Spectral information from T>0 lattice QCD and in-medium quarkonium

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University of Stavanger

References:

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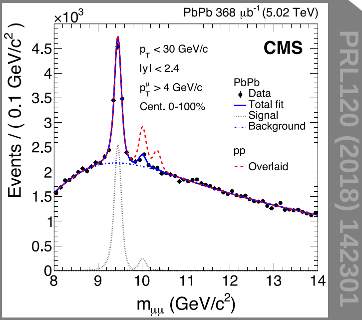
P. Petreczky, A.R., J. Weber, NPA982 (2019) 735
A.R. arXiv:1903.02293

Exploring strong interactions at T>0



Heavy quarkonium

- Heavy quarkonium: simplest QCD bound state (theory & experiment)

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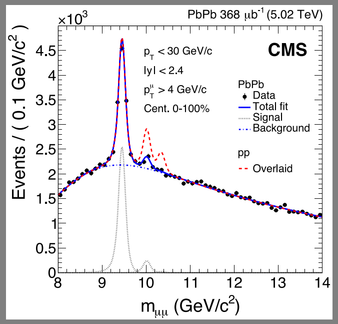
Equilibrium QCD



PRL120 (2018) 142301

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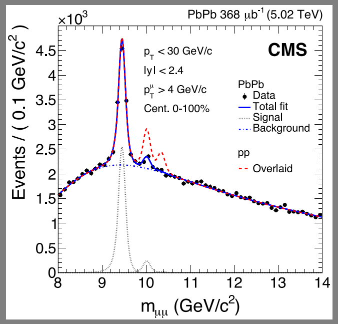
Static potential



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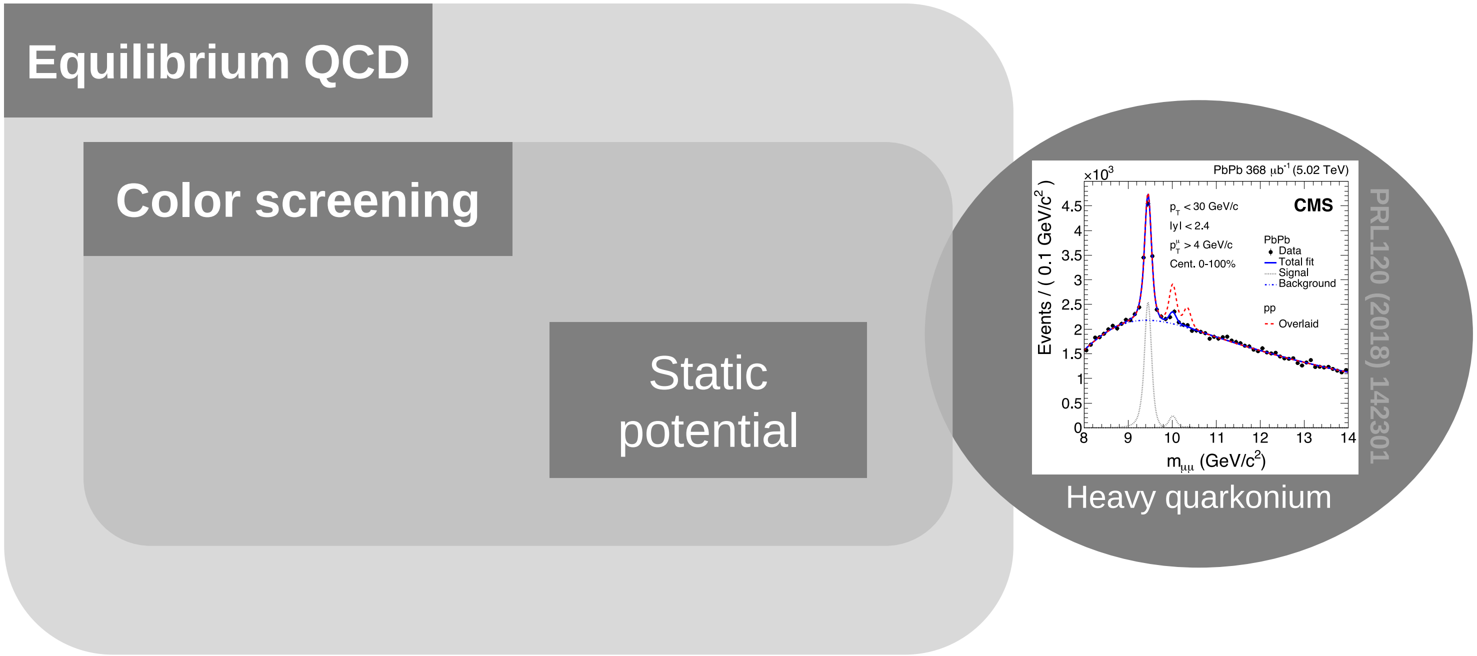

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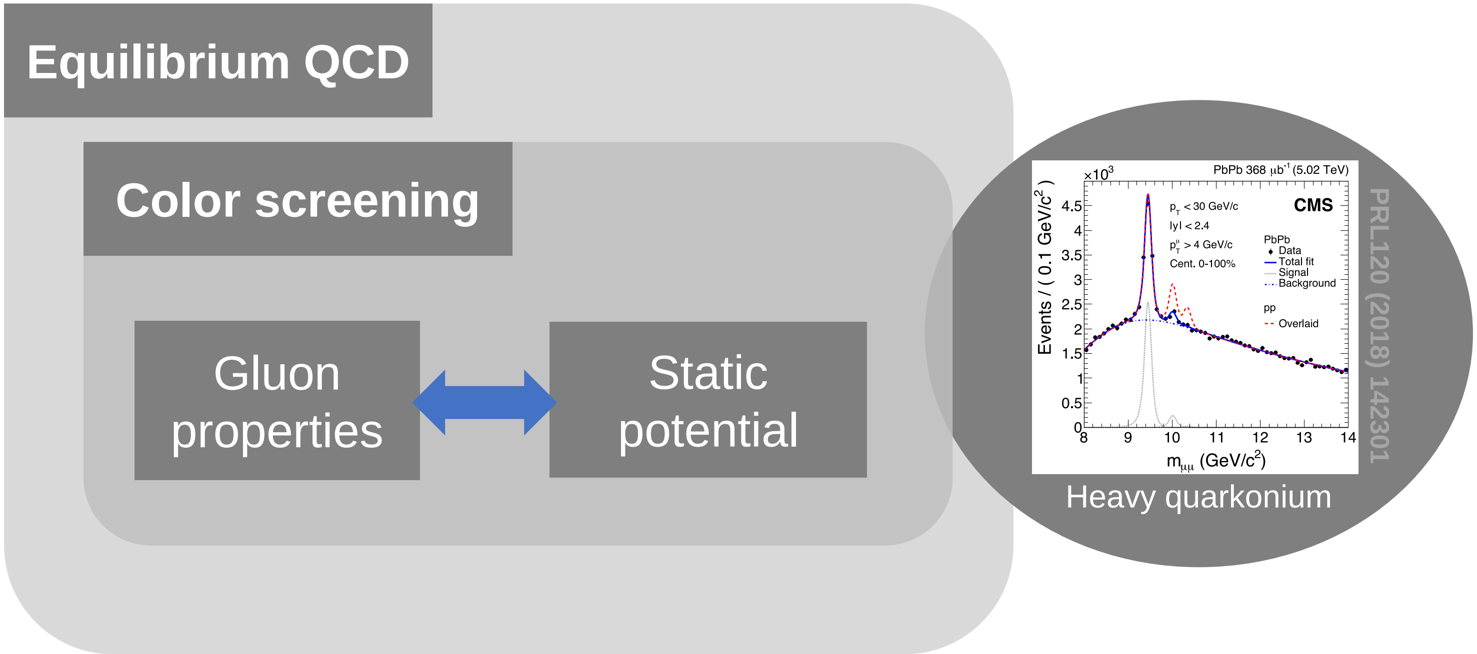

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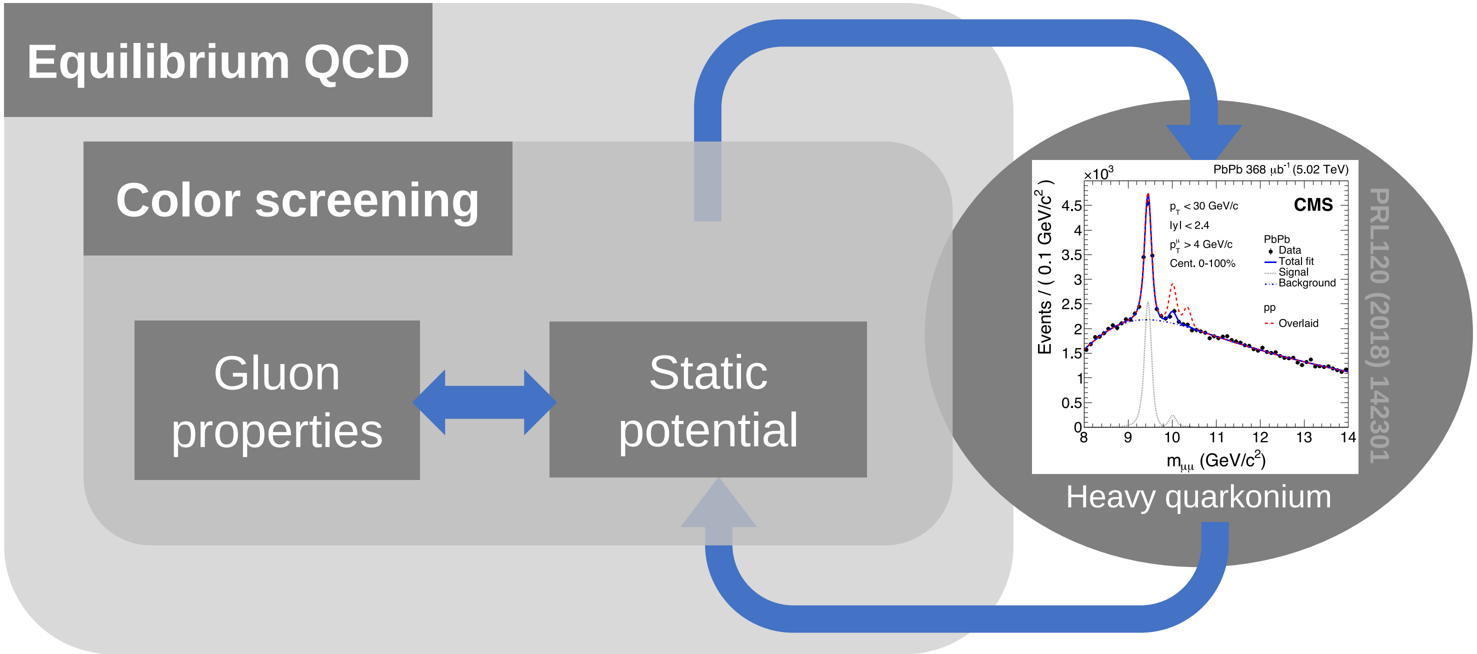

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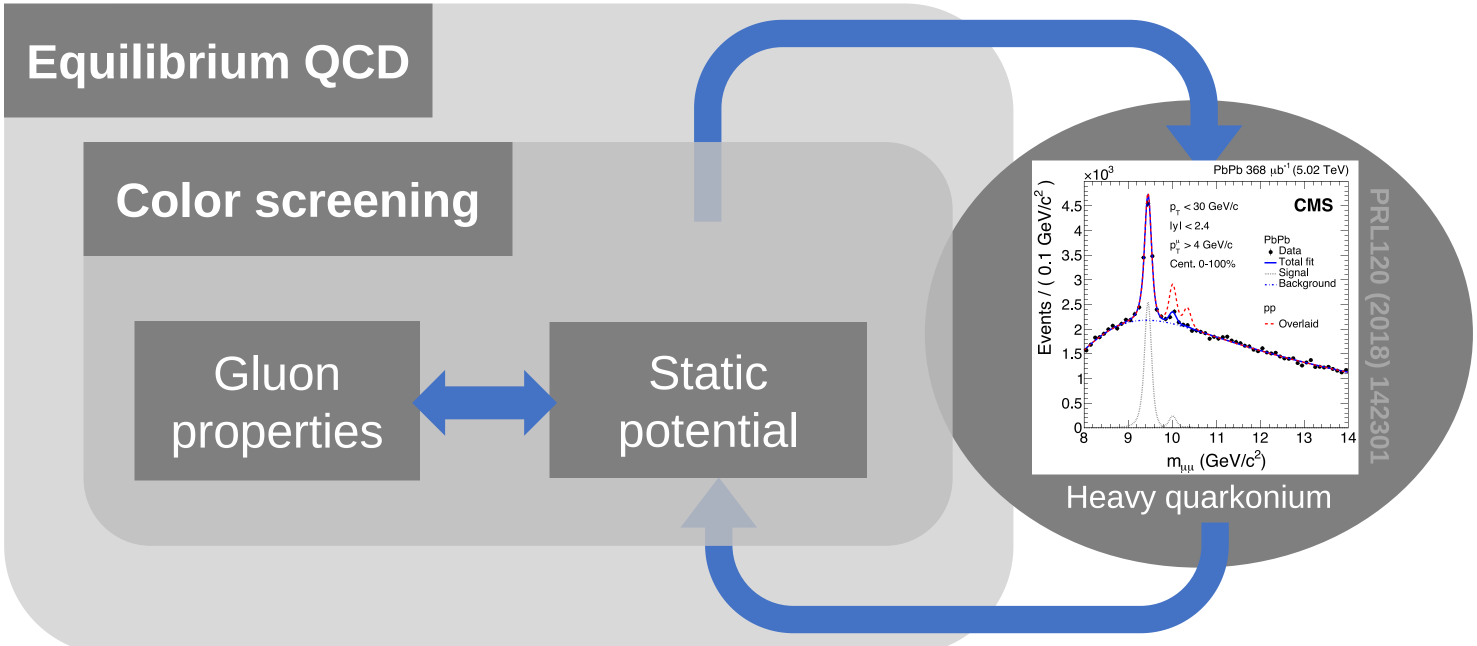

- Heavy quarkonium: simplest QCD bound state (theory & experiment)
- Spectral functions plays two crucial roles:
  - Provides direct insight into single-particle (gluon) or bound state ($Q\bar{Q}$) properties at T>0
  - Provides a bridge between the Minkowski (physics) and Euclidean (simulation) domain

- Spectral information from the lattice & inverse problems
- T>0 quarkonium in-medium spectral functions on the lattice
- The complex static potential from lattice QCD
- Gluon spectral functions from T>0 lattice QCD
- Conclusion

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A robust tool: lattice QCD

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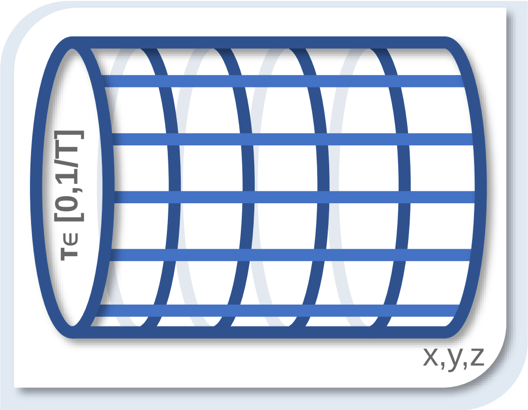
- Non-perturbative 1st principles approach to Quantum Chromo Dynamics



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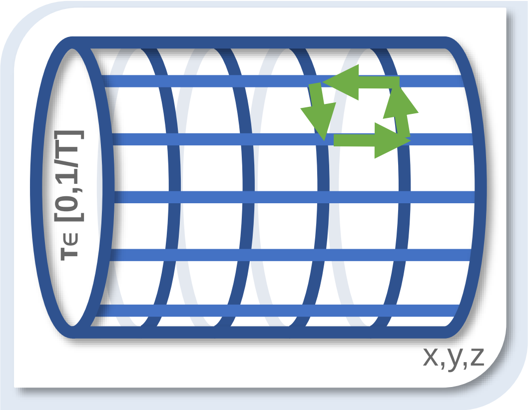

- Gauge fields as links: $U_\mu(x)=\exp[\, i\, g\, \Delta x_\mu\, A_\mu(x)\,]$

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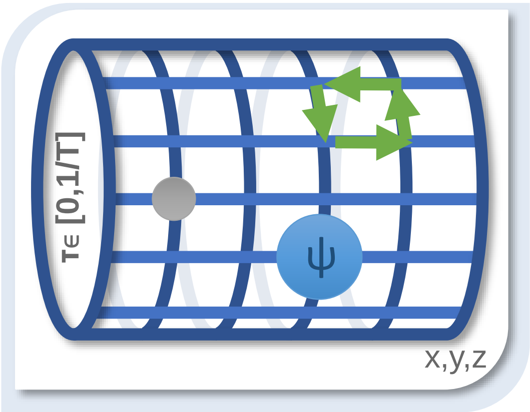

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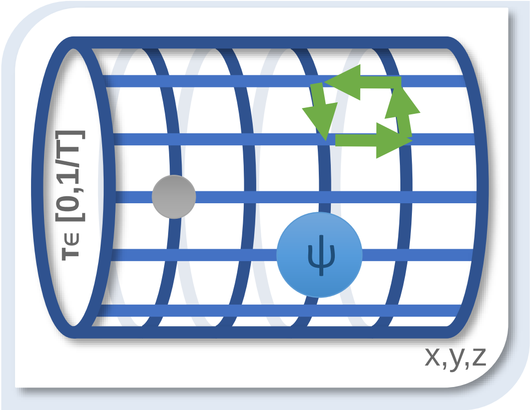

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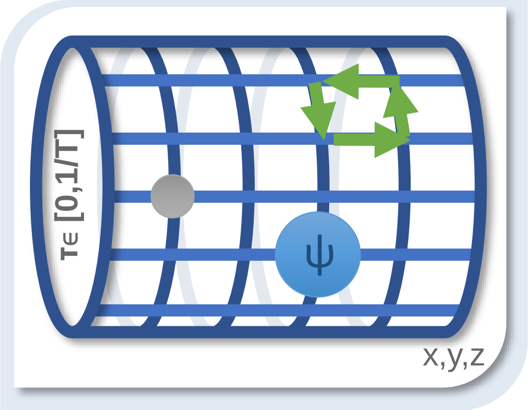

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$$\langle O(U)\rangle = \int \mathcal{D}U\, O(U)\, e^{-S_E^{QCD}[U]}$$

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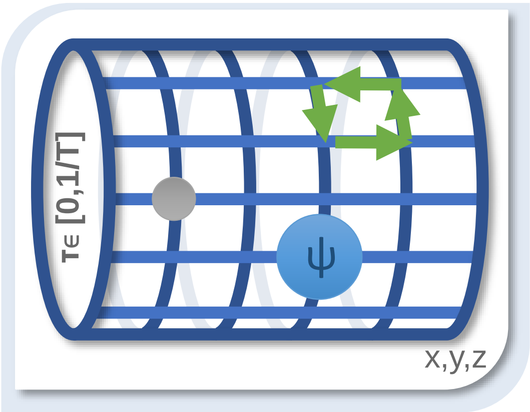

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$$\langle O \rangle = \frac{1}{N} \lim_{N\to\infty} \sum_{k=1}^{N} O(U^k) \qquad P[U] \propto e^{-S_E[U,\psi,\bar\psi]}$$

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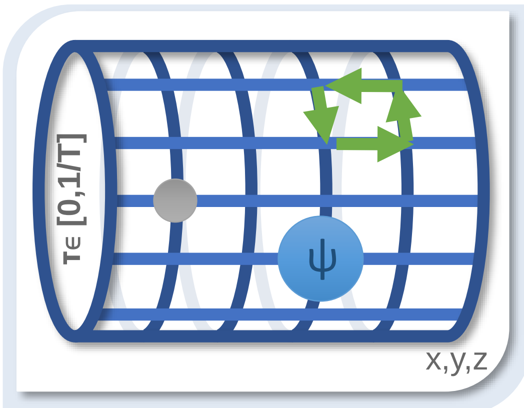

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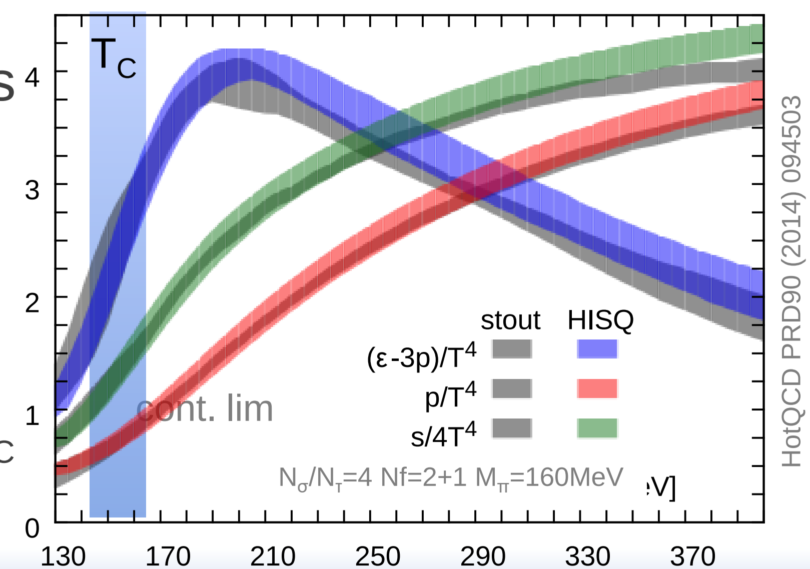
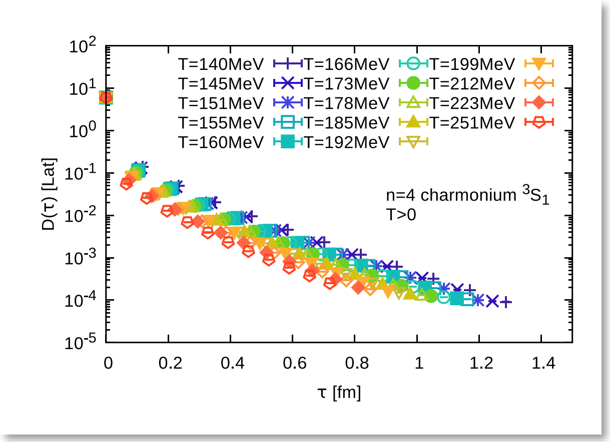
- Successful at T>0: QCD static properties
  - (Pseudo)critical temperature: 154±9 MeV
    WB JHEP 1009 (2010) 073 - HotQCD PRD85 (2012) 054503
  - Equation of state as input for hydro-dynamics
  - Trace anomaly $T^{\mu\mu}$ = ε-3p strong coupling at T_C
    HotQCD PRD90 (2014) 094503 - WB PLB730 (2014) 99-104



The Minkowski-time challenge



$$D(\tau) = \int d\omega K(\omega,\tau)\rho(\omega)$$

LQCD correlation function
in Euclidean time: **D**

The Minkowski-time challenge

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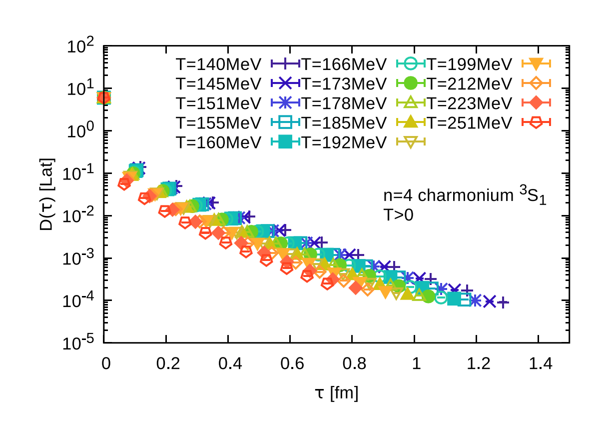

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$$D(\tau) = \int d\omega K(\omega, \tau)\rho(\omega)$$

$$D(t) = \int d\omega K(\omega, \tau \to it)\rho(\omega)$$

The Minkowski-time challenge

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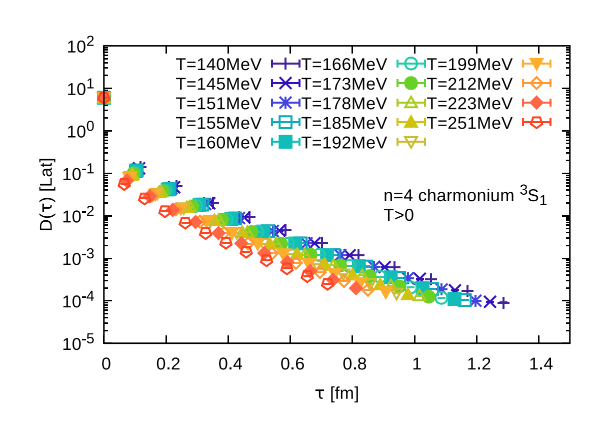

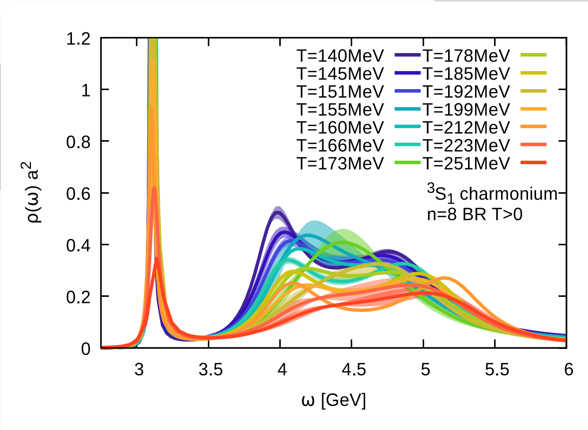

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LQCD correlation function
in Euclidean time: **D**

An ill-posed inverse problem

real-time quantity
in momenta: **ρ**

The Minkowski-time challenge

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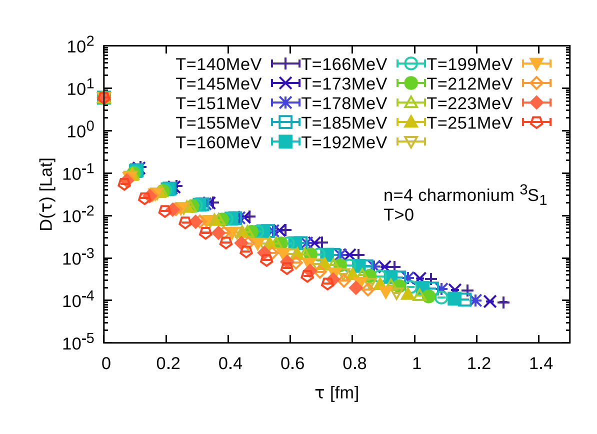

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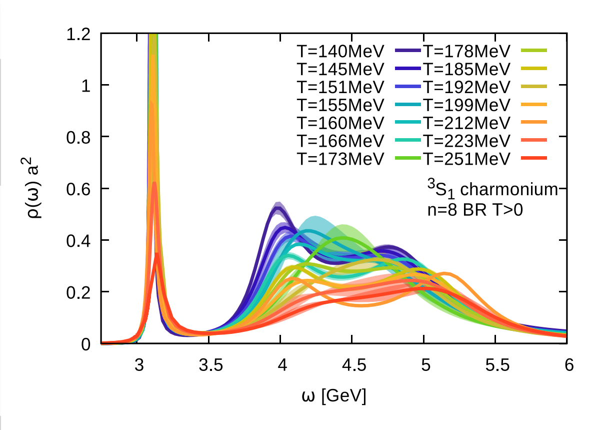

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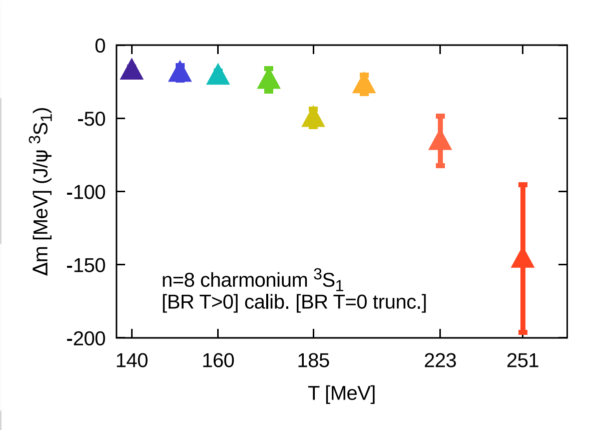

An ill-posed inverse problem

real-time quantity
in momenta: **ρ**

extracted piece of
dynamical information

The two main challenges

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- Invert the spectral representation of the correlator **D**: $\mathbf{D} = \hat{K}\mathbf{r}$

$$K^{T=0}_{\omega\tau} = e^{-\omega\tau} \qquad K^{T>0}_{\omega\tau} = \frac{cosh(\omega(\tau-\beta/2))}{sinh(\omega\beta/2)} \qquad K_{\omega q_4} = \frac{1}{\pi}\frac{\omega}{q_4^2+\omega^2}$$

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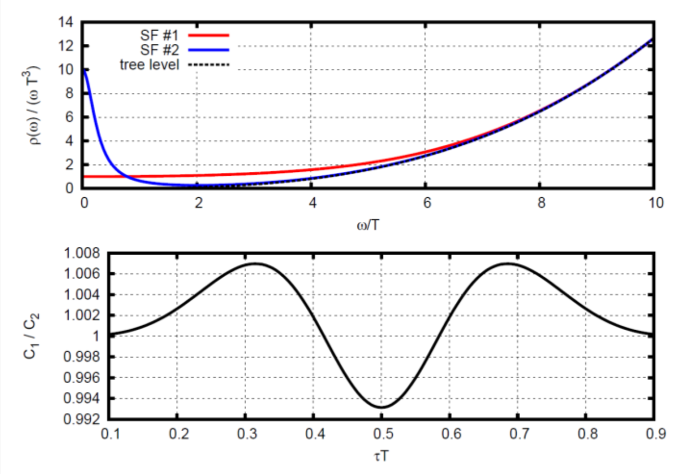

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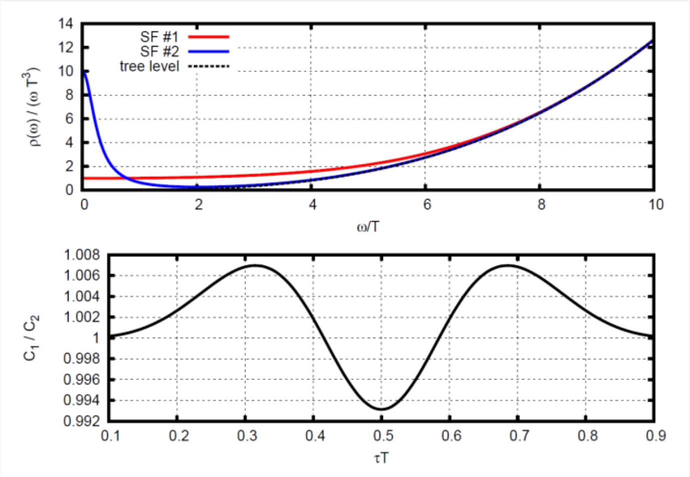

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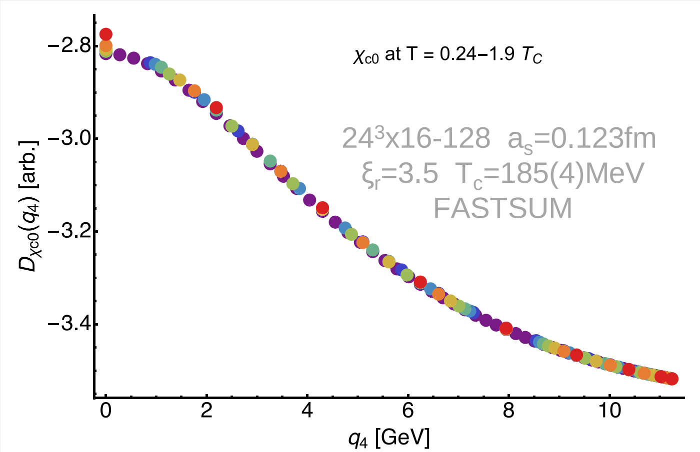

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- Cont. limit: resolve only large w_n behavior

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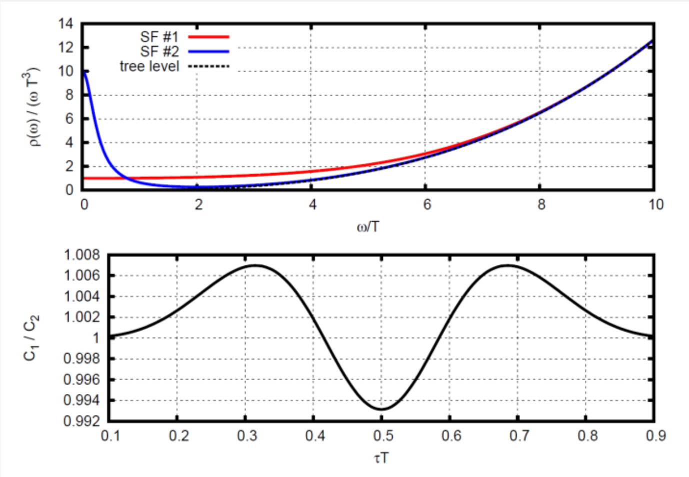

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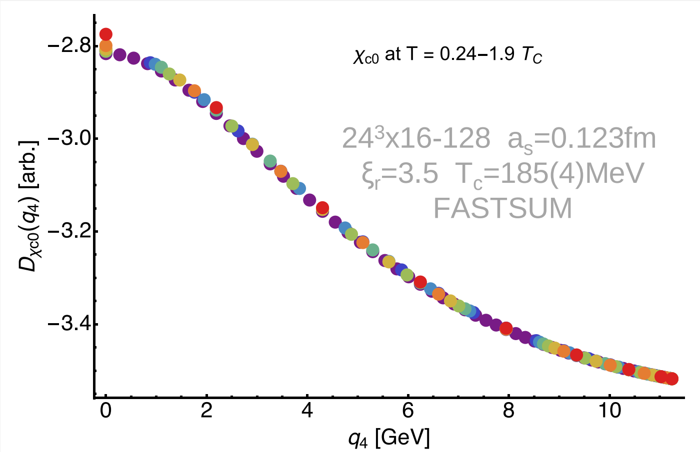

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- How to extract most accurately the information inside lattice data?

The Bayesian strategy

- Extraction of real-time quantities: inversion of ill-conditioned linear transformation

$$D(\tau) = \int_{-2m_Q}^{\infty} d\omega e^{-\omega\tau} \rho(\omega)$$

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2. data D_i has finite precision

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- Regularize this task using prior information – Bayes introduces prior **P[ρ|I]=exp[-S]**

M. Jarrell, J. Gubernatis, Physics Reports 269 (3) (1996)

$$P[\rho|D,I] \propto P[D|\rho,I]\,P[\rho|I]$$

posterior **likelihood** **prior**

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$$\partial_\rho P[\rho|\alpha,m]\big|_{\rho=m} = 0 \qquad \lim_{\alpha\to 0} P[\rho|\alpha,m] = 1$$

α quantifies confidence in the values of m

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- **Bayesian continuum limit** $N_\tau \to \infty$ & $\Delta D \to 0$: all version converge to same result

Choice of S influences how efficiently one converges to this limit

Bayes for positive spectra

- Differences in **prior information** that is incorporated & how to **find extremum**.

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  - MEM: $$S_{SJ}[m,\alpha] = \alpha \int d\omega \Big(\rho - m - \rho log\Big[\frac{\rho}{m}\Big]\Big)$$

    Enforces positivity.
    Flat directions for ρ -> 0.

  Artificially restricts solution space to N_t dimensions around default model. Works well for accurate m.

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- BR: $$S_{BR} = \alpha \int d\omega \Big(1 - \frac{\rho}{m} + log\Big[\frac{\rho}{m}\Big]\Big)$$

Enforces positivity. Derived from a smoothness/scale inv. axiom
Y.Burnier, A.R. PRL 111 (2013) 18, 182003

Designed to minimize influence of default model on final answer, weakest curvature of S among priors

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- Several strategies to handle the **parameter α** (weighting data vs prior information)

  - Historic MEM & Morozov: tune α to get χ^2/d.o.f. = 1 preventing overfitting of errors
  - BR method: integrate out α apriori assuming P[α]=1

Ringing from local regulators

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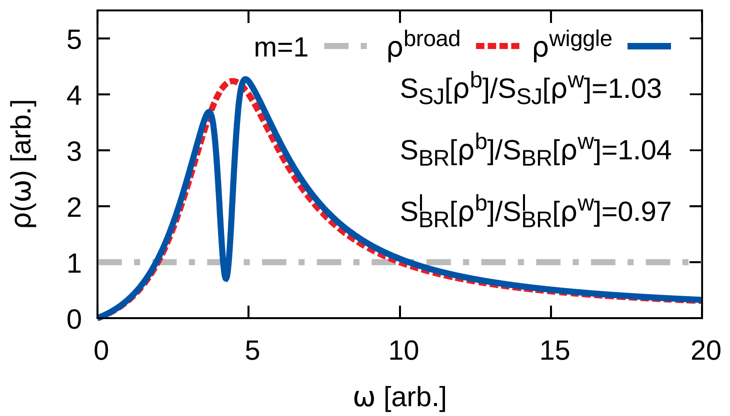
- MEM and BR incorporate positivity but no further info on e.g. analytic structure

  Both methods are susceptible to favor unphysical oscillatory solutions



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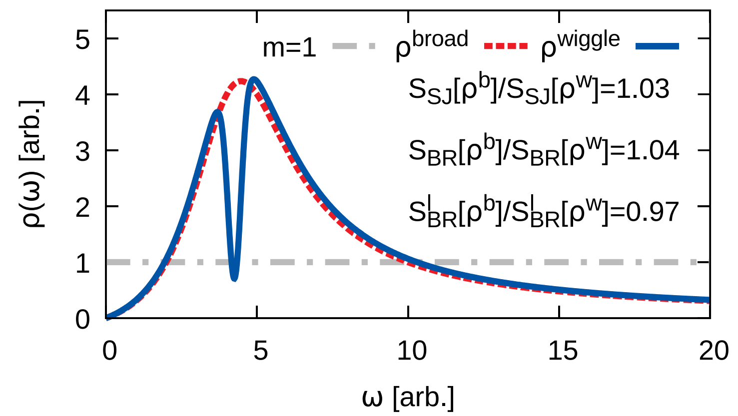

- Wiggliness of solution related to its arc length: introduce naïve penalty

C.Fischer, J. Pawlowski, A.R., C. Welzbacher PRD98 (2018) 014009

$$S_{BR}^{smooth} = \alpha \int d\omega \Big(\kappa \left(\frac{\partial \rho}{\partial \omega} \right)^2 + 1 - \frac{\rho}{m} + log\left[\frac{\rho}{m} \right] \Big)$$

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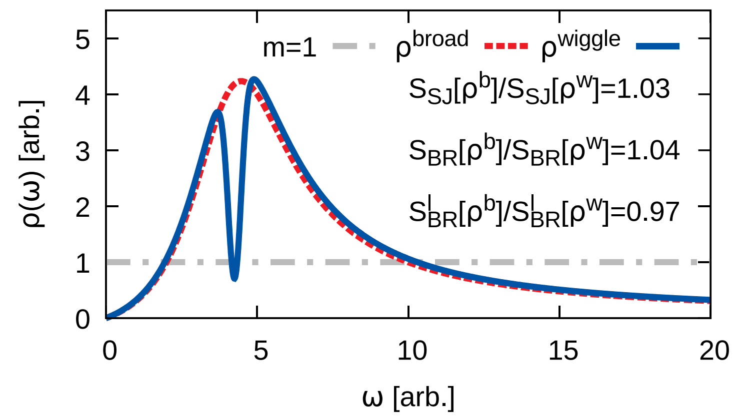

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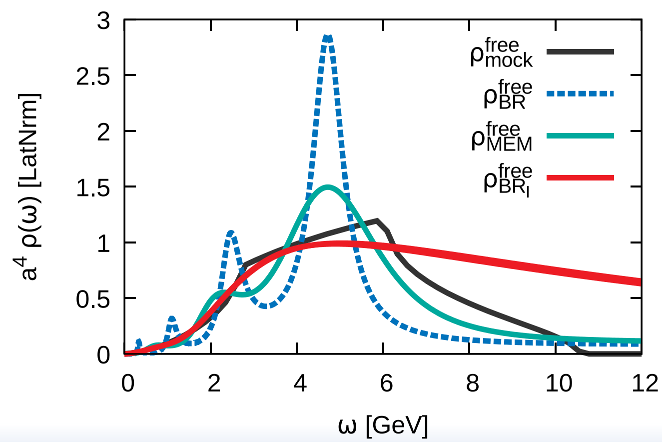
C.Fischer, J. Pawlowski, A.R., C. Welzbacher PRD98 (2018) 014009

$$S_{BR}^{smooth} = \alpha \int d\omega \left(\kappa \left(\frac{\partial \rho}{\partial \omega} \right)^2 + 1 - \frac{\rho}{m} + log\left[\frac{\rho}{m}\right] \right)$$

- Value of smoothing hyperparameter κ self consistently set via appropriately discretized mock data



Bayes for non-positive spectra

University of Stavanger

- Bayesian proposals to treat non-positive definite spectra in the literature:
  - Tikhonov: $$S_T[m,\alpha] = \alpha \int d\omega(\rho - m)^2$$ see David's talk Wed. 14:20h
    Dudal, Oliveira, Silva PRD89 (2014) 014010
  - decompose ρ into $\rho^+ > 0$ and $\rho^- < 0$ and use standard MEM — beyond prior information
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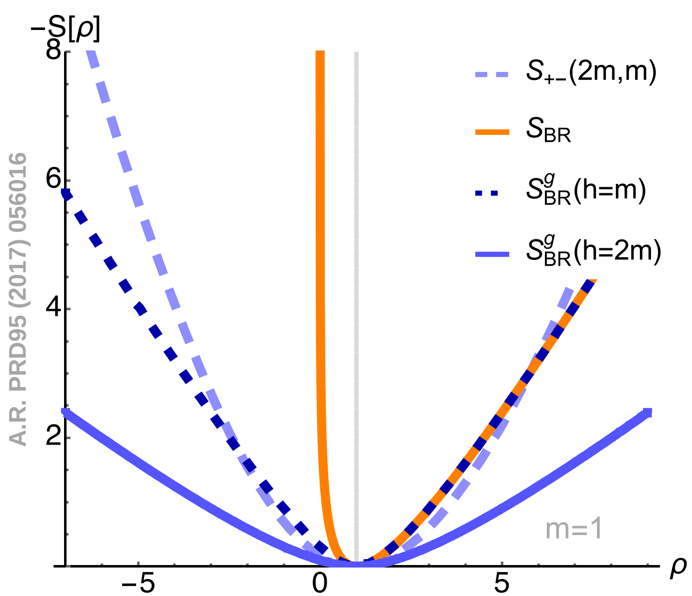

- Here instead generalized BR prior: A.R. PRD95 (2017) 056016

$$S^g_{BR} = \alpha \int d\omega \Big(- \frac{|\rho - m|}{h} + \log\Big[\frac{|\rho - m|}{h} + 1\Big]\Big)$$

- absolute deviation |ρ-m| vs. previously ρ/m
- new default model function h: confidence in m
- weakest amongst different priors: *let the data speak*

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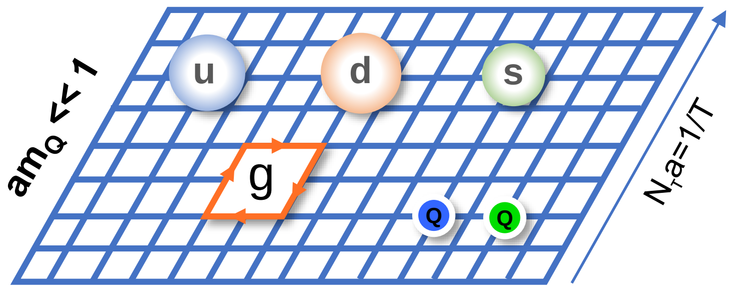
Heavy quarks on the lattice

University of Stavanger

- Exploit separation of scales to treat heavy quarks non-relativistically



Full Lattice QCD simulation incl. $Q\bar{Q}$
(still too costly)

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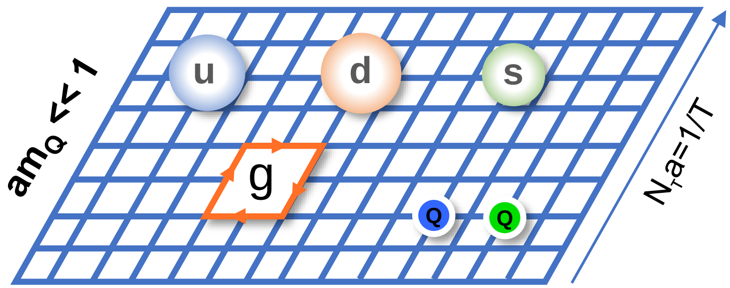

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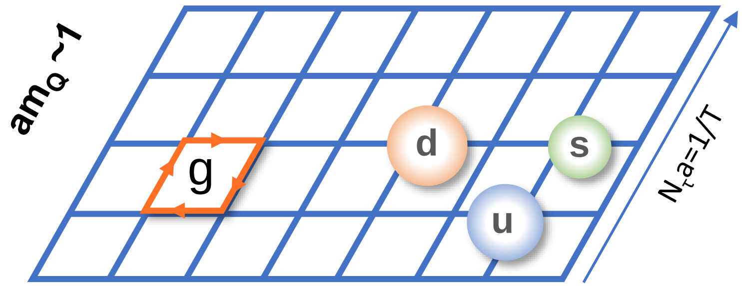
$$\frac{\Lambda_{QCD}}{m_Q} \ll 1$$

$$\frac{T}{m_Q} \ll 1$$



Lattice QCD simulation without $Q\bar{Q}$

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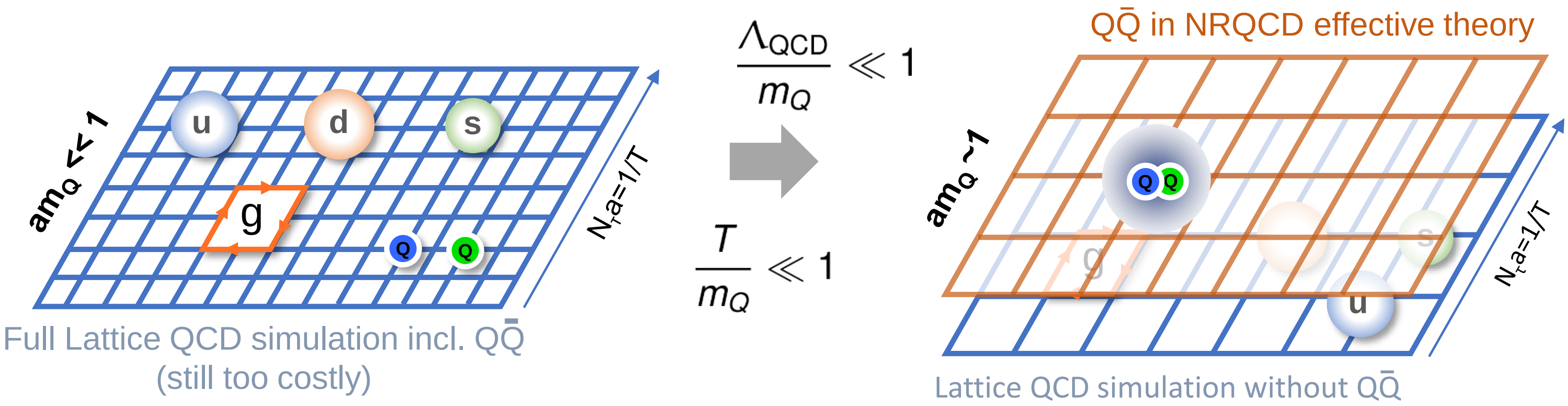

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  - no modeling, systematic expansion of QCD action in $1/m_Q a$, includes $v\neq 0$ contributions
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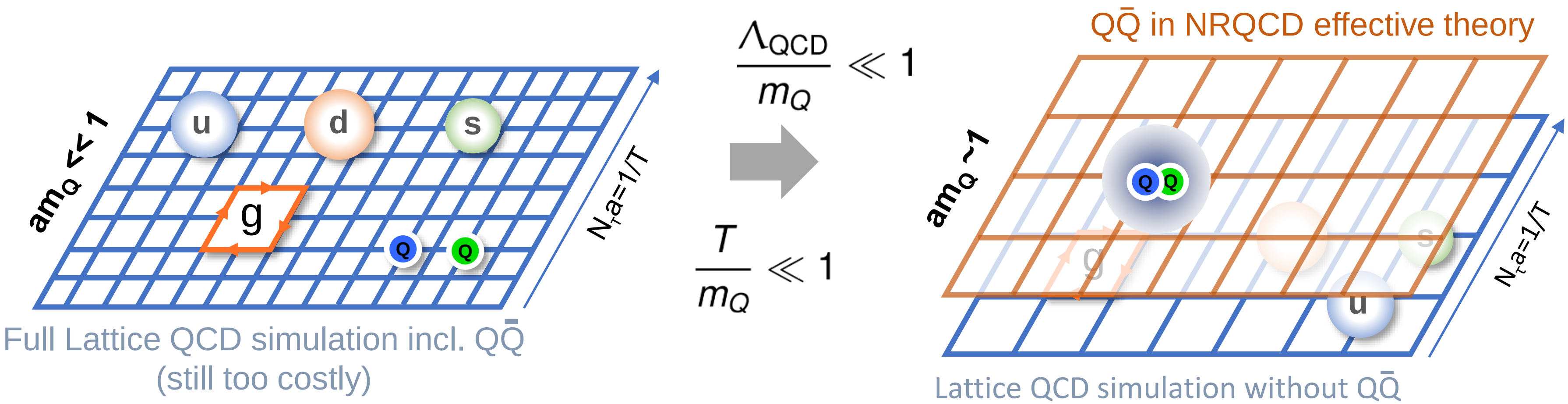

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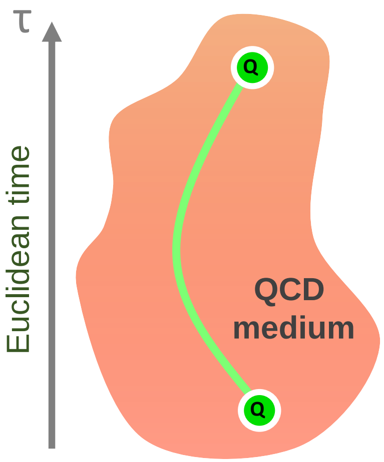
- **Realistic** & **high statistics** simulations of the QCD medium by HotQCD
  HotQCD PRD85 (2012) 054503, PRD90 (2014) 094503
  - **m_π=161MeV** T=0 N_τ=32-64 T>0 N_τ =12 — T= [**140 – 407**] MeV $m_b a$= [2.759 – 1.559] Lepage n_b=4

NRQCD Euclidean correlators



Non-rel. propagator of
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Davies, Thacker Phys.Rev. D45 (1992)

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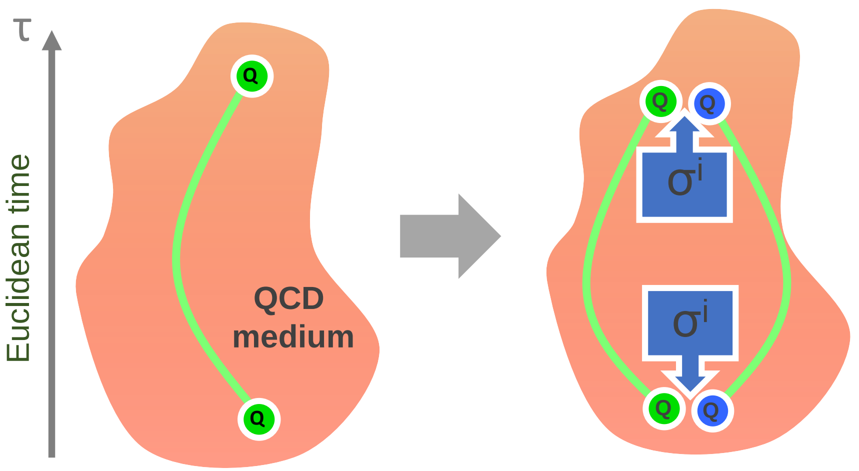

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QQ propagator
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$D_{J/\psi}(\tau) \triangleq \langle \psi_{J/\psi}(\tau)\psi^{\dagger}_{J/\psi}(0)\rangle$ “

Brambilla et. al. Rev.Mod.Phys. 77 (2005) 1423

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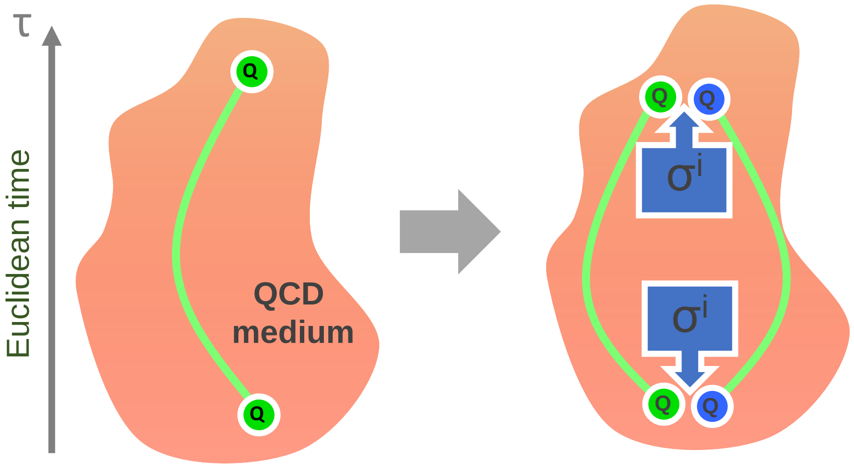

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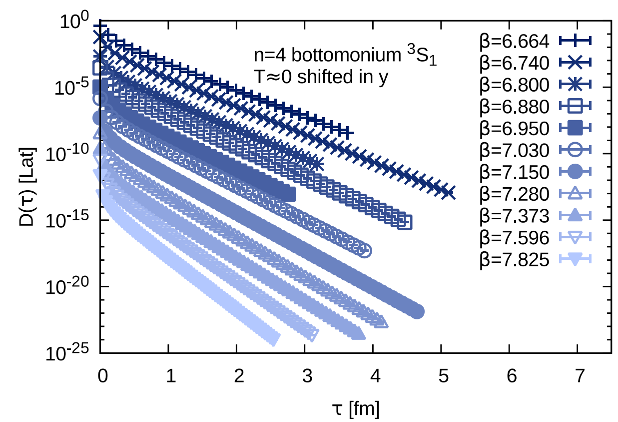
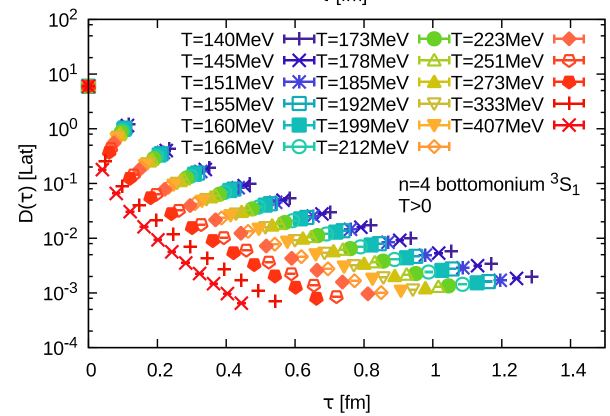
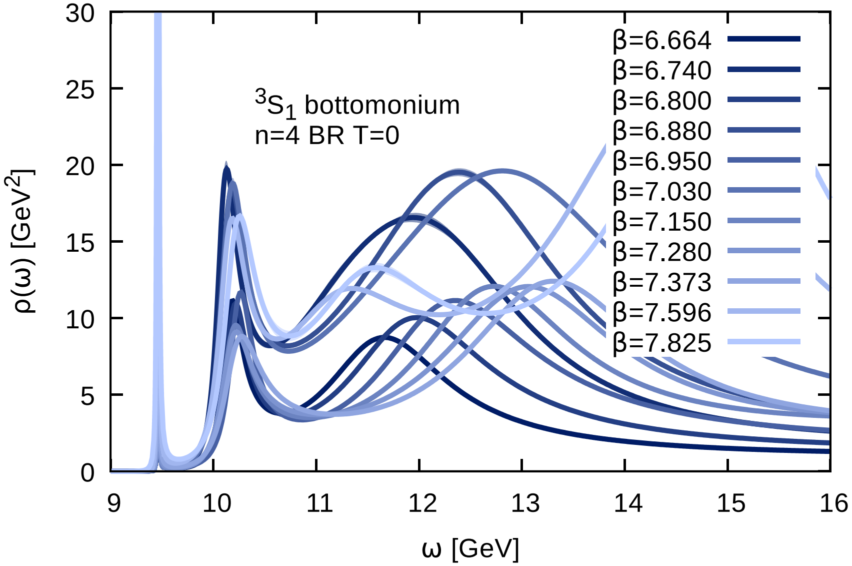
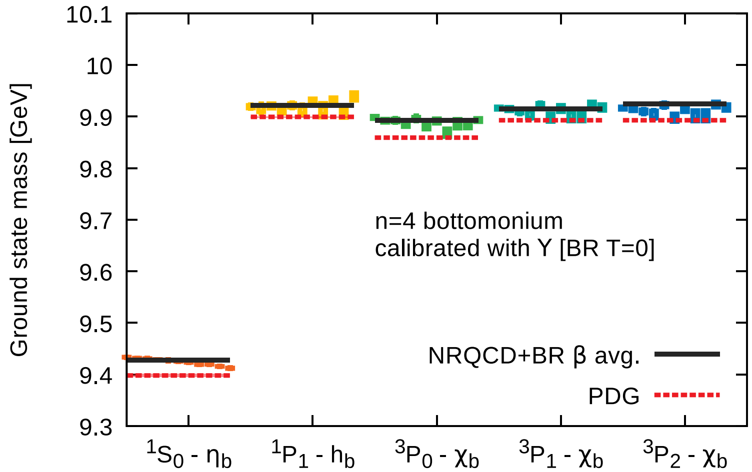




Euclidean correlation functions at T=0 and T>0

T=0 spectral reconstructions

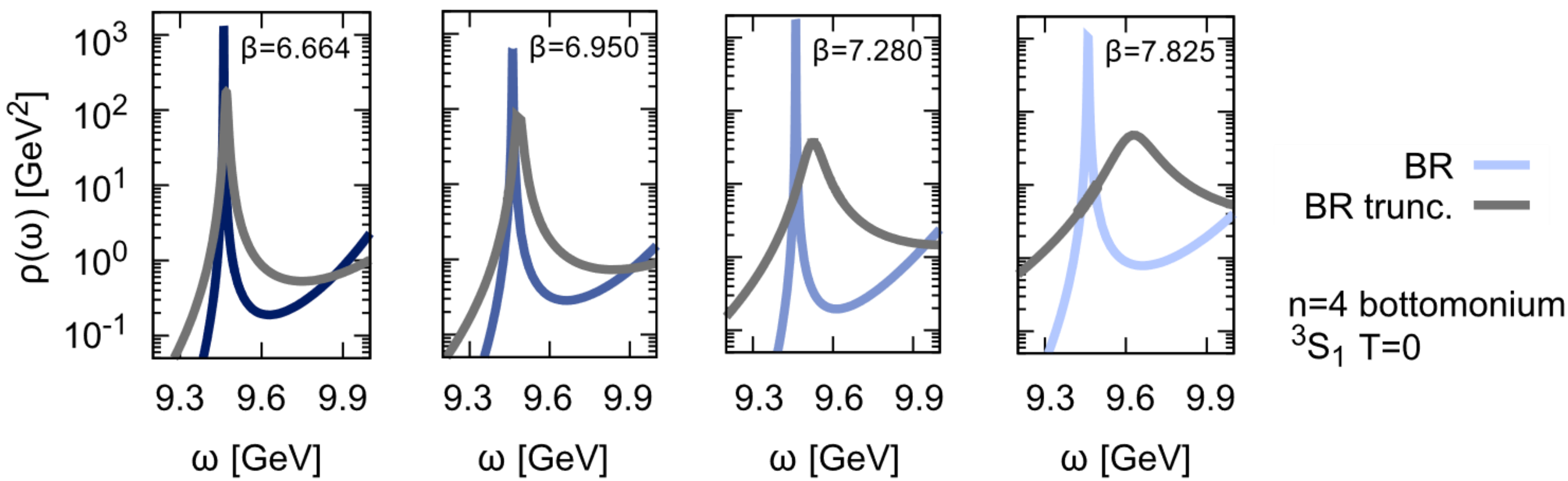




- Ground state width similar to uncertainty of effective mass analysis
- NRQCD mass splitting: maximum deviation from PDG is ~35MeV

Towards finite temperature spectra

University of Stavanger



- Standard BR method resolves T=0 ground state very well from N_T=48-64 points
- How does accuracy suffer from limited Euclidean extent at T>0 (N_T=12) ?

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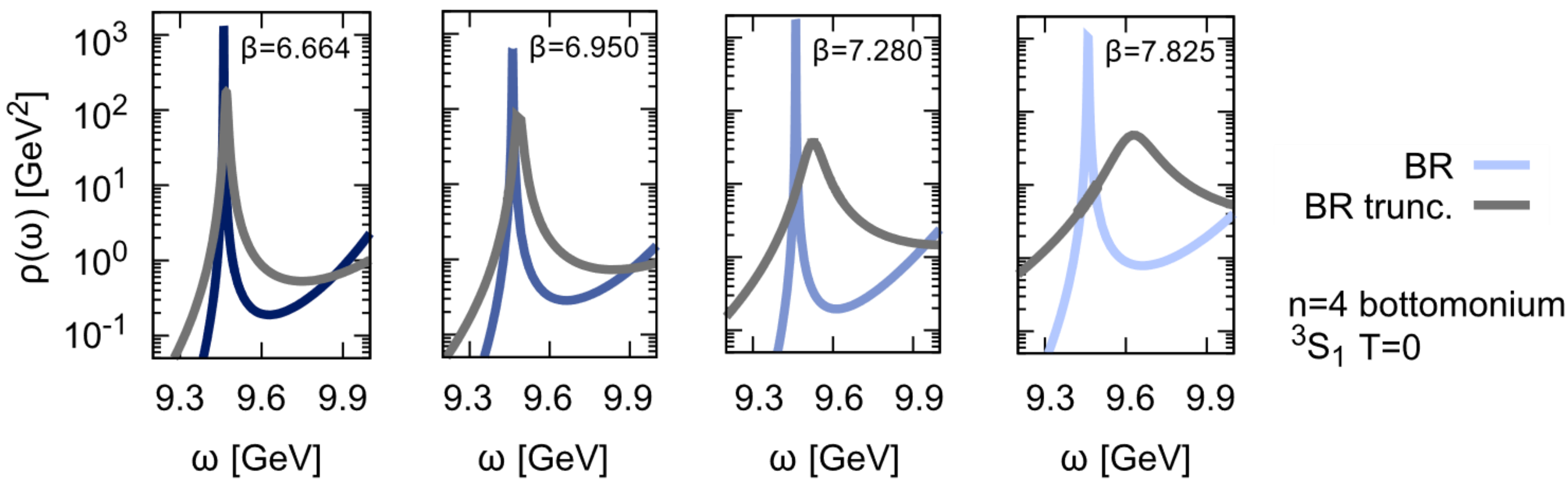

- Standard BR method resolves T=0 ground state very well from N_T=48-64 points
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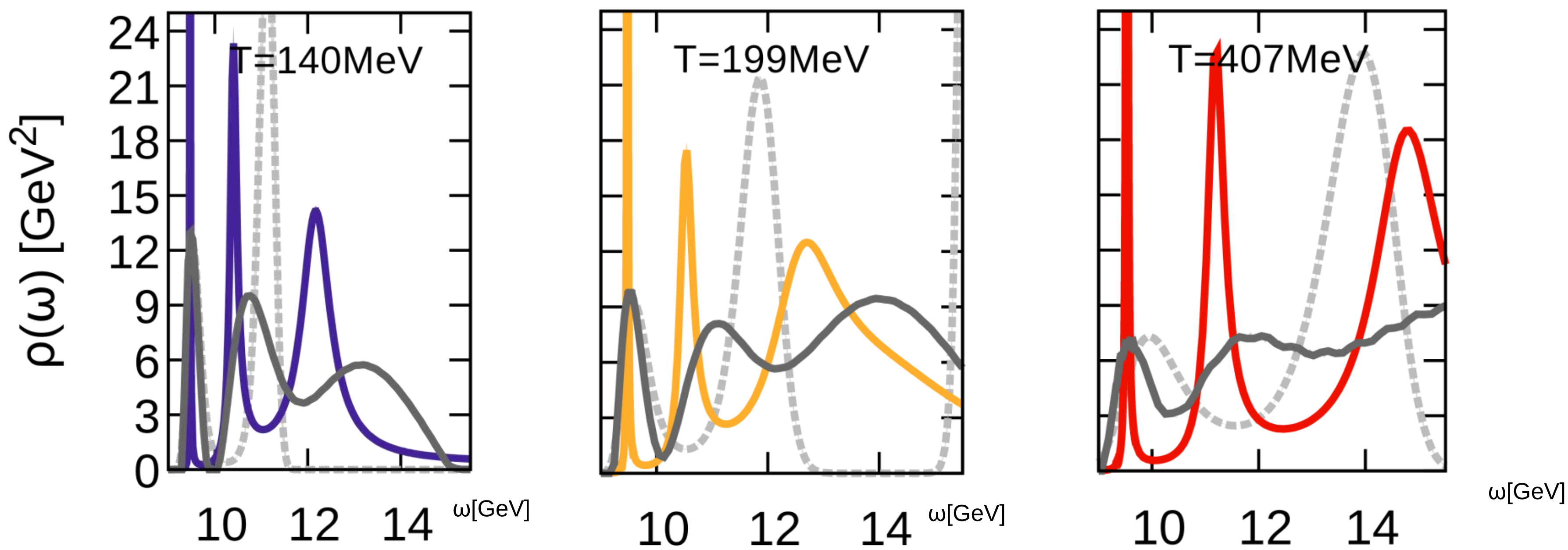
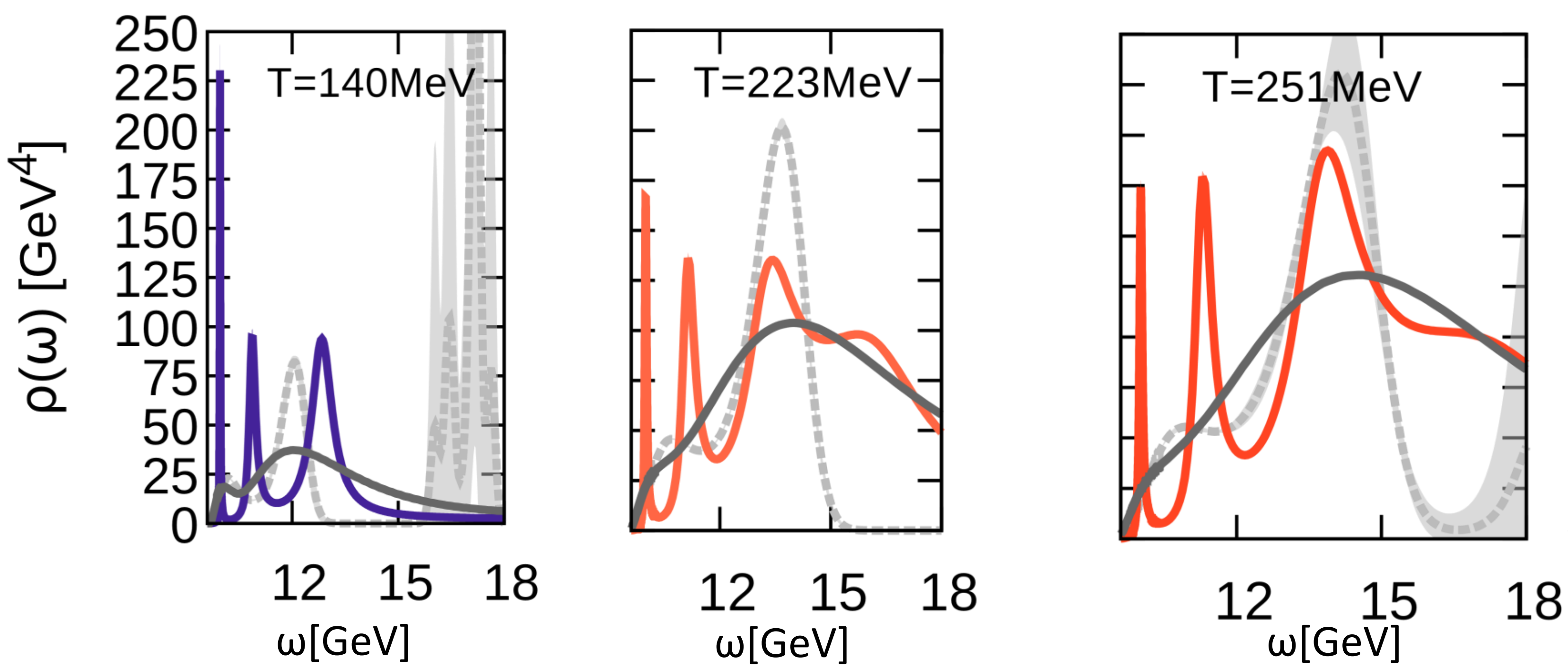
➡ Systematic shift of peaks to higher frequencies, as well as broadening. needs to be accounted for when analyzing T>0 spectra

Bottomonium S-wave at T>0



- Three methods: BR (colored), smooth BR (gray solid) & MEM (gray dashed)
  - lower bound: all methods show peak remnant up to certain temperature
  - melting regime: at least one method shows disappearance of peak
  - melted: at least two methods show GS peak weaker than higher lying structures

Bottomonium P-wave melting at T>0

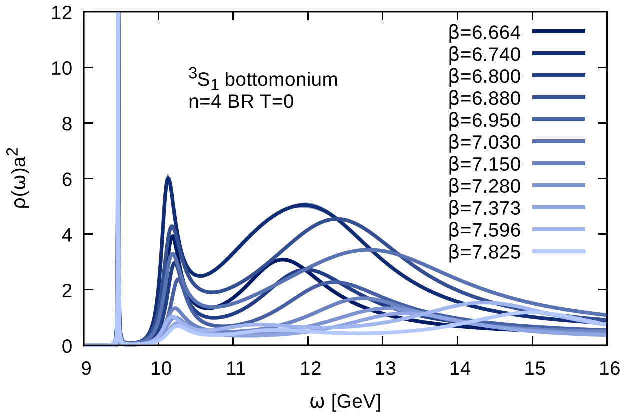


- Three methods: BR (colored), smooth BR (gray solid) & MEM (gray dashed)
  - Expectation from models: single GS peak + continuum
  - Both MEM and standard BR show ringing - smooth BR well controlled UV
  - At T=223MeV lowest peak in BR method identified as not physical

Lattice NRQCD spectral functions

University of Stavanger

$b\bar{b}$ S-wave T=0 spectra



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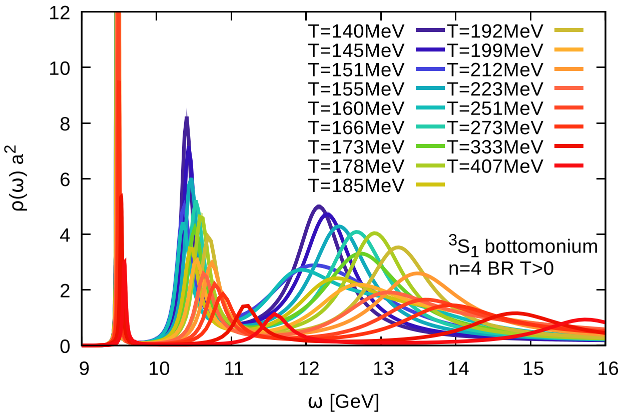

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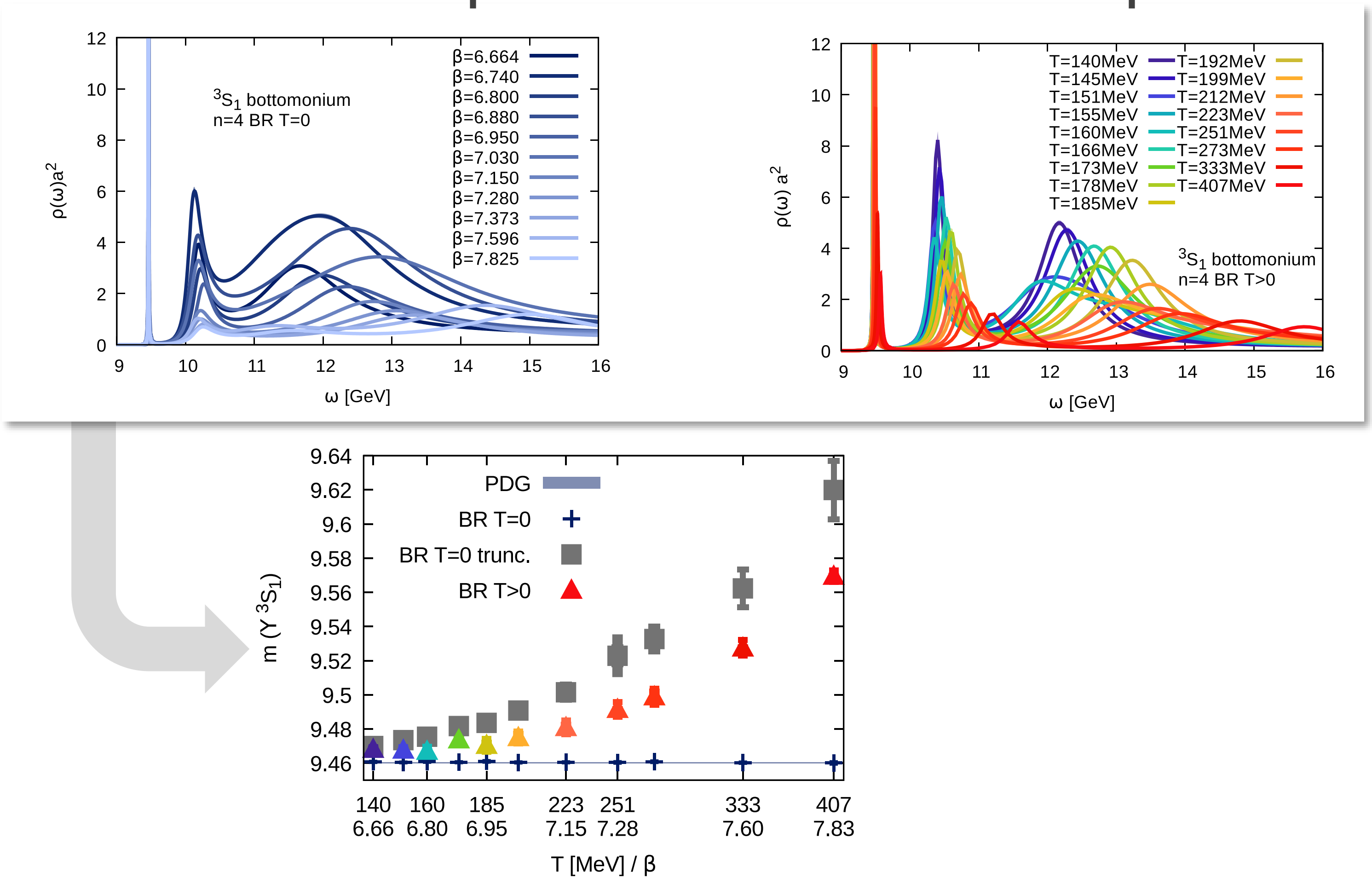

- Crucial step: **defining correct T=0 baseline** in presence of methods artifacts

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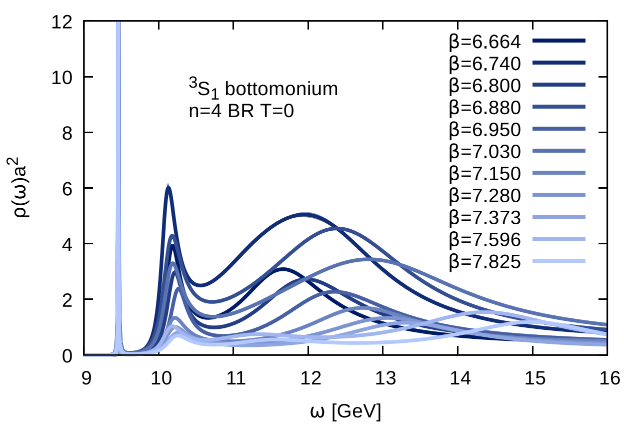

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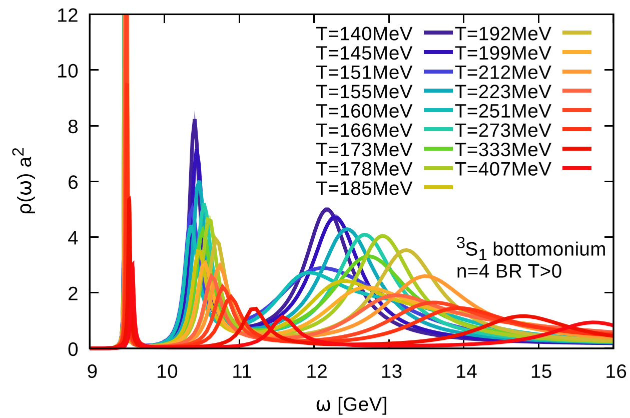

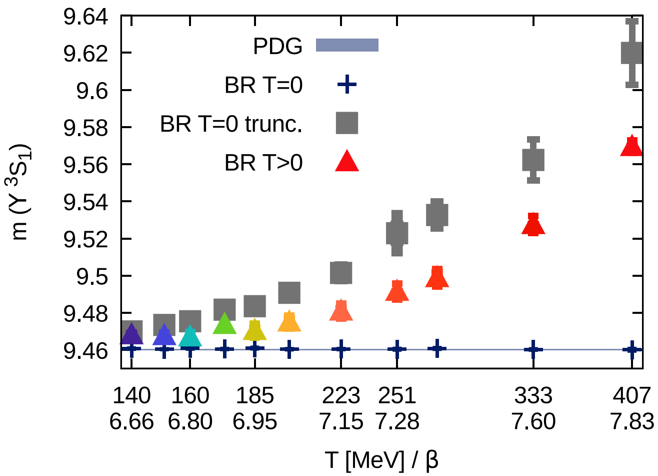

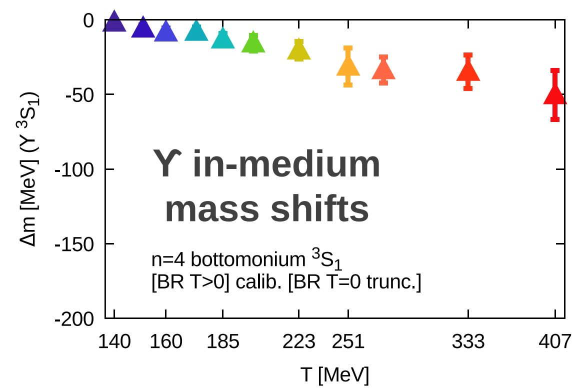

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The real-time interquark potential

University of Stavanger

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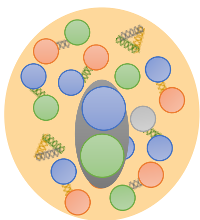
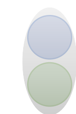
Relativistic T>0 field theory



Brambilla et.al. Rev.Mod.Phys. 77 (2005) 1423

QCD	NRQCD	pNRQCD
Dirac fields	**Pauli fields**	**Singlet/Octet**
$Q(x), \bar{Q}(x)$	$\psi, \psi^\dagger, \chi, \chi^\dagger$	$\psi_S(R,t), \psi_O(R,t)$



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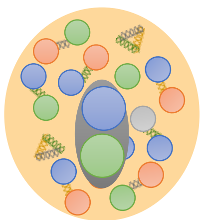

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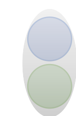

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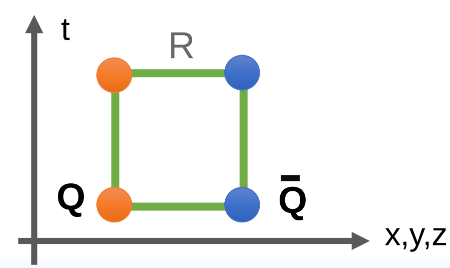

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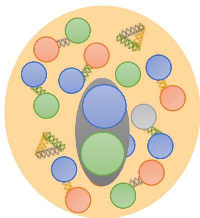

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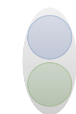

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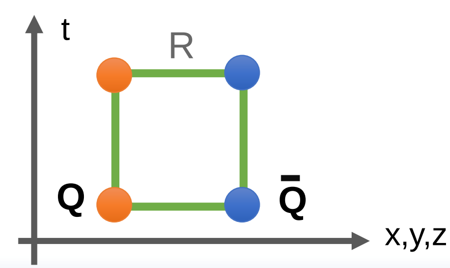

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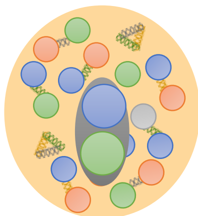

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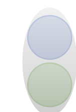

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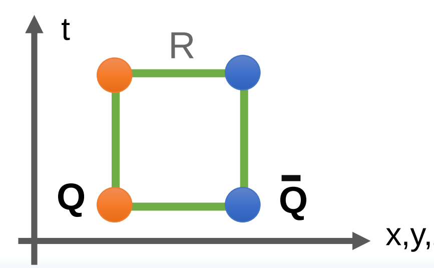
Im[V]: Laine et al. JHEP03 (2007) 054;
Beraudo et. al. NPA 806:312,2008
Brambilla et.al. PRD 78 (2008) 014017



Non-perturbative evaluation of V(R)

University of Stavanger

- How to connect to the Euclidean domain: **spectral functions**

A.R., T.Hatsuda & S.Sasaki
PRL 108 (2012) 162001

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A.R., T.Hatsuda & S.Sasaki PoS LAT2009 (2009) 162

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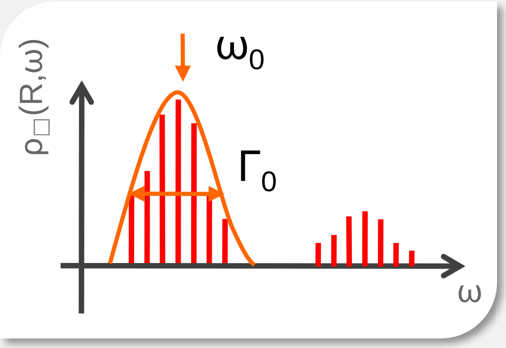
A.R., T.Hatsuda & S.Sasaki PoS LAT2009 (2009) 162



well defined V(R) if low lying Breit-Wigner present in Wilson loop spectral function

For technical details see Y.B., A.R. PRD86 (2012) 051503

$$V(R) = \omega_0(R) - i\Gamma_0(R)$$

Non-perturbative evaluation of V(R)

University of Stavanger

- How to connect to the Euclidean domain: **spectral functions**

A.R., T.Hatsuda & S.Sasaki PRL 108 (2012) 162001

$$W_\square(R,t) = \int_{-\infty}^{\infty} d\omega\, e^{-i\omega t} \rho_\square(R,\omega) \quad \Longleftrightarrow \quad W_\square(R,\tau) = \int_{-\infty}^{\infty} d\omega\, e^{-\omega\tau} \rho_\square(R,\omega)$$

Spectral Decomposition

$$V^{QCD}(R) = \lim_{t\to\infty} \frac{\int_{-\infty}^{\infty} d\omega\, \omega\, e^{-i\omega t} \rho_\square(R,\omega)}{\int_{-\infty}^{\infty} d\omega\, e^{-i\omega t} \rho_\square(R,\omega)}$$

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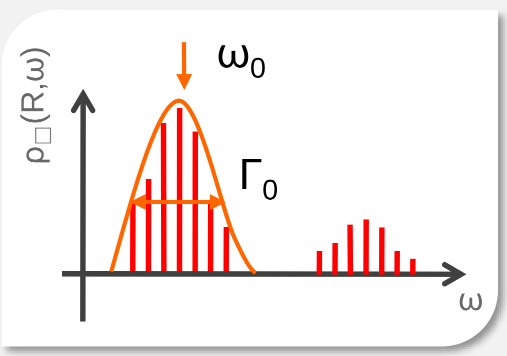

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Spectral Reconstruction

- In case of usual $\Delta W/W=10^{-2}$ statistical uncertainty in $W_\square$: **Bayesian inference**

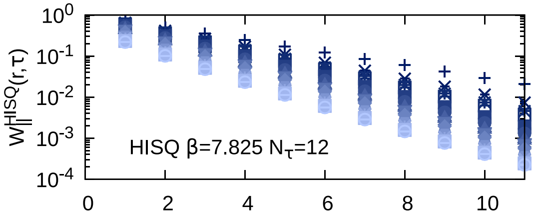
incorporate prior information to regularize the inversion task (BR method)

- In case of **small $\Delta W/W<10^{-3}$** statistical uncertainty in $W_\square$ also **Pade approximation**

exploit the analyticity of the Wilson correlator to extract spectra

The Pade projection method (I)

- Non-Bayesian reconstruction method via direct projection (no default model)



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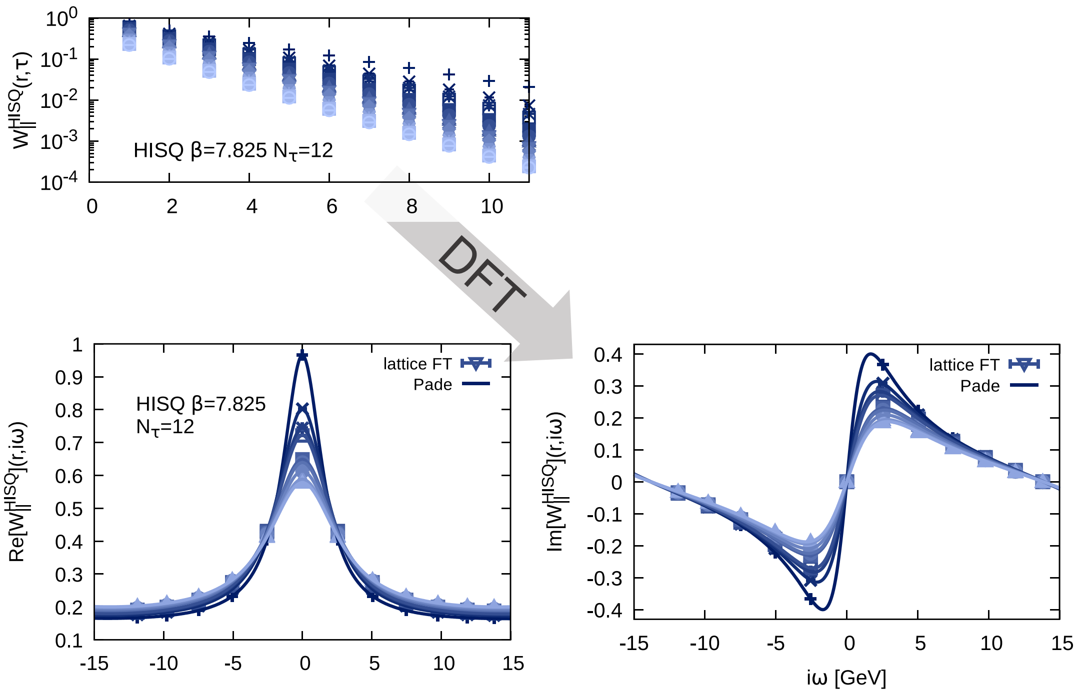

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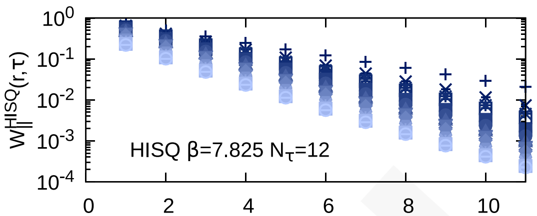
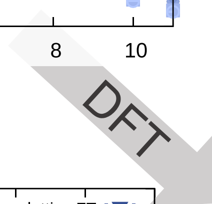
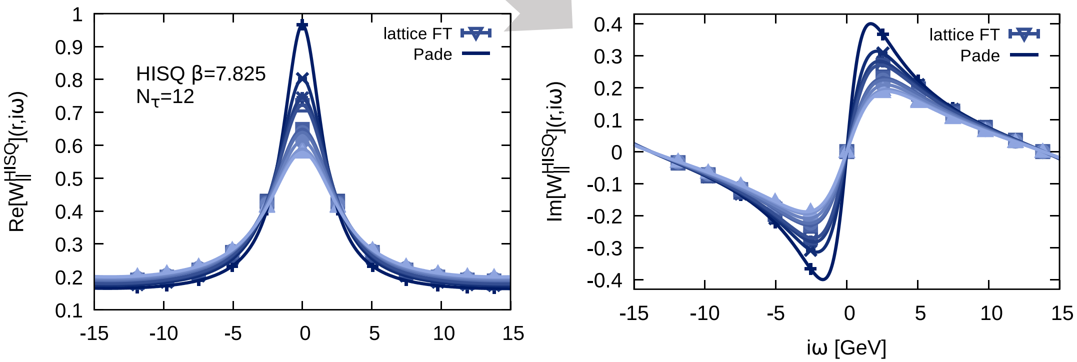




- Optimal Pade (n-1,n) approximation via Schlessinger formula

$$R_{N_\tau}(i\omega) = \frac{W_\square(r_k,\omega_0)}{1+}\,\frac{a_0(\omega-\omega_0)}{1+}\,\frac{a_1(\omega-\omega_1)}{1+}\cdots\frac{a_{N_\tau-1}(\omega-\omega_{N_\tau-1})}{1+}$$

$$a_l(\omega_{l+1}-\omega_l) = -\left\{1+\frac{a_{l-1}(\omega_{l+1}-\omega_{l-1})}{1+}+\frac{a_{l-2}(\omega_{l+1}-\omega_{l-2})}{1+}\cdots\frac{a_0(\omega_{l+1}-\omega_0)}{1-[W_\square(r_k,\omega_0)-W_\square(r_k,\omega_{l+1})]}\right\}$$



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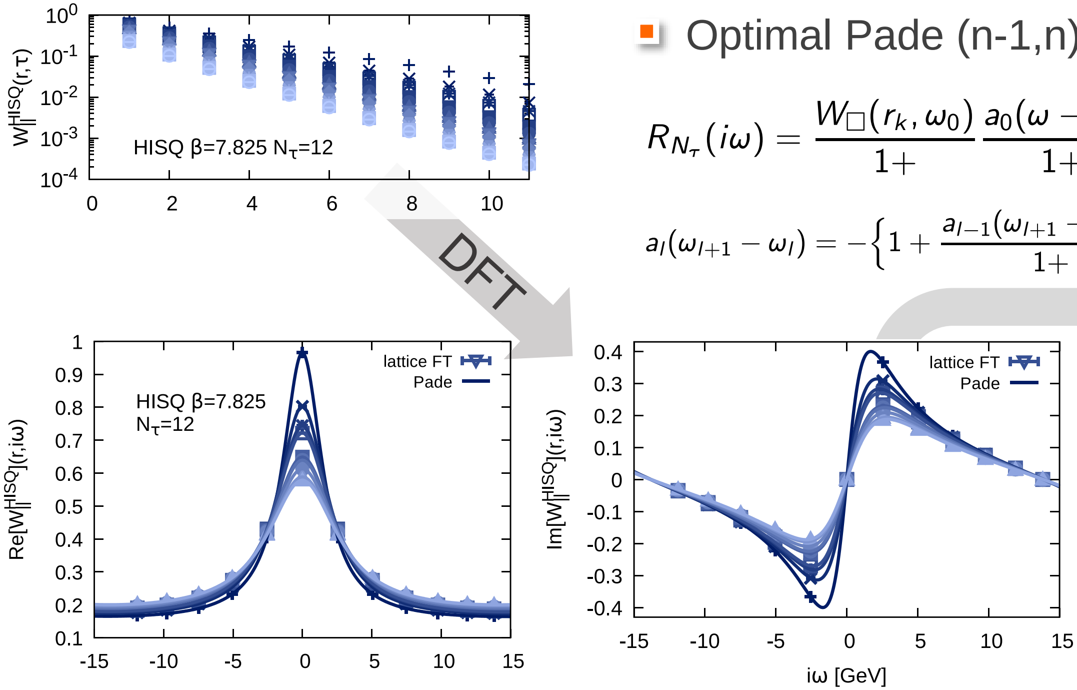

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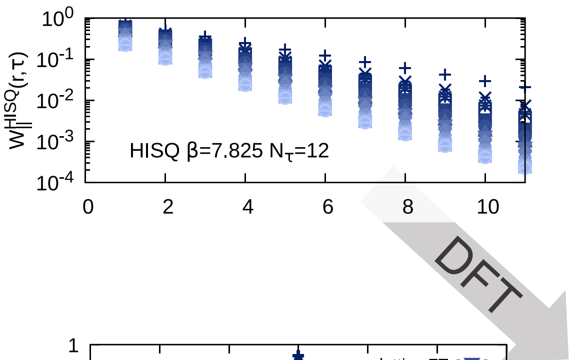

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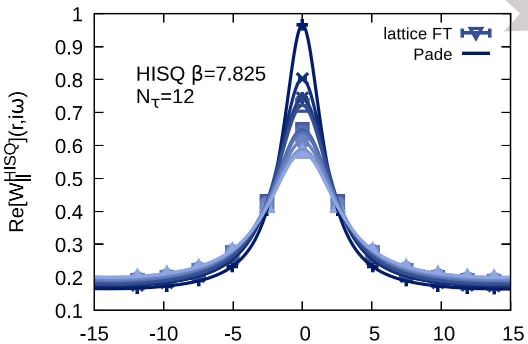

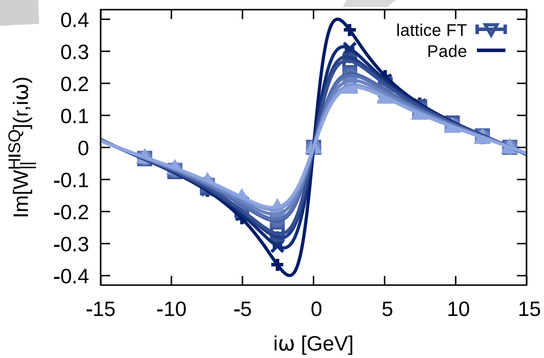

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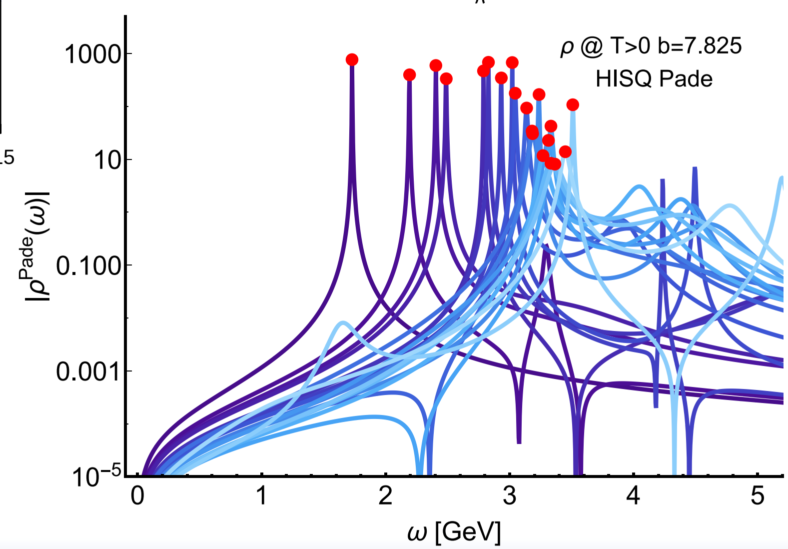

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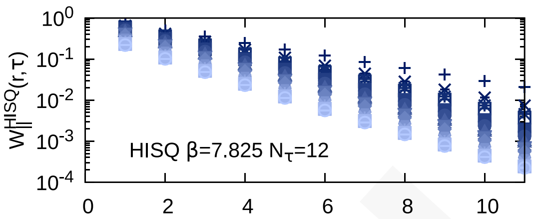

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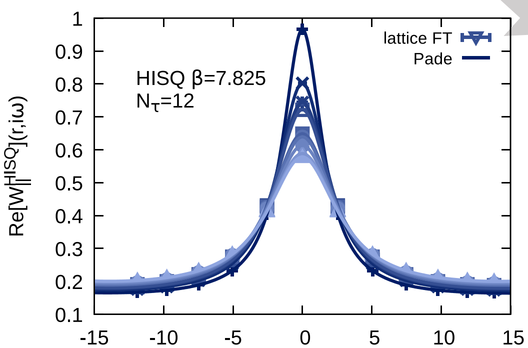

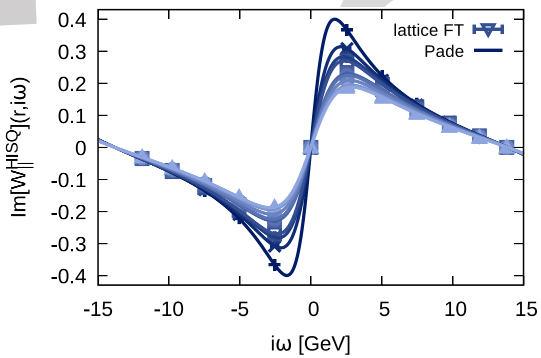

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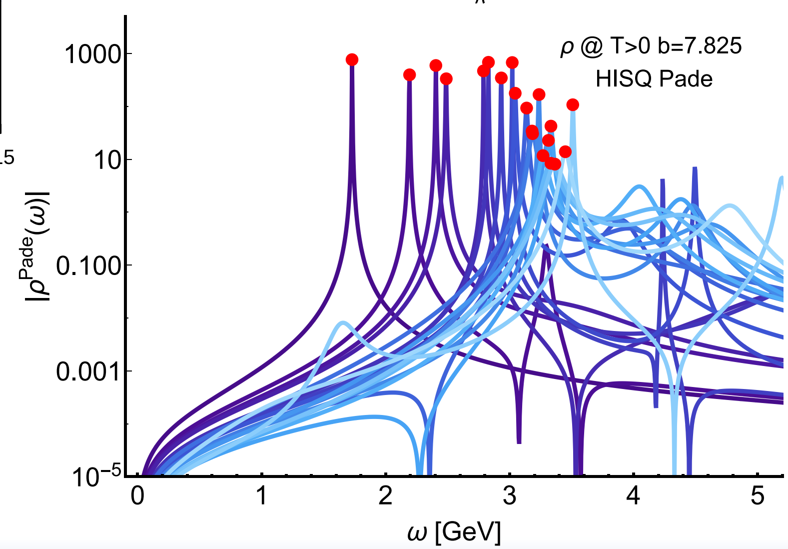
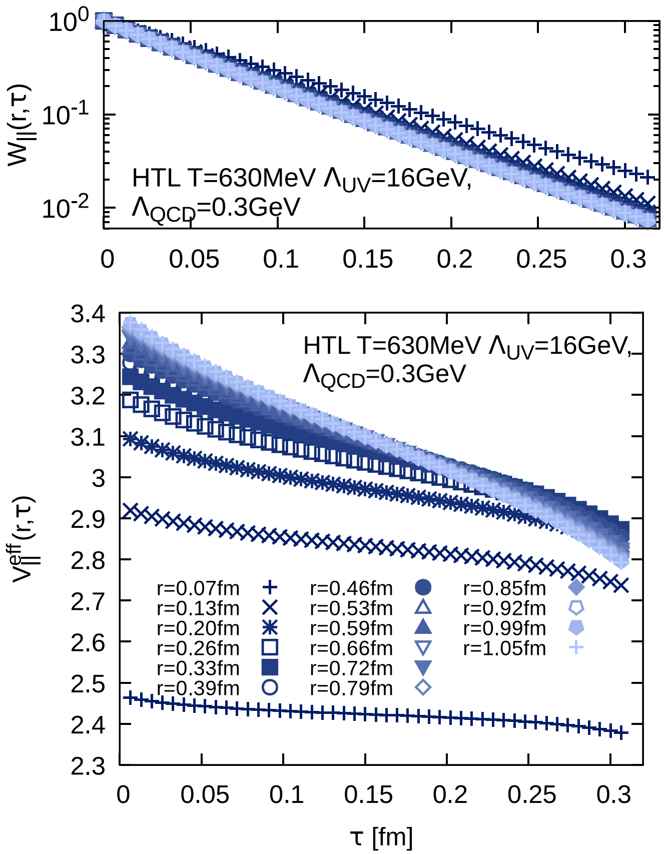


- In practice usually violates the spectral decom-position but individual structures well reproduced

The Pade projection method (II)

University of Stavanger

- Perform mock data analysis with resummed perturbative Wilson correlators (HTL)



No plateau in effective potential
V_{eff}=-log[W_i/W_{i+1}]: large spectral widths

HTL correlators: Y.Burnier, A.R. PRD87 (2013) 114019

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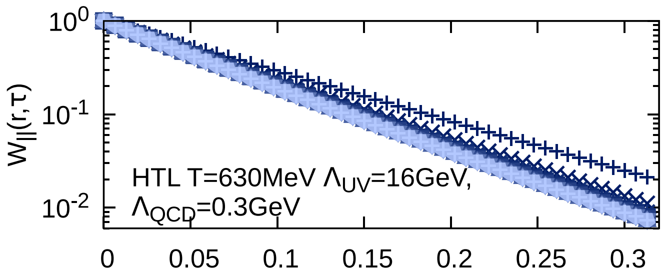

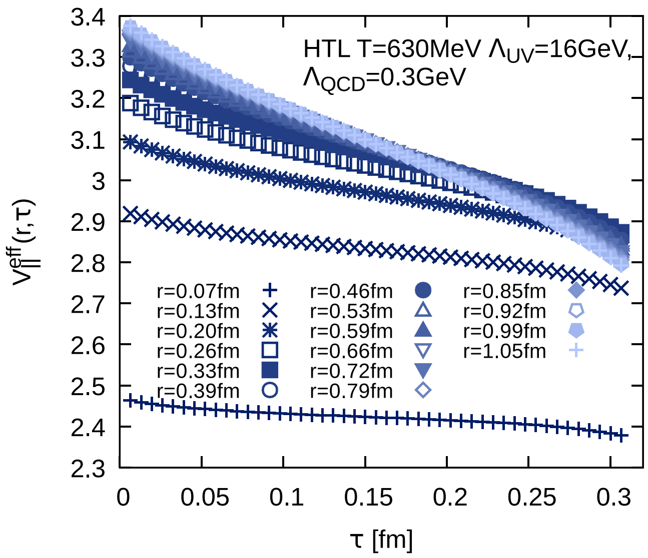

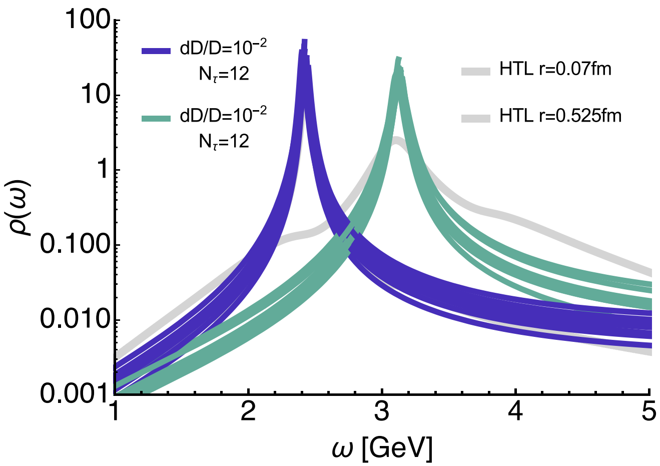

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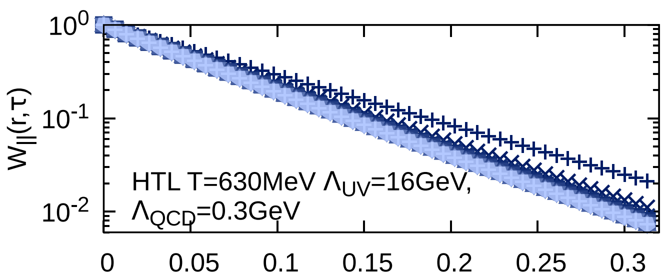

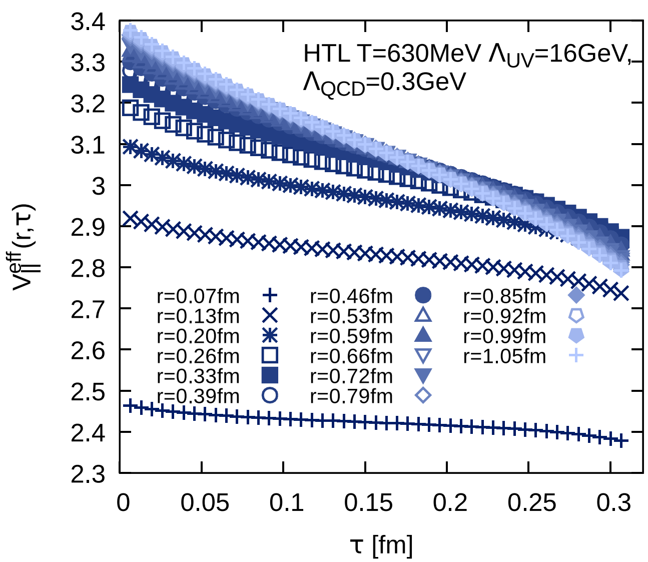

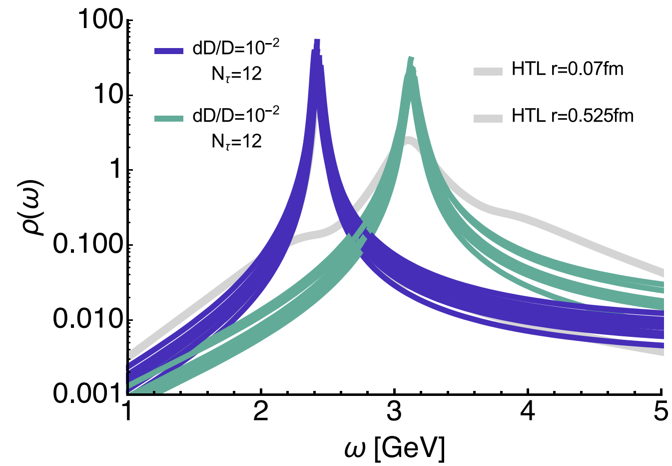

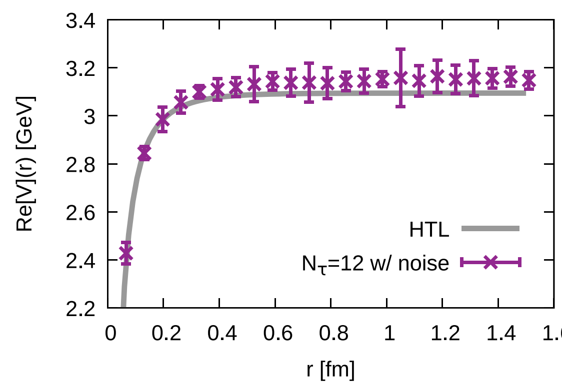

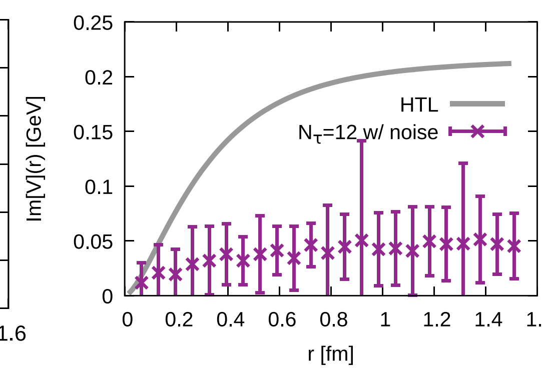

No plateau in effective potential $V_{eff}=-\log[W_i/W_{i+1}]$: large spectral widths

At $N_\tau=12$, errors of $dD/D=10^{-2}$ detrimental to Im[V] but Re[V] well reconstructed

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Latest results on the lattice potential

University of Stavanger

- Lattices with dynamical u,d,s quarks (HISQ action, HotQCD & TUMQCD)

A. Bazavov et.al. PRD97 (2018) 014510, HotQCD PRD90 (2014) 094503

  - realistic m_π~161MeV (T=151-1451MeV)
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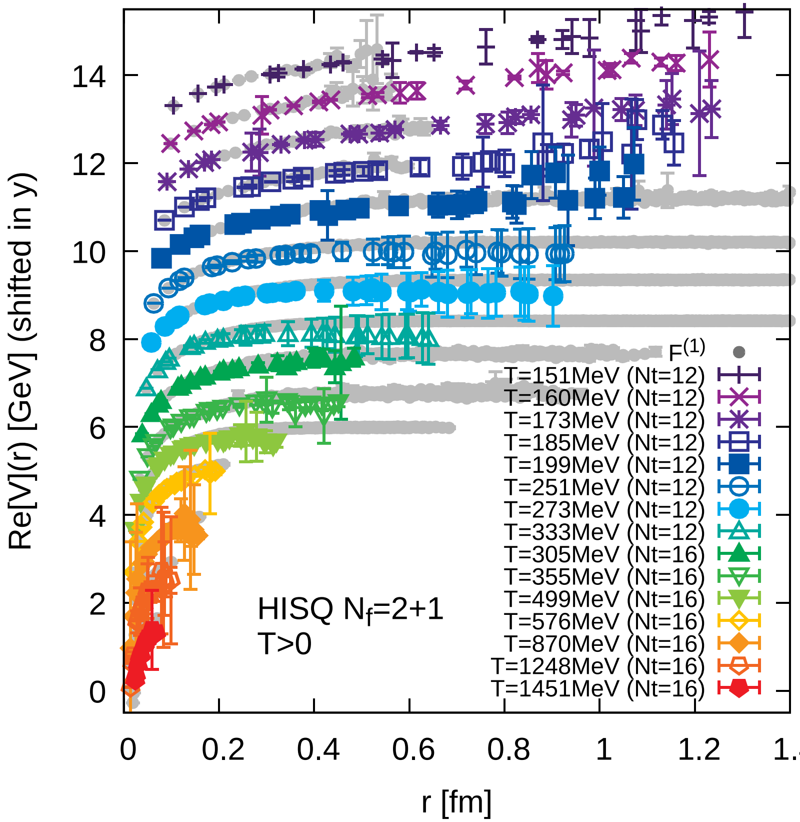

P. Petreczky, A.R., J. Weber, NPA982 (2019) 735

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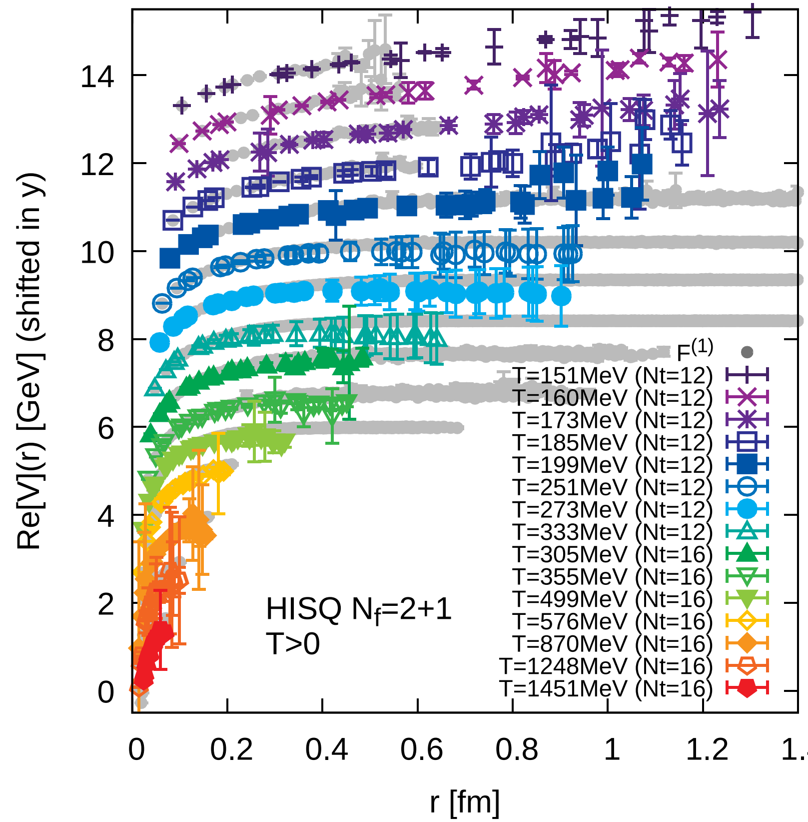

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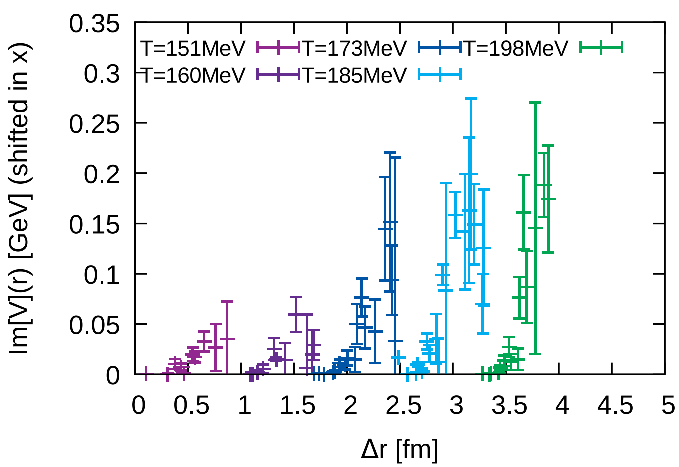
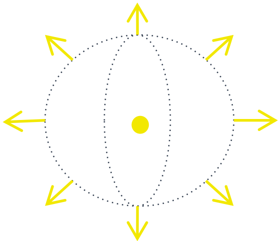


- Smooth transition from Cornell @ T=0 to Debye screened @ $T>T_C$
- Finite Im[V] above T_c present

An improved Gauss law approach

- For use in phenomenology applications: analytic expression for Re[V] and Im[V]



$$V^{T=0}_{Q\bar{Q}}(R) = V_C(R) + V_S(R) = -\frac{\alpha_S}{r} + \sigma r + c$$

Strategy:

α_s,σ and c are vacuum prop. and do not change with T

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University of Stavanger

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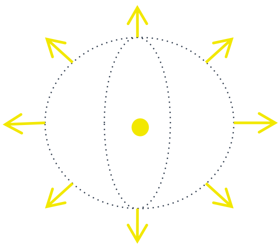

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V. V. Dixit,
Mod. Phys. Lett. A 5, 227 (1990)

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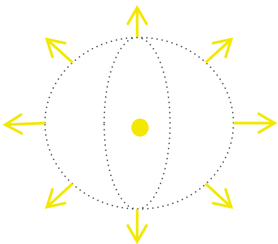

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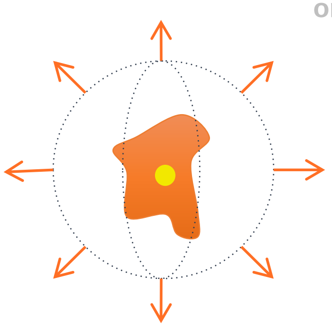
- Immerse non-perturbative charge in weak coupling HTL medium: permittivity ε

original idea: Y.Burnier, A.R. Phys.Lett. B753 (2016) 232 improved derivation D.Lafferty and A.R. arXiv:1906.00035



$$V^{med}(\mathbf{p}) = V^{vac}(\mathbf{p})/\epsilon(\mathbf{p}) \qquad \epsilon^{-1}(\vec{p}, m_D) = \frac{p^2}{p^2+m_D^2} - i\pi T\frac{p m_D^2}{(p^2+m_D^2)^2}$$

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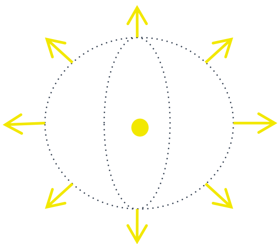

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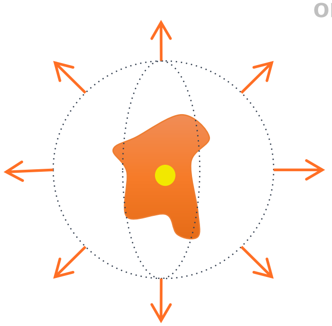

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An improved Gauss law approach

University of Stavanger

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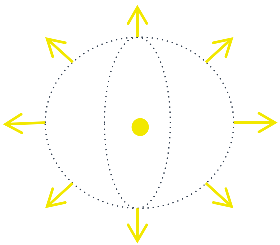

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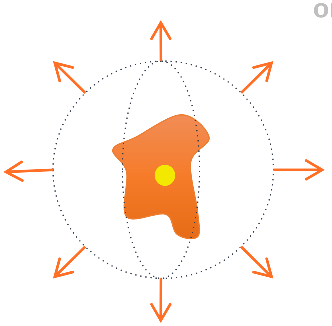
- Immerse non-perturbative charge in weak coupling HTL medium: permittivity ε

original idea: Y.Burnier, A.R. Phys.Lett. B753 (2016) 232 improved derivation D.Lafferty and A.R. arXiv:1906.00035



$$V^{med}(\mathbf{p}) = V^{vac}(\mathbf{p})/\epsilon(\mathbf{p}) \qquad \epsilon^{-1}(\vec{p}, m_D) = \frac{p^2}{p^2+m_D^2} - i\pi T\frac{pm_D^2}{(p^2+m_D^2)^2}$$

$$\mathcal{G}_a[V^{med}(\mathbf{r})] = \mathcal{G}_a\int d^3y\left(V^{vac}(\mathbf{r}-\mathbf{y})\epsilon^{-1}(\mathbf{y})\right) = 4\pi q\epsilon^{-1}(\mathbf{r}, m_D)$$

- 3 vacuum parameters and 1 temperature dependent m_D fix both Re[V] and Im[V].

Gauss-law solution to Re[V] & Im[V]

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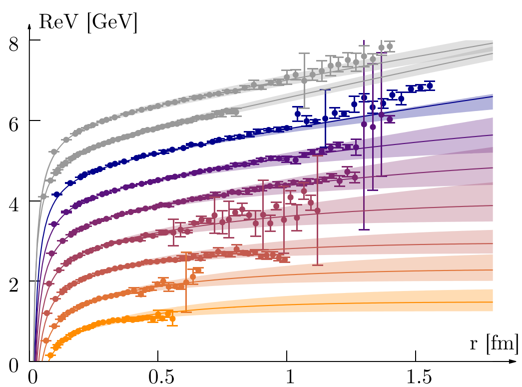
- A single T-dependent parameter m_D interpolates between T=0 and high T results

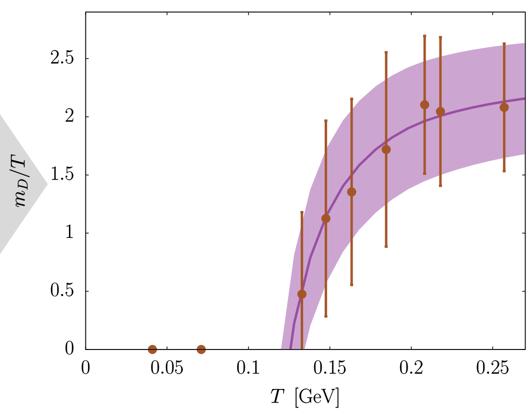
$$Re[V^{med}](r) = \frac{2\sigma}{m_D}\left(1 - e^{-m_D r}\right) - \sigma r e^{-m_D r} - \frac{\alpha_s}{r} e^{-m_D r}$$

  - for m_D ->0 recovers Cornell, for m_D large recovers the coulombic HTL result

- Gauss-Law result allows to fit Re[V] data even in the non-perturbative regime

D.Lafferty and A.R. arXiv:1906.00035





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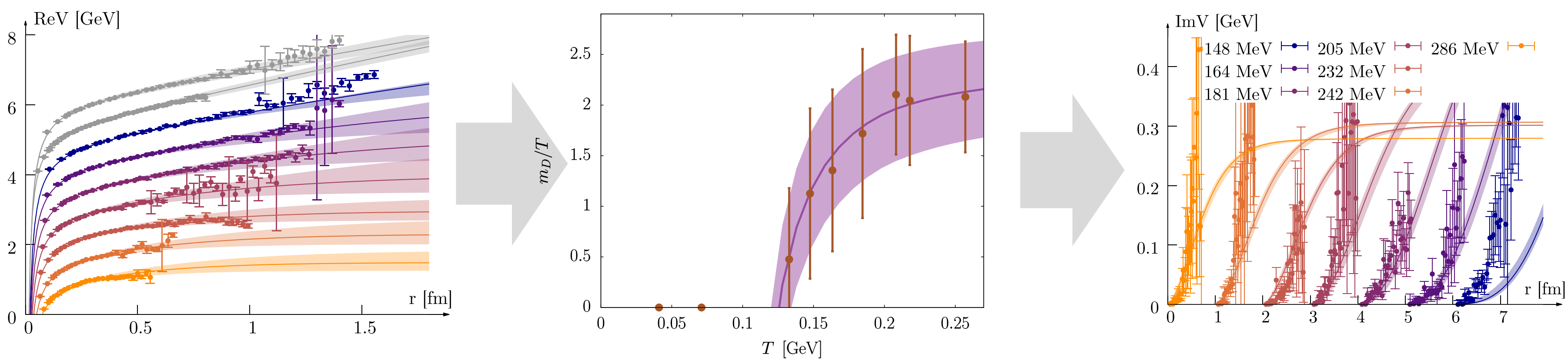

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Lattice setup for the T>0 gluon

- N_f=2+1+1 flavors of twisted Mass Wilson fermions in the thermal QCD medium

R. Baron et al. PoS LAT2010, 123 (2010) and F. Burger et al. (tmft) PoS LAT2013 (2013) 153

ETMC ens. $(T=0)$	D45.32
tmfT ens. $(T \neq 0)$	D370
β	2.10
a [fm]	0.0646
m_π [MeV]	369(15)
$T_{\rm deconf}$ [MeV]	193(13)(2)
$N_\tau = N_{q4}$ range	4-20

$D370\ N_\tau$	4	6	8	10	11	12	14	16	18	20
T MeV	762	508	381	305	277	254	218	191	170	152
N_s	32	32	32	32	32	32	32	32	40	48
$N_{\rm meas}$	310	400	120	410	420	380	790	610	590	280

- Gluon correlator $D^{ab}_{\mu\nu}(\mathbf{q}) = \langle A^a_\mu(-\mathbf{q})A^b_\nu(\mathbf{q})\rangle$ requires gauge fixing (e.g. $\partial_\mu A^\mu = 0$)
  - Minimize $F_U[g] = \frac{1}{3}\sum_{x,\mu} {\rm ReTr}\left(g^\dagger_x U_{x\mu} g_{x+\mu}\right)$ via gauge transf. $U_{x\mu} \overset{g}{\mapsto} U^g_{x\mu} = g^\dagger_x U_{x\mu} g_{x+\mu}$
- At T>0 separation into longitudinal (electric) & transversal (magnetic) mode

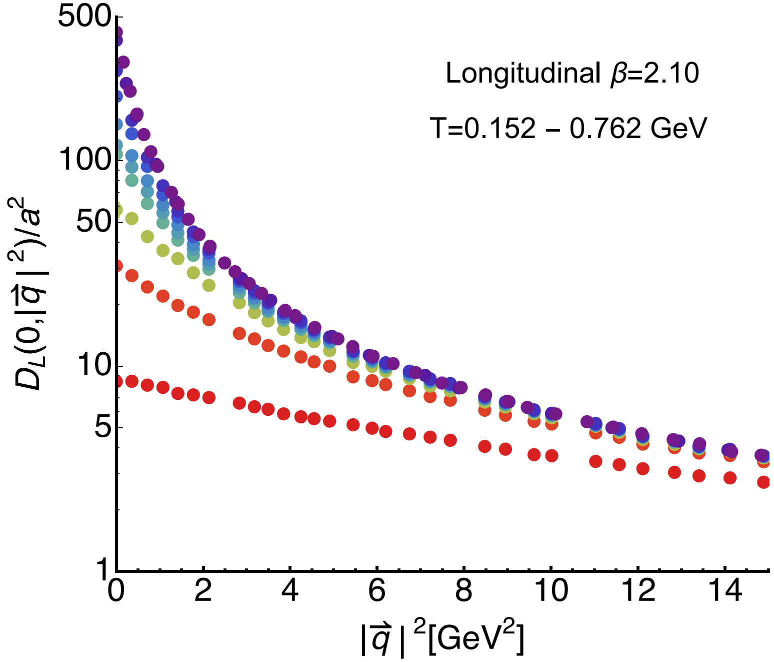
$$P^T_{\mu\nu} = (1-\delta_{\mu 4})(1-\delta_{\nu 4})\left(\delta_{\mu\nu} - \frac{q_\mu q_\nu}{\vec{q}^{\,2}}\right),$$

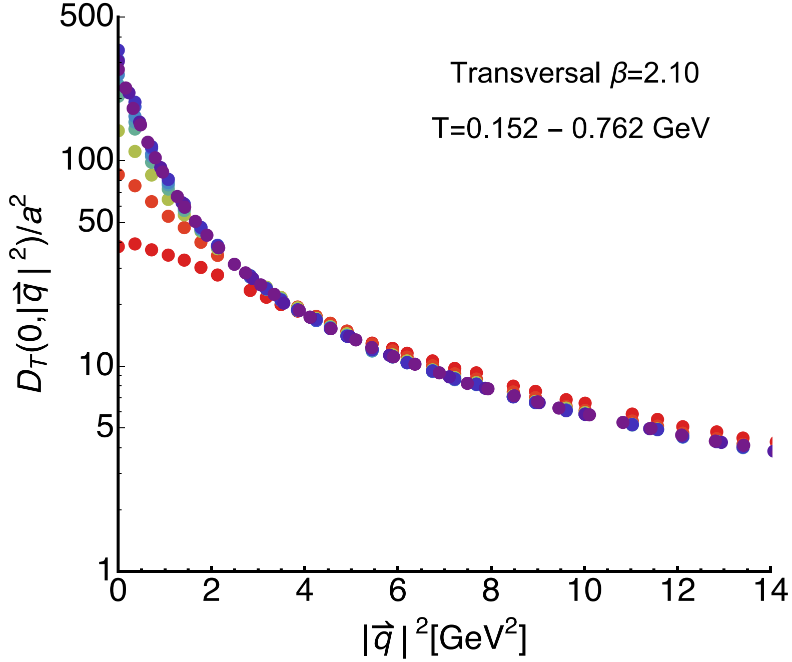
$$P^L_{\mu\nu} = \left(\delta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right) - P^T_{\mu\nu}\,.$$

$$D_T(q) = \frac{1}{2N_g}\left\langle \sum_{i=1}^{3} A^a_i(q)A^a_i(-q) - \frac{q_4^2}{\vec{q}^{\,2}}A^a_4(q)A^a_4(-q)\right\rangle$$

$$D_L(q) = \frac{1}{N_g}\left(1+\frac{q_4^2}{\vec{q}^{\,2}}\right)\langle A^a_4(q)A^a_4(-q)\rangle$$

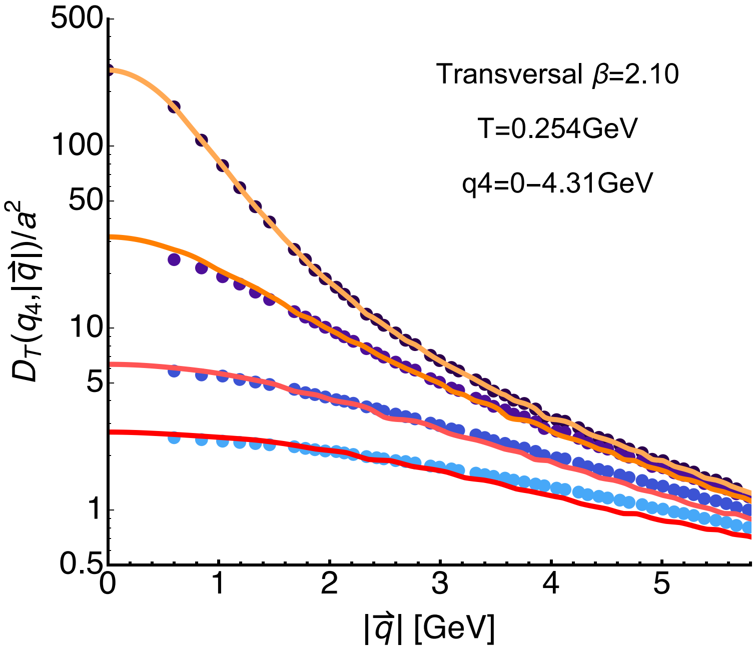
The gluon correlator in Landau gauge

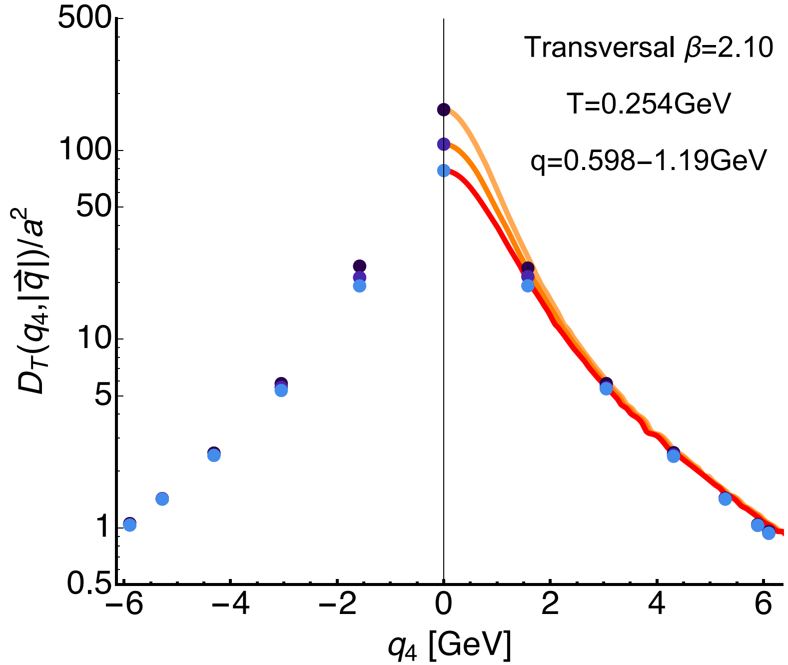




- Thermal effects are much more pronounced in longitudinal propagator
- An electric and magnetic mass visible at q_4=0 **q**=0 : D(0,**0**)=1/M^2 increase with T

The fate of O(4) invariance





- Well established trick in functional computations: exploitation of O(4) invariance

$$D(q_4, \mathbf{q}) \approx D(0, \sqrt{q_4^2 + \mathbf{q}^2})$$

- Here: explicitly compute finite q4 and observe range of validity of assumption
  - Close to q4=0 ok but already deviations at q4~2πT and end of Brillouin zone problematic

The raw input data

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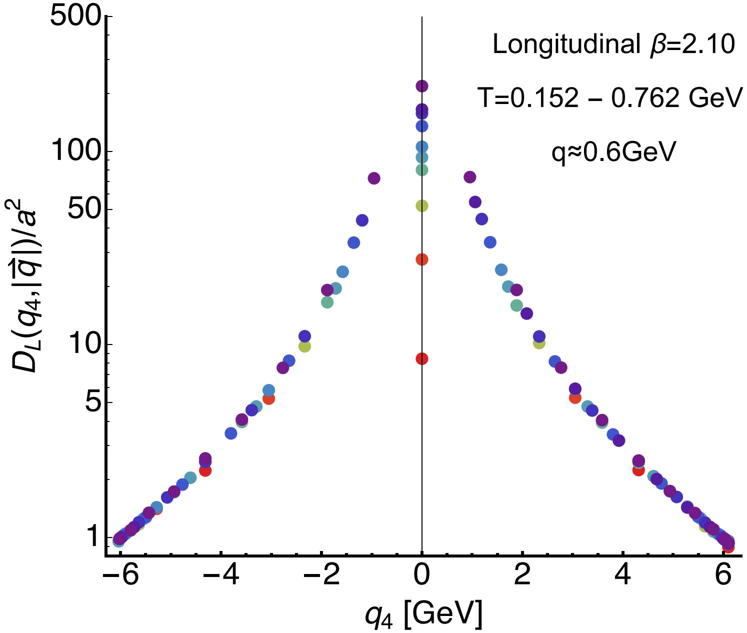

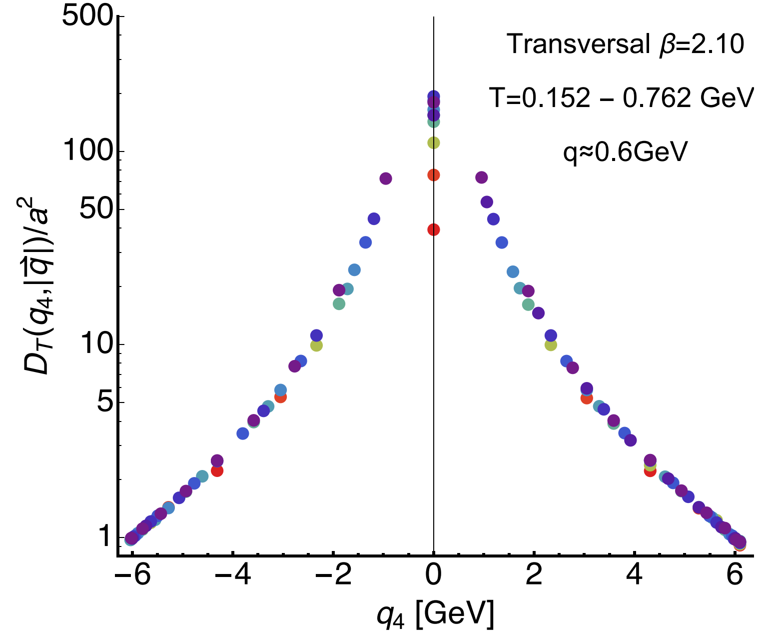

- Central challenge: the T>0 effects are hidden between low Matsubara frequencies
- Except for highest T data, UV regime q_4 > 3 GeV virtually indistinguishable

Low temperature spectral functions

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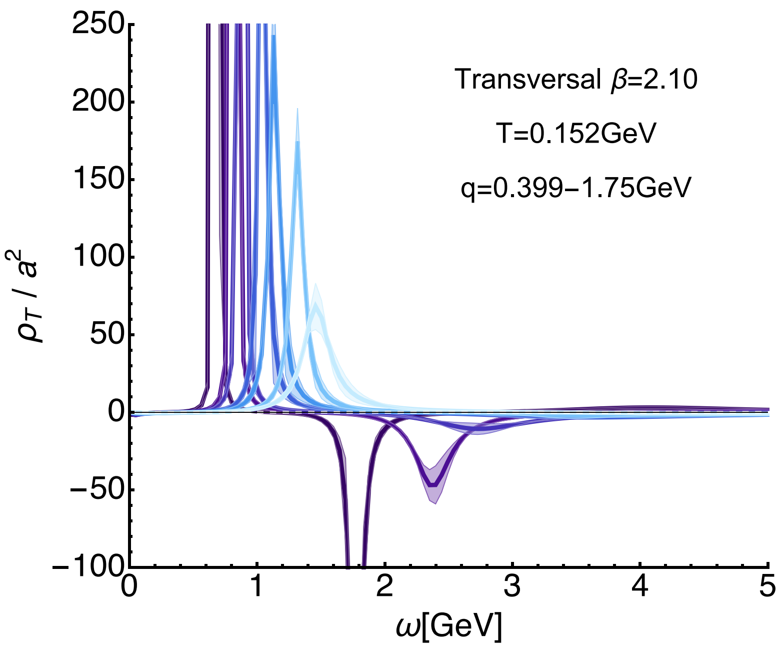

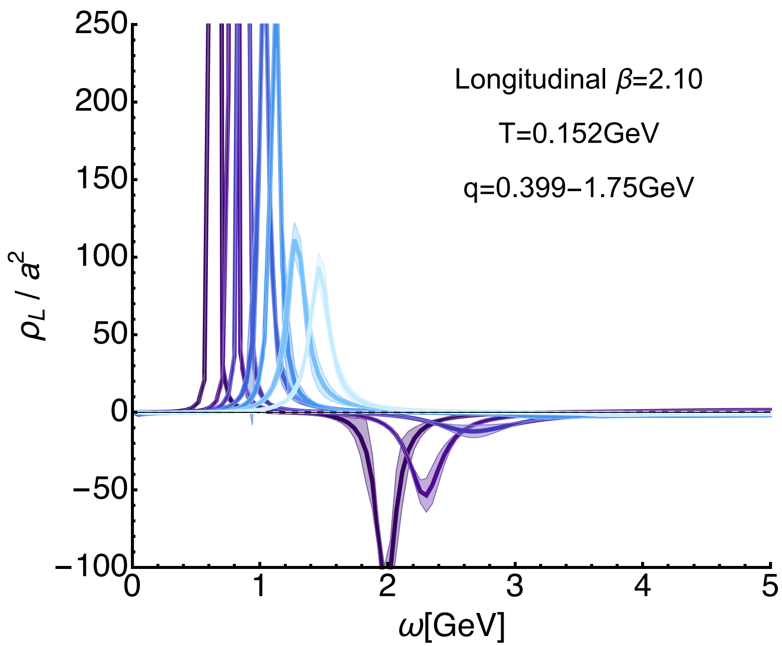

- Clear signs of positivity violation at intermediate frequencies
- Consistent sign of positivity violation at small frequencies too
- Dominant quasi-particle like peak structure at all spatial momenta

Intermediate T spectral functions

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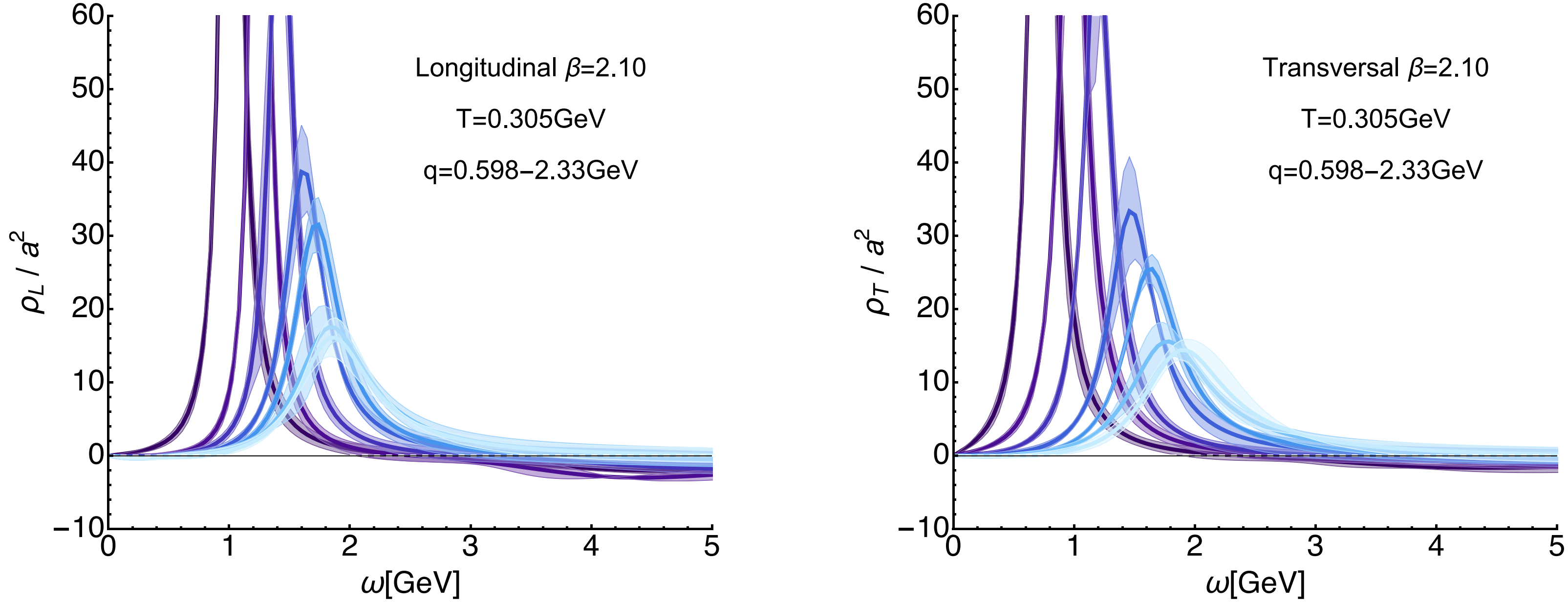

- Take overall shape with a grain of salt: # of datapoints reduces
- Qualitative trend: the higher the temperature, the less positivity violation

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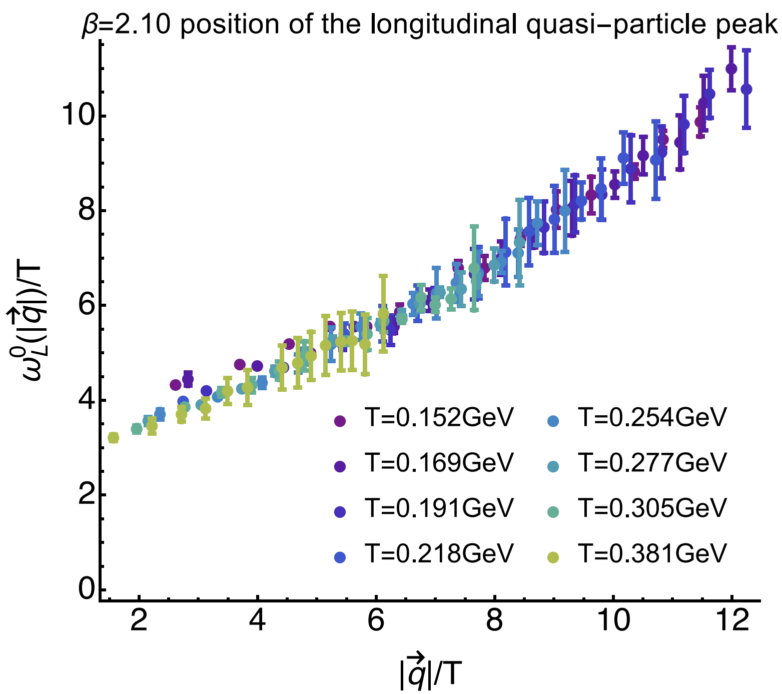
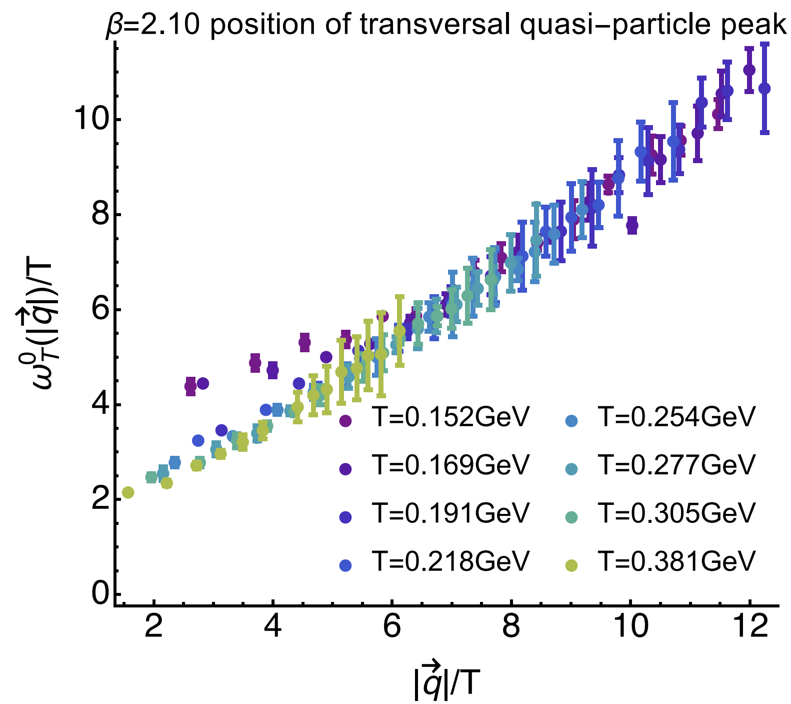
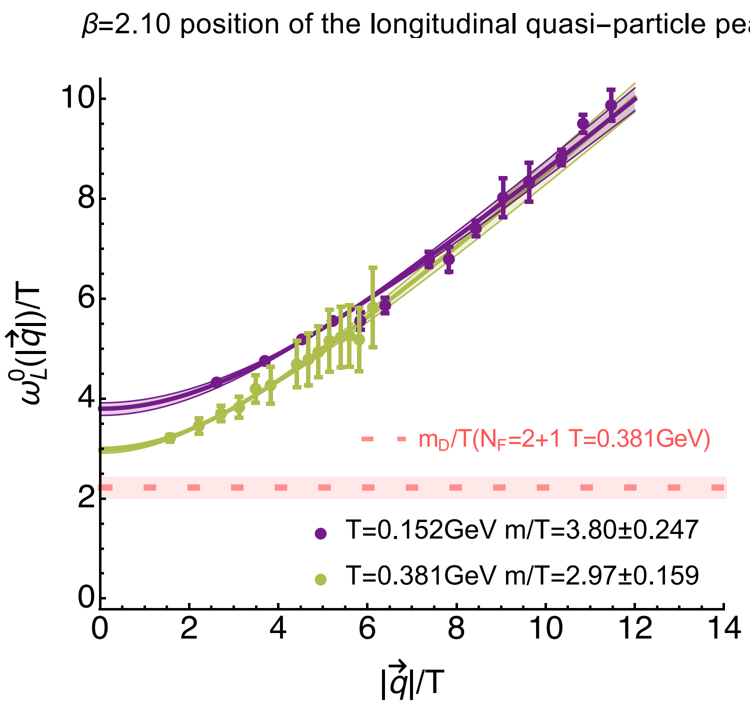
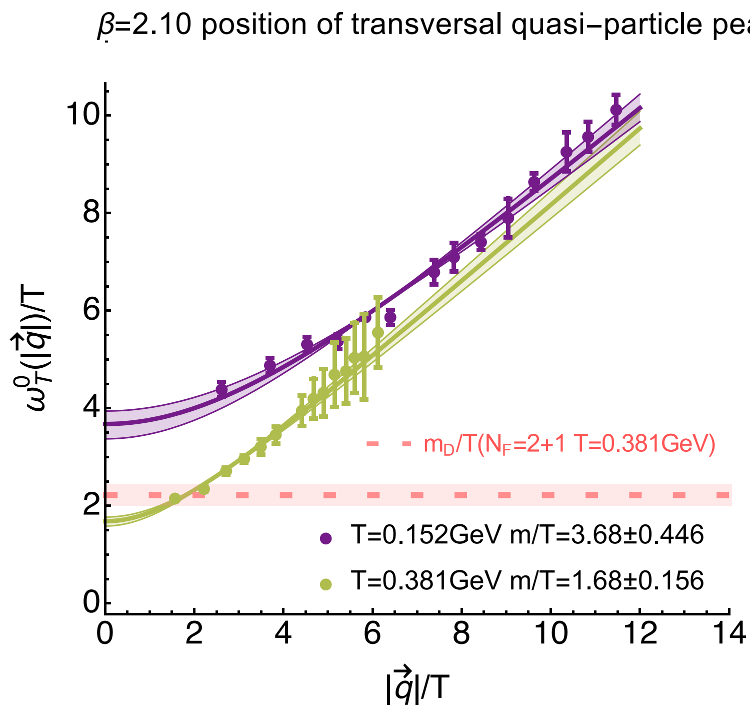




- Dispersion relation from the quasiparticle peak position – agreement in UV

$$\omega_0(\mathbf{q}) = A\sqrt{B^2 + |\mathbf{q}|^2}$$





- Dispersion relation from the quasiparticle peak position – agreement in UV
- Use as fit ansatz: $\omega_0(\mathbf{q}) = A\sqrt{B^2 + |\mathbf{q}|^2}$ extrapolated masses:

$$m_L/T|_{T=0.152\text{GeV}} = 3.80 \pm 0.25$$
$$m_L/T|_{T=0.381\text{GeV}} = 2.97 \pm 0.16$$

$$m_T/T|_{T=0.152\text{GeV}} = 3.68 \pm 0.45$$
$$m_T/T|_{T=0.381\text{GeV}} = 1.68 \pm 0.16$$

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Conclusion

- T>0 lattice QCD spectral functions: insight on dynamics of gauge (in)variant d.o.f.
- Bayesian inference viable strategy to attack underlying ill-posed inverse problem
- Recent progress for lattice NRQCD quarkonium spectral functions:
  - Melting regime now consistent among different lattice groups
  - Robust results on in-medium ground state masses: states become lighter as T increases
- Complex heavy-quark potential accessible on realistic lattices close to continuum
- Exploratory work on gluon spectral functions revealing
- Need for improved nonlocal regulators to combat oscillatory behavior, finite T simulations for gluons with smaller lattice spacing …

The XIVth Quark confinement and the Hadron spectrum conference

Quark Confinement
The XIVth
the Hadron
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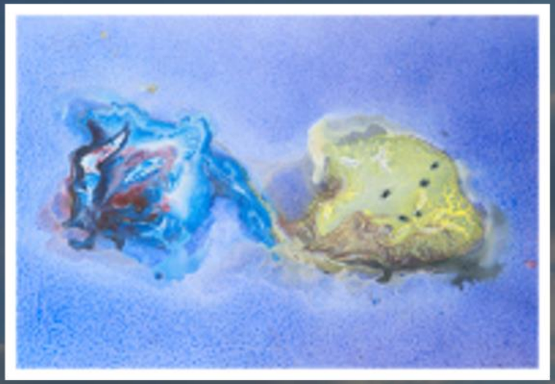

dates: 27th July - 1st August, 2020

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webpage: ux.uis.no/confxiv

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