-1}\, iS_{\Lambda\alpha\beta}(x-y)$$

Will work in momentum space

$$S_{\alpha\beta}(x-y) = \int \frac{d^4p}{(2\pi)^4}\, e^{-ip\cdot(x-y)}\, S_{\alpha\beta}(p)$$

Lorentz + parity symmetries

$$S_\Lambda(p) = \frac{1}{A_\Lambda(p^2)\not{p} - B_\Lambda(p^2) + i\varepsilon} = \frac{1}{A_\Lambda(p^2)}\frac{1}{\not{p} - M_\Lambda(p^2) + i\varepsilon}$$

$$= Z_\psi S(p) = Z_\psi \frac{1}{A(p^2)\not{p} - B(p^2) + i\varepsilon}$$

$$= \frac{Z_\psi}{A(p^2)}\frac{1}{\not{p} - M(p^2) + i\varepsilon}$$

$$M_\Lambda(p^2) = \frac{B_\Lambda(p^2)}{A_\Lambda(p^2)} \quad \begin{cases} B(p^2) = Z_\psi B_\Lambda(p^2) \\ A(p^2) = Z_\psi A_\Lambda(p^2) \end{cases} \quad \rightarrow \quad M(p^2) = M_\Lambda(p^2)$$

Spectral representation

— CPT & Lorentz symm. + unitarity

$$S_\Lambda(p) = \int_0^\infty ds^2 \frac{\rho_{1\Lambda}(s^2)\not{p} + \rho_{2\Lambda}(s^2)}{p^2 - s^2 + i\varepsilon}$$

Positivity constraints

$$\rho_{1\Lambda}(s^2) \geq 0$$

$$s\rho_{1\Lambda}(s^2) - \rho_{2\Lambda}(s^2) \geq 0$$

Instead of two, one spectral function

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$$S_\Lambda(p) = \int_{-\infty}^{+\infty} d\kappa\, \rho_\Lambda(\kappa) \frac{\not{p} + \kappa}{p^2 - \kappa^2 + i\varepsilon}$$

$$\rho_{\Lambda 1}(\kappa^2) = \frac{\rho_\Lambda(\kappa) + \rho_\Lambda(-\kappa)}{2\kappa}$$

$$\rho_{\Lambda 2}(\kappa^2) = \frac{\rho_\Lambda(\kappa) - \rho_\Lambda(-\kappa)}{2}$$

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$$\rho_\Lambda(\kappa) \geq 0$$

Getting rid of Dirac structure

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Projection operators

Getting rid of Dirac structure

Projection operators $P_{\pm}(p) = \frac{1}{2}\left(1 \pm \frac{\not{p}}{w(p)}\right)$ where $w(p) \equiv \begin{cases} \sqrt{p^2} = \sqrt{(p^0)^2 - \mathbf{p}^2}, & p^2 > 0 \\ i\sqrt{-p^2} = i\sqrt{\mathbf{p}^2 - (p^0)^2}, & p^2 < 0 \end{cases}$

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$$S_\Lambda(p) = P_+(p)\,\widetilde{S}_\Lambda(w(p) + i\varepsilon) + P_+(p)\,\widetilde{S}_\Lambda(-w(p) - i\varepsilon)$$

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$$S_\Lambda(p) = P_+(p)\,\widetilde{S}_\Lambda(w(p) + i\varepsilon) + P_+(p)\,\widetilde{S}_\Lambda(-w(p) - i\varepsilon)$$

$$\widetilde{S}_\Lambda(z) = \int_{-\infty}^{+\infty} d\kappa\, \frac{\rho_\Lambda(\kappa)}{z - \kappa}$$

$$z = \pm(w(p) + i\varepsilon)$$

Renormalized x unrenormalized spectral functions

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From anticommutator $\{\psi_{\Lambda\alpha}(x^0,\boldsymbol{x}),\overline{\psi}_{\Lambda\beta}(y^0,\boldsymbol{y})\}_{x^0=y^0} = i\delta^{(3)}(\boldsymbol{x}-\boldsymbol{y})(\gamma^0)_{\alpha\beta}$

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$$0 \le Z_\psi < 1$$

Self-energy

$$S_\Lambda^{-1}(p) = [S_\Lambda^{(0)}(p)]^{-1} - \Sigma_\Lambda(p)$$

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Renormalized self-energy

$$\widetilde{S}^{-1}(z) = Z_\psi \widetilde{S}_\Lambda^{-1}(z) = Z_\psi(z - Z_m m) - \int_{-\infty}^{+\infty} d\kappa\, \frac{\sigma(\kappa)}{z-\kappa}$$

$$\sigma_\Lambda(\kappa) = Z_\psi^{-1}(\mu)\,\sigma(\kappa,\mu)$$

Propagator has no zeros or poles off the real axis

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No poles off real axis

$\widetilde{S}^{-1}(z)$ does not have zeros off real axis

Renormalization

— set renormalisation condition

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— set renormalisation condition

1) At some spacelike point

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2) On mass-shell, a timelike point

$$p^2 = \mu^2 \equiv M_p^2 > 0$$

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Using the projection operators

$$Z_\psi(\mu) = 1 - \int_{-\infty}^{+\infty} d\kappa \frac{\sigma(\kappa,\mu)}{\kappa^2+\mu^2}$$

$$Z_\psi(\mu)Z_m(\mu)m(\mu) = m(\mu) + \int_{-\infty}^{+\infty} d\kappa \frac{\kappa\sigma(\kappa,\mu)}{\kappa^2+\mu^2}$$

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Eliminate the renormalization constants

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Eliminate the renormalization constants

$$\tilde{S}^{-1}(z,\mu) = z - m(\mu) - (z^2+\mu^2)\int_{-\infty}^{+\infty} d\kappa \frac{\sigma(\kappa,\mu)}{(z-\kappa)(\kappa^2+\mu^2)}$$

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$$Z_\psi^{\rm os}(M_{\rm p}) = 1 - \int_{-\infty}^{+\infty} d\kappa \, \frac{\sigma(\kappa, M_p)}{(M_{\rm p} - \kappa)^2}$$

$$Z_\psi^{\rm os}(M_{\rm p})[M_{\rm p} - Z_m^{\rm os}\, m(M_{\rm p})] = \int_{-\infty}^{+\infty} d\kappa \, \frac{\sigma(\kappa, \mu)}{M_{\rm p} - \kappa}$$

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$$\widetilde{S}_{\rm os}^{-1}(z, M_p) = (z - M_p)\left[1 - (z - M_p)\int_{-\infty}^{+\infty} d\kappa \, \frac{\sigma(\kappa, M_p)}{(z-\kappa)(\kappa - M_p)^2}\right]$$

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Spectral function of the self-energy

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$$\sigma(\kappa) = \frac{1}{2\pi i}\left[\widetilde{S}^{-1}(\kappa + i\varepsilon) - \widetilde{S}^{-1}(\kappa - i\varepsilon)\right]$$

$$= |\widetilde{S}^{-1}(\kappa + i\varepsilon)|^2 \rho(\kappa)$$

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Spectral function of the propagator

For numerics:

Spectral function of the self-energy

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Spectral function of the propagator

$$\rho(\kappa) = \frac{i}{2\pi}\left[\widetilde{S}(\kappa+i\varepsilon) - \widetilde{S}(\kappa-i\varepsilon)\right] = \frac{i}{2\pi}\left\{\left[\widetilde{S}^{-1}(\kappa+i\varepsilon)\right]^{-1} - \left[\widetilde{S}^{-1}(\kappa-i\varepsilon)\right]^{-1}\right\}$$

$$= R(M_p)\,\delta(\kappa - M_p) + \overline{\rho}(\kappa)$$

$$\overline{\rho}(\kappa) = |\widetilde{S}^{-1}(\kappa+i\varepsilon)|^{-2}\sigma(\kappa)$$

An explicit calculation

— use a toy model

1. Dyson-Schwinger equation
2. Model for quark-gluon kernel
3. Positivity violation
4. No complex poles
5. Perturbation theory

Toy model

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Dyson-Schwinger equation for the quark propagator

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Dyson-Schwinger equation for the quark propagator

$$S_{\Lambda}^{-1}(p) = \not{p} - m_{\Lambda} - i \int \frac{d^4q}{(2\pi)^4} g_{\Lambda}^2 \gamma_{\mu} D_{\Lambda}^{\mu\nu}(q) S_{\Lambda}(p-q) T^a \Gamma_{\Lambda\,\nu}^a(q, p-q, p)$$

Toy model

Dyson-Schwinger equation for the quark propagator

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Model quark-gluon kernel

Toy model

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Model quark-gluon kernel

$$g_\Lambda^2 D_\Lambda^{\mu\nu}(q)\Gamma^a_{\Lambda\,\nu}(q,p-q,p) = -g^2 T^a F(q,p-q,p)\gamma^\mu$$

$$F(q,p-q,p) = \frac{R(q,p-q,p)}{q^2-\varsigma^2+i\varepsilon}$$

singularity-free form-factor

Dyson-Schwinger equation for the model

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$$\widetilde{S}^{-1}(w(p)+i\varepsilon)=Z_\psi(\mu)\left[w(p)-Z_m(\mu)\,m(\mu)\right]+C_F\left(\frac{g}{4\pi}\right)^2\int_{-\infty}^{+\infty}d\kappa\,K(w(p),\kappa)\,\rho(\kappa,\mu)$$

$$C_F=T^aT^a=3/4$$

$$K(w(p),\kappa)=\frac{2}{w(p)}\,\frac{i}{\pi^2}\int d^4q\left[\frac{2w(p)\kappa-p\cdot(p-q)}{(p-q)^2-\kappa^2+i\varepsilon}\right]\frac{R(q,p-q,p)}{q^2-\varsigma^2+i\varepsilon}$$

Dyson-Schwinger equation for the model

$$\widetilde{S}^{-1}(w(p)+i\varepsilon) = Z_\psi(\mu)\left[w(p) - Z_m(\mu)\, m(\mu)\right] + C_F \left(\frac{g}{4\pi}\right)^2 \int_{-\infty}^{+\infty} d\kappa\, K(w(p),\kappa)\, \rho(\kappa,\mu)$$

$$C_F = T^a T^a = 3/4$$

$$K(w(p),\kappa) = \frac{2}{w(p)}\,\frac{i}{\pi^2}\int d^4q \left[\frac{2w(p)\kappa - p\cdot(p-q)}{(p-q)^2 - \kappa^2 + i\varepsilon}\right]\frac{R(q,p-q,p)}{q^2 - \varsigma^2 + i\varepsilon}$$

Unknown is $\rho(\kappa,\mu)$

Solve by iteration

Iteration procedure

1. Make ansatz for $\rho(\kappa,\mu)$ and use it in:

$$\sigma(\kappa) = \frac{1}{2\pi i}\left[\widetilde{S}^{-1}(\kappa+i\varepsilon) - \widetilde{S}^{-1}(\kappa-i\varepsilon)\right]$$

$$\sigma(\kappa,\mu) = C_F\left(\frac{g}{4\pi}\right)^2 \int_{-\infty}^{+\infty} d\kappa' \, \frac{1}{2\pi i}\left[K(\kappa,\kappa') - K^*(\kappa,\kappa')\right] \rho(\kappa',\mu)$$

$$= \frac{\alpha_s}{\pi}\frac{1}{3}\int_{-\infty}^{+\infty}\frac{d\kappa'}{|\kappa|^3}\left[\left(\kappa^2-\kappa'^2\right)^2 - \left(\kappa^2+\kappa'^2\right)+\varsigma^4\right]^{1/2}\left[(\kappa-\kappa')^2 - 2\kappa\kappa' - \varsigma^2\right]$$

$$\times\,\theta(\kappa^2 - (|\kappa'|+\varsigma)^2)\, R(\varsigma,\kappa',\kappa)\,\rho(\kappa',\mu)$$

2. Find new $\rho(\kappa,\mu)$ from

$$\rho(\kappa) = \frac{i}{2\pi}\left[\widetilde{S}(\kappa+i\varepsilon) - \widetilde{S}(\kappa-i\varepsilon)\right] = \frac{i}{2\pi}\left\{\left[\widetilde{S}^{-1}(\kappa+i\varepsilon)\right]^{-1} - \left[\widetilde{S}^{-1}(\kappa-i\varepsilon)\right]^{-1}\right\}$$

$$= R(M_p)\,\delta(\kappa - M_p) + \overline{\rho}(\kappa)$$

$$\overline{\rho}(\kappa) = |\widetilde{S}^{-1}(\kappa+i\varepsilon)|^{-2}\sigma(\kappa)$$

Need find pole mass $M_p(p)$ and residue $R(M_p)$

3. Cycle to convergence

Parameters

$$R(q, p-q, p) = f(q) f(p-q) f(p)$$

$$f(p) = \exp(-|p^2|/\omega^2)$$

$$\mu = 100 \text{ GeV}, \quad m(\mu) = 0.005 \text{ GeV}, \quad \alpha_s/\pi = 1.25$$

$$\varsigma = 0.6 \text{ GeV}, \quad \omega = 2.5 \text{ GeV}$$

Parameters

Form-factor in quark-gluon kernel

$$F(q,p-q,p) = \frac{R(q,p-q,p)}{q^2-\varsigma^2+i\varepsilon}$$

$$R(q,p-q,p) = f(q)f(p-q)f(p)$$

$$f(p) = \exp(-|p^2|/\omega^2)$$

Numerical values

$$\mu = 100\ \mathrm{GeV}, \quad m(\mu) = 0.005\ \mathrm{GeV}, \quad \alpha_s/\pi = 1.25$$

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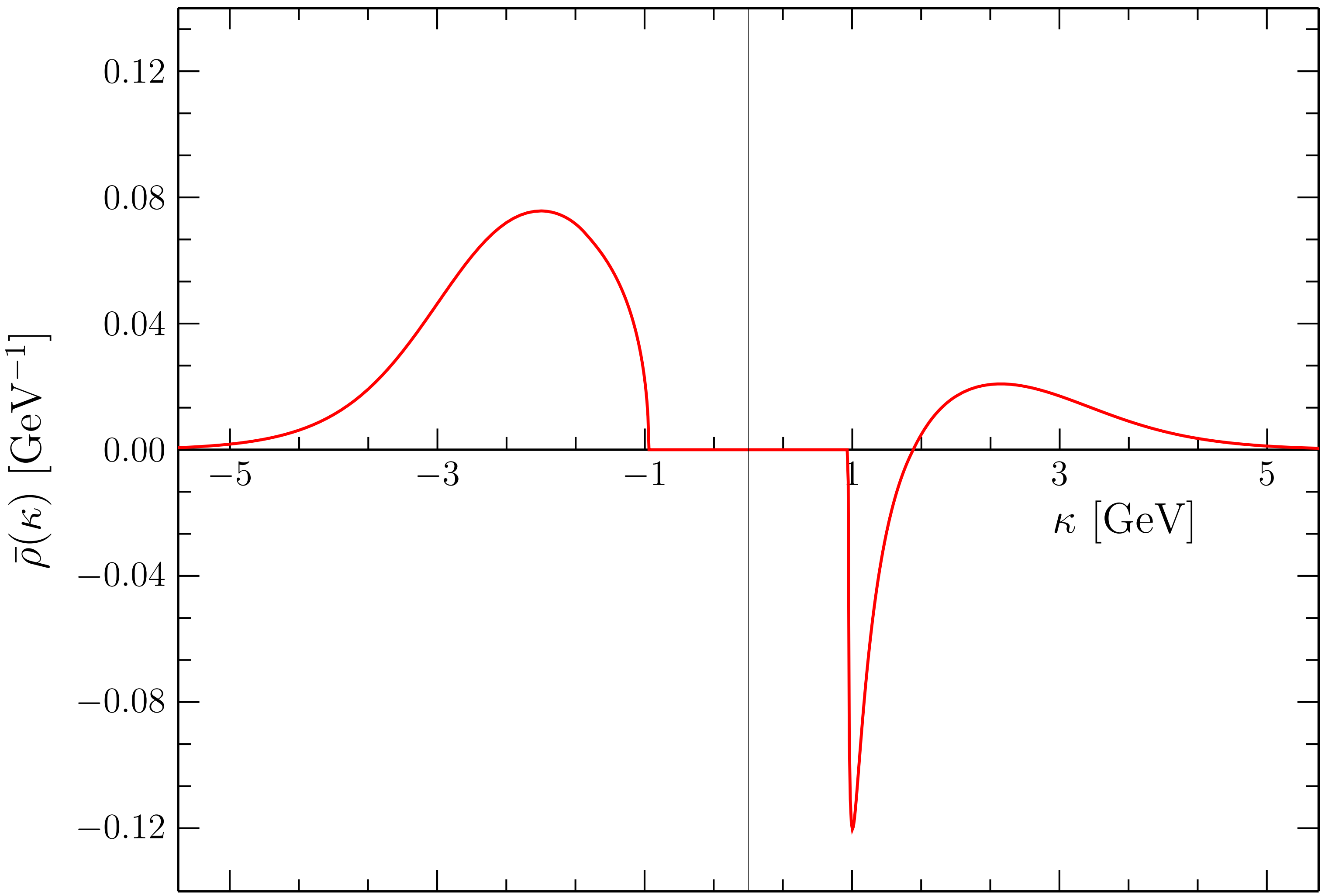
Spectral function of the propagator

$$\rho(\kappa) = R(M_p)\,\delta(\kappa - M_p) + \bar{\rho}(\kappa)$$



Pole mass and residue: $M_p = 0.36\,\mathrm{GeV}$ $R(M_p) = 0.83$

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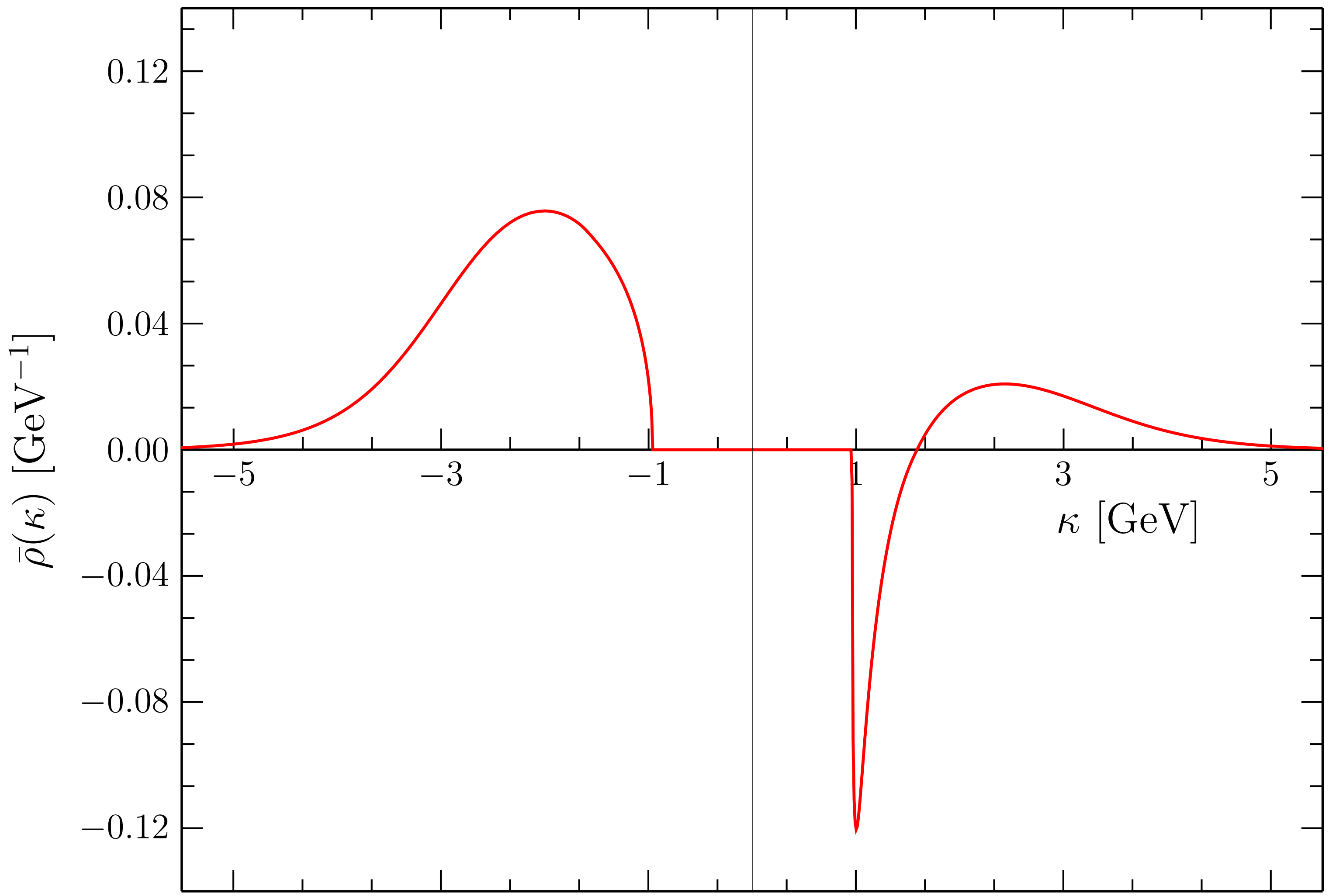
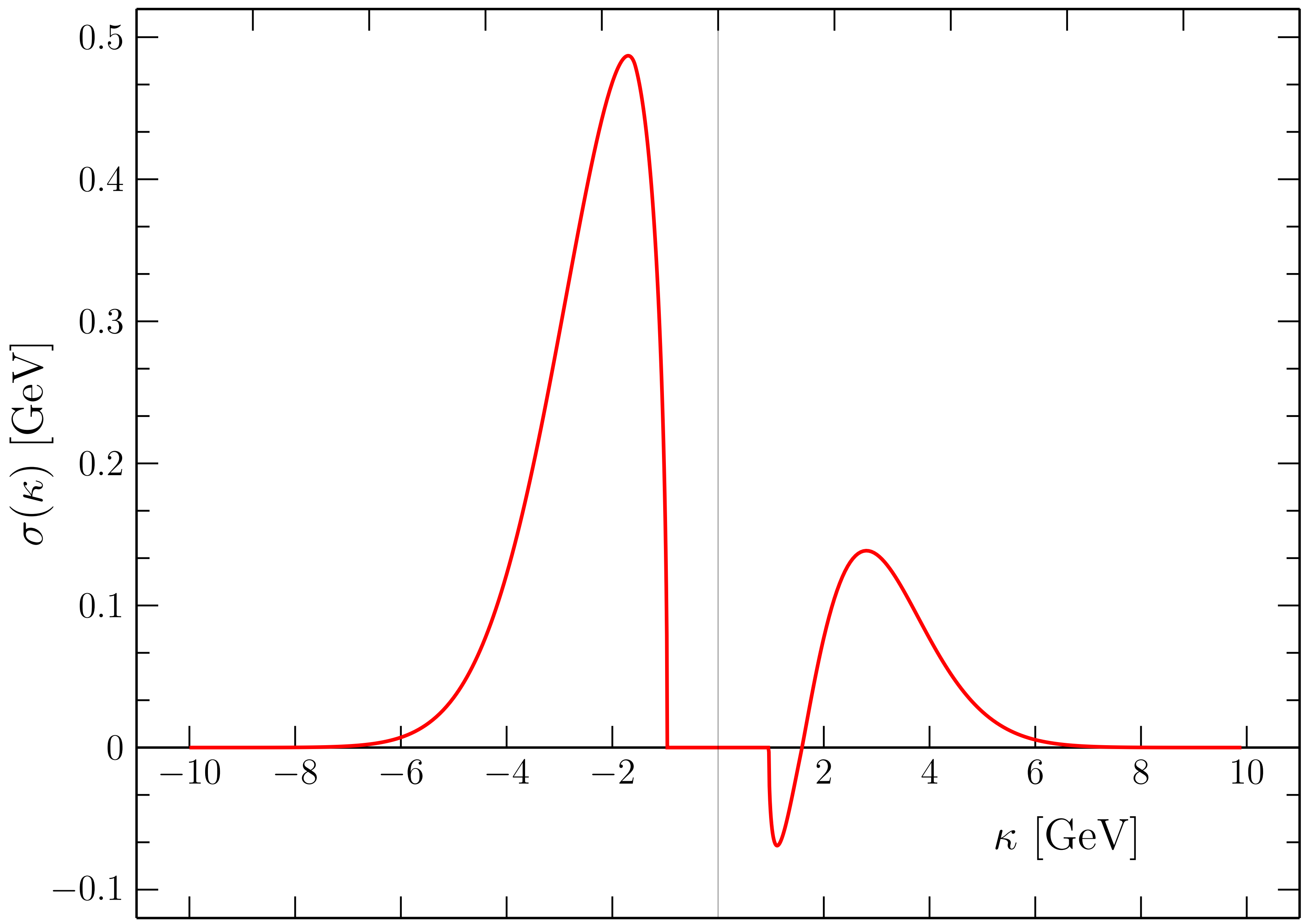


Positivity Violation

Pole mass and residue: $M_p = 0.36\,\mathrm{GeV}$ $R(M_p) = 0.83$

NO complex-mass poles

Spectral function of the self-energy



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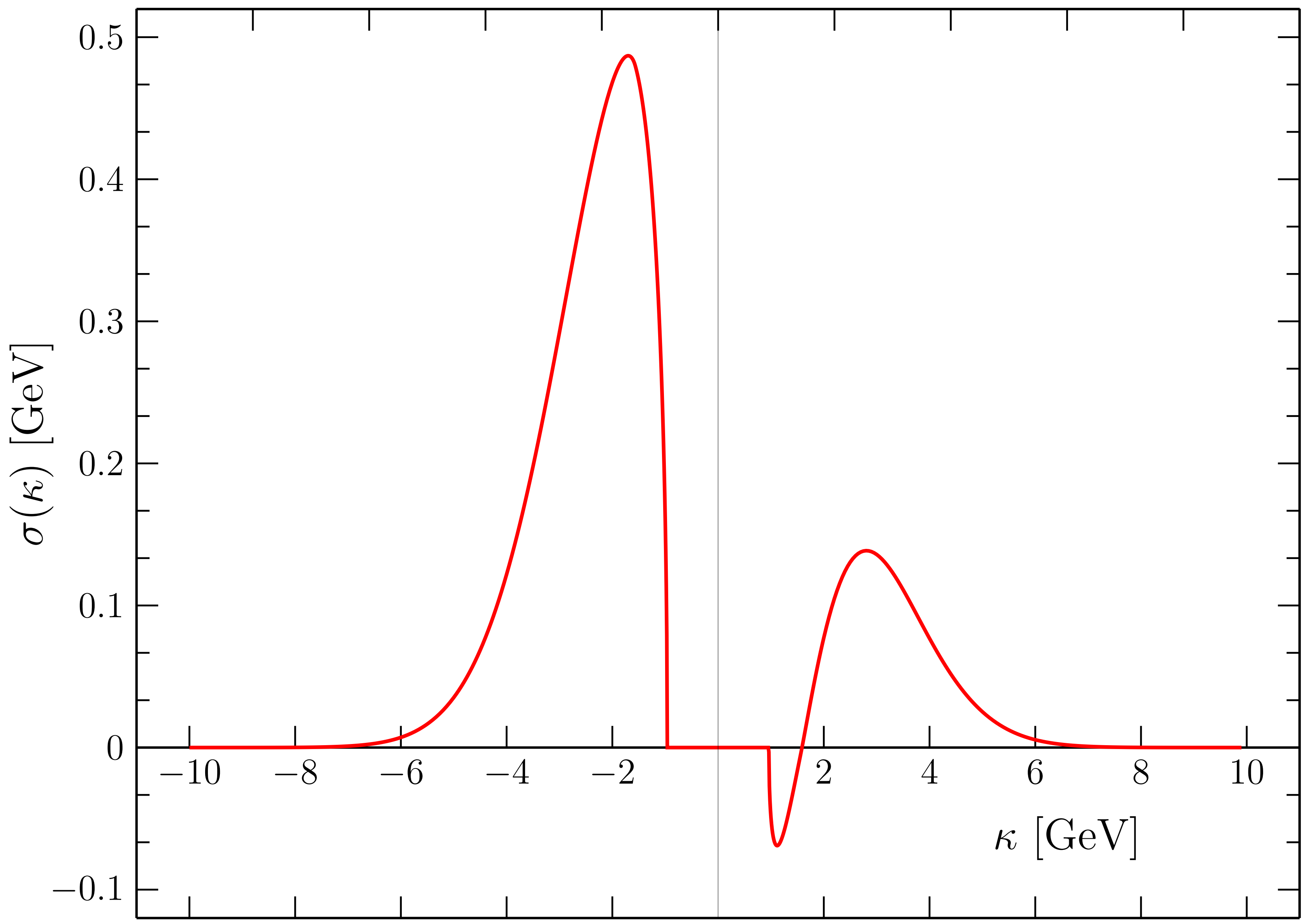
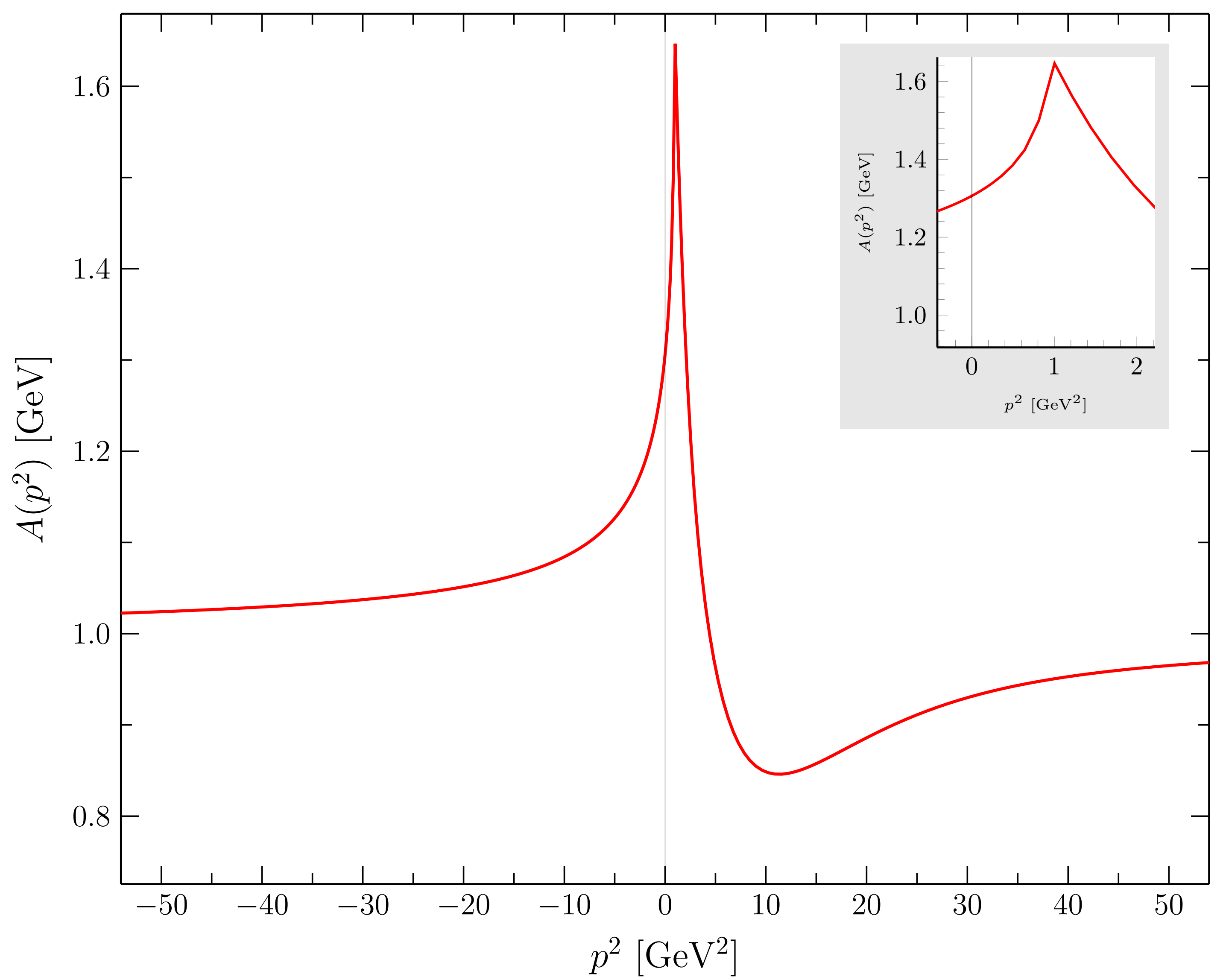


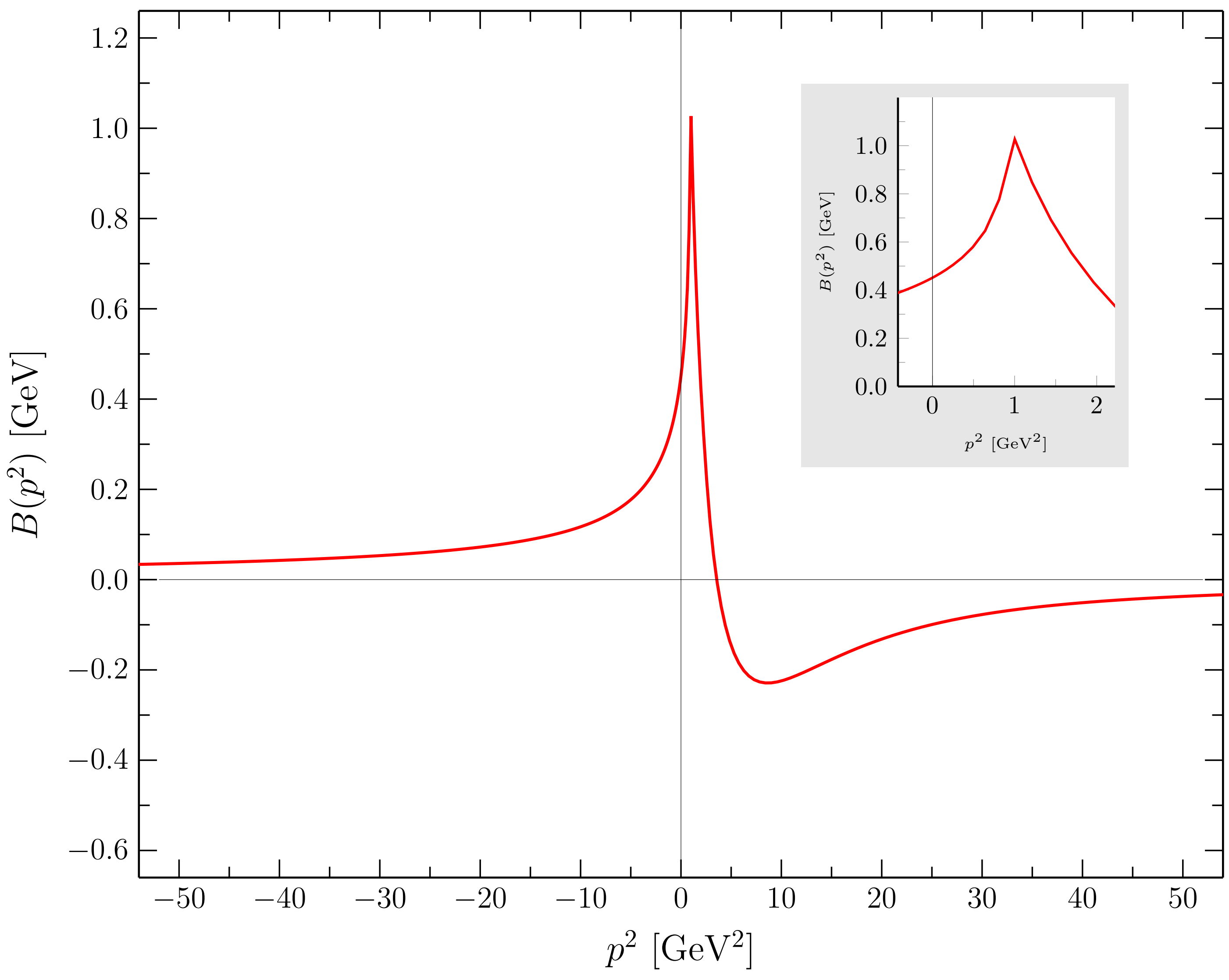
Positivity Violation

$$S(p) = \frac{1}{A(p^2)\not{p} - B(p^2) + i\varepsilon} = \frac{1}{A(p^2)}\frac{1}{\not{p} - M(p^2) + i\varepsilon}$$

$A(p^2)$

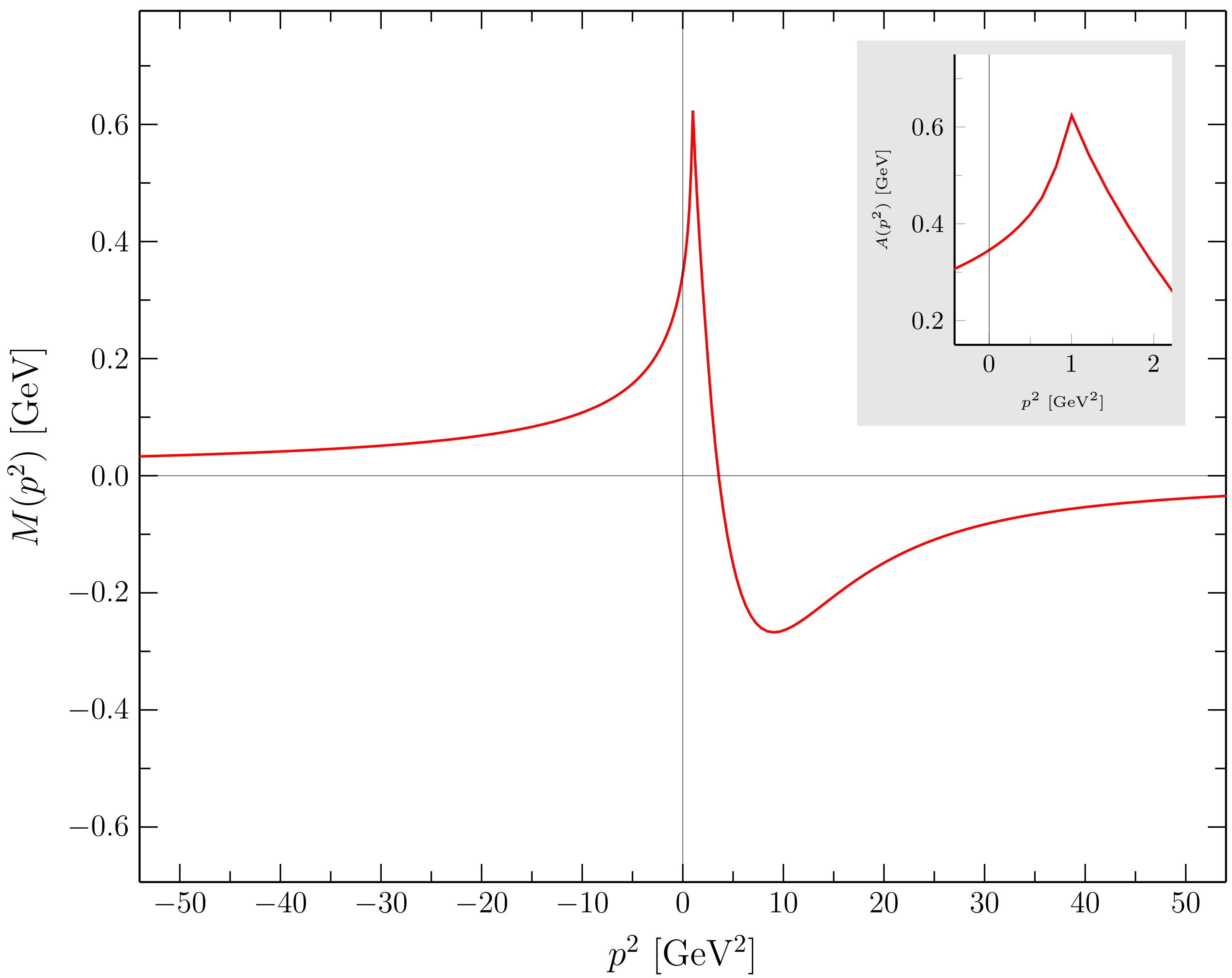


$B(p^2)$



1.2
1.0
0.8
0.6
0.4
0.2
0.0
−0.2
−0.4
−0.6
$B(p^2)$ [GeV]
−50
−40
−30
−20
−10
0
10
20
30
40
50
p^2 [GeV^2]
1.0
0.8
0.6
0.4
0.2
0.0
$B(p^2)$ [GeV]
0
1
2
p^2 [GeV^2]

$M(p^2)$



0.6
0.4
0.2
0.0
−0.2
−0.4
−0.6
$M(p^2)$ [GeV]
−50
−40
−30
−20
−10
0
10
20
30
40
50
p^2 [GeV^2]
$A(p^2)$ [GeV]
0.6
0.4
0.2
0
1
2
p^2 [GeV^2]

Where is positivity violation coming from?

In the present model from the $-2\kappa\kappa'$ term in

$$\sigma(\kappa,\mu) = C_F\left(\frac{g}{4\pi}\right)^2 \int_{-\infty}^{+\infty} d\kappa' \, \frac{1}{2\pi i}\left[K(\kappa,\kappa') - K^*(\kappa,\kappa')\right] \rho(\kappa',\mu)$$

$$= \frac{\alpha_s}{\pi}\frac{1}{3}\int_{-\infty}^{+\infty}\frac{d\kappa'}{|\kappa|^3}\left[\left(\kappa^2-\kappa'^2\right)^2 - \left(\kappa^2+\kappa'^2\right)+\varsigma^4\right]^{1/2}\left[(\kappa-\kappa')^2 - 2\kappa\kappa' - \varsigma^2\right]$$

$$\times\, \theta(\kappa^2 - (|\kappa'|+\varsigma)^2) R(\varsigma,\kappa',\kappa)\, \rho(\kappa',\mu)$$

it comes from the γ^μ in the quark-gluon kernel

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$$\times\,\theta(\kappa^2 - (|\kappa'|+\varsigma)^2)\,R(\varsigma,\kappa',\kappa)\,\rho(\kappa',\mu)$$

it comes from the γ^μ in the quark-gluon kernel

$$g_\Lambda^2 D_\Lambda^{\mu\nu}(q)\Gamma_{\Lambda\,\nu}^a(q,p-q,p) = -g^2 T^a F(q,p-q,p)\,\gamma^\mu$$

One-loop calculation

$$S_\Lambda^{-1}(p) = \not{p} - m_\Lambda - i\int \frac{d^4q}{(2\pi)^4} g_\Lambda^2 \gamma_\mu D_\Lambda^{\mu\nu}(q) S_\Lambda(p-q) T^a \Gamma^a_{\Lambda\,\nu}(q, p-q, p)$$

On the r.h.s. use:

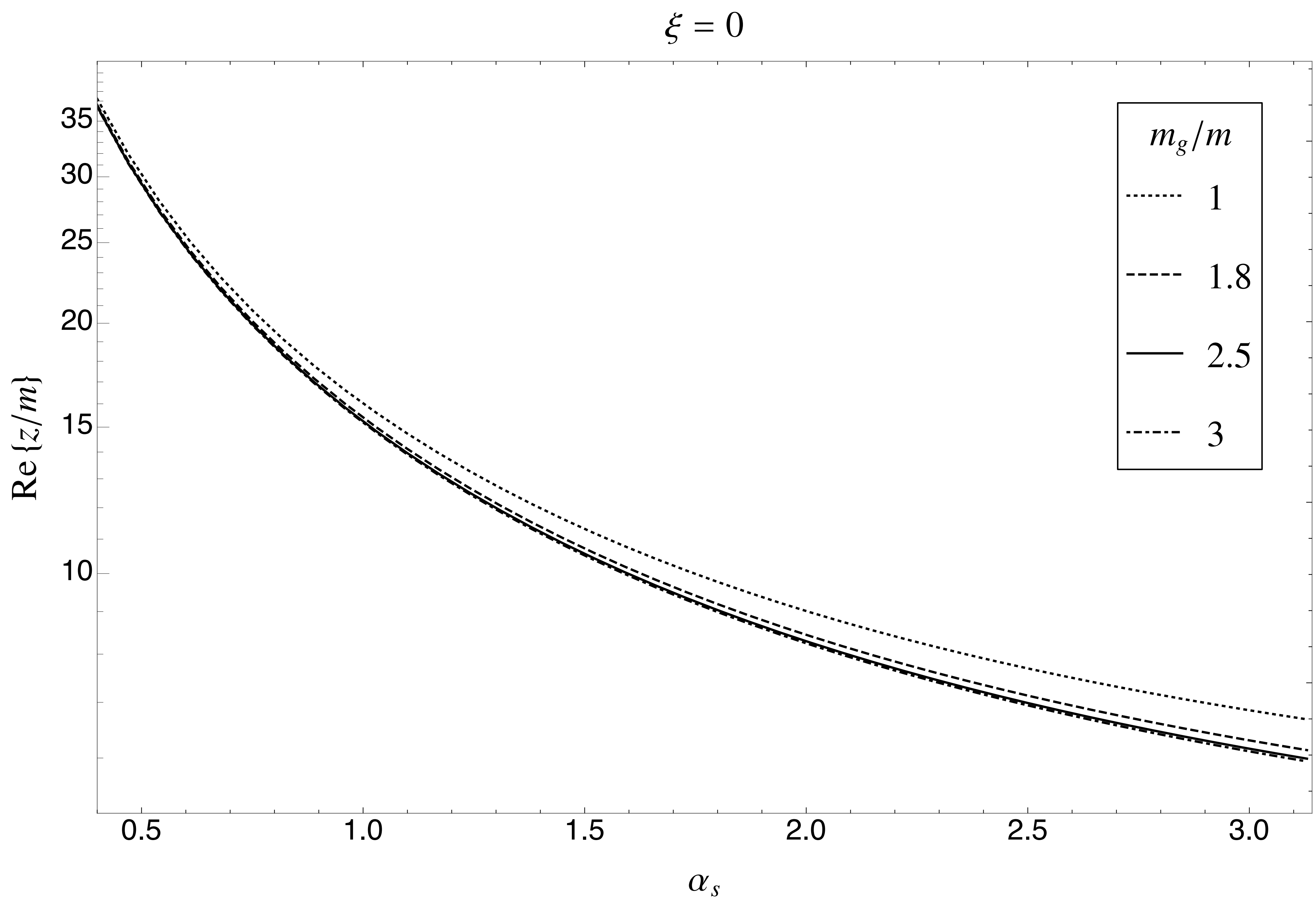
$$D^{\mu\nu}(q) = \left(-g^{\mu\nu} + \xi \frac{q^\mu q^\nu}{q^2}\right) \frac{1}{q^2 - m_g^2 + i\varepsilon}$$

$$\Gamma^a_\mu = g T^a \gamma_\mu$$

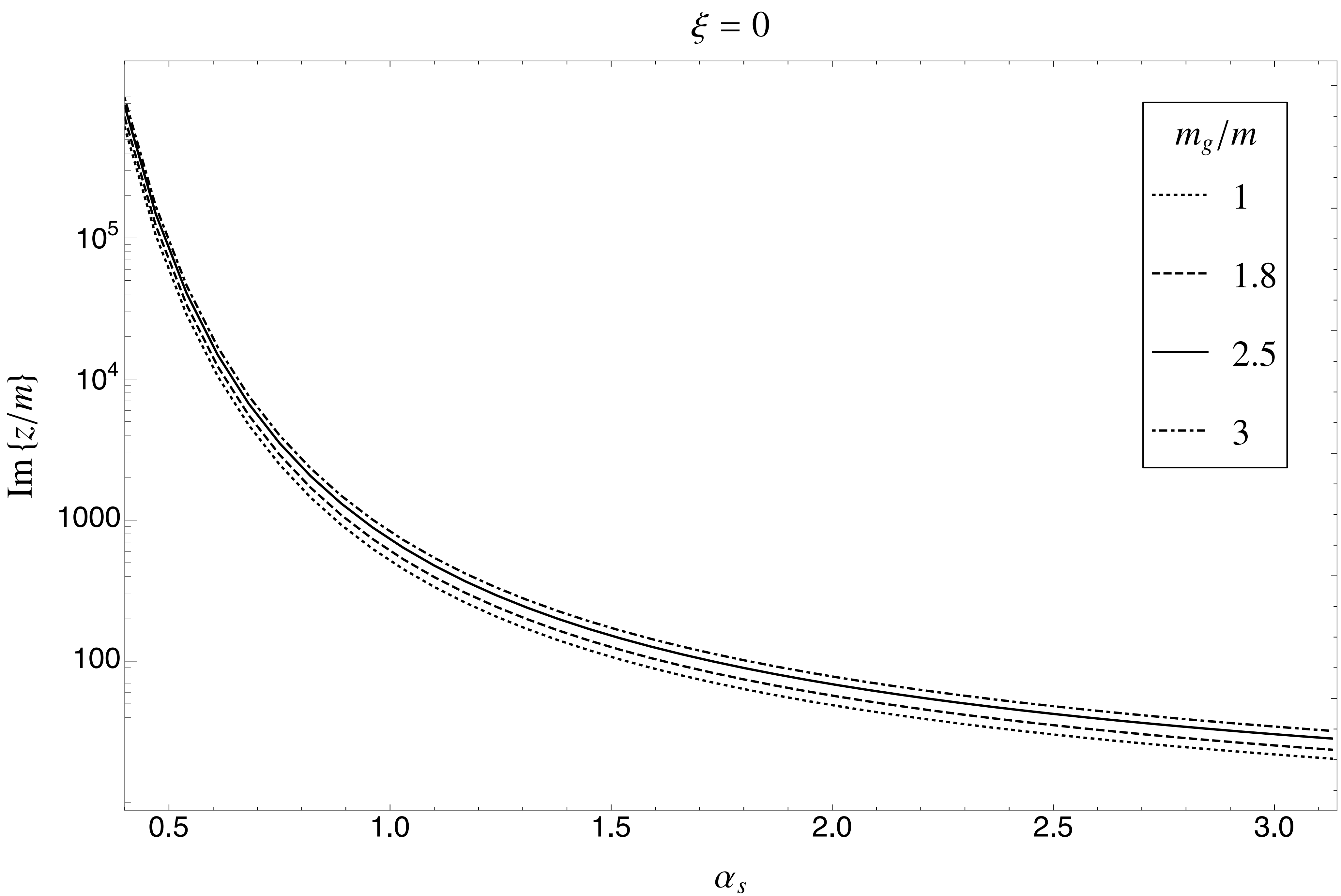
$$S(p) = \frac{1}{\not{p} - M + i\varepsilon} \to \rho(\kappa) = \delta(\kappa - M)$$

Positivity violation + complex-mass poles

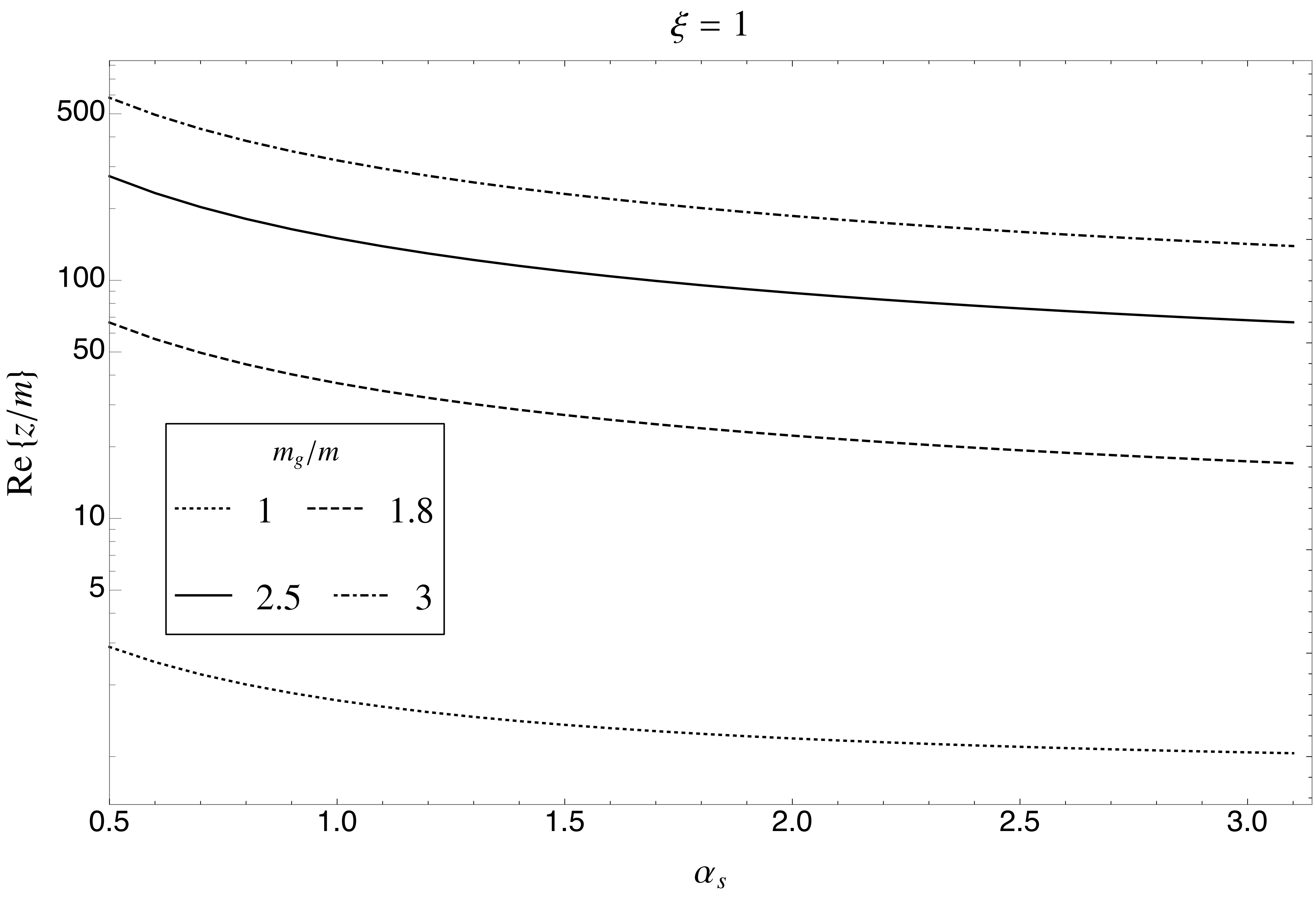
Real part of complex mass



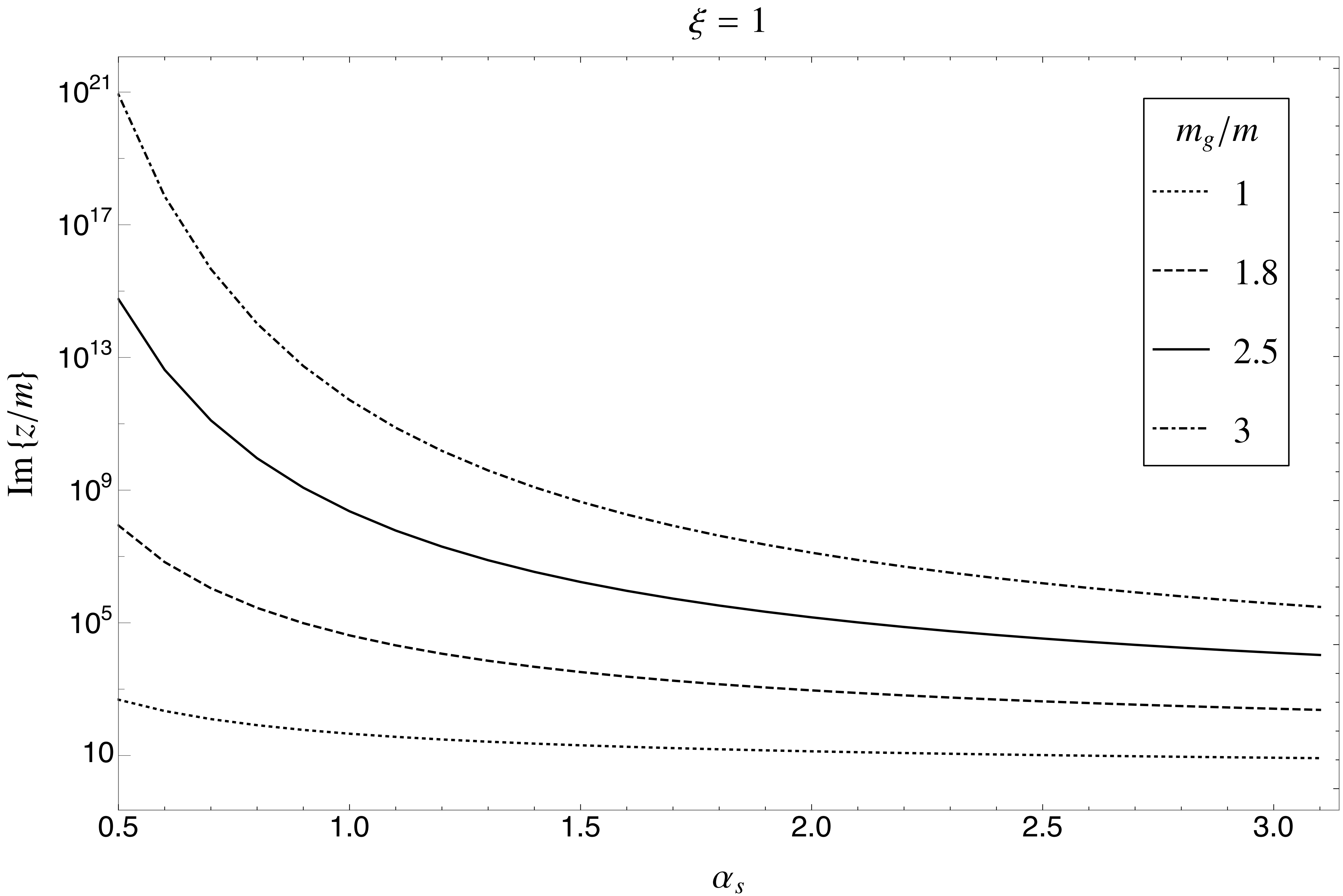
Imaginary part of complex mass



Real part of complex mass



Imaginary part of complex mass



Complex-mass poles in propagators

Known since 1942

— P.A.M. Dirac, Proc. R. Soc. London, Ser. A 180, 1 (1942)

— W. Pauli and F. Villars, Rev. Mod. Phys. 15, 175 (1943); 21, 21 (1949)

— T.D. Lee, Phys. Rev. 95, 1329 (1954)

Perturbative corrections to propagators introduce

complex poles — ghosts (phantoms)

Baryon-meson Yukawa coupling

— 25 years back*

Spin-1/2 field Yukawa coupled to spin-0 and spin-1 meson fields

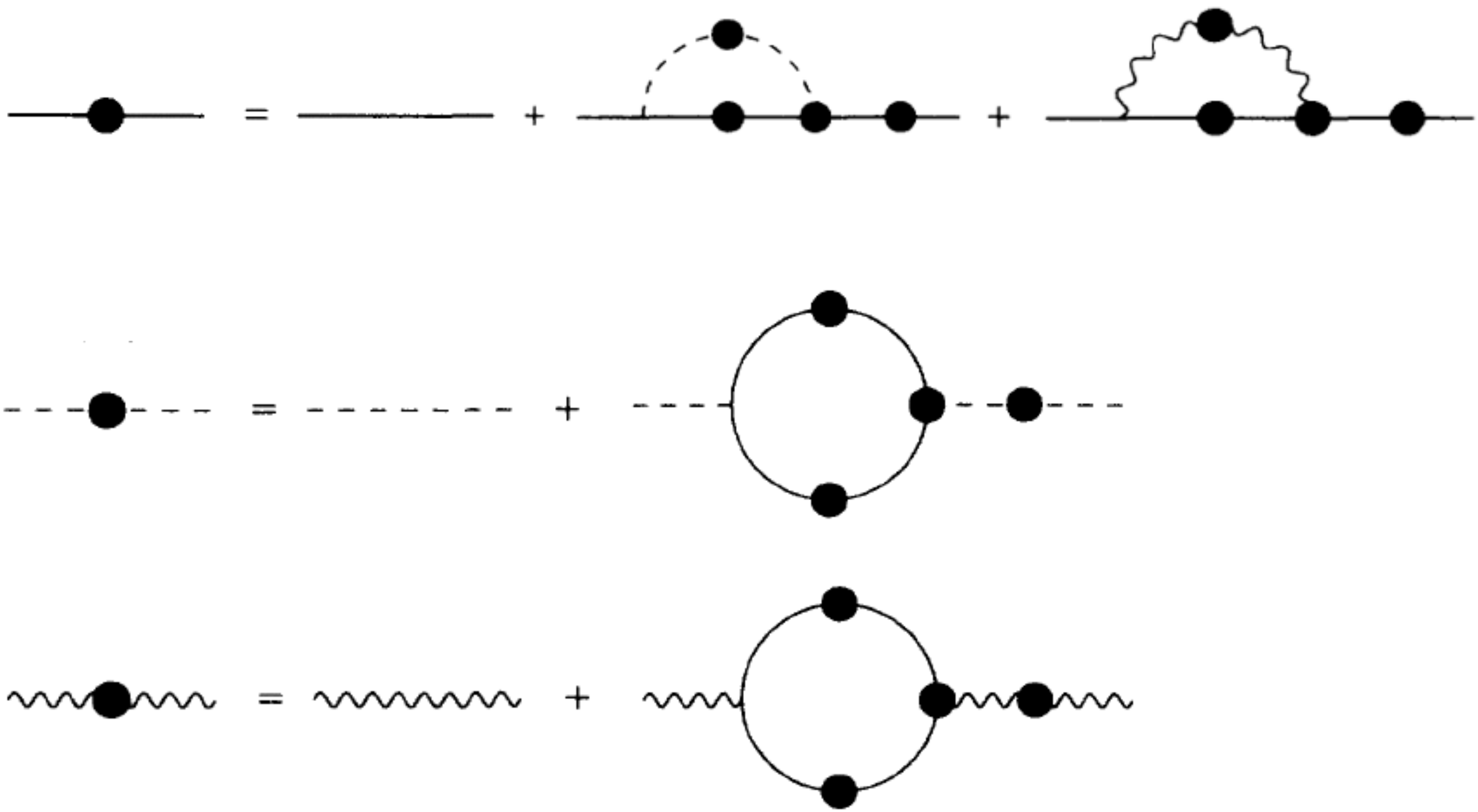
$$\mathcal{L} = \bar{\psi}(i\gamma_\mu\partial^\mu - ig_{0\pi}\gamma_5\boldsymbol{\tau}\cdot\boldsymbol{\pi} - g_{0\omega}\gamma_\mu\omega^\mu)\psi$$
$$-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}m_\omega^2\omega_\mu\omega^\mu + \frac{1}{2}\partial_\mu\boldsymbol{\pi}\cdot\partial^\mu\boldsymbol{\pi} - \frac{1}{2}m_\pi^2\boldsymbol{\pi}\cdot\boldsymbol{\pi}$$

$$F^{\mu\nu} = \partial^\mu\omega^\nu - \partial^\nu\omega^\mu$$

Model is renormalizable because massive vector mesons couple to a conserved current (baryon current)

* C.A. da Rocha,G.K., L.Wilets, NPA 616, 625 (1997)
M.E. Bracco,A. Eiras, G.K., L.Wilets, PRC 49,1299 (1994)
G.K., M. Nielsen, R.D. Puff, L.Wilets, PRC 47, 2485 (1993)

Coupled system of DSE



1. Rainbow approximation for the fermion

— use bare meson propagators, bare baryon-meson vertices

$$D_\pi(p^2) = \frac{1}{p^2 - m_\pi^2 + i\epsilon}$$

$$D_\omega^{\mu\nu}(p^2) = \left(-g^{\mu\nu} + \frac{p^\mu p^\nu}{m_\omega^2}\right)\frac{1}{p^2 - m_\omega^2 + i\epsilon}$$

Hadron physics scale

$$\frac{g_\pi^2}{4\pi} = 14.4 \qquad m_\pi = 0.144\, M$$

$$\frac{g_\omega^2}{4\pi} = 6.36 \qquad m_\omega = 0.833\, M$$

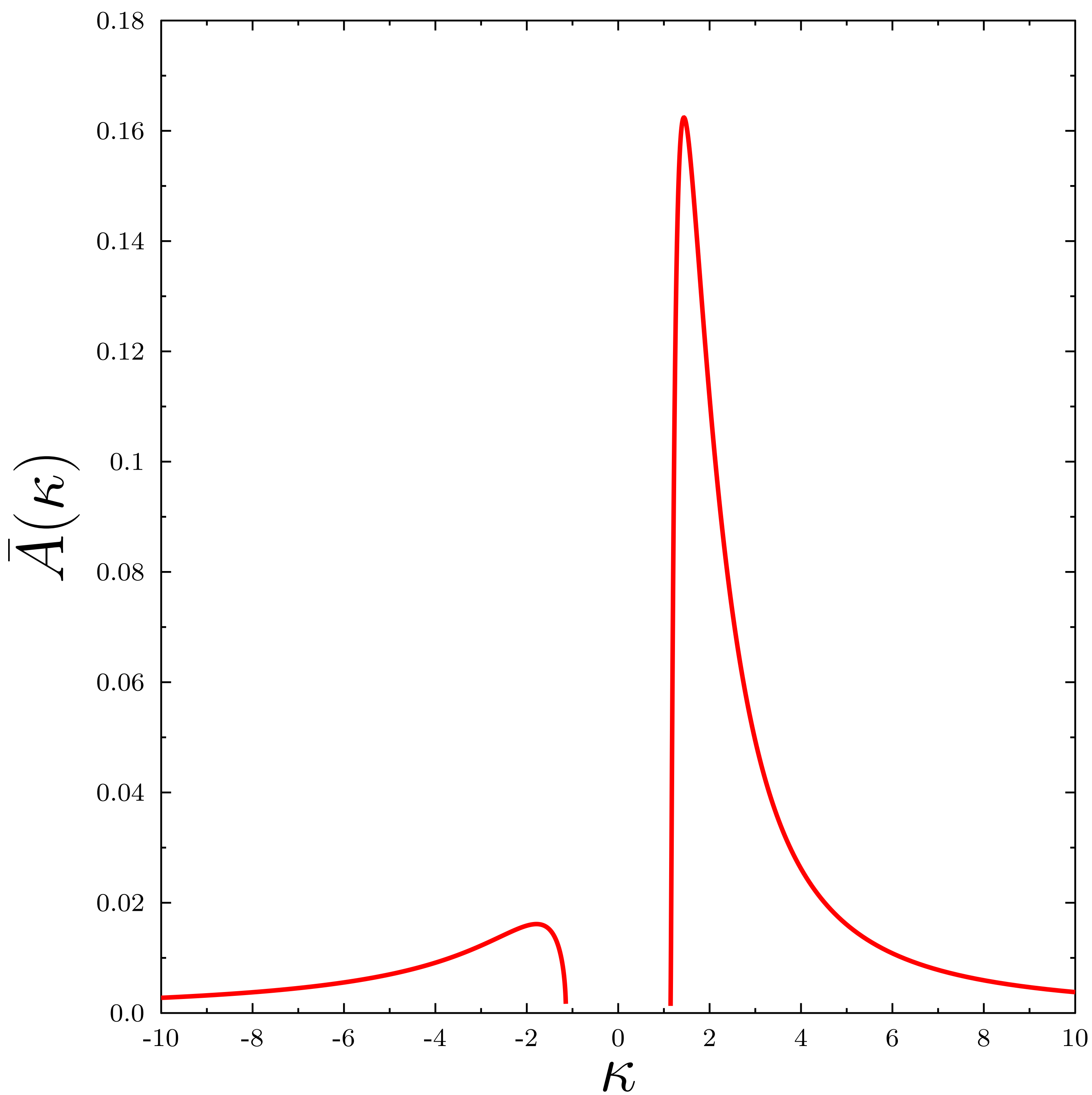
Change in notation:

$$\overline{\rho}(\kappa) \rightarrow \overline{A}(\kappa)$$

Spin-0 meson only

Change in notation:

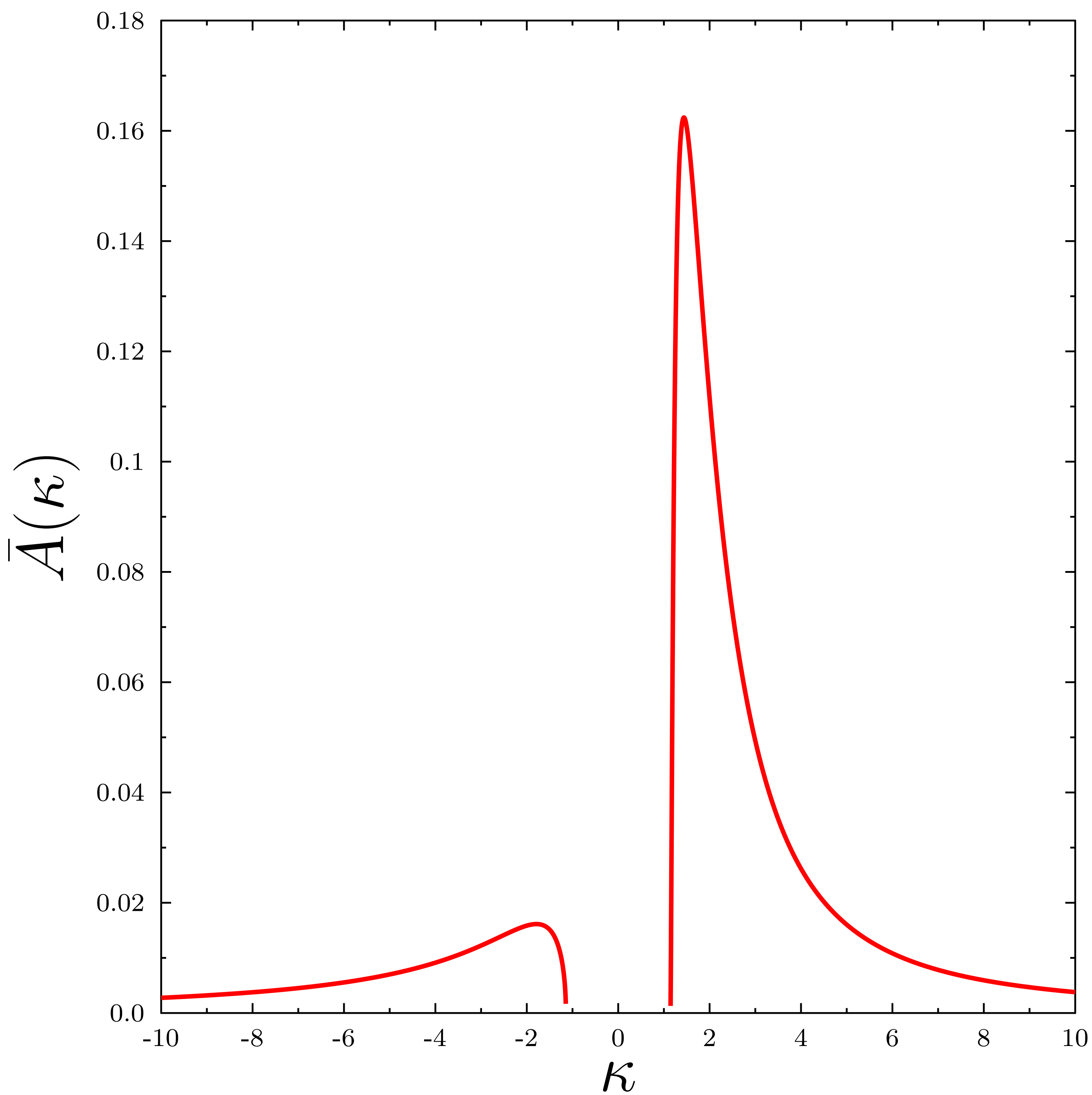
$$\overline{\rho}(\kappa) \rightarrow \overline{A}(\kappa)$$



Spin-0 meson only

Change in notation:

$$\overline{\rho}(\kappa) \to \overline{A}(\kappa)$$

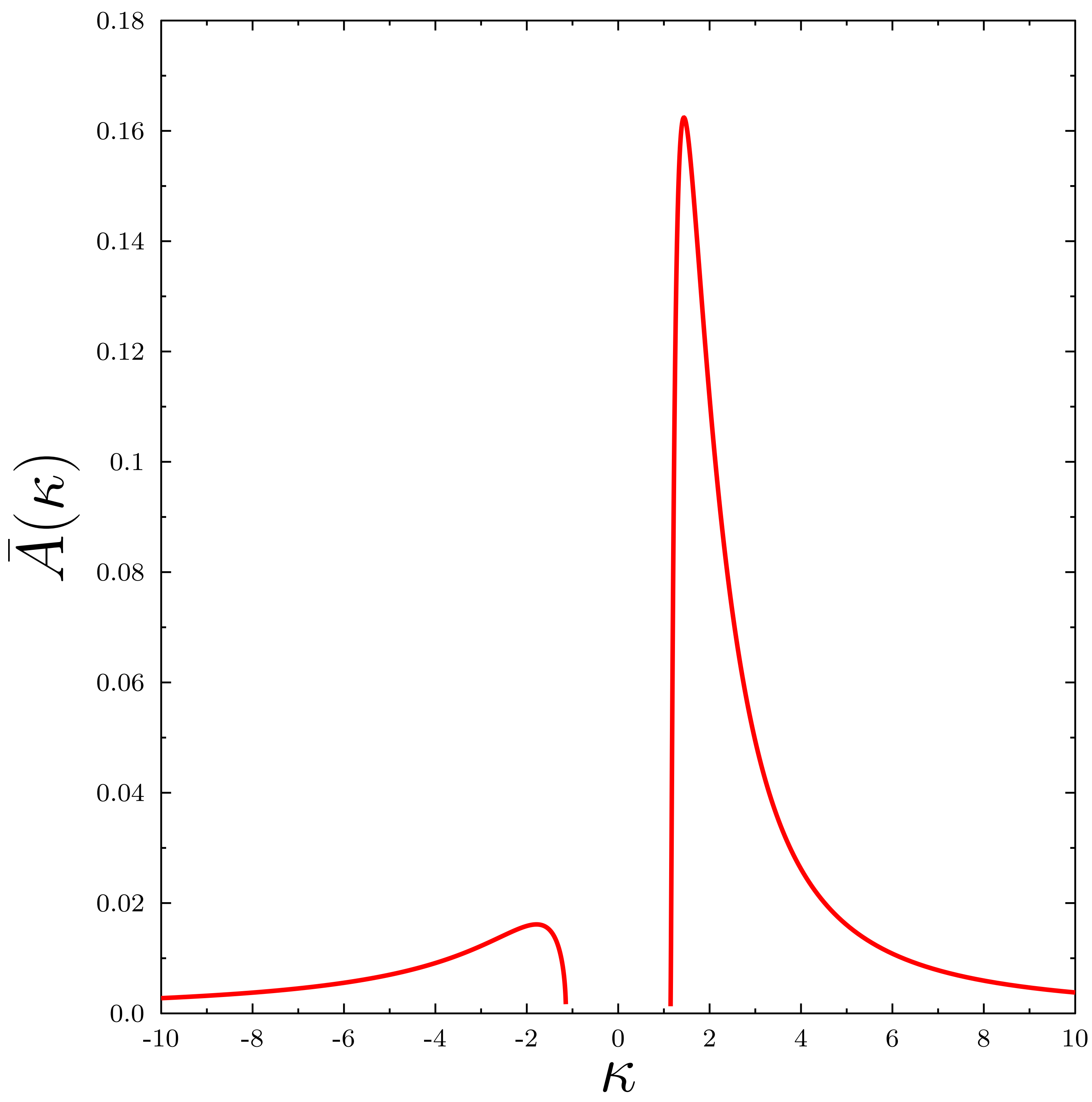


Perfect!

Spin-0 meson only

Change in notation:

$$\overline{\rho}(\kappa) \to \overline{A}(\kappa)$$



Perfect!

NOT QUITE

In addition to the pole and branch cut on the real axis

$$z/M = 0.73 \pm 1.25\, i$$

$$\mathrm{Res}(z) = -0.75 \pm 0.32\, i$$

Spin-0 meson only

In addition to the pole and branch cut on the real axis

— a pair of complex-mass poles

$$z/M = 0.73 \pm 1.25\, i$$

$$\mathrm{Res}(z) = -0.75 \pm 0.32\, i$$

Change in notation:

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Spin-1 meson only

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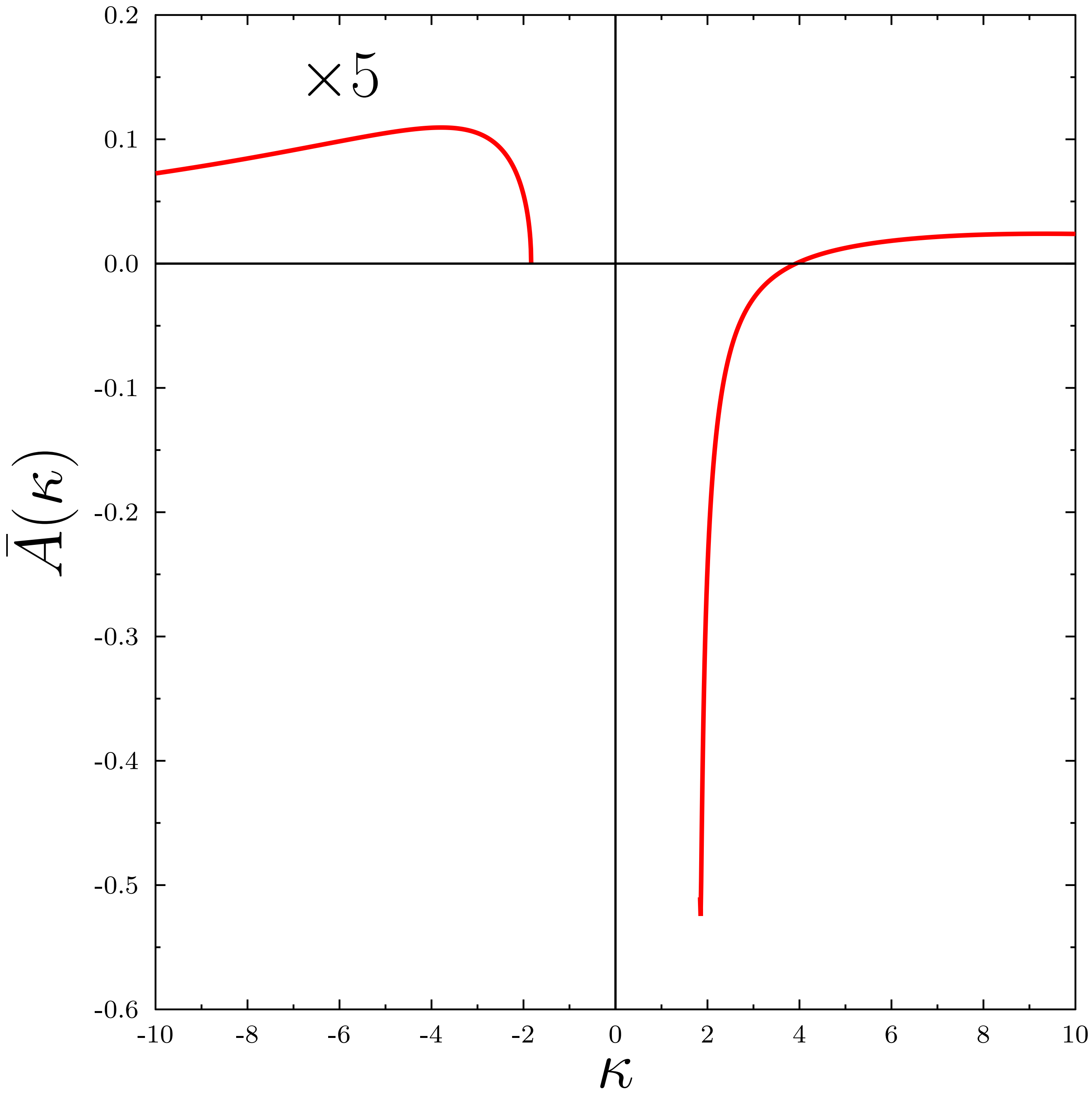
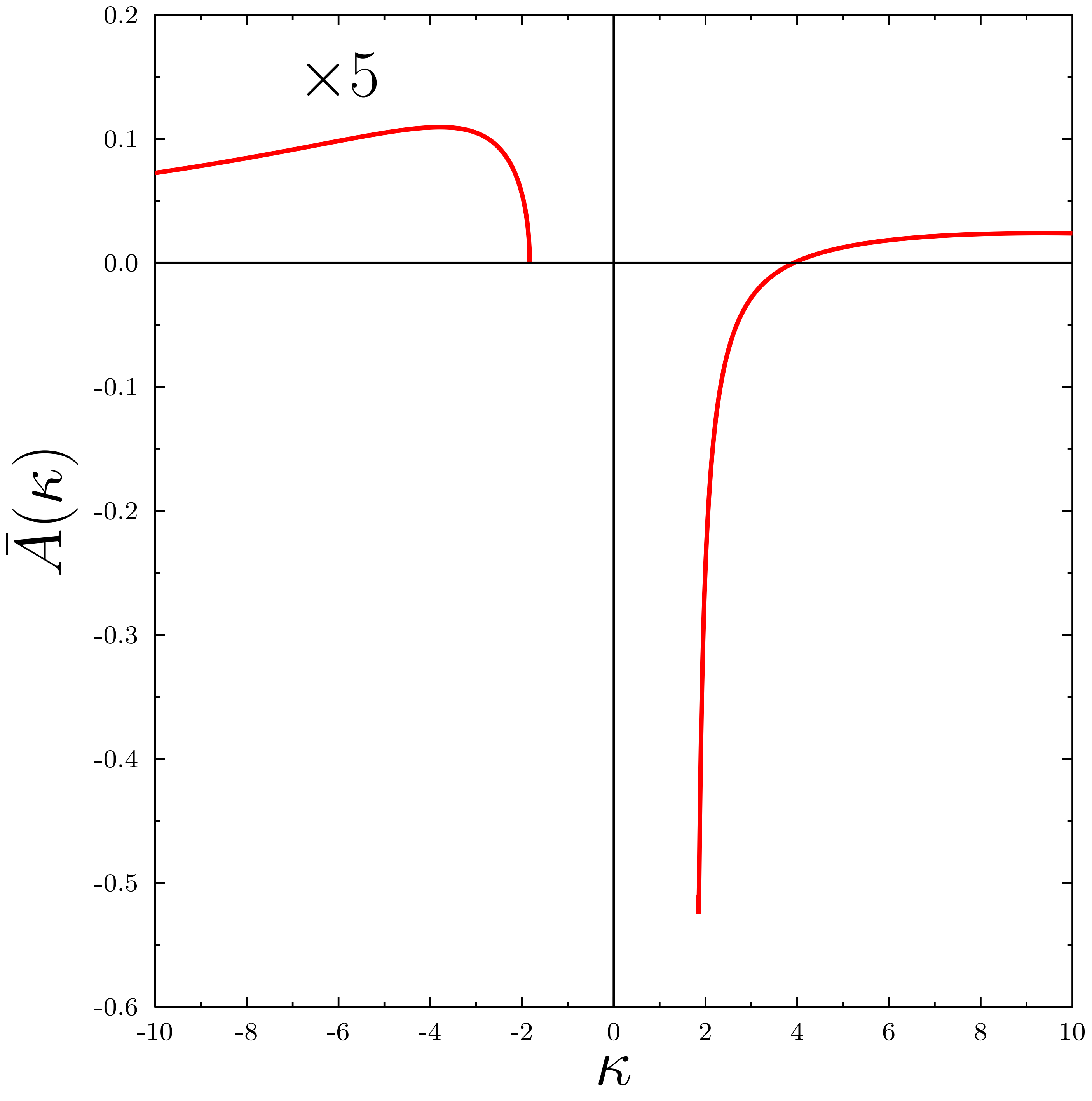


Spin-1 meson only

Change in notation:

$$\overline{\rho}(\kappa) \to \overline{A}(\kappa)$$

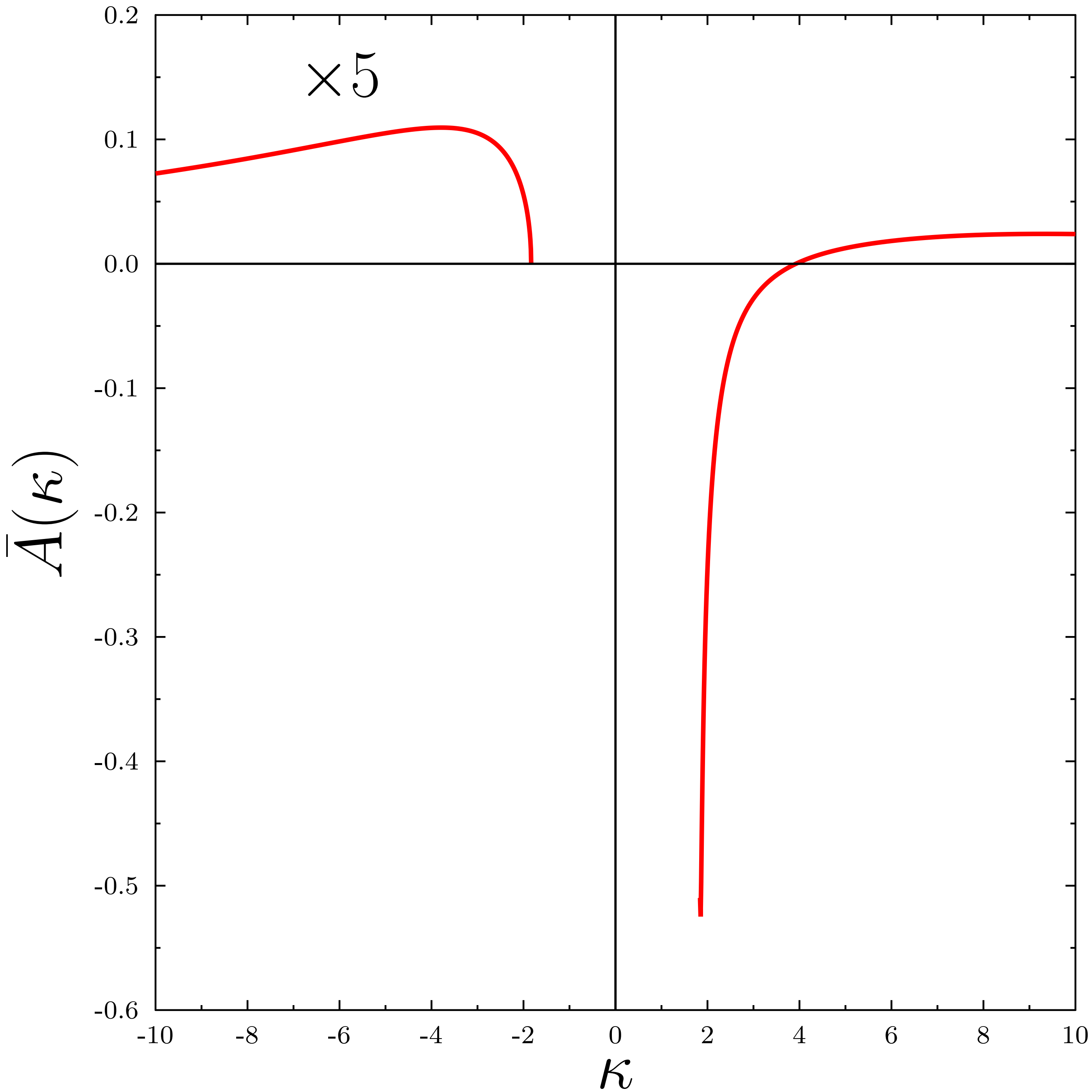


Spectral function is negative

Spin-1 meson only

Change in notation:

$$\overline{\rho}(\kappa) \to \overline{A}(\kappa)$$



Spectral function is negative

Positivity violation!

In addition to the pole and branch cut
on the real axis

In addition to the pole and branch cut on the real axis

— again pair of complex-conjugated poles

In addition to the pole and branch cut on the real axis

— again pair of complex-conjugated poles

$$z/M = 5.7 \pm 11.8\,i$$

In addition to the pole and branch cut on the real axis

— again pair of complex-conjugated poles

$$z/M = 5.7 \pm 11.8\, i$$

$$\mathrm{Res}(z) = -1.04 \pm 0.22\, i$$

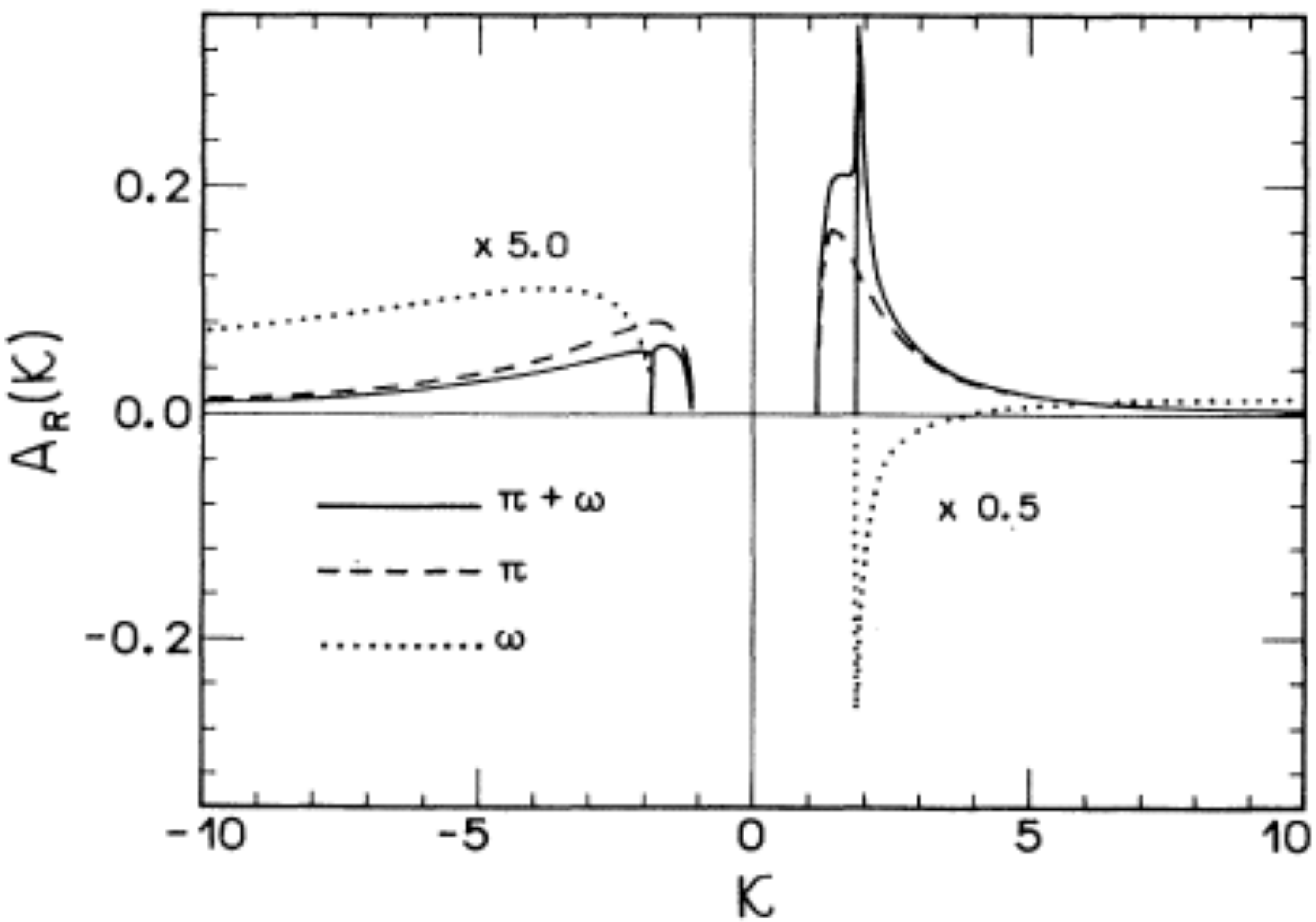
Spin-1 meson only

In addition to the pole and branch cut on the real axis

— again pair of complex-conjugated poles

$$z/M = 5.7 \pm 11.8\, i$$

$$\mathrm{Res}(z) = -1.04 \pm 0.22\, i$$

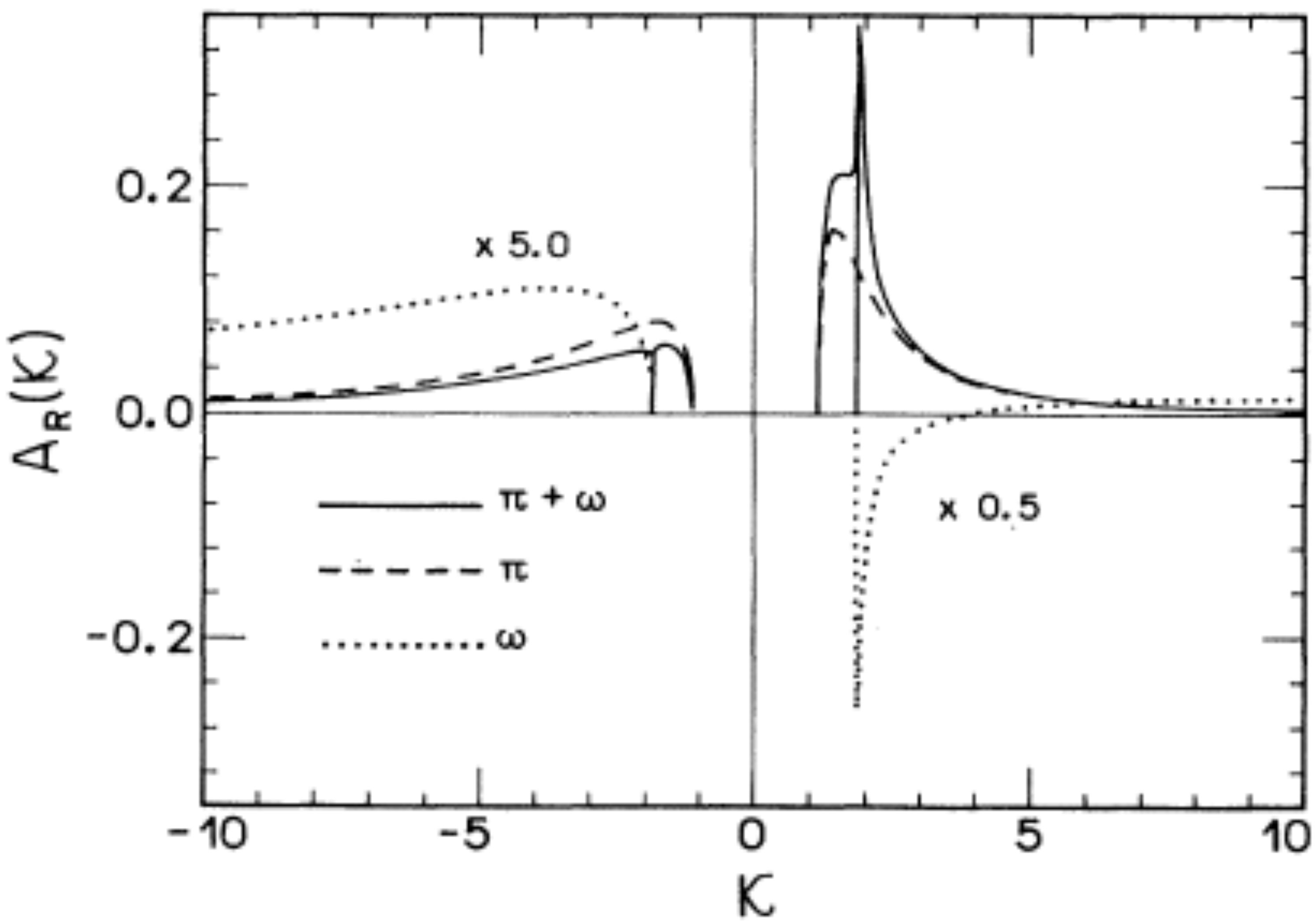


Complex-mass poles

$$z/M = 1.05 \pm 1.26\, i$$

$$\mathrm{Res}(z) = -0.77 \mp 0.20\, i$$

Including both mesons



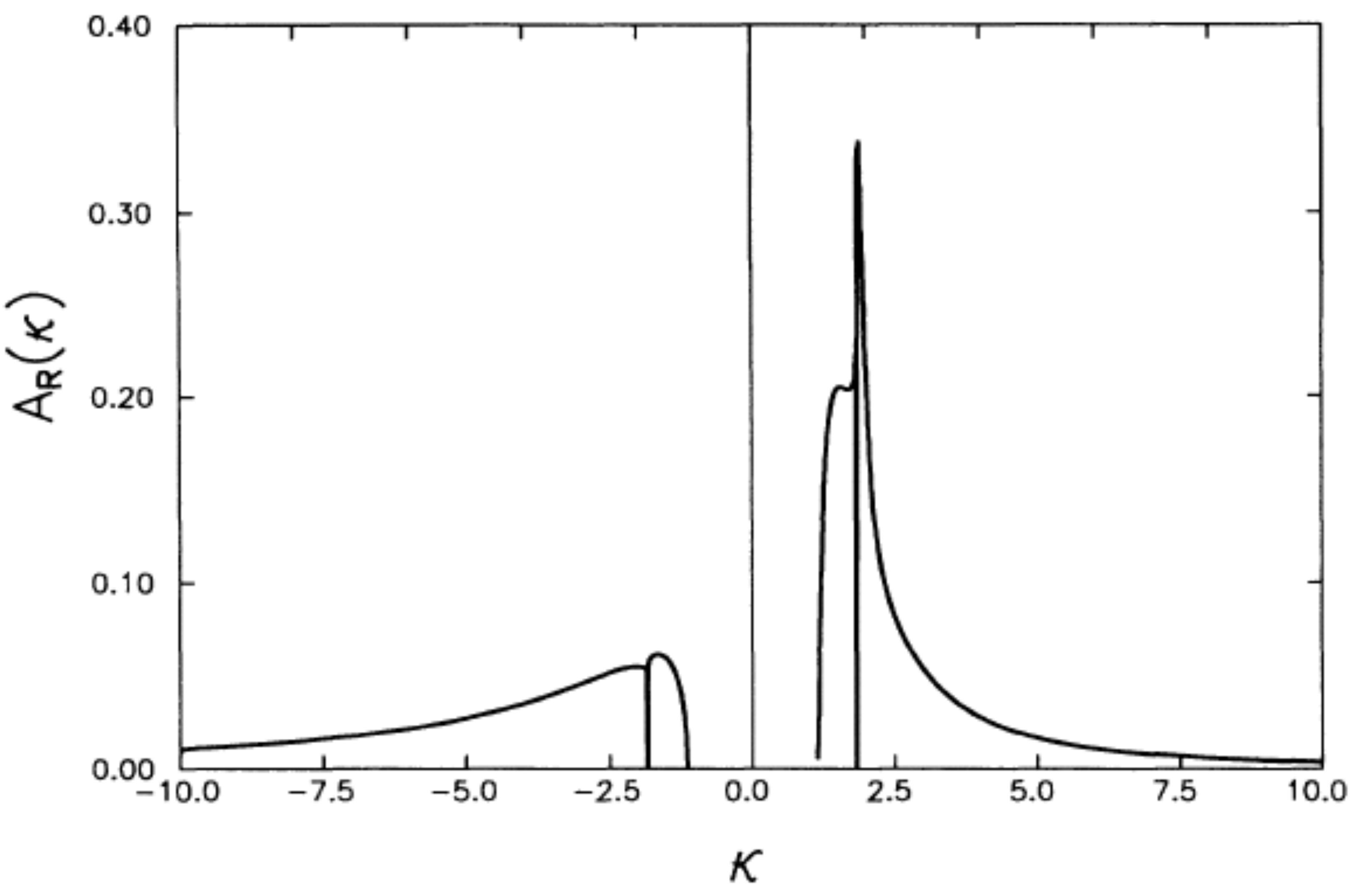
Complex-mass poles

$$z/M = 1.05 \pm 1.26\,i$$

$$\mathrm{Res}(z) = -0.77 \mp 0.20\,i$$

2. Coupled DSE baryon + meson — use bare vertices

NO positivity violation



	Self-consistent		Not self-consistent	
B	$1.06 \pm 1.25i$	$-0.77 \pm 0.20i$	$1.05 \pm 1.26i$	$-0.77 \pm 0.20i$
M0	-1.04	-1.08	-1.44	-1.13
M1	-3.50	-1.30	-5.68	-1.49

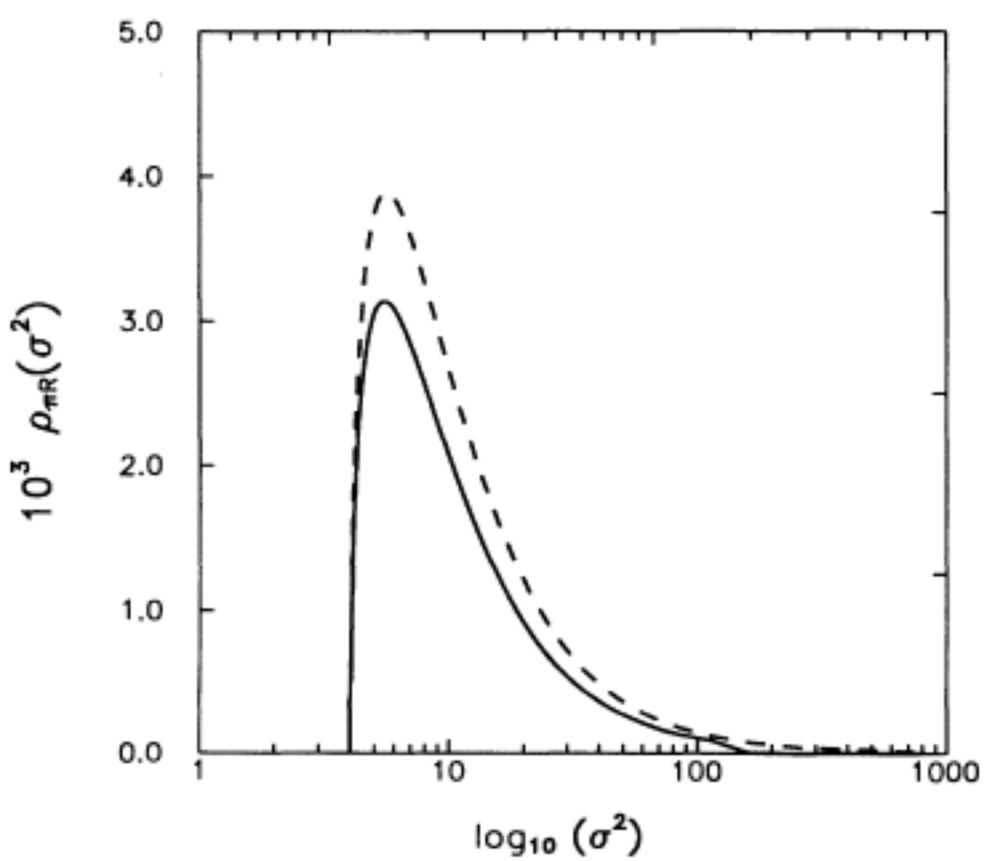


FIG. 3. Self-consistent (solid curve) and not self-consistent (dashed curve) π spectral function $\rho_{\pi R}(\sigma^2)$. σ^2 is in units of M^2 and $\rho_{\pi R}(\sigma^2)$ is in units of M^{-2}.

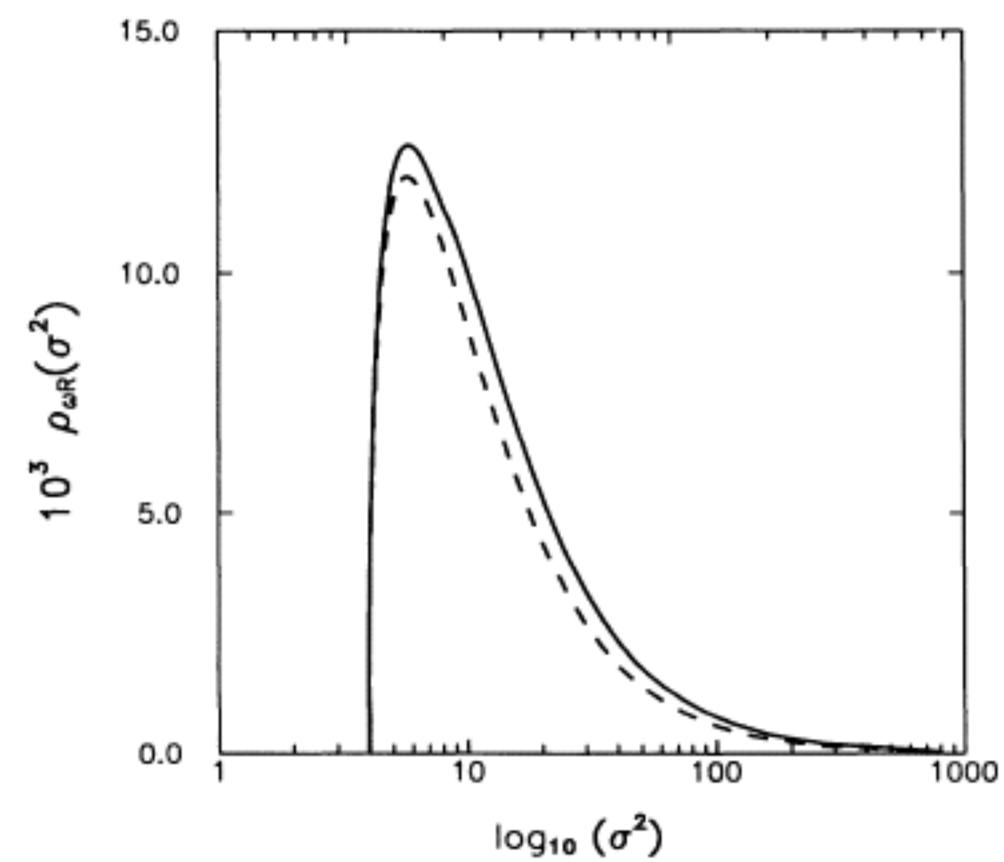
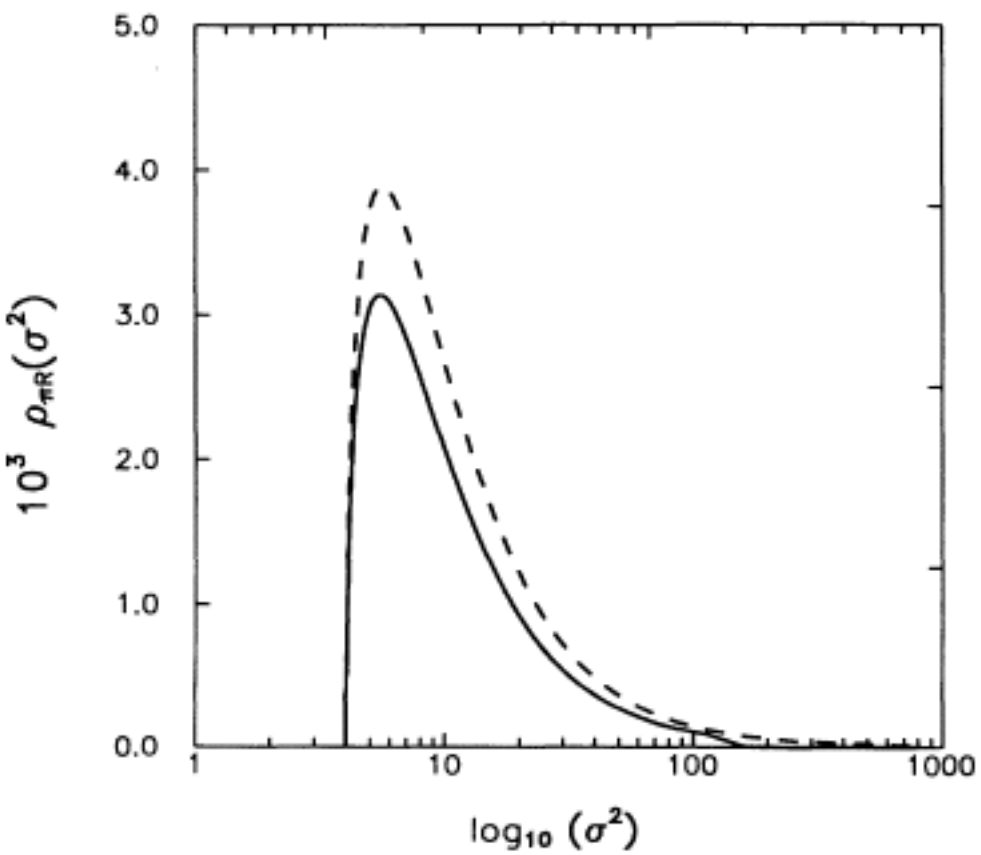


FIG. 4. Self-consistent (solid curve) and not self-consistent (dashed curve) ω spectral function $\rho_{\omega R}(\sigma^2)$. The units are the same as in Fig. 3.

Complex-mass poles in all propagators

	Self-consistent		Not self-consistent	
B	$1.06 \pm 1.25i$	$-0.77 \pm 0.20i$	$1.05 \pm 1.26i$	$-0.77 \pm 0.20i$
M0	-1.04	-1.08	-1.44	-1.13
M1	-3.50	-1.30	-5.68	-1.49



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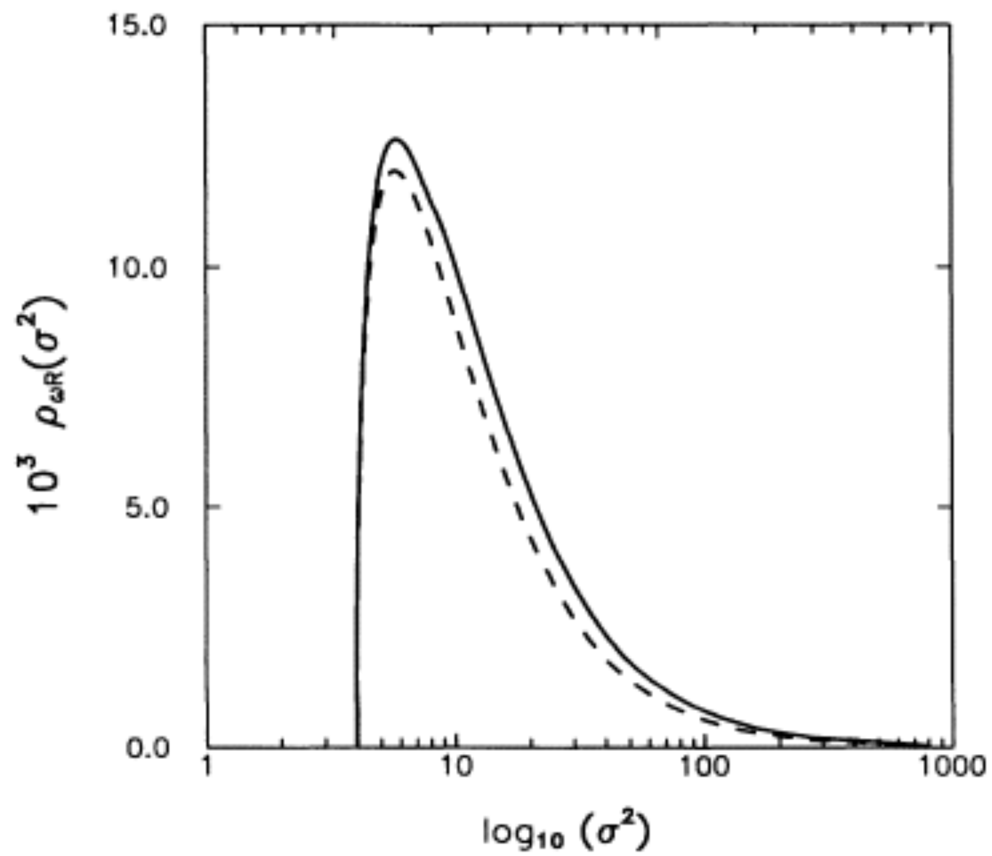


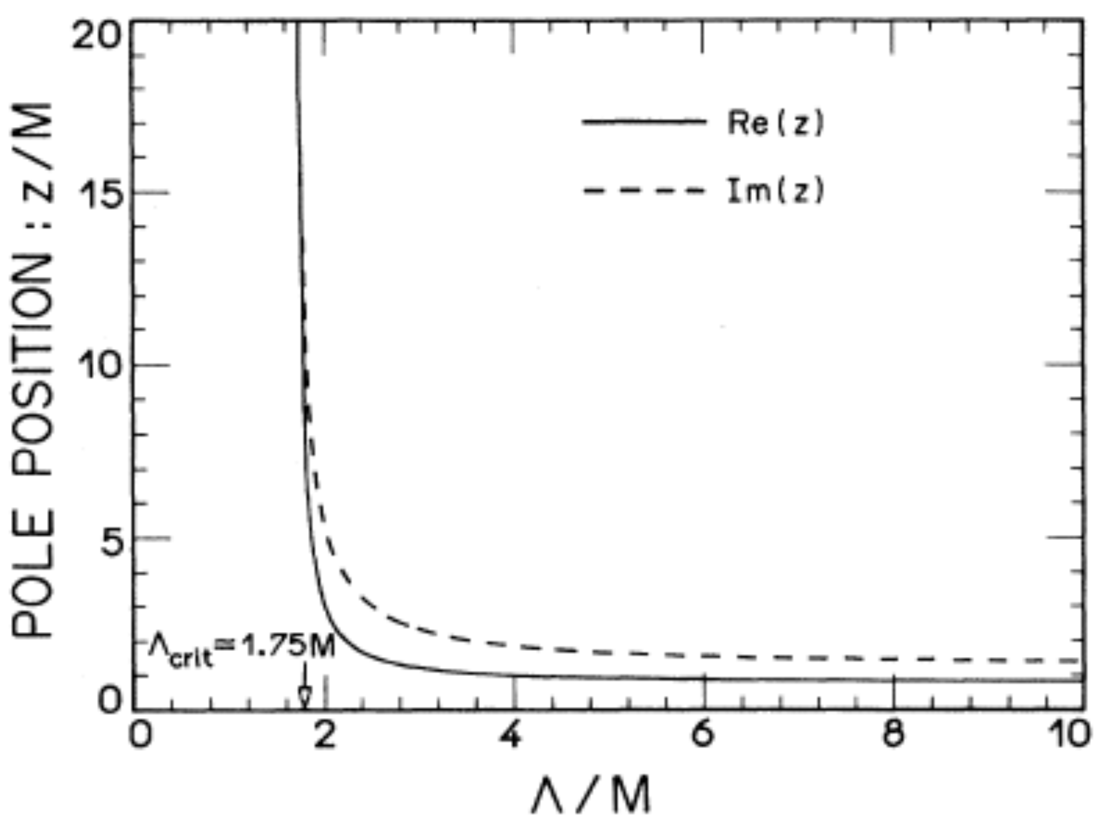
FIG. 4. Self-consistent (solid curve) and not self-consistent (dashed curve) ω spectral function $\rho_{\omega R}(\sigma^2)$. The units are the same as in Fig. 3.

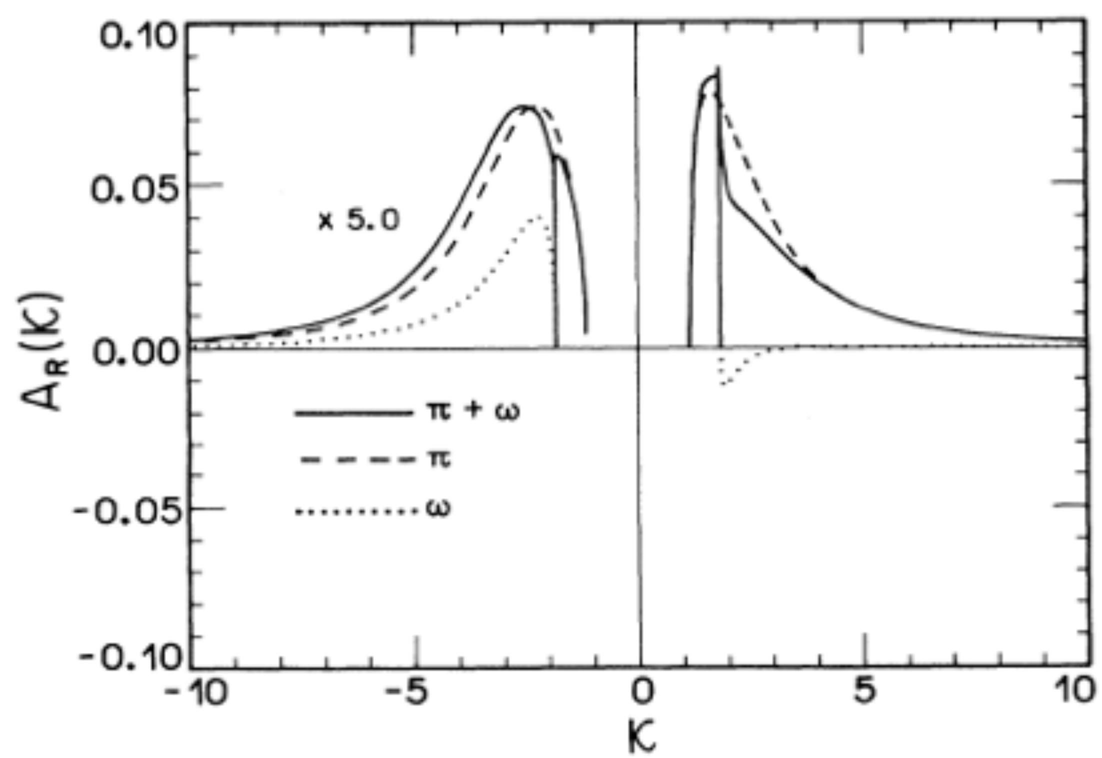
Can one kill the complex-mass poles?

YES - use form factors that soften the ultraviolet

$$\sigma(\kappa) \xrightarrow[\kappa\to\infty]{} |\kappa| \qquad \rho(\kappa) \xrightarrow[\kappa\to\infty]{} \frac{1}{|\kappa|\,\ln^2|\kappa|}$$

$$F(p_1,p_2,q) = \frac{1}{1+|p_1^2/\Lambda^2|}\,\frac{1}{1+|q^2/\Lambda^2|}\,\frac{1}{1+|p_2^2/\Lambda^2|}$$





G.K., M. Nielsen, R.D. Puff, L. Wilets, PRC 47, 2485 (1993)

Conclusions

— Can get positivity violation with a model whose relation to QCD is very remote, to say the least

— Can get positivity violation and complex-mass poles in a one-loop calculation (can fit lattice data)

— Can get positivity violation and complex-mass poles in meson-baryon models

Suppose one finds positivity violation and/or complex-mass poles in a QCD model/truncation

— how can one tell whether they are real features of QCD or are due to approximation/truncation used?

Need detailed comparisons with lattice (when possible), gauge symmetry constraints, if physical are there observables related to complex poles (fragmentation)?

Funding

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