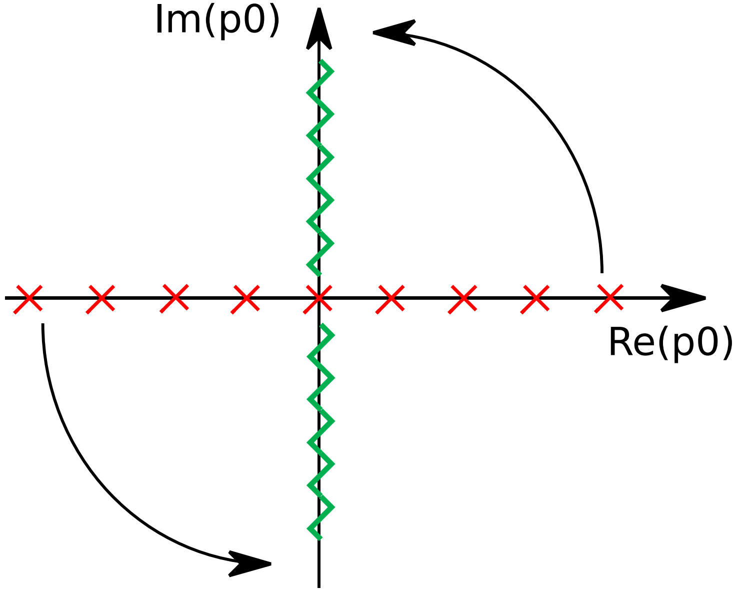
Spectral functions
&
the Functional Renormalization Group

Non-Perturbative QFT in Euclidean and Minkowski

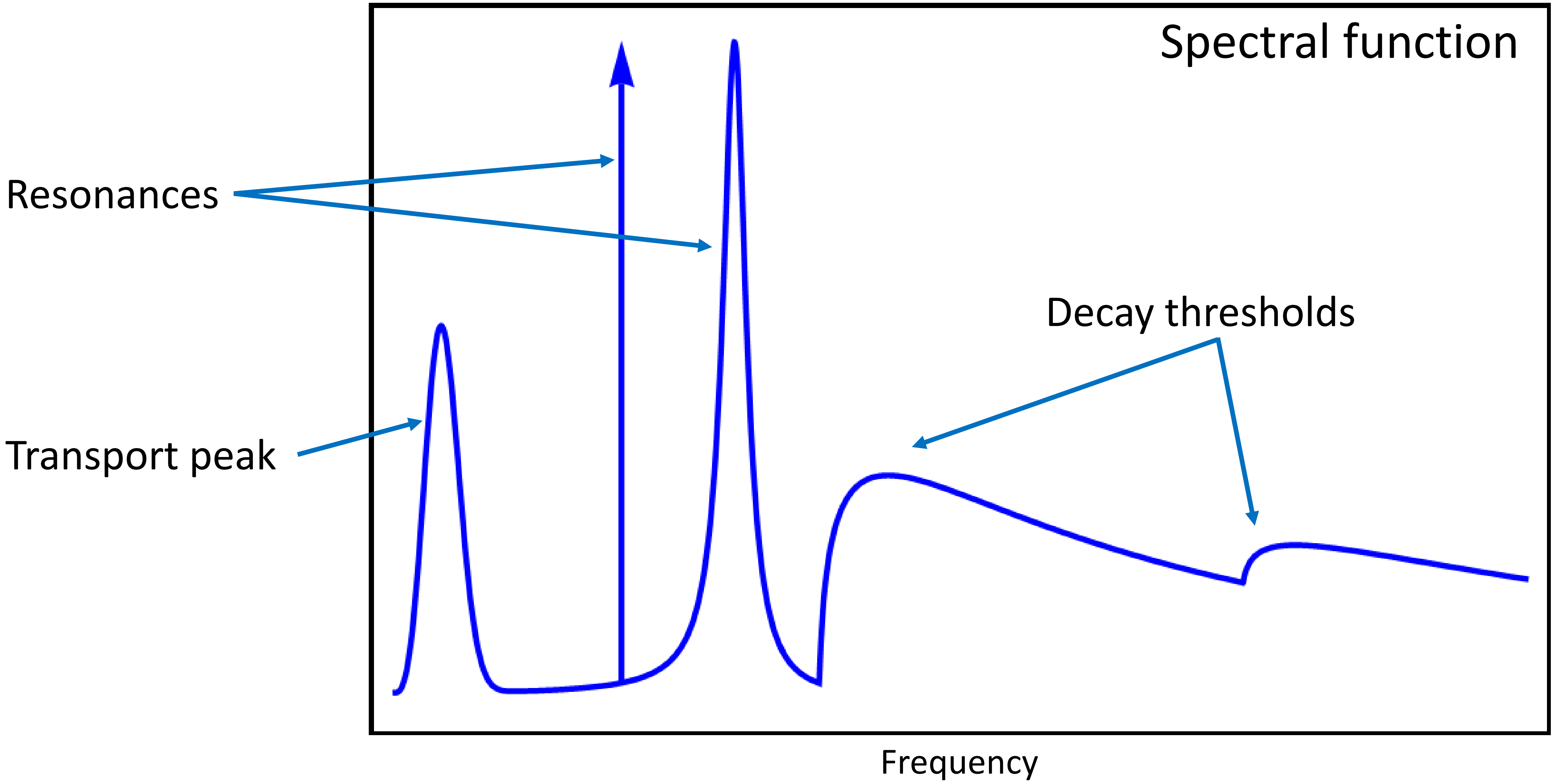
Nicolas Wink

Work in collaboration with:

M. Bluhm, A. K. Cyrol,
J. Horak, Y. Jiang,
M. Nahrgang, J. M. Pawlowski,
F. Rennecke, A. K. Rothkopf, …



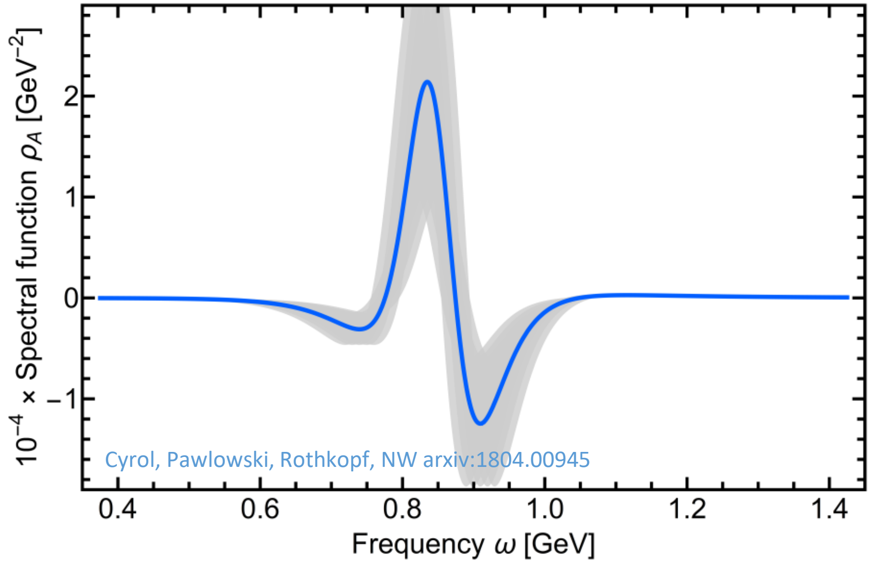
Spectral functions in QCD



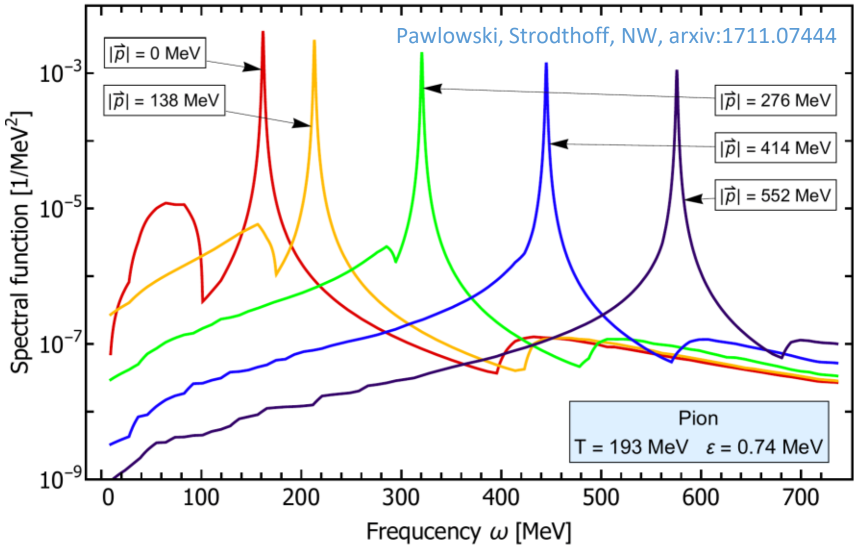
Spectral functions in QCD

How to get non-perturbative correlation functions in Minkowski space-time

Reconstruction from Euclidean data



Direct calculation

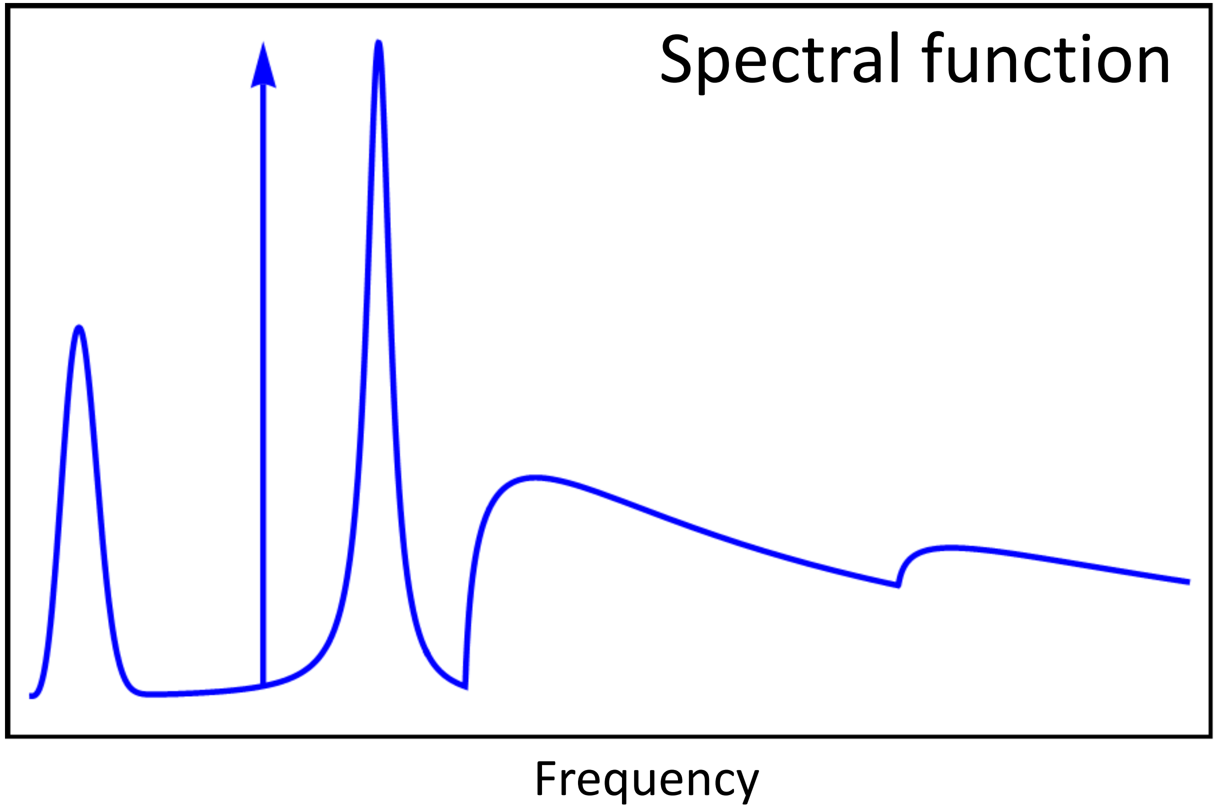


➡ Applications

Spectral functions

Spectral representation

What are spectral functions



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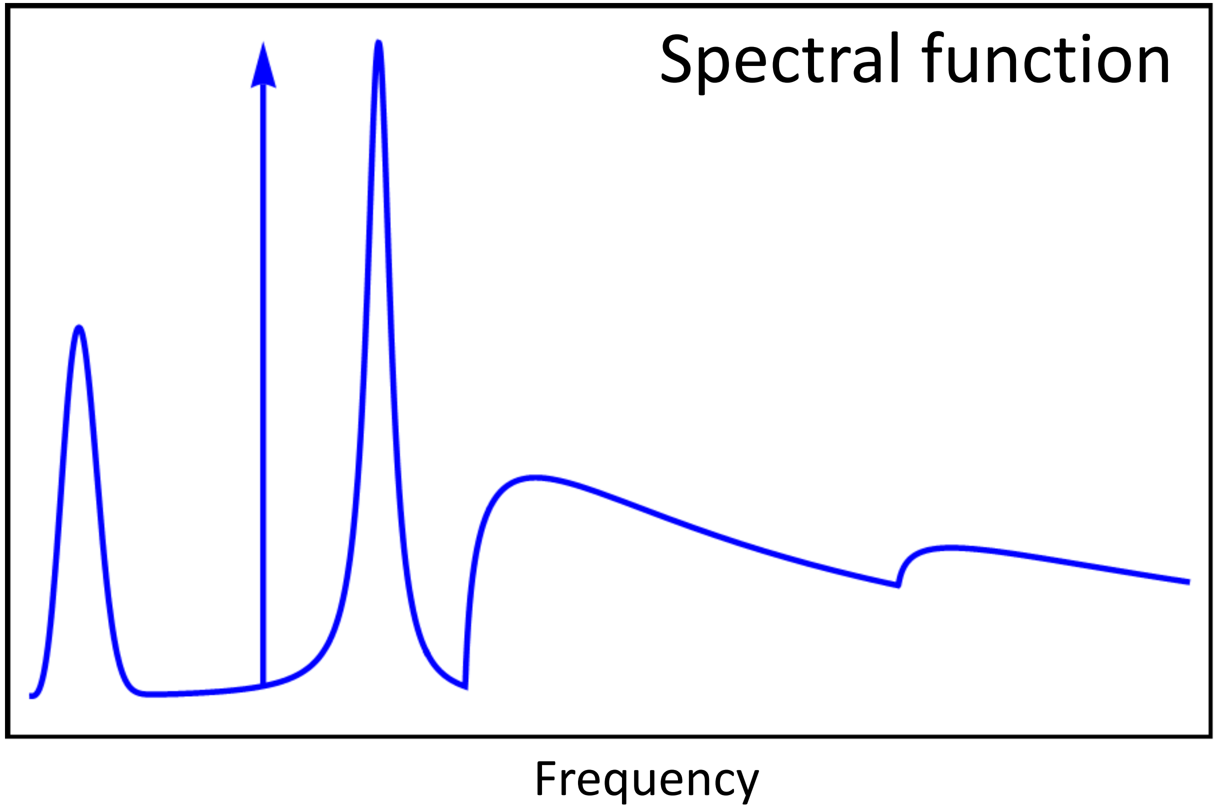
Physical picture :

- Encodes the spectrum of the theory
- Linear response functions



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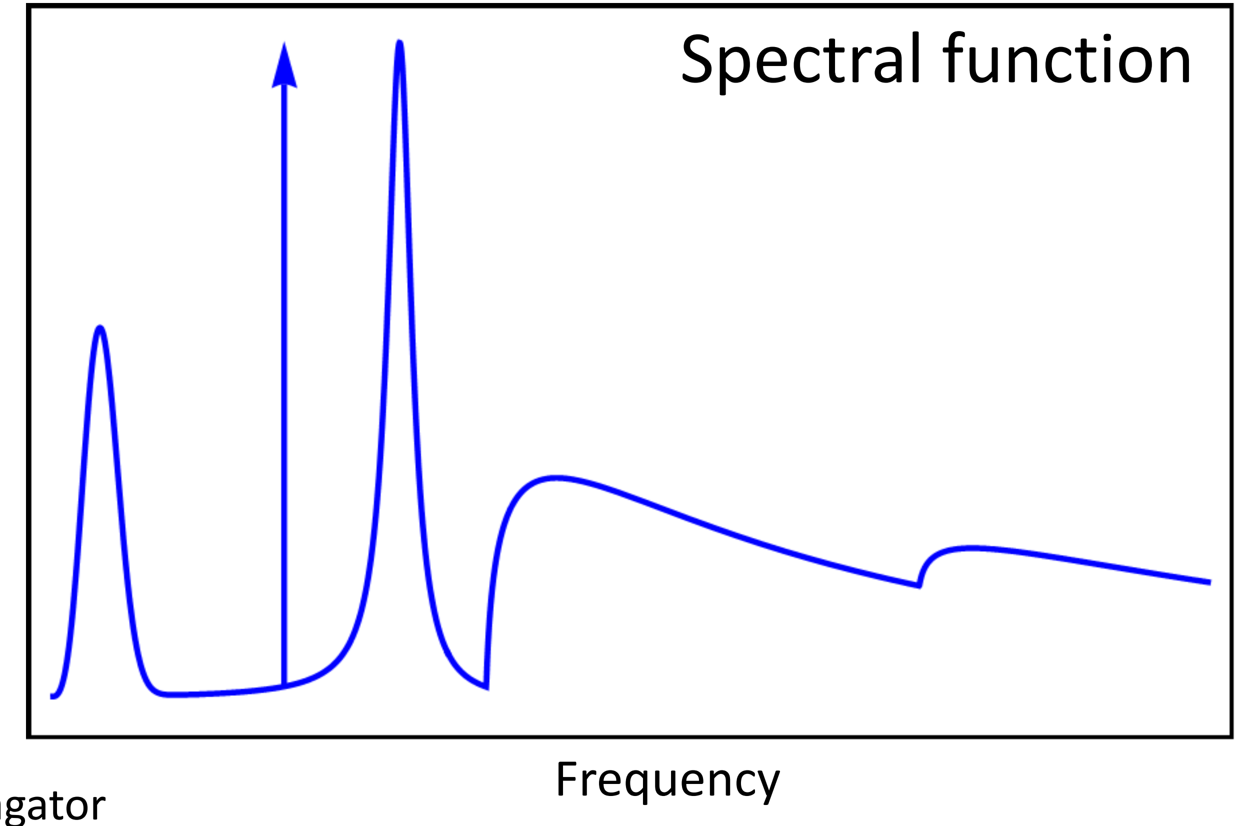
- $$G_E(p_0, \mathbf{p}) = \int \frac{\mathrm{d}\eta}{2\pi} \frac{\rho(\eta, \mathbf{p})}{\eta - \mathrm{i}p_0}$$

  Integral representation of the (Euclidean) propagator

- Statement about the analytic structure of the propagator



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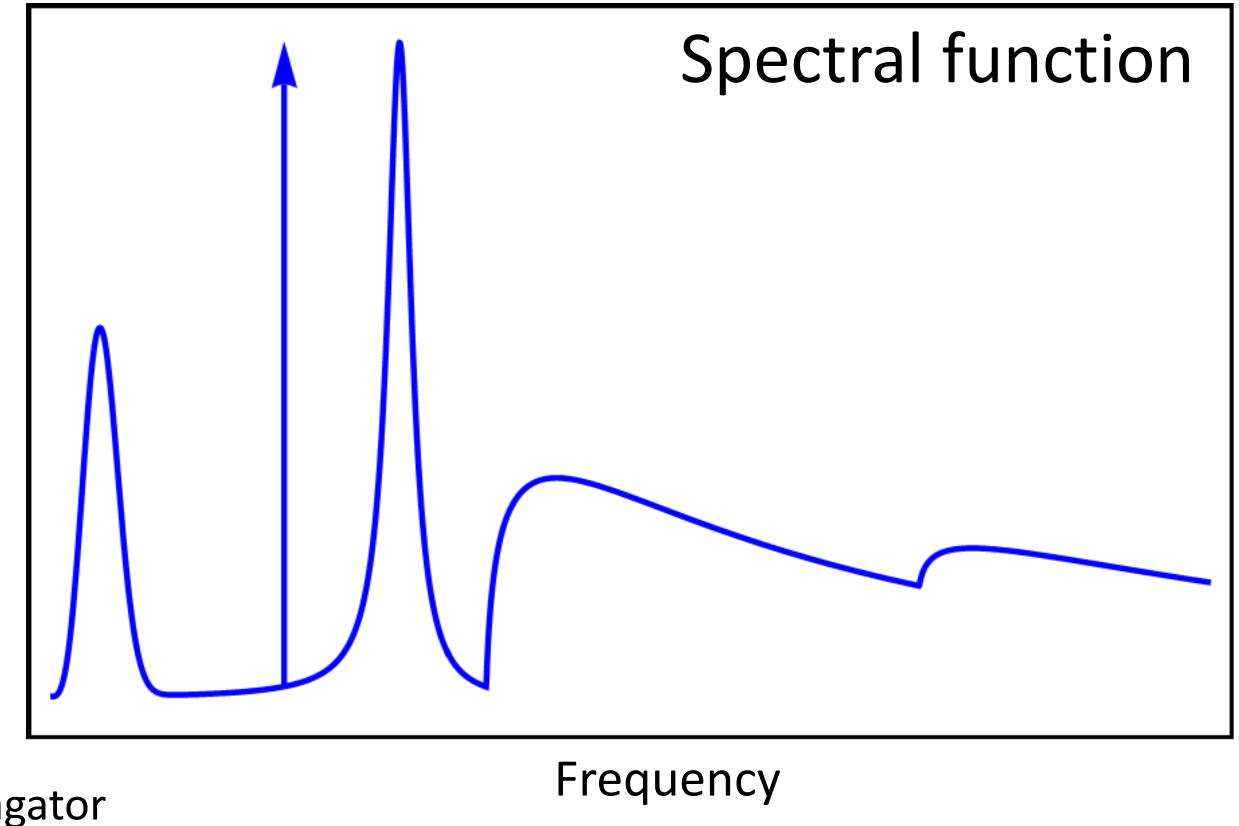

- Statement about the analytic structure of the propagator

Axiomatic/Mathematical picture :

- Existence linked to a restriction of the underlying functional space

c.f. talk of Peter Lowdon

Higher order spectral representations

What about vertices?

→ Vertices admit a spectral representation!

Evans, Phys.Lett. B249 (1990)
Evans, Nucl.Phys. B374 (1992)
Bodeker, Sangel, JCAP 1706 (2017)
Pawlowski, NW, work in progress

Higher order spectral representations

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→ **Vertices admit a spectral representation!**

Consider

$$\Gamma^{(n)}(p_1, p_2, \ldots, p_n)$$

Constrained by $\sum \varepsilon_i = 0$

Analytically continue with $p_i \to -\mathrm{i}(\omega_i + \mathrm{i}\varepsilon_i)$

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Three-point function

	$\varepsilon_1/\varepsilon$	$\varepsilon_2/\varepsilon$	$\varepsilon_3/\varepsilon$
$\Gamma^{(2)}_{\mathrm{RAA}}$	$+2$	-1	-1
$\Gamma^{(2)}_{\mathrm{ARA}}$	-1	$+2$	-1
$\Gamma^{(2)}_{\mathrm{AAR}}$	-1	-1	$+2$
$\Gamma^{(2)}_{\mathrm{ARR}}$	-2	$+1$	$+1$
$\Gamma^{(2)}_{\mathrm{RAR}}$	$+1$	-2	$+1$
$\Gamma^{(2)}_{\mathrm{RRA}}$	$+1$	$+1$	-2

Identities:

$$\Gamma^{(3)}_{\alpha\alpha\alpha} = 0 \quad \text{and} \quad \Gamma^{(3)}_{\alpha\beta\gamma} = \left(\Gamma^{(3)}_{\bar{\alpha}\bar{\beta}\bar{\gamma}}\right)^*$$

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preliminary

Spectral functions:

$$\rho_1 = 2\,\mathrm{Re}\left(\Gamma^{(3)}_{ARA} + \Gamma^{(3)}_{AAR}\right)$$

$$\rho_2 = 2\,\mathrm{Re}\left(\Gamma^{(3)}_{RAA} + \Gamma^{(3)}_{AAR}\right)$$

Degenerate for identical fields:

$$\rho_1(\eta_1,\eta_2) = \rho_2(\eta_2,\eta_1)$$

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Generalizes to n-point functions

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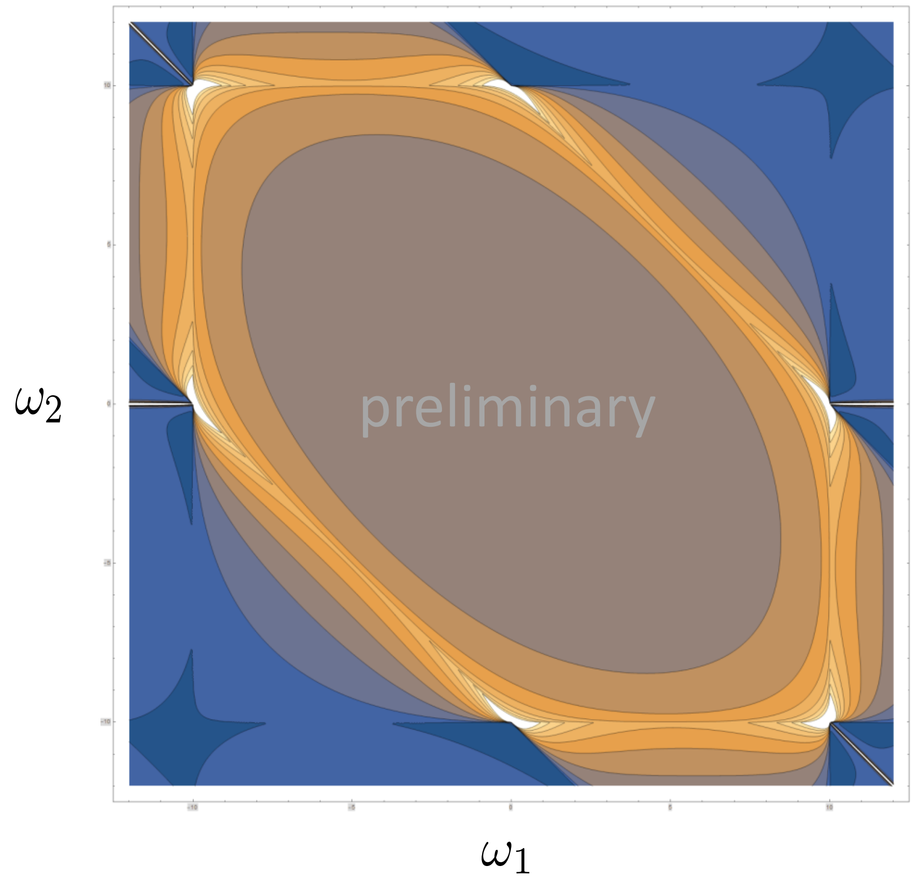
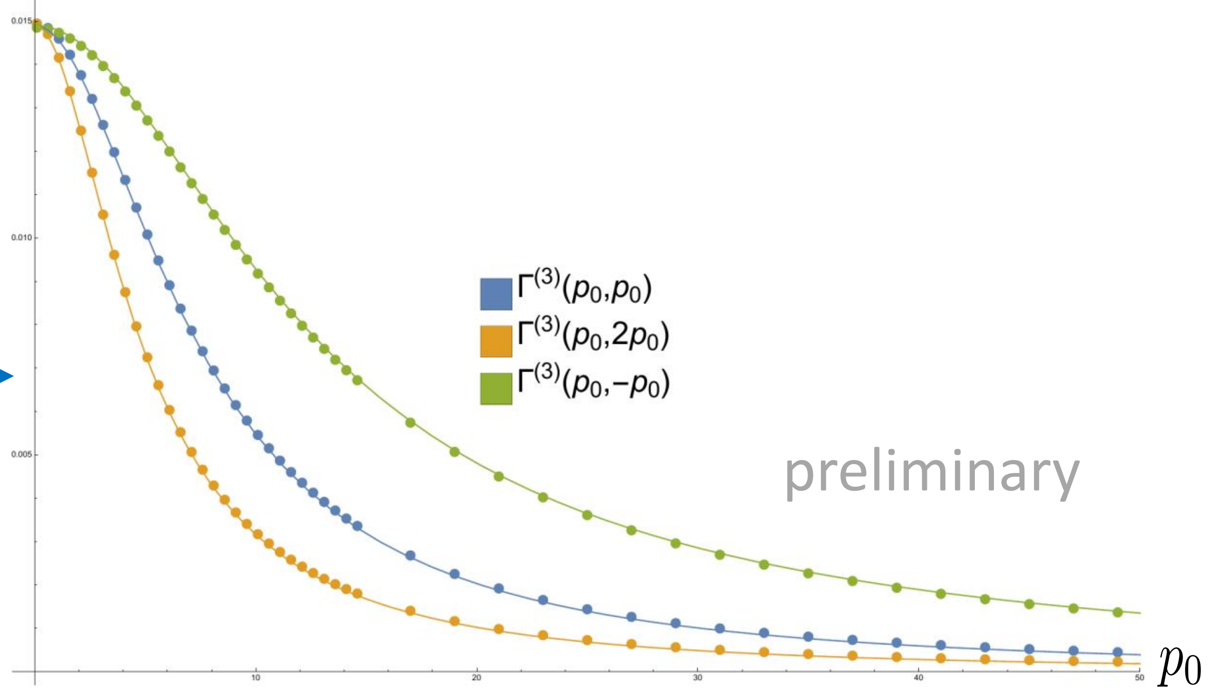
Application to scalar field

ϕ³-theory, 1-loop Perturbation theory from the FRG

<u>Spectral function</u>



<u>Euclidean Dressing</u>



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Direct Calculation

Implications of the analytic structure

Applies to all functional methods (e.g. pert. theory, FRG, DSE, 2PI, …)

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c.f. talk of Gernot Eichmann

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- Unique integration prescription can be obtained by a smooth deformation of the Euclidean path (keep all residues)

$$G_E(p_0, \mathbf{p}) = \int_0^\infty \frac{\mathrm{d}\eta}{\pi} \eta \frac{\rho(\eta, \mathbf{p})}{\eta^2 + p_0^2}$$

Map cuts to poles via their spectral representations

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Map cuts to poles via their spectral representations

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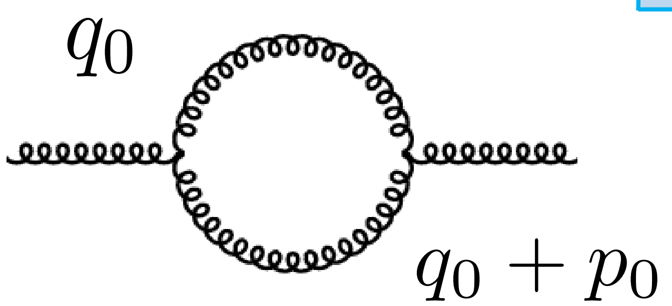
- ⇒ Unique integration prescription can be obtained by a smooth deformation of the Euclidean path (keep all residues)
- ⇒ Analytic continuation problem at finite temperature resolved by demanding preservation of this structure

Baym, Mermin, Journal of Mathematical Physics 2, 232 (1961)

Evans, Nucl.Phys. B374 (1992)

With spectral representation

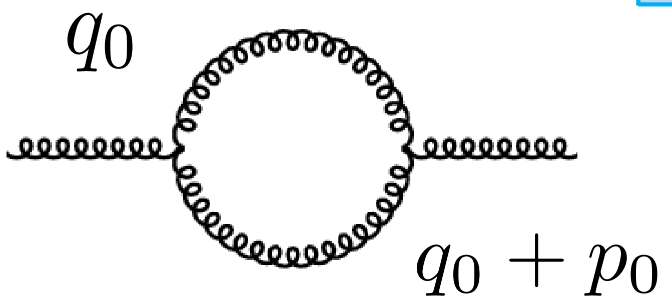


Vacuum polarization

With spectral representation

Sufficient to consider frequency dependence:

- Contains full information in vacuum due to Lorentz invariance
- Independent, relevant variable at finite Temperature/chemical Potential



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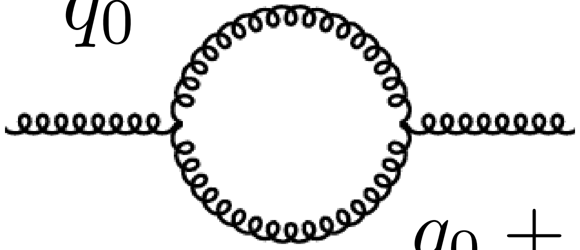

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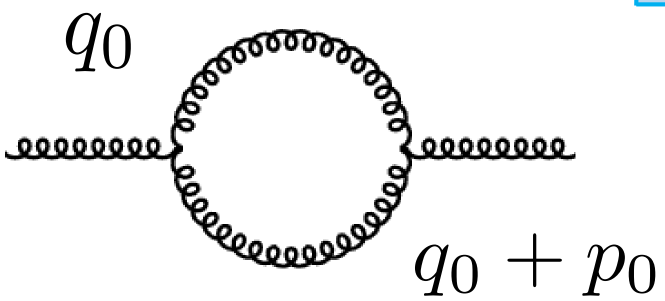
$$\mathcal{D}(p_0) = (2\pi)^{-d} \int \mathrm{d}^d q \; G(q) G(p+q)$$

With spectral representation



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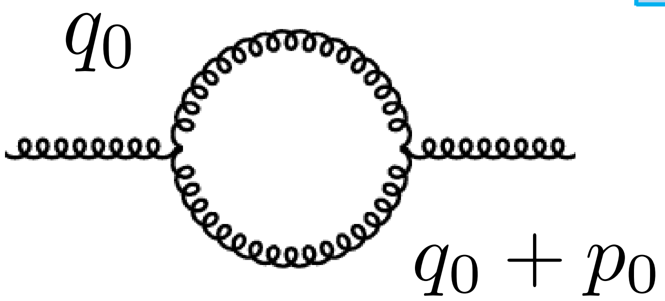

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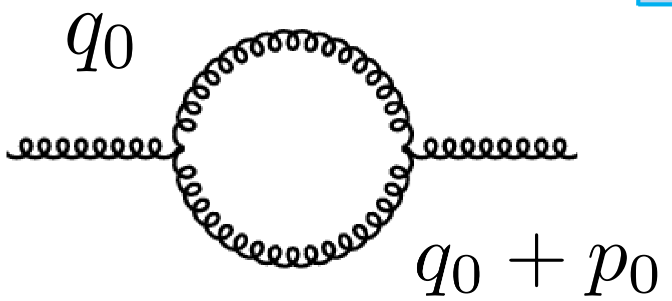
$$\mathcal{D}(p) = 4\,(2\pi)^{-d-2} \int_{\eta_1, \eta_2 > 0} \eta_1 \eta_2 \rho(\eta_1) \rho(\eta_2) \underbrace{\int \mathrm{d}^d q \; \frac{1}{q^2 + \eta_1^2} \frac{1}{(q+p)^2 + \eta_2^2}}$$

Perturbative integral with arbitrary masses

Without spectral representation



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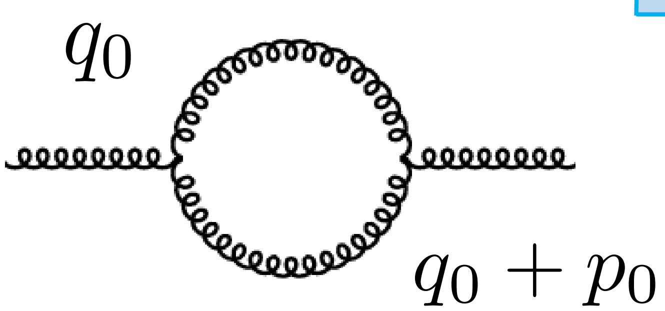

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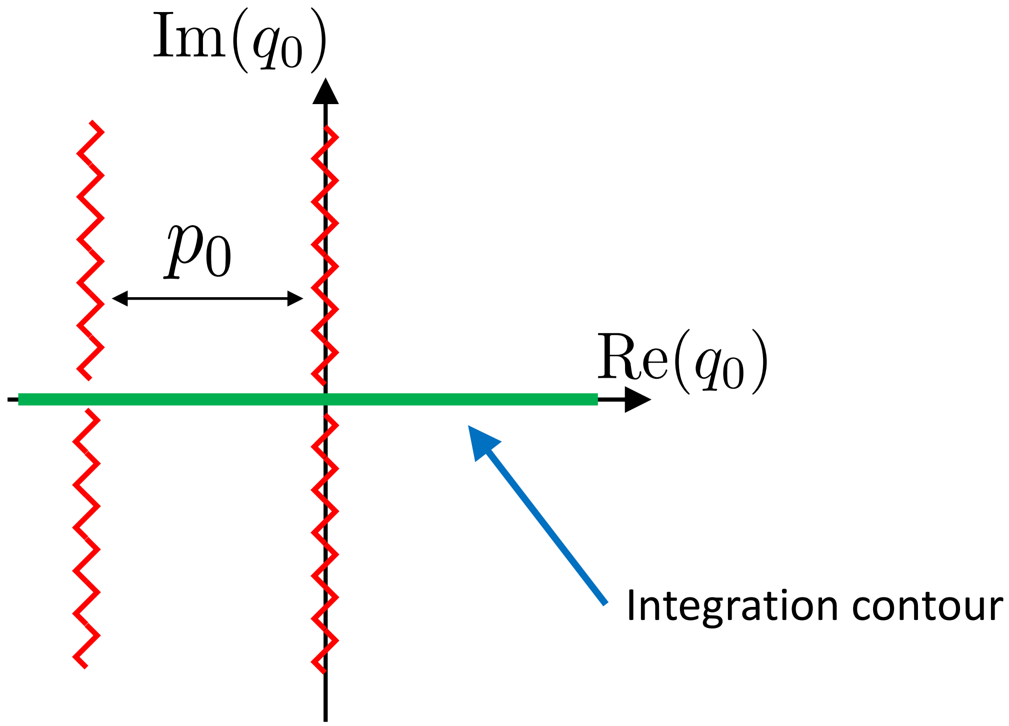

Analytic structure polarization diagram

Without spectral representation

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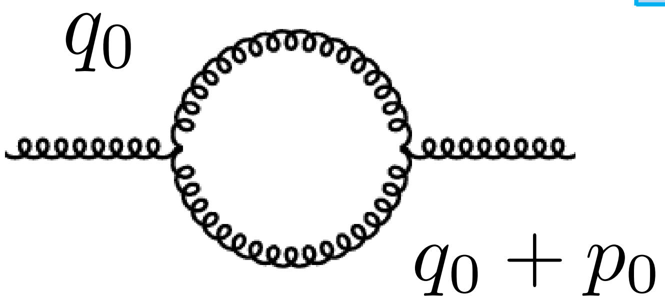

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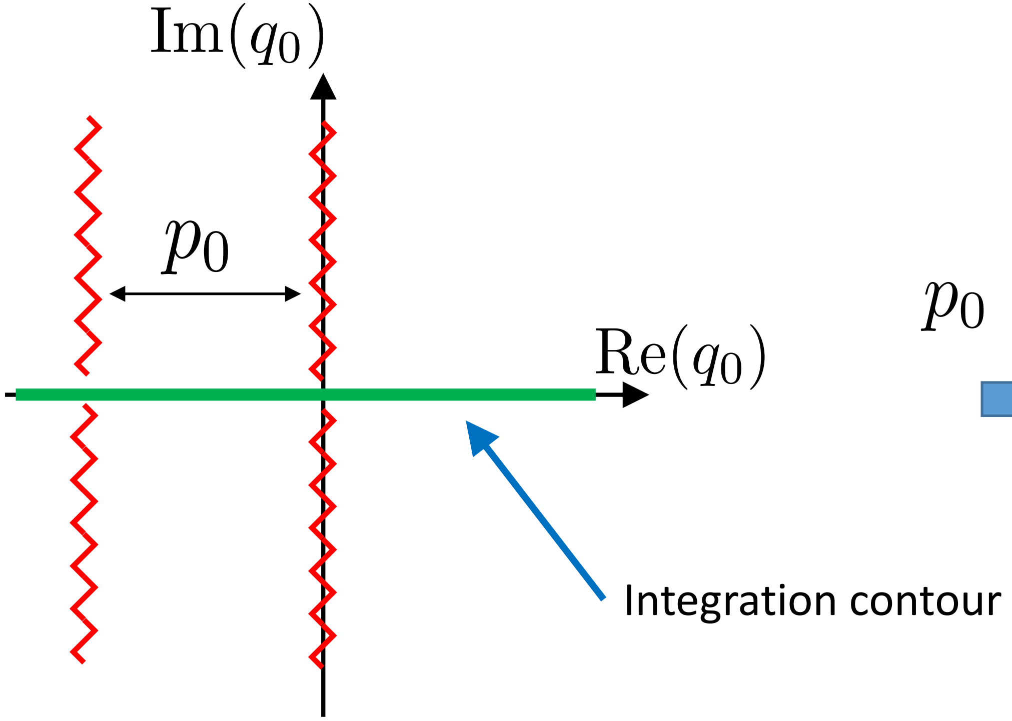
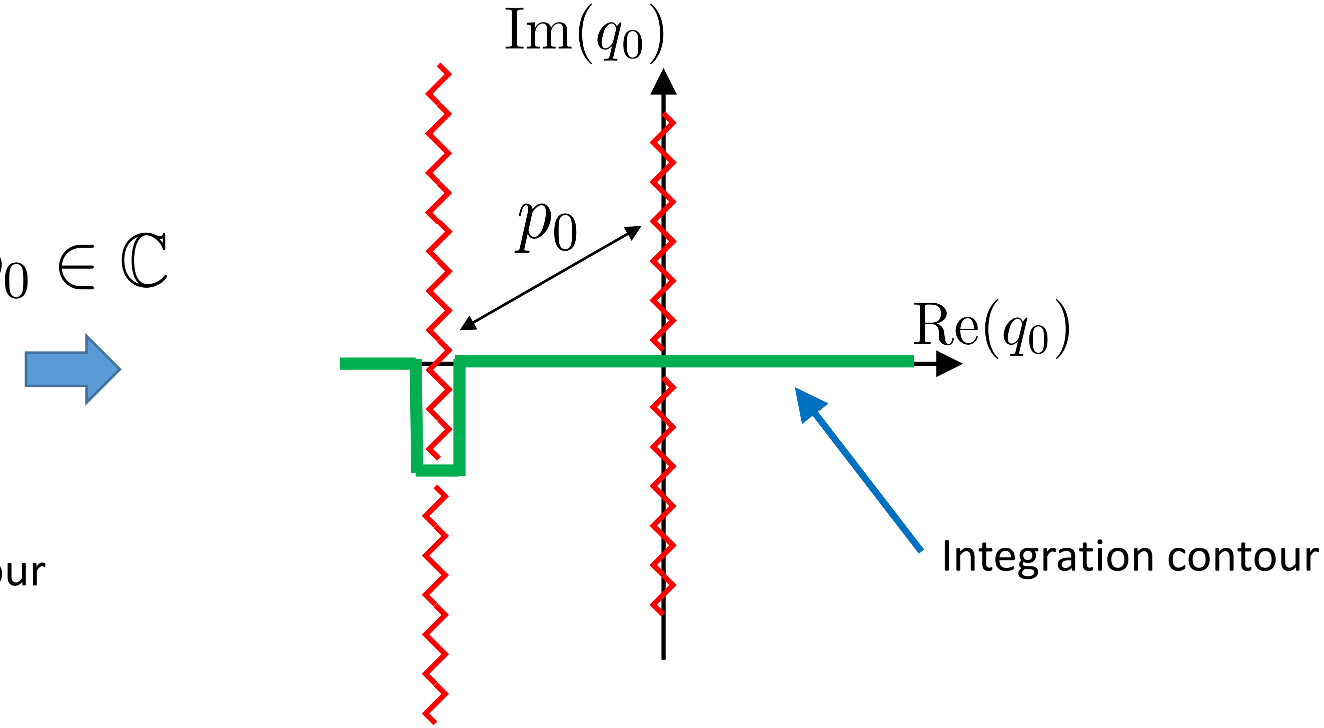


Analytic structure polarization diagram

$p_0 \in \mathbb{C}$



c.f. talk of Gernot Eichmann

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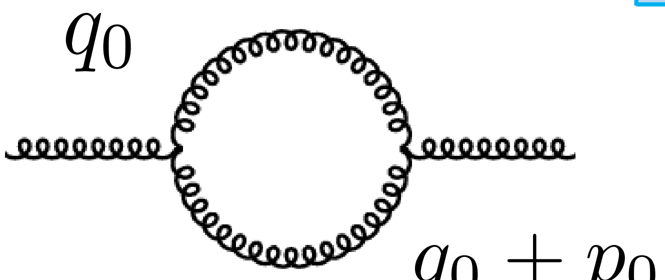

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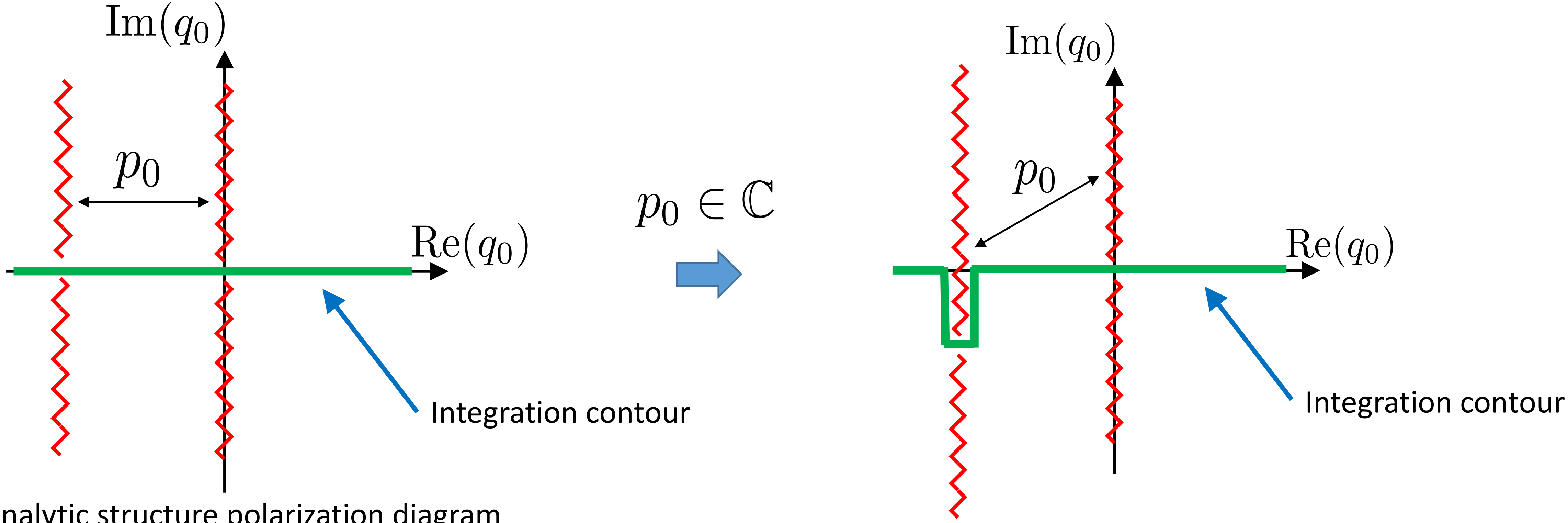

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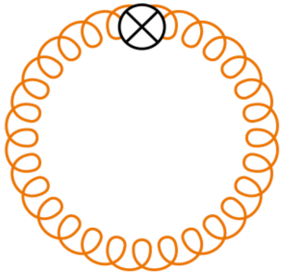
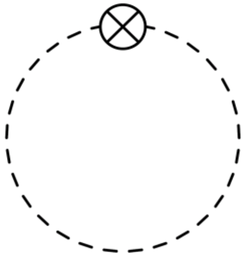
$$\partial_t \Gamma_k[\Phi] = \frac{1}{2}$$  $$-$$ 

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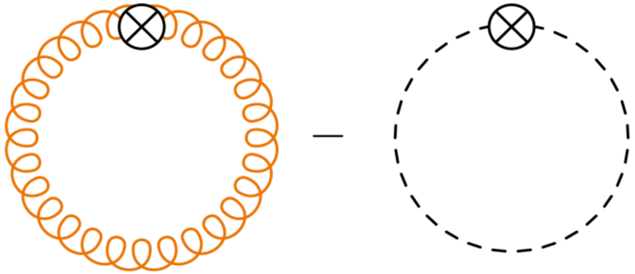
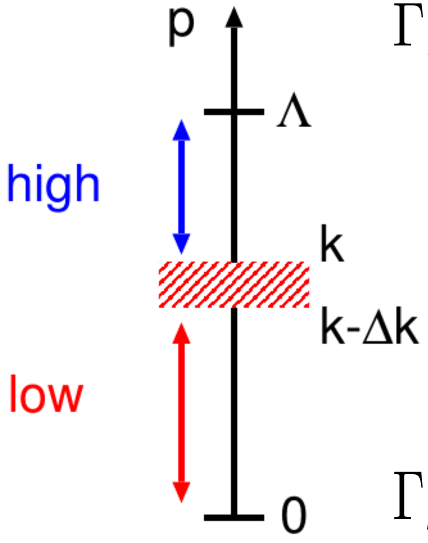
$\partial_t \Gamma_k[\Phi] = \frac{1}{2}$ 



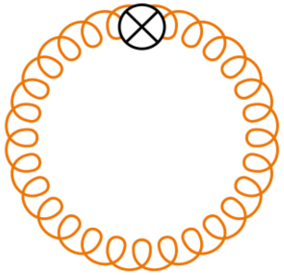
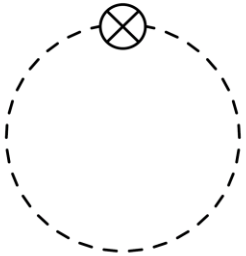
$\Gamma_{k=\Lambda} = S$

- Non-perturbative first principle method
- Access to physical mechanisms
- No sign problem
  - Chemical potential
  - Real time

$\Gamma_{k=0} = \Gamma$

Functional Renormalization Group

Regulator derivative

$\partial_t \Gamma_k[\Phi] = \frac{1}{2}$  $-$ 

Dressed regulated propagator ⟷ All quantities fully dressed

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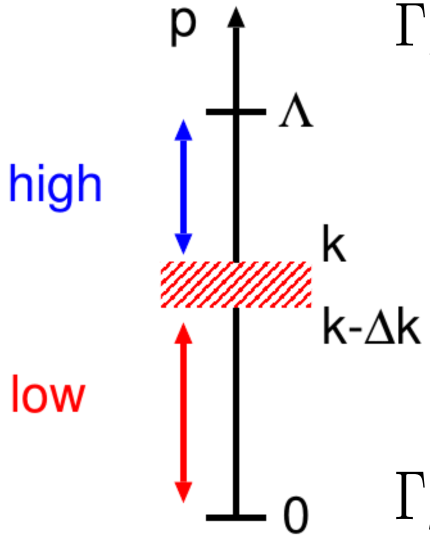

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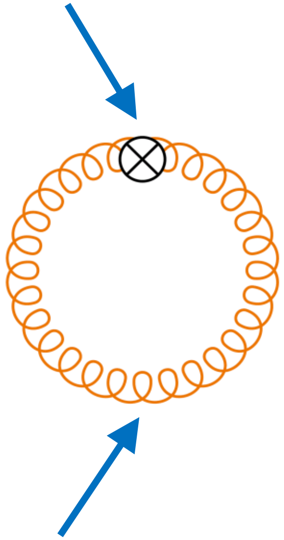
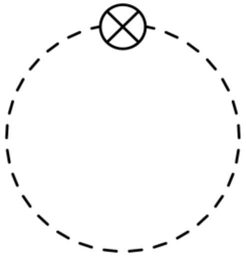
$\partial_t \Gamma_k[\Phi] = \frac{1}{2}$  $-$ 

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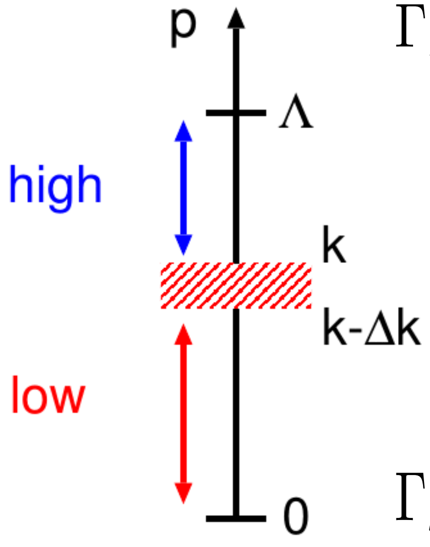

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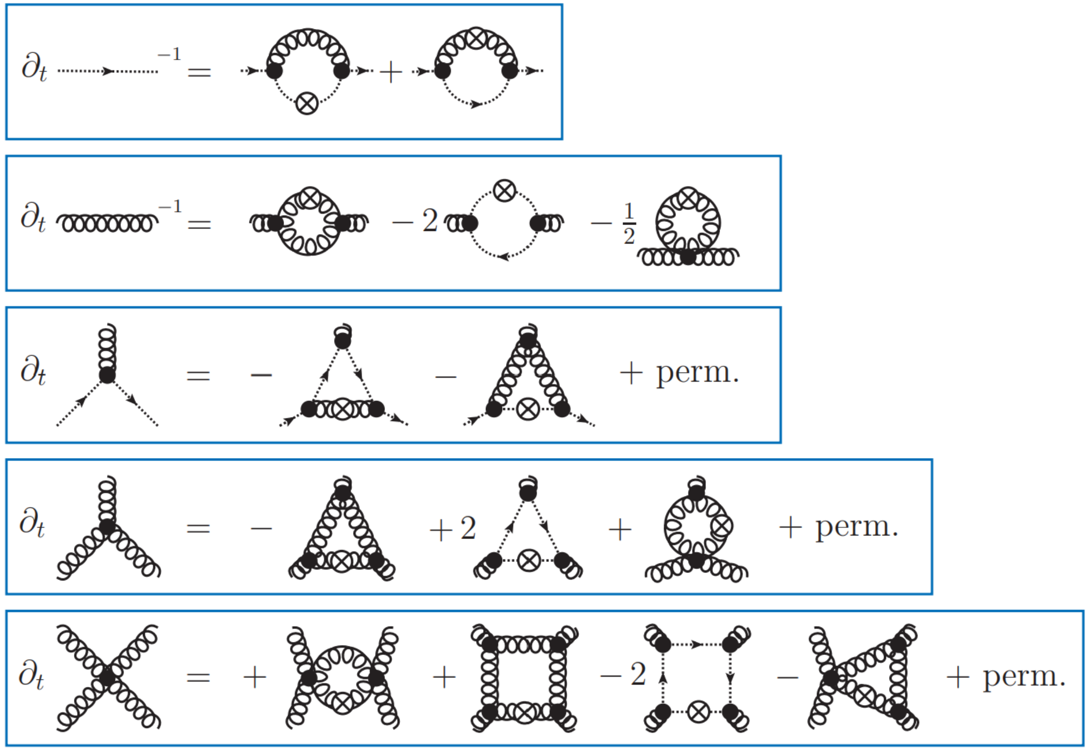
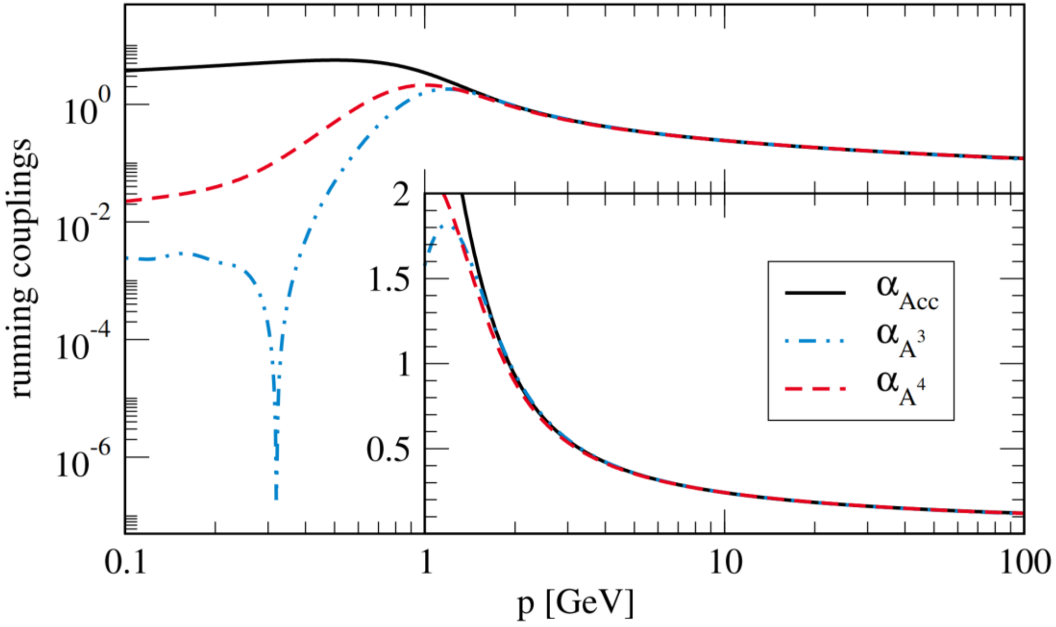
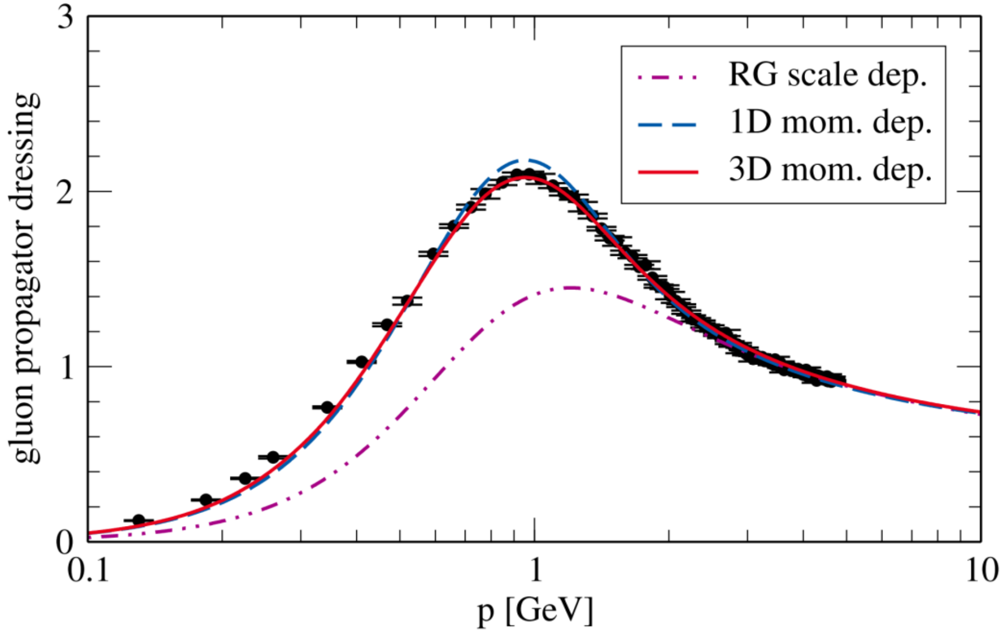
No new (major) conceptual problems

Yang-Mills theory







Cyrol, Fister, Mitter, Pawlowski, Strodthoff, Phys.Rev. D94 (2016)

Regulator

Regulator introduces inconviences

→ Analytic structure of (full) regularized propagators altered

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see e.g. Foerchinger, JHEP 1205 (2012)
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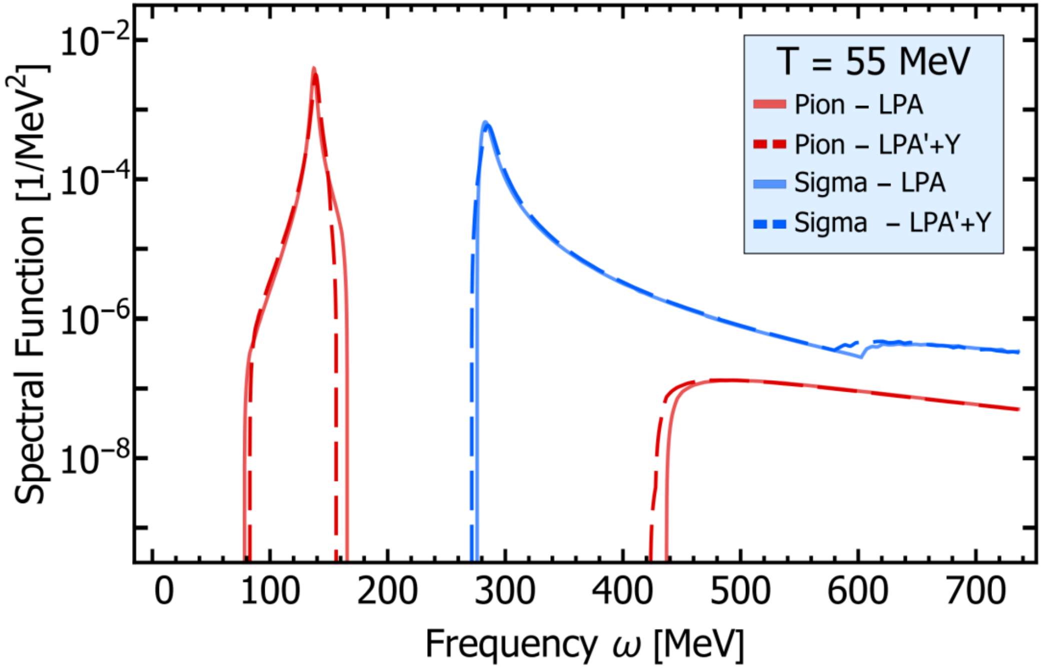

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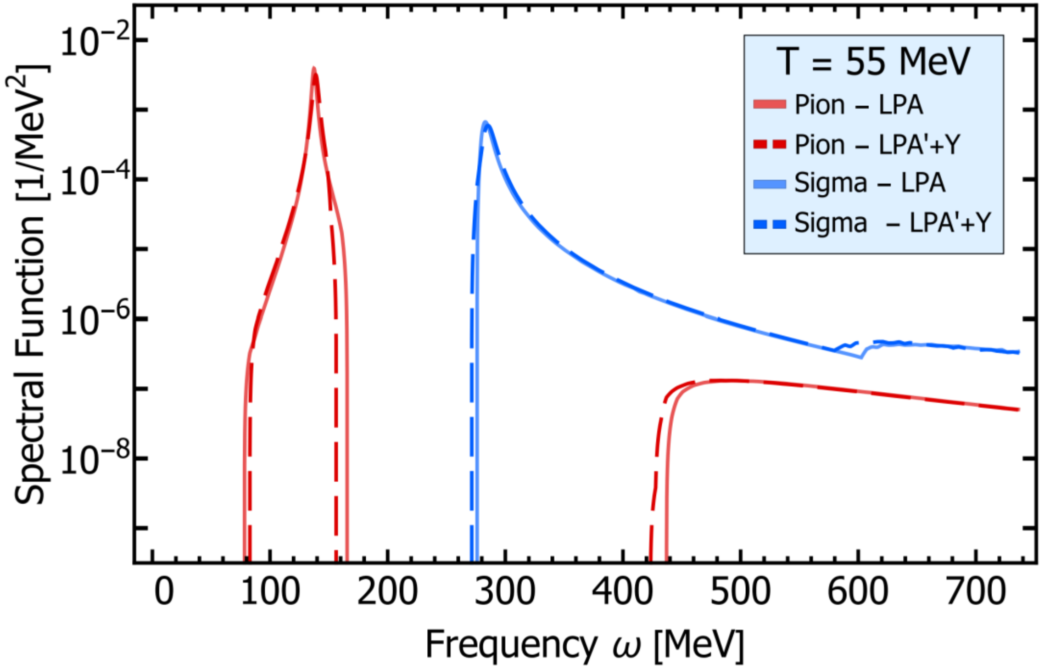

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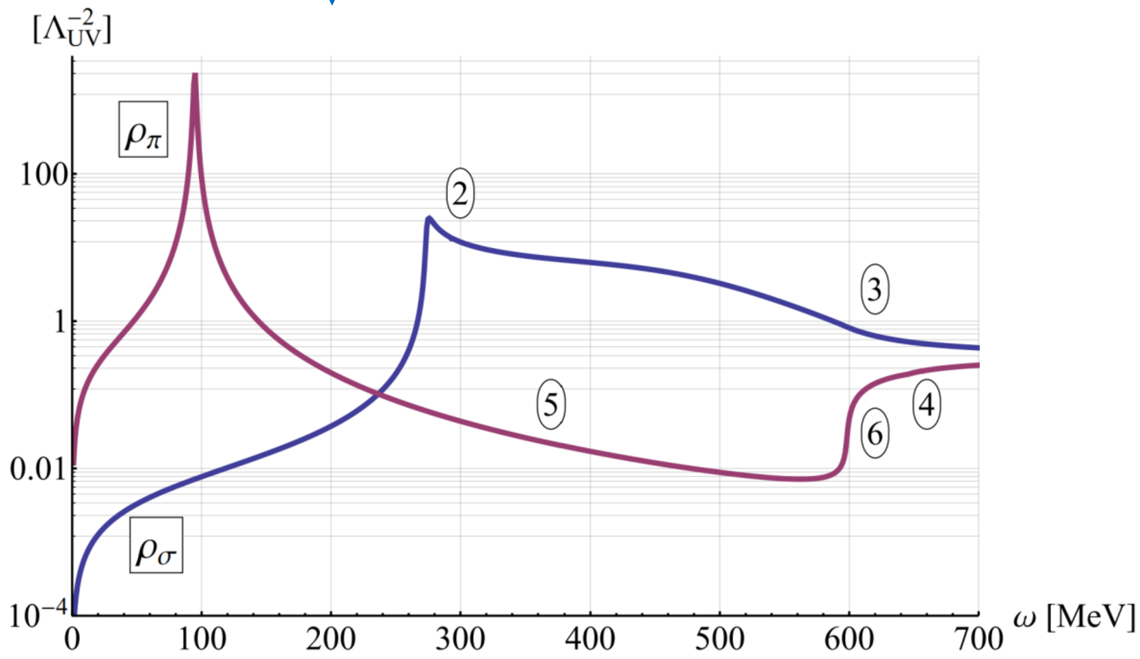

Tripolt, Strodthoff , von Smekal, Wambach, PRD89 (2014)

Application 1

-

Low energy effective theory of QCD

Transport approach to QCD

Transport approach to QCD

➡ Describe non-equilibrium QCD in the linear response regime around an equilibrium state

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- Describe non-equilibrium QCD in the linear response regime around an equilibrium state
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$$\frac{\delta\Gamma}{\delta\sigma} = \xi$$

Quantum equation of motion

Noise field (Dissipation-Fluctuation)

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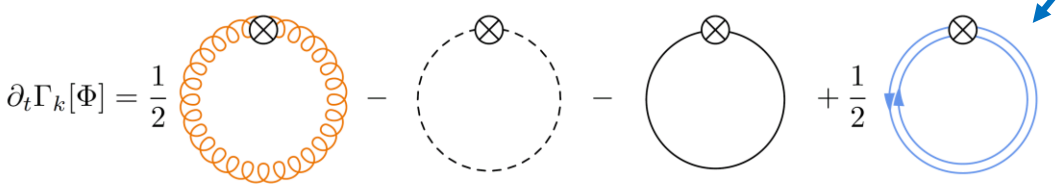
Bound states efficiently taken into account via Dynamical Hadronization

Flow equation for QCD



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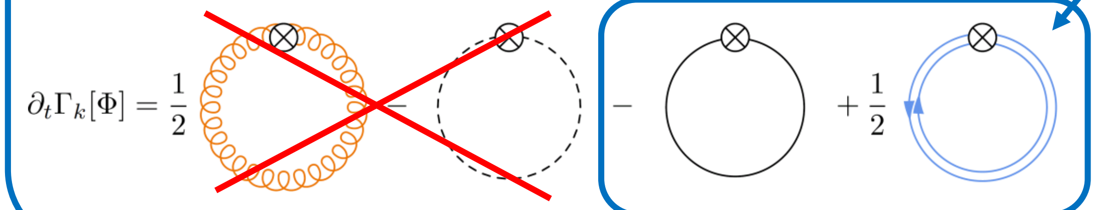
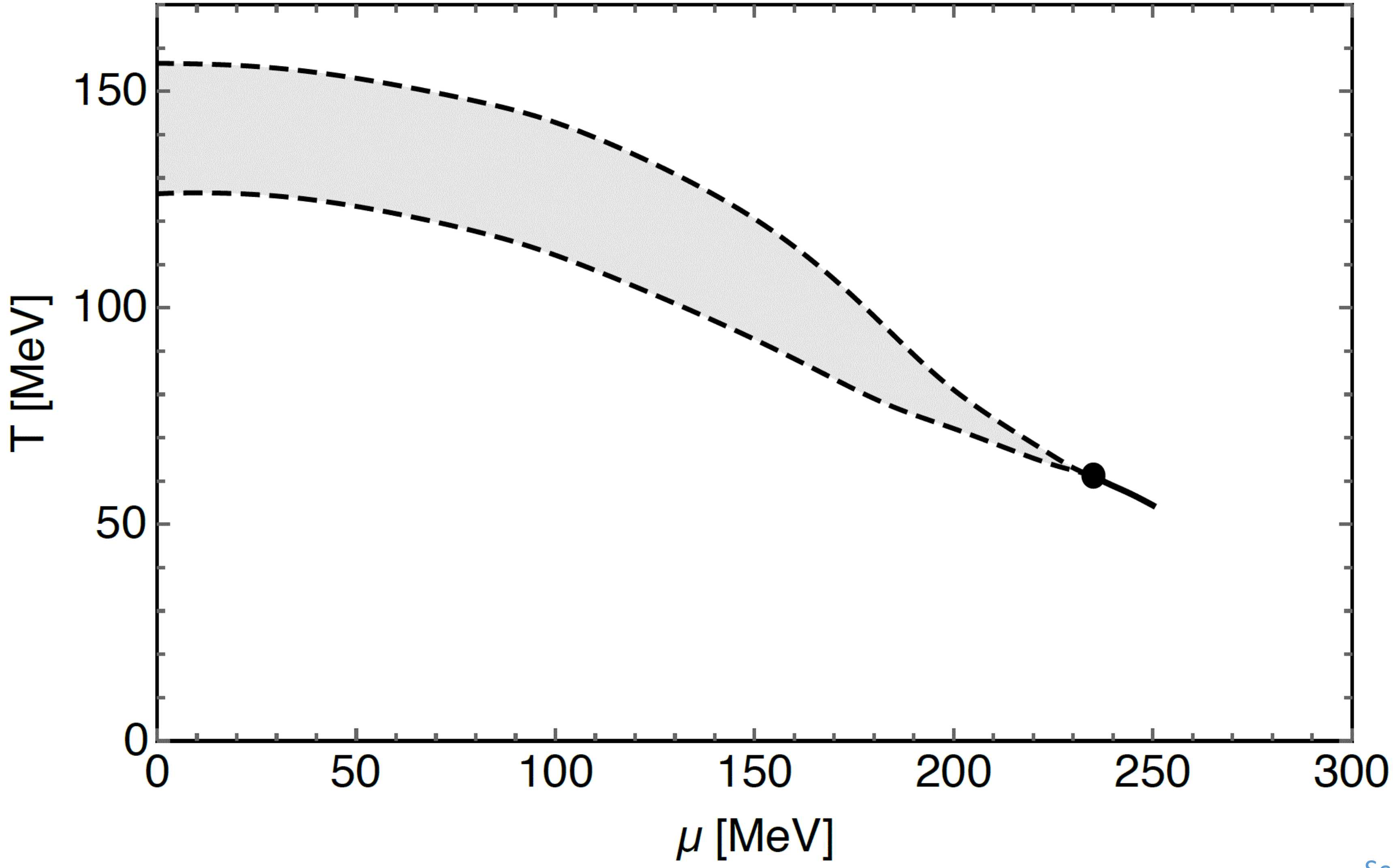


Low-energy effective theory of QCD

Phase structure contains a critical endpoint



Schaefer, Rennecke, PRD96 (2017)

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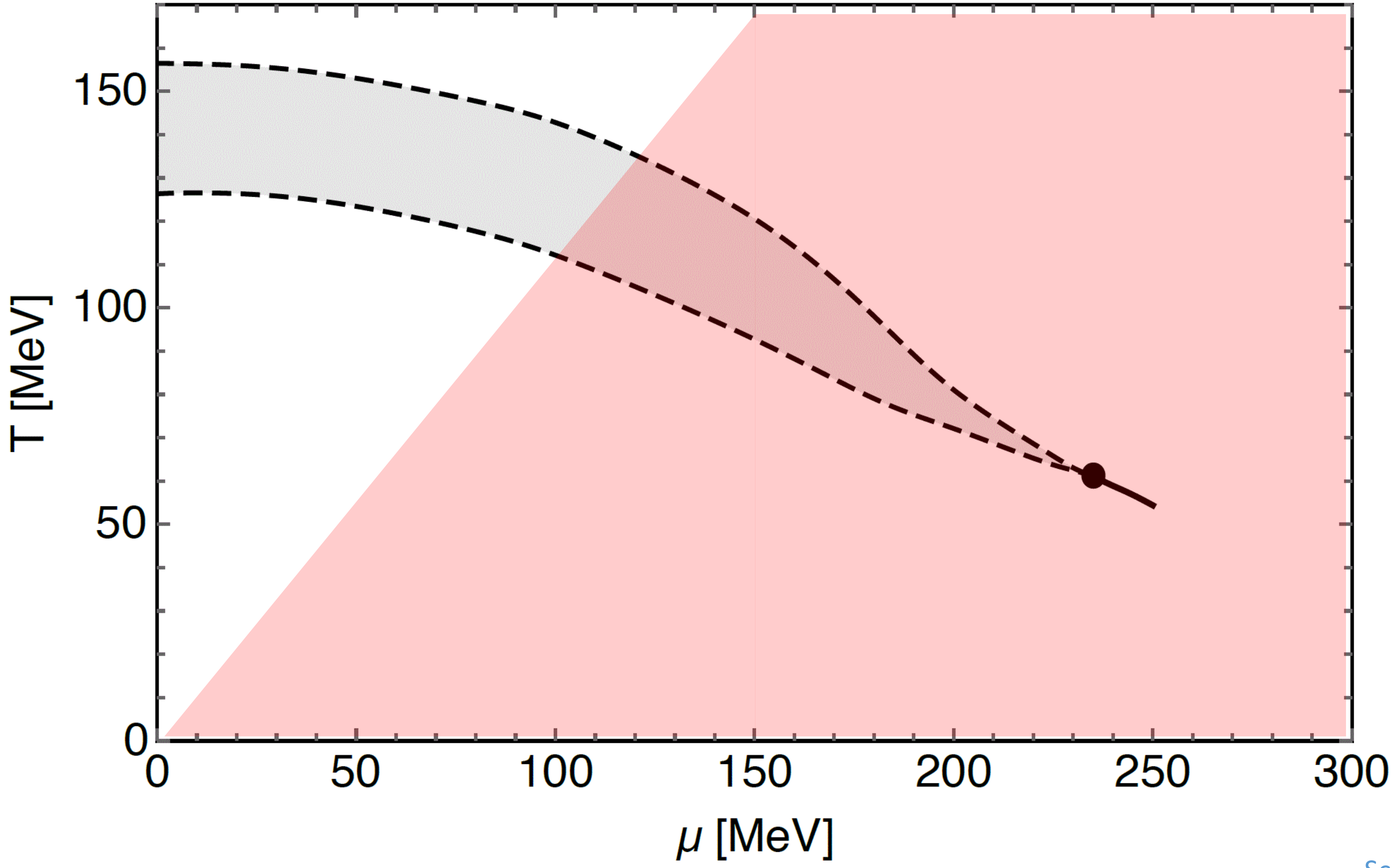
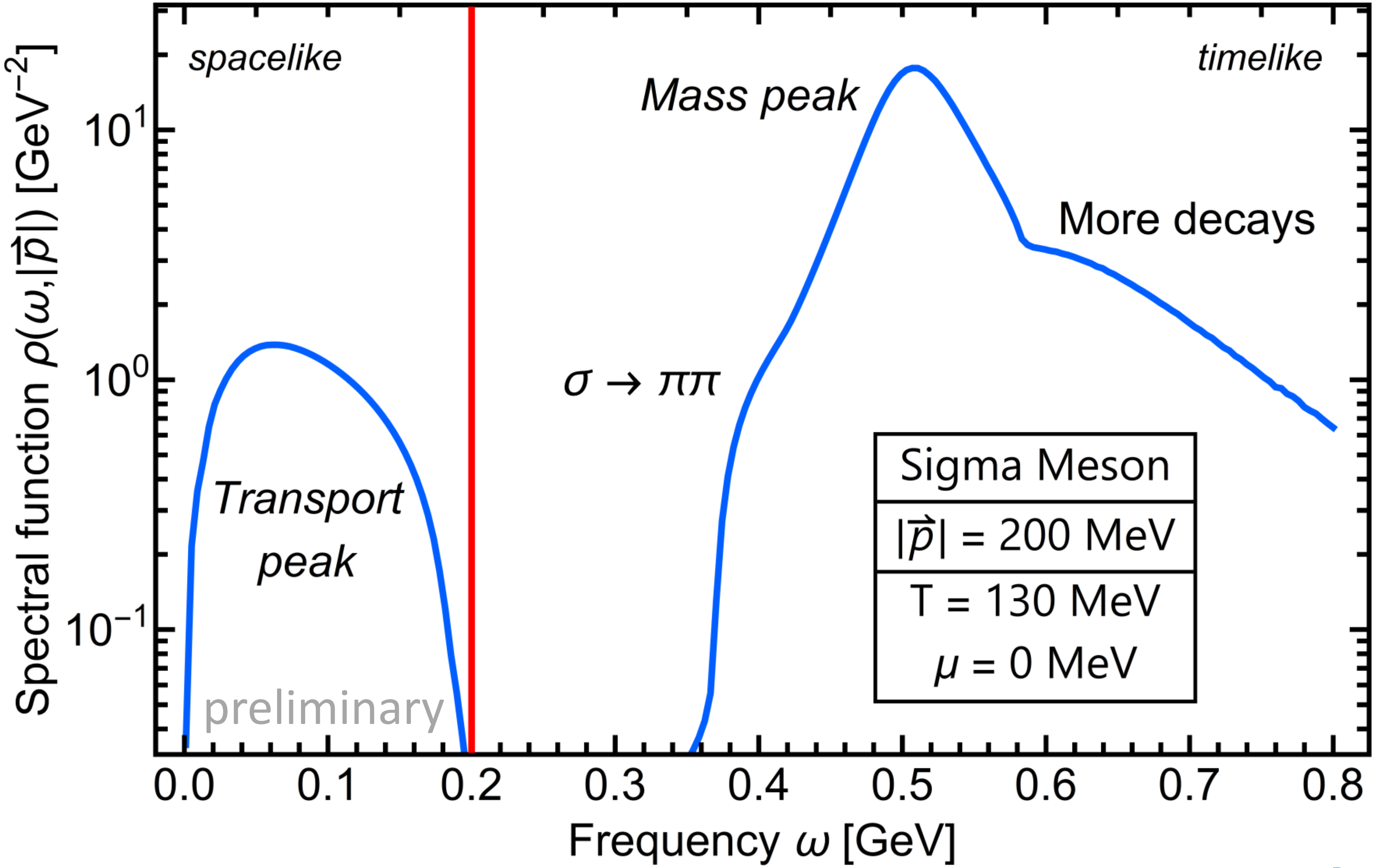


Schaefer, Rennecke, PRD96 (2017)

Linear response function

Sigma meson spectral function at T = 130 MeV and vanishing chemical potential



Pawlowski, Rennecke, NW, in prep.

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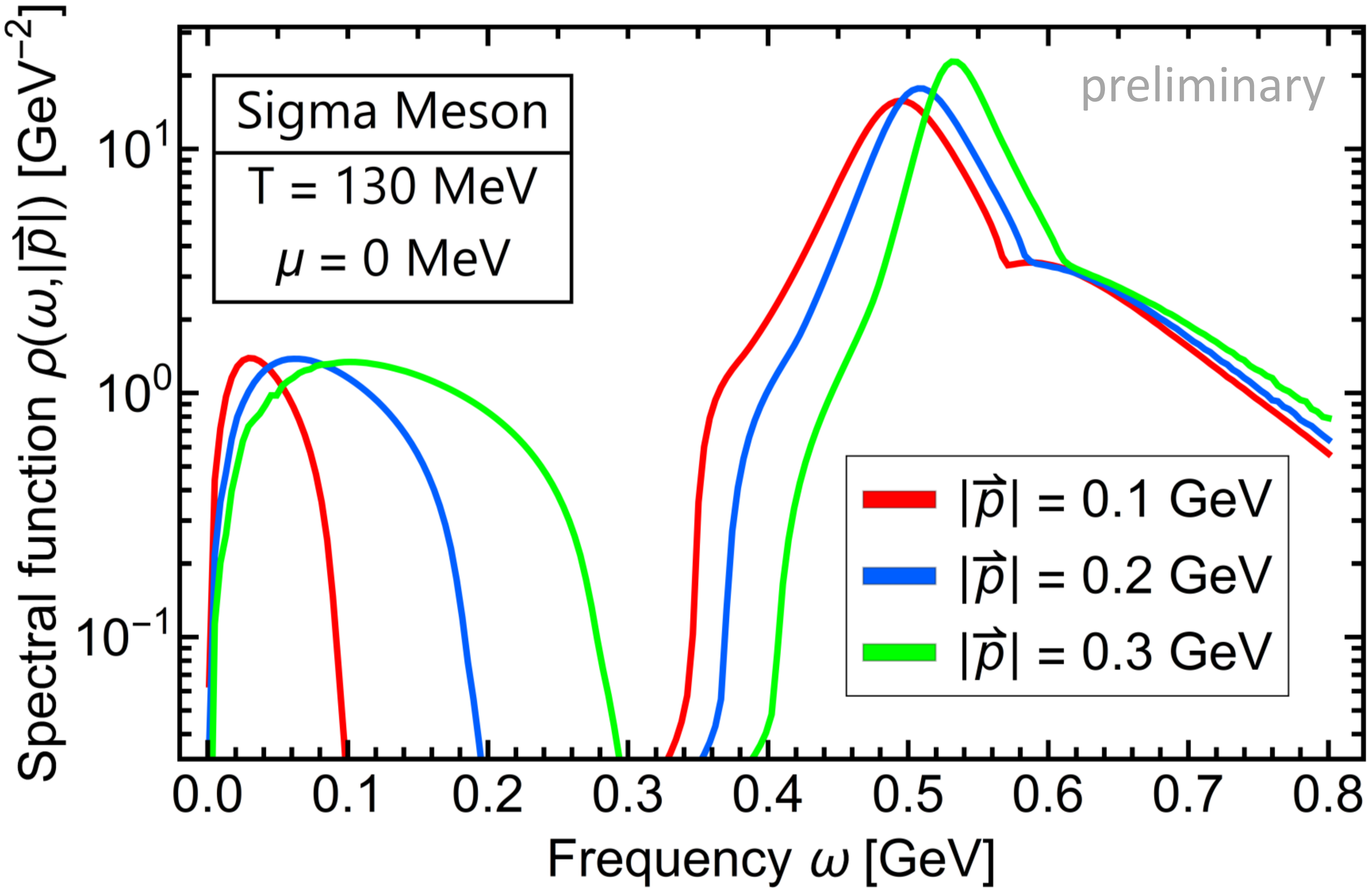

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Transport equation

Evolution governed by transport equation:

$$\frac{\delta\Gamma}{\delta\sigma} = \xi$$

with

$$\left\{ \mathrm{Re}\,\Gamma^{(2)}_{\sigma}(\omega,\vec{p}),\ \mathrm{Im}\,\Gamma^{(2)}_{\sigma}(\omega,\vec{p}),\ U(\sigma) \right\} \in \Gamma$$

$$\sigma(r,t) = \sigma_0 + \delta\sigma(r,t)$$

Split into equilibrium and fluctuation part

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Split into equilibrium and fluctuation part

White noise approximation:

$$\langle \xi(t) \rangle = 0$$

$$\langle \xi(t)\xi(t') \rangle = \frac{1}{V}\delta(t-t')m_\sigma \eta \coth\left(\frac{m_\sigma}{2T}\right)$$

Spatial isotropy approximation:

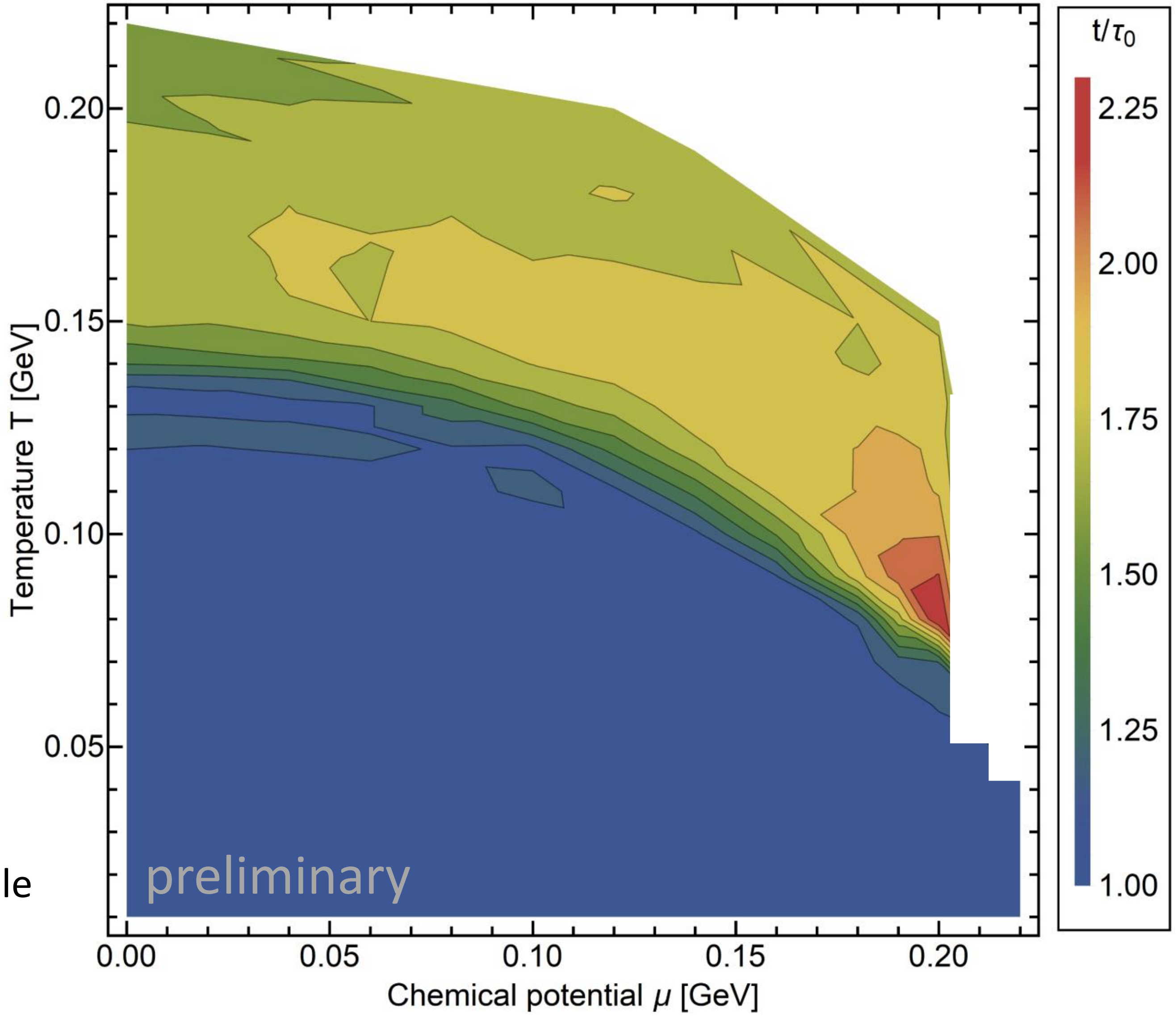
$$\sigma(\vec{x}) = \sigma(r)$$

Initial conditions:

Quench from „high temperature“

$$\sigma(r) = 0 = \partial_t \sigma(r)$$

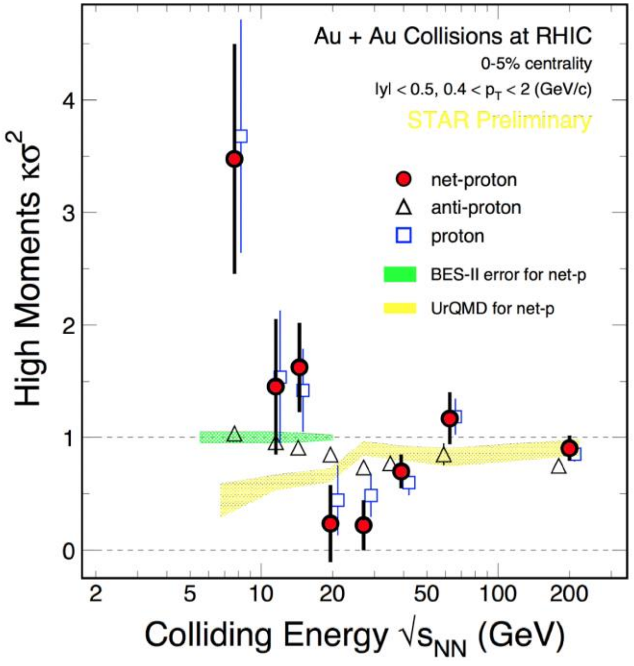
Equilibration time



Reference time scale
$\tau_0 \approx 0.4$ fm/c

Bluhm, Jiang, Nahrgang, Pawlowski, Rennecke, NW, in prep.

- Critical endpoint and phase boundary clearly identifiable
- Critical slowing down at the critical endpoint
- Impact on observables?



Luo, Xu, Nucl.Sci.Tech. 28 (2017)

Application 2
-
Dyson-Schwinger equations

Dyson-Schwinger equations

Scalar ϕ^4-theory (in the broken phase)

Work with Jan Horak, Jan M. Pawlowski

Horak, Pawlowski, NW, wip

Dyson-Schwinger equations

→ Scalar ϕ^4-theory (in the broken phase)

Truncation:

- Full two-point function
- Classical vertices

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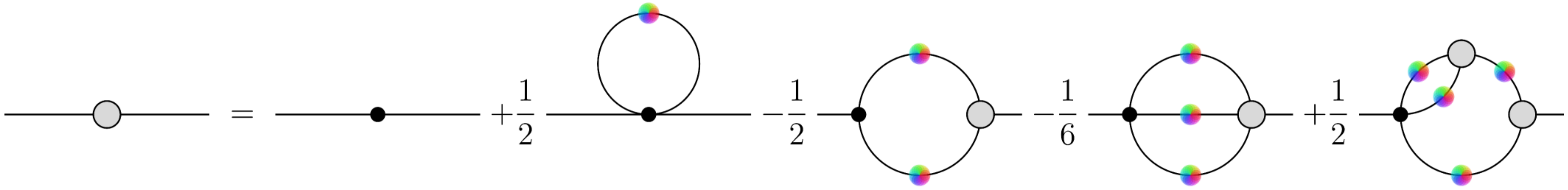

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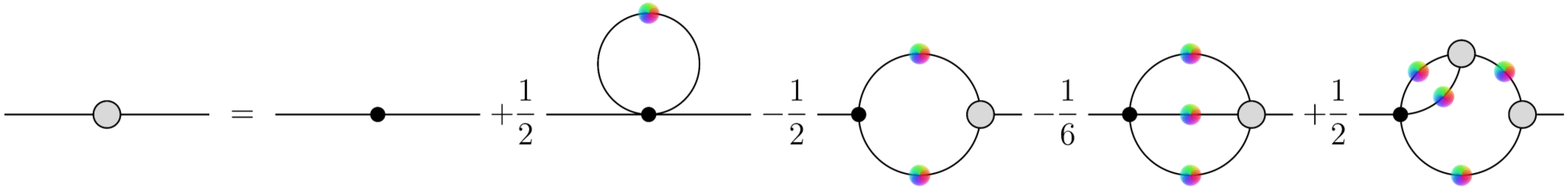

Insert spectral represenations for all propagators

$$\mathcal{D}(p) = 4\,(2\pi)^{-d-2} \int_{\eta_1,\eta_2>0} \eta_1\eta_2\rho(\eta_1)\rho(\eta_2) \underbrace{\int \mathrm{d}^d q \, \frac{1}{q^2+\eta_1^2}\frac{1}{(q+p)^2+\eta_2^2}}$$

Perturbative integral with arbitrary masses

Horak, Pawlowski, NW, wip

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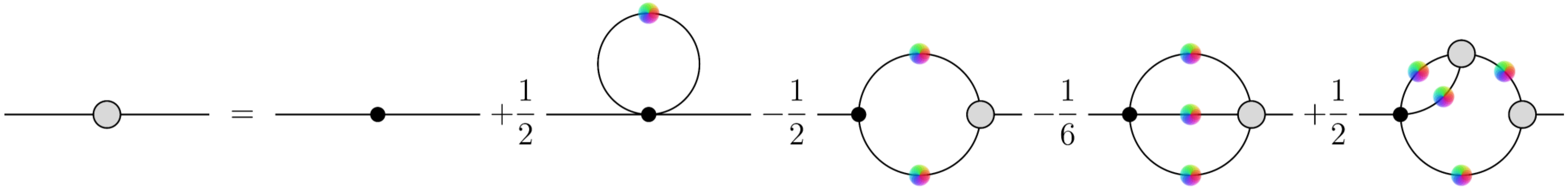

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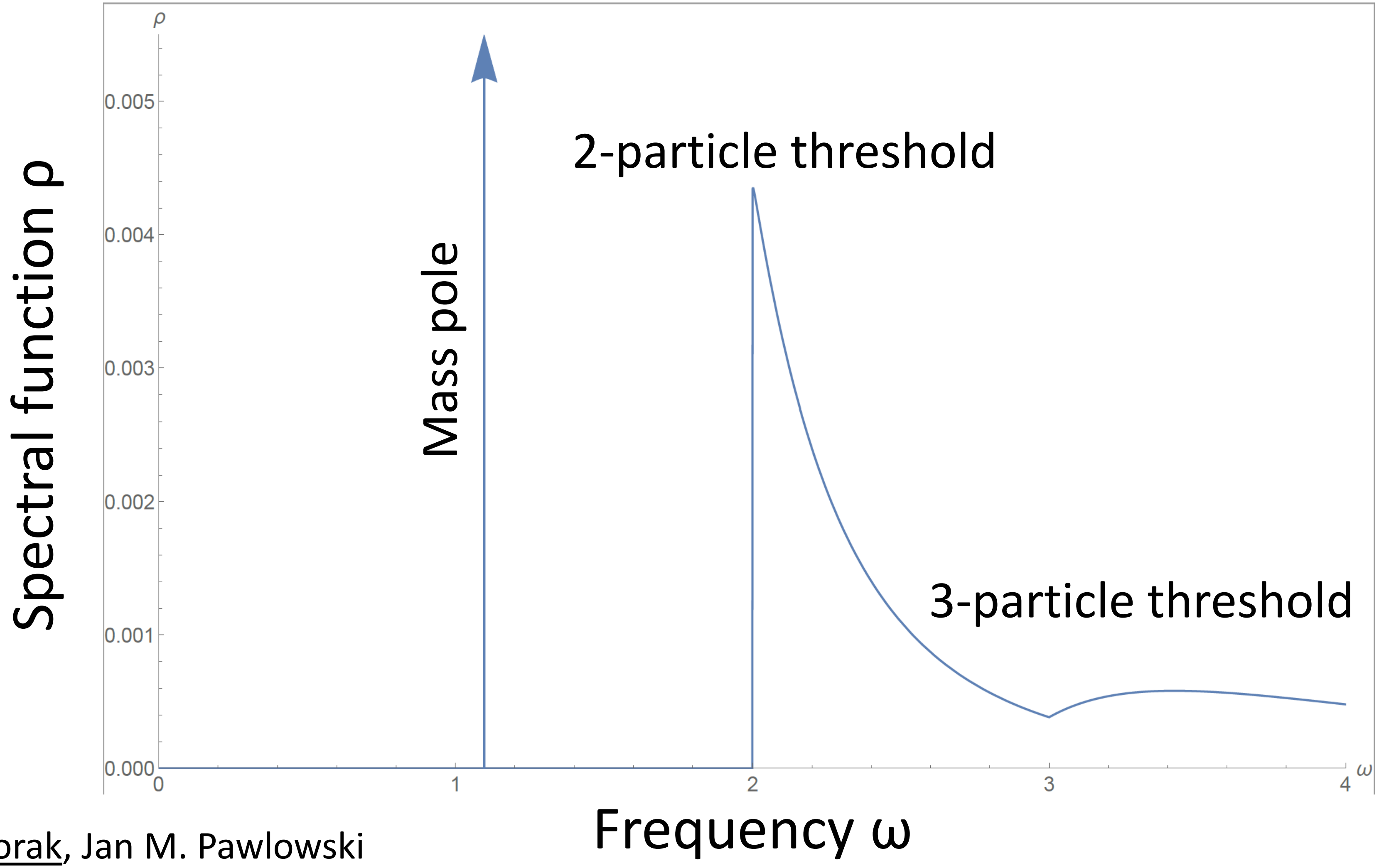
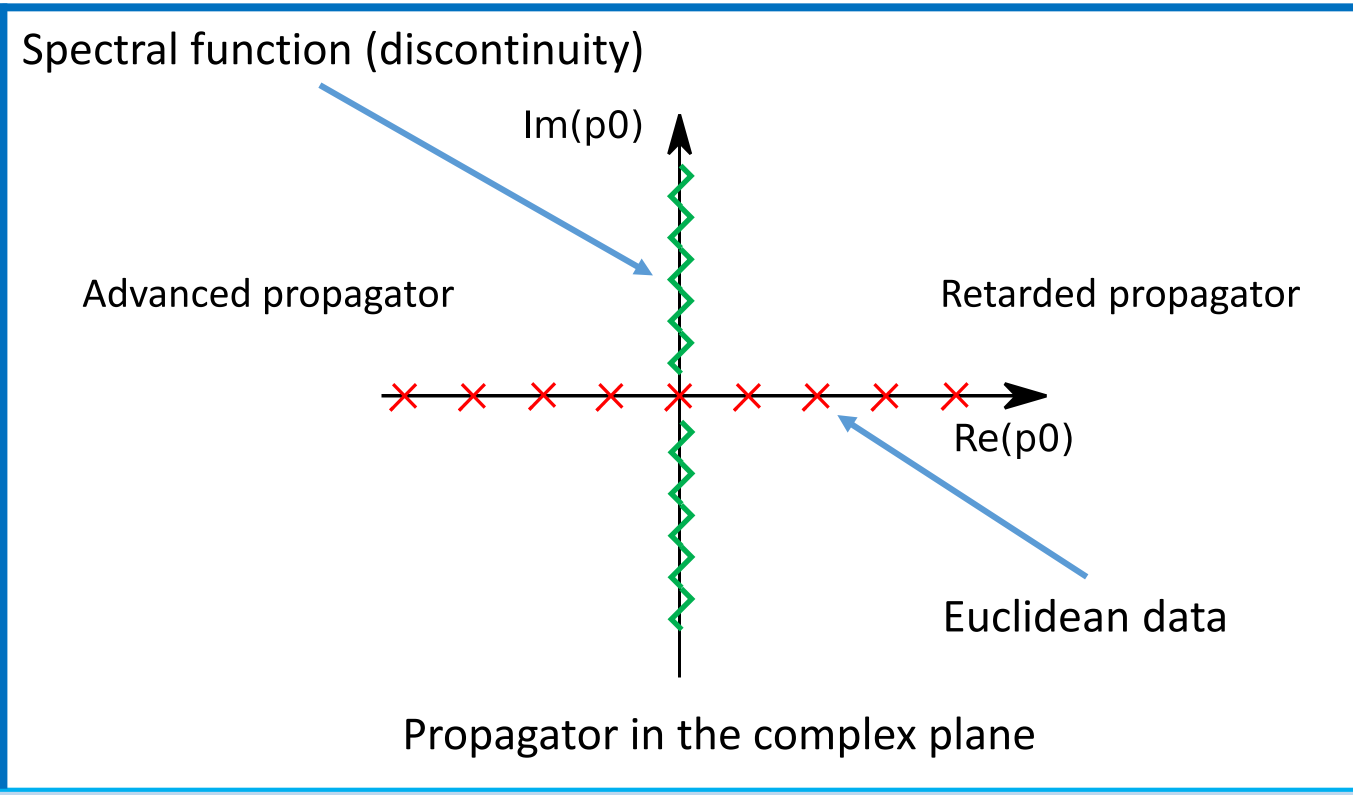


Work with Jan Horak, Jan M. Pawlowski

Horak, Pawlowski, NW, wip

Reconstruction

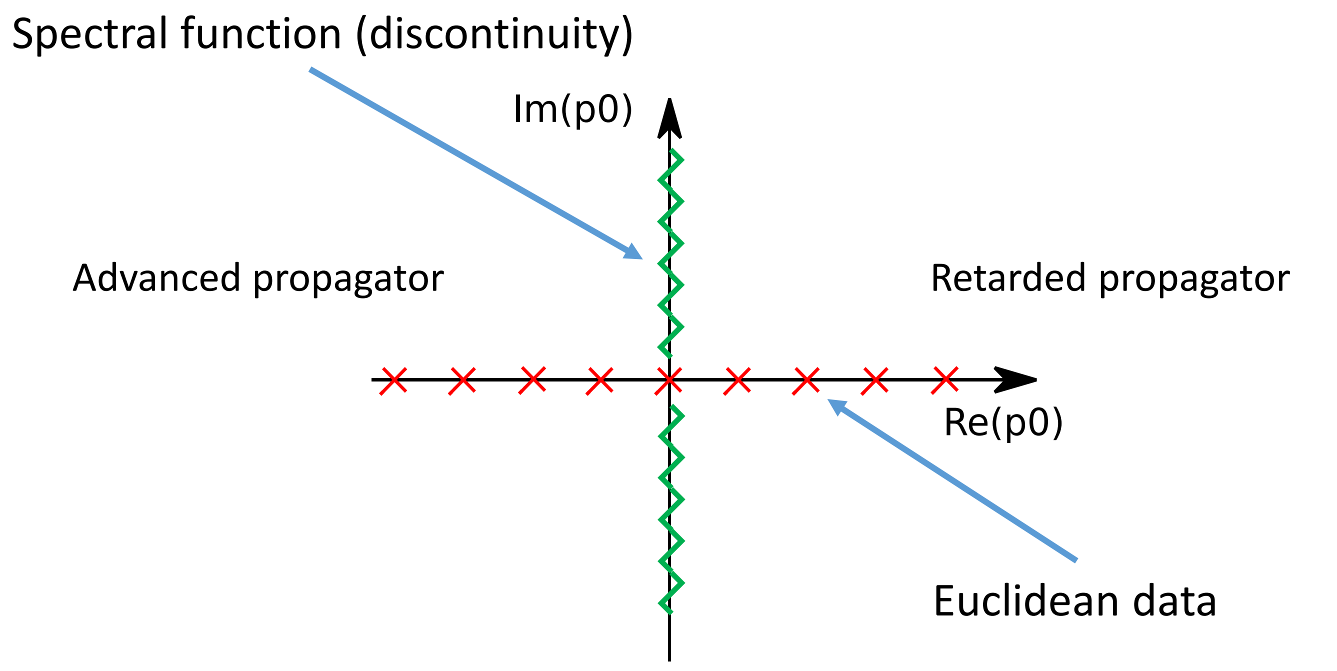
Spectral reconstruction



Spectral reconstruction

Invert: $$G_E(p_0,\mathbf{p}) = \int \frac{\mathrm{d}\eta}{2\pi}\frac{\rho(\eta,\mathbf{p})}{\eta - \mathrm{i}p_0}$$

⇒ more convenient $$G_E(p_0,\mathbf{p}) = \int_0^\infty \frac{\mathrm{d}\eta}{\pi}\eta\frac{\rho(\eta,\mathbf{p})}{\eta^2+p_0^2}$$



Propagator in the complex plane

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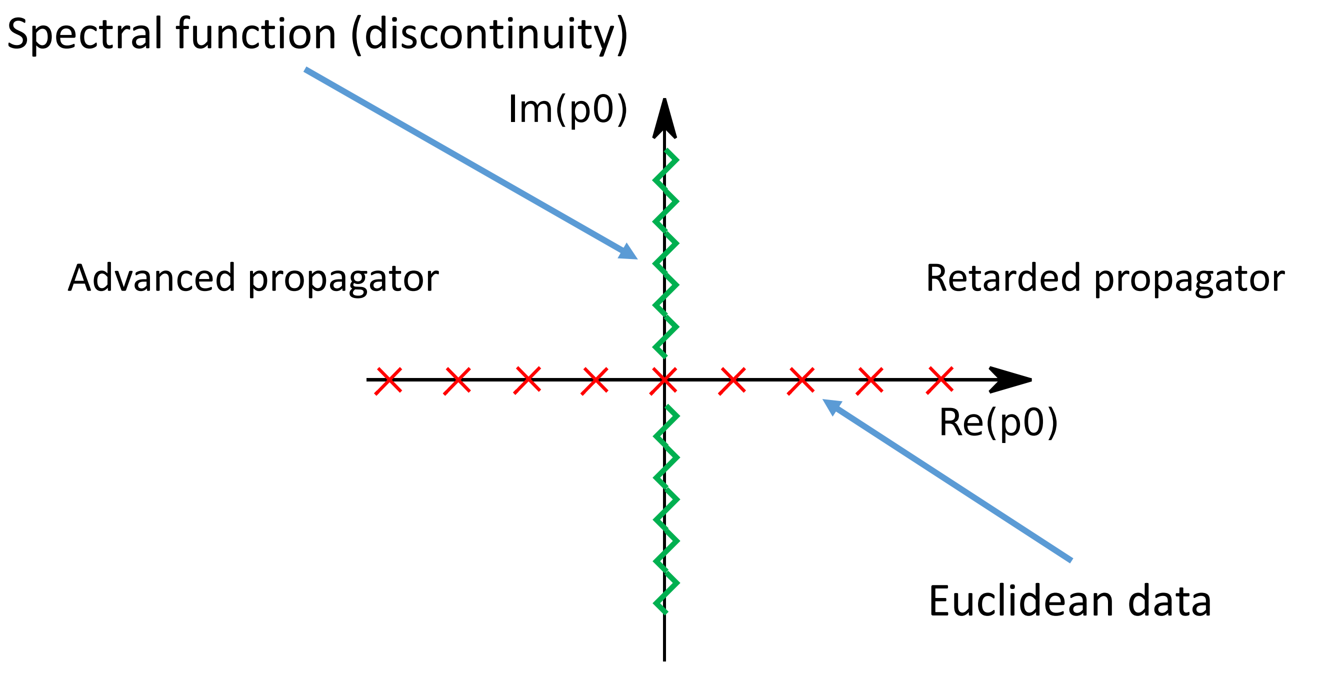
Consider finite temperature (includes vacuum as special case)

➡ Reconstruct analytic function from equally spaced points in one half-plane — Matsubara modes



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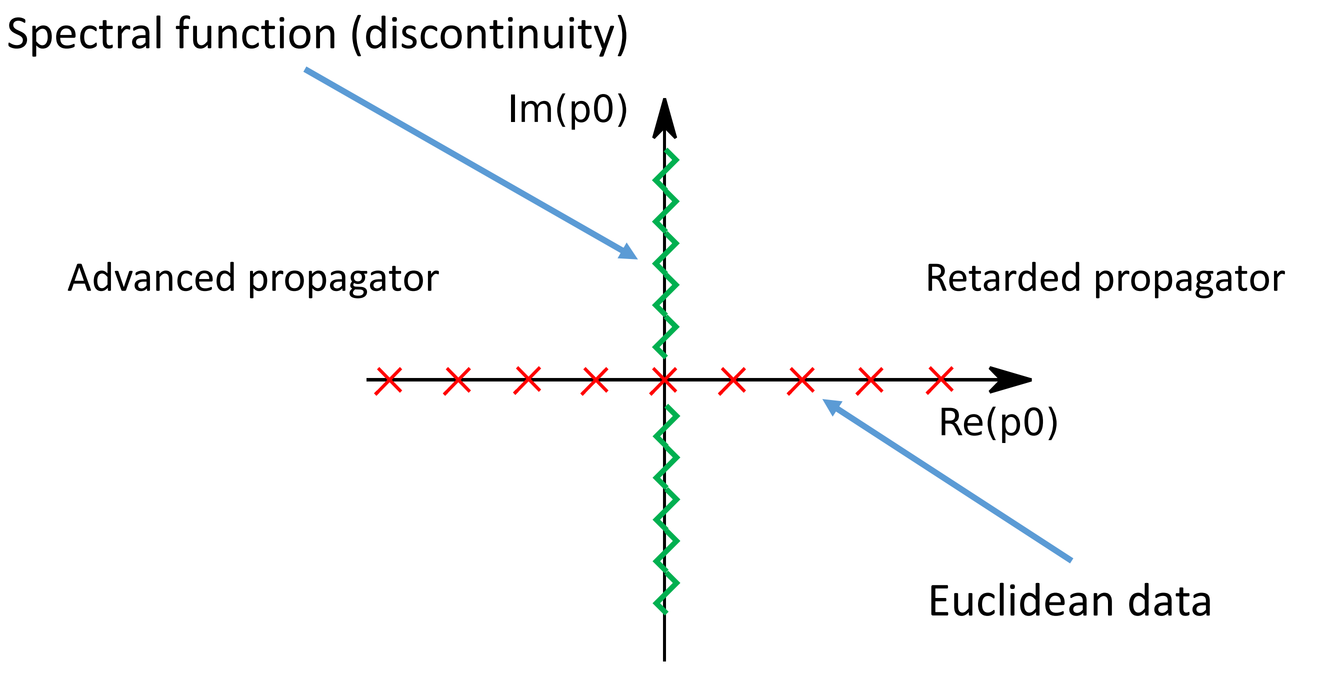
➡ Uniqueness by Carlson's theorem

➡ Explicit construction of spectral function possible, however the problem is ill-conditioned

Cuniberti, De Micheli, Viano, Commun.Math.Phys. 216 (2001)



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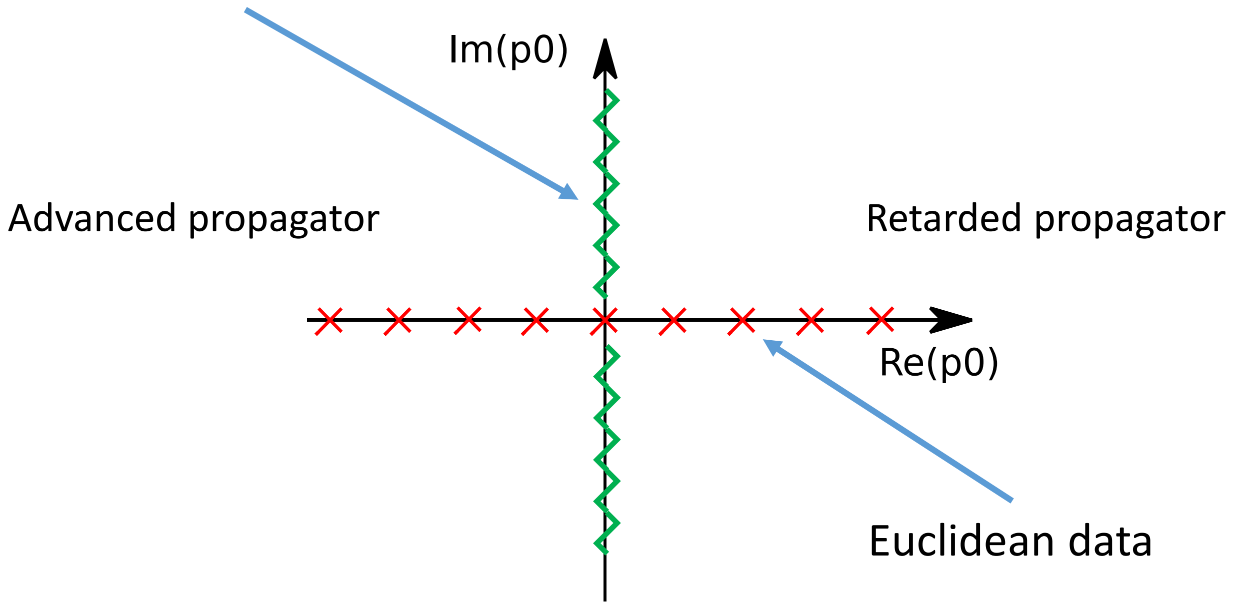
Usual reconstructions:

➡ Linked functional basis and determination of coefficients

Idea: Physically inspired basis that resprects analytic structure of the propagator



Propagator in the complex plane

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Guiding principles:

Cyrol, Pawlowski, Rothkopf, NW arxiv:1804.00945

Spectral reconstruction

$$\sim \left(\frac{1}{(p_0 + \Gamma)^2 + M^2} \right)^{\delta}$$

Guiding principles:

➡ Chose a suitable functional basis ⟷ Utilize structures with a physics picture

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Analytic relation for IR asymptotic

- Determine coefficients in a reliable way ⟷ Different levels of quality

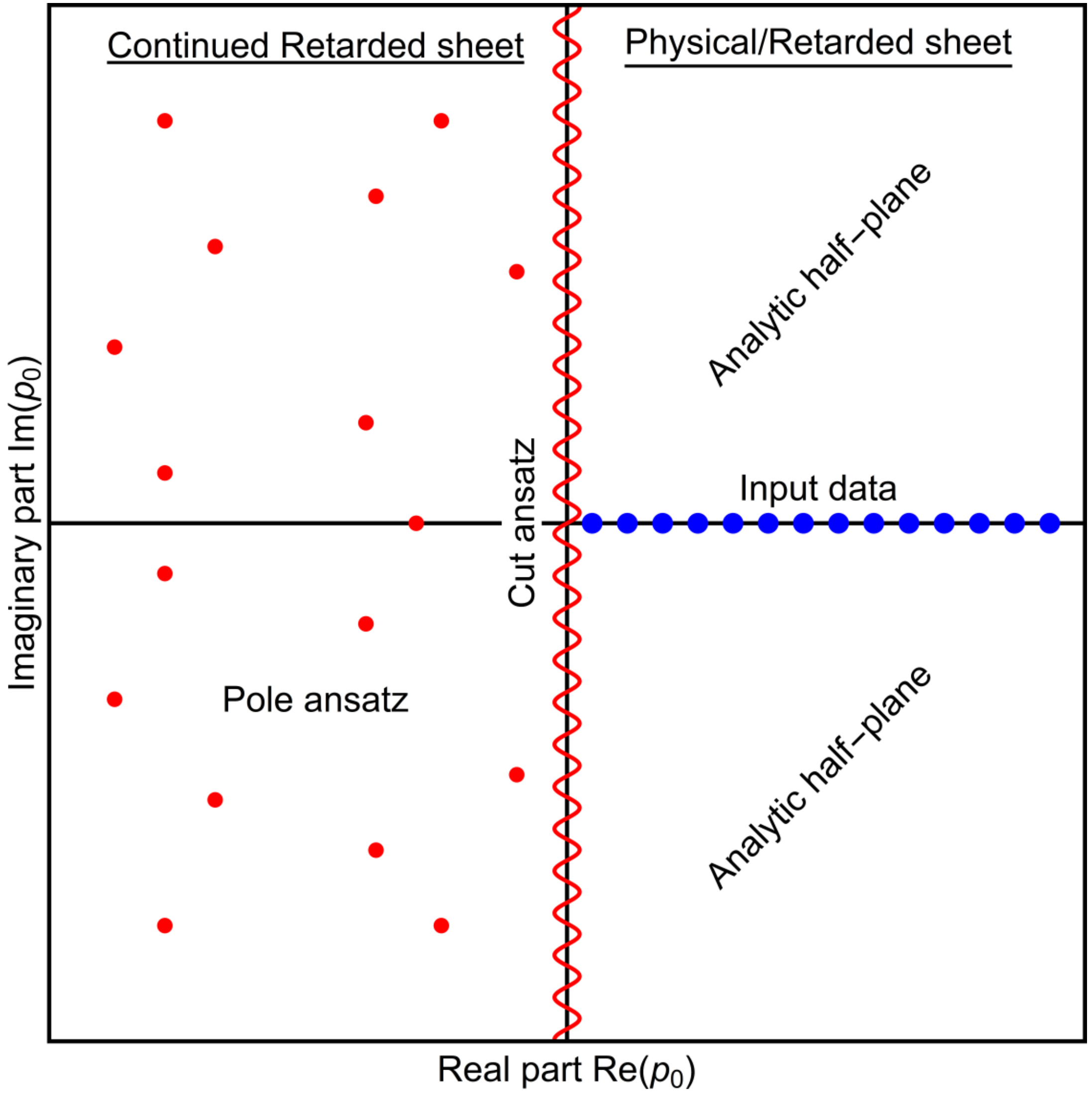
χ^2-fit ⇒ Bayesian Inference (Hamiltonian Monte-Carlo)

Cyrol, Pawlowski, Rothkopf, NW arxiv:1804.00945

Spectral reconstruction

Connection to the analytic structure

➡ Consider the analytically continued retarded propagator



Cyrol, Pawlowski, Rothkopf, NW arxiv:1804.00945

Spectral reconstruction

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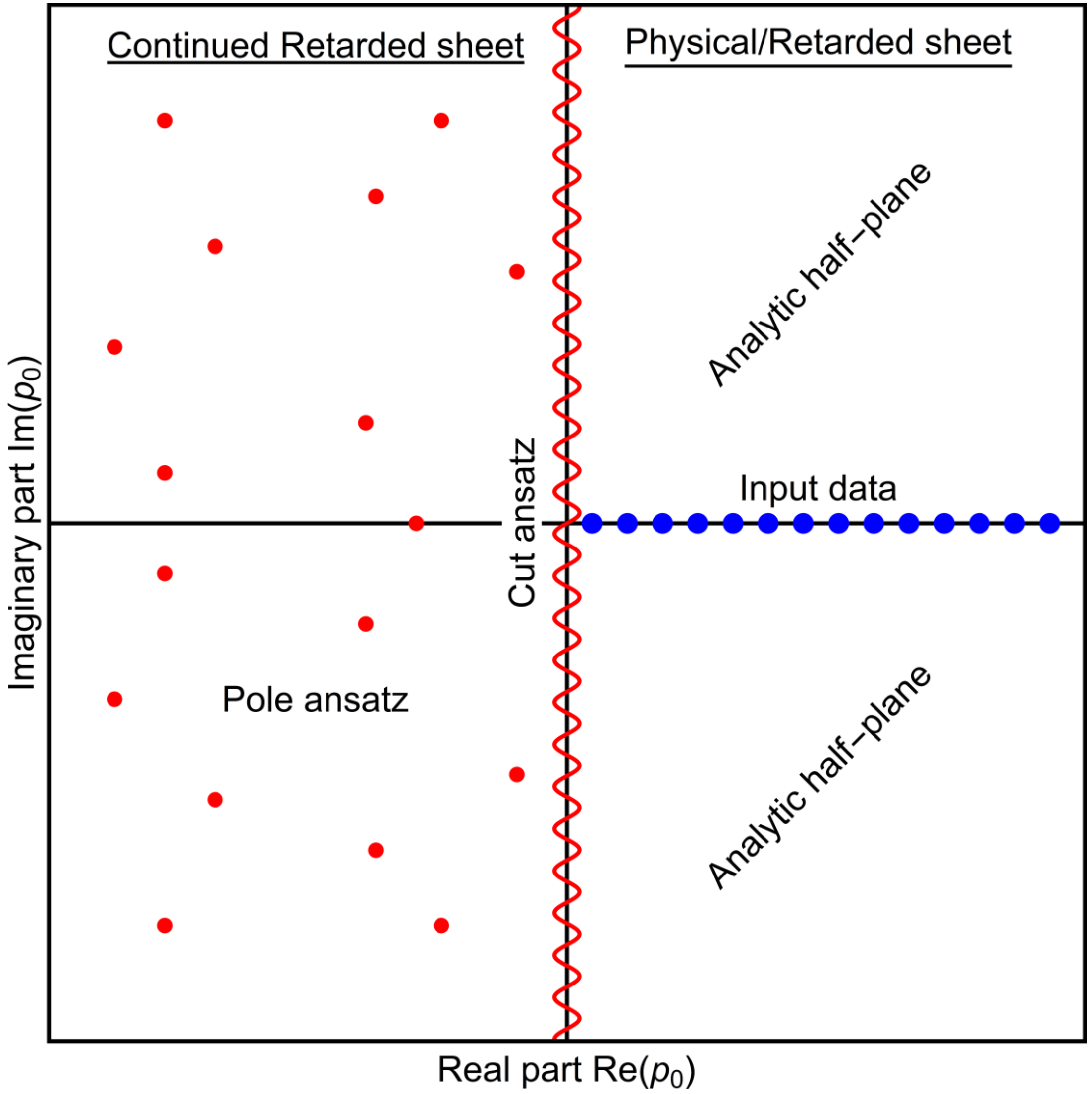

Cyrol, Pawlowski, Rothkopf, NW arxiv:1804.00945

Spectral reconstruction

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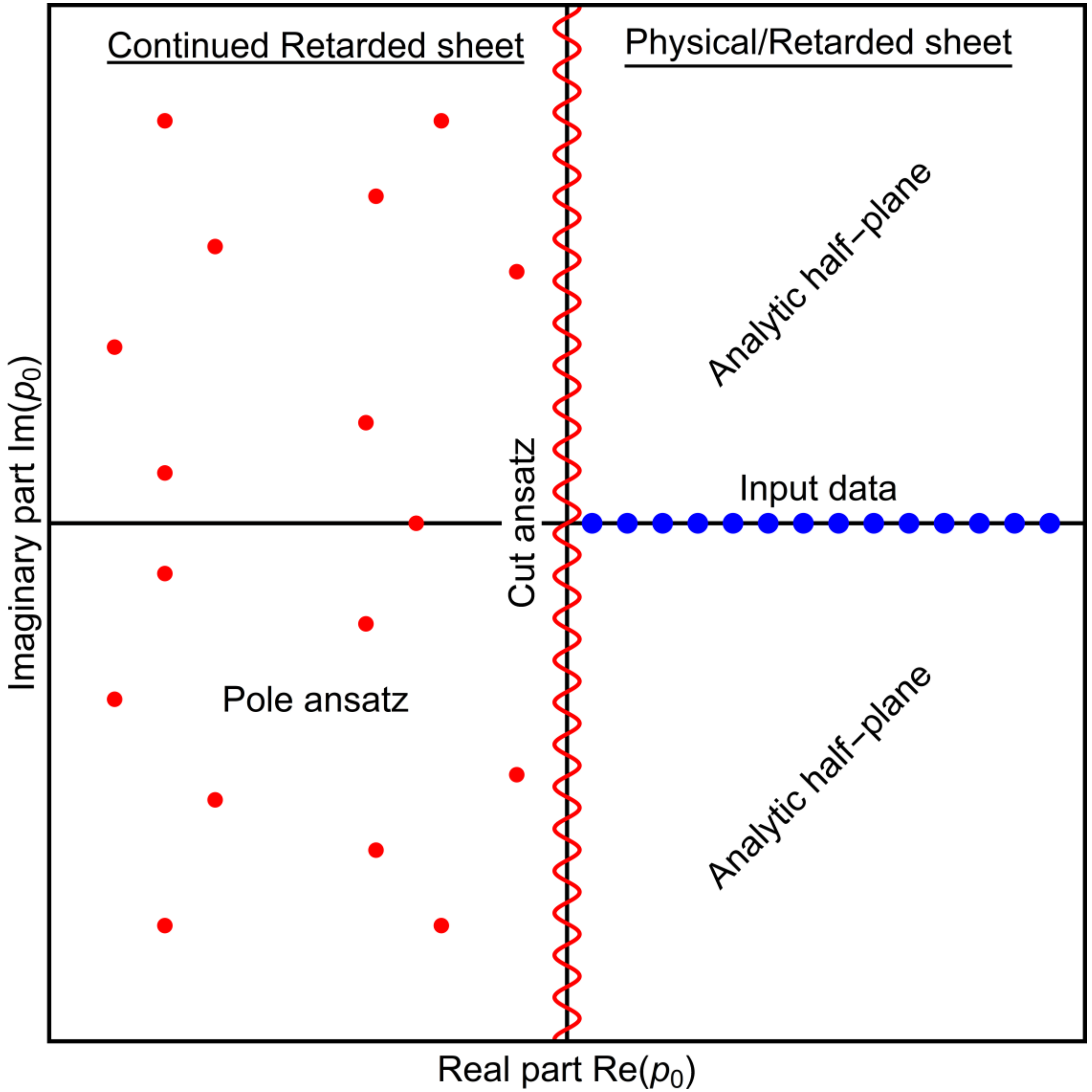
- The other half-plane is necessarily meromorphic
- Ansatz for the complex structure of the retarded propagator



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Spectral reconstruction

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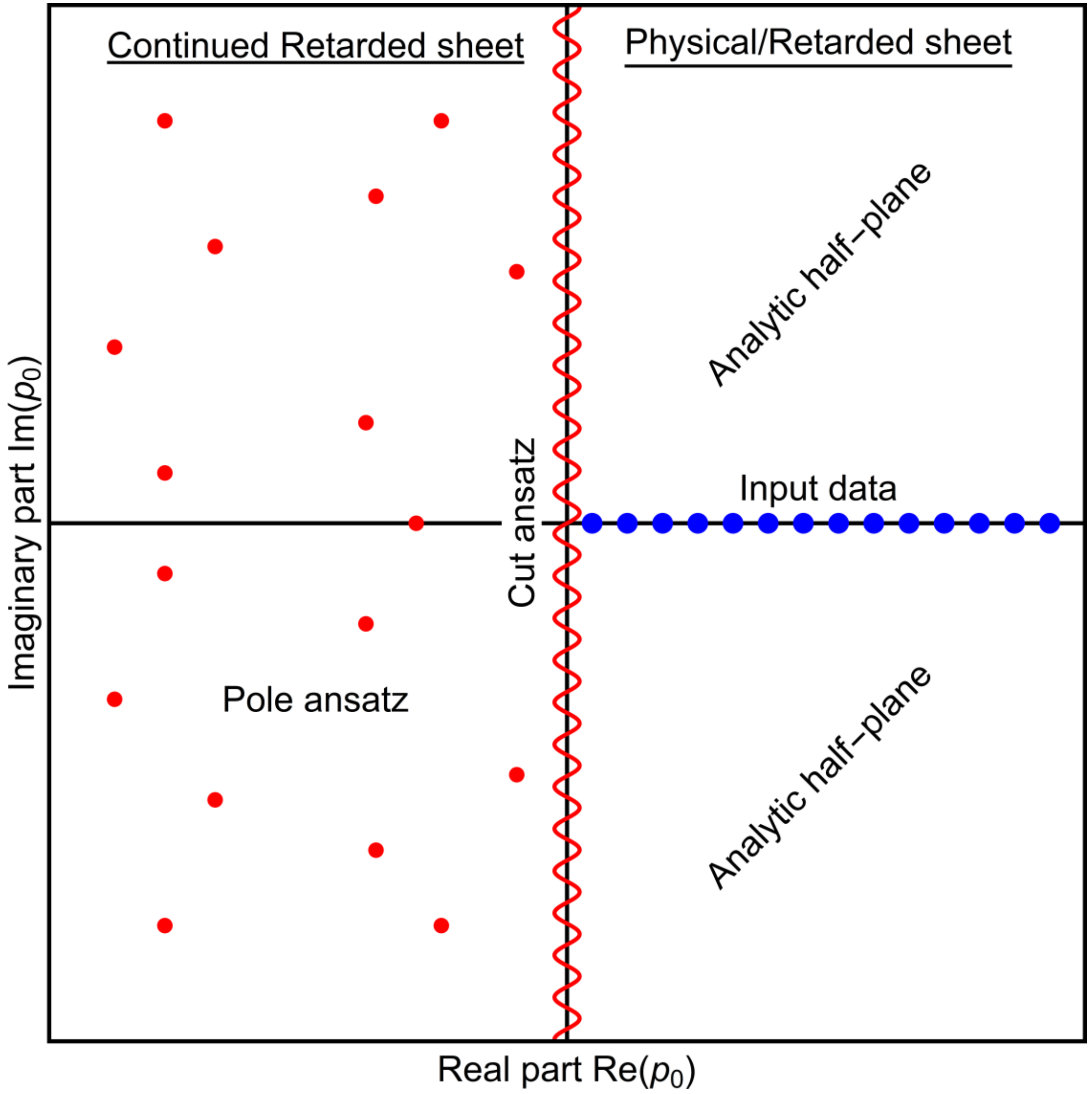
- Ansatz for the complex structure of the retarded propagator
- Previous knowledge easily included



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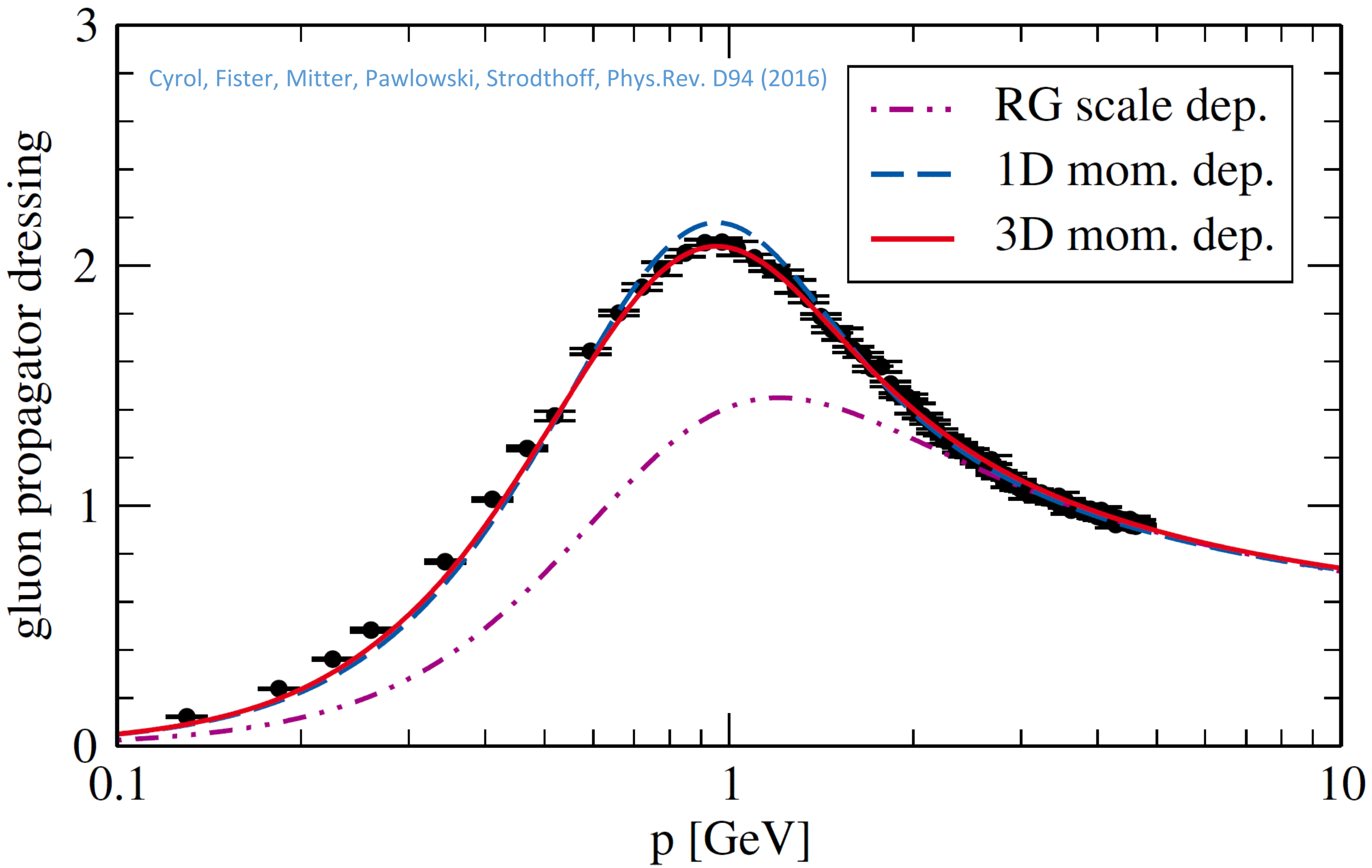
Reconstructing the gluon



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Reconstructing the gluon

- Gluon admits positivity violation
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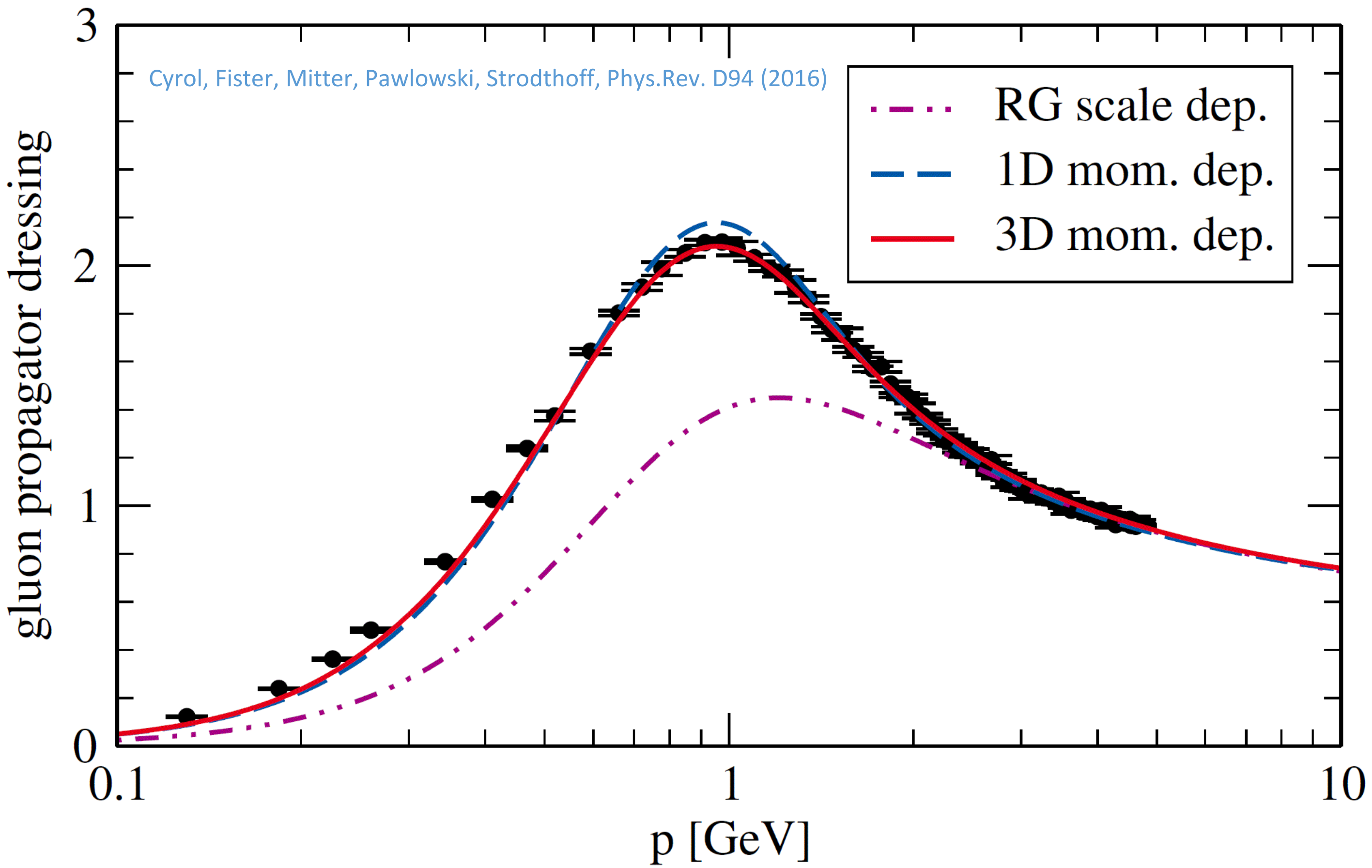

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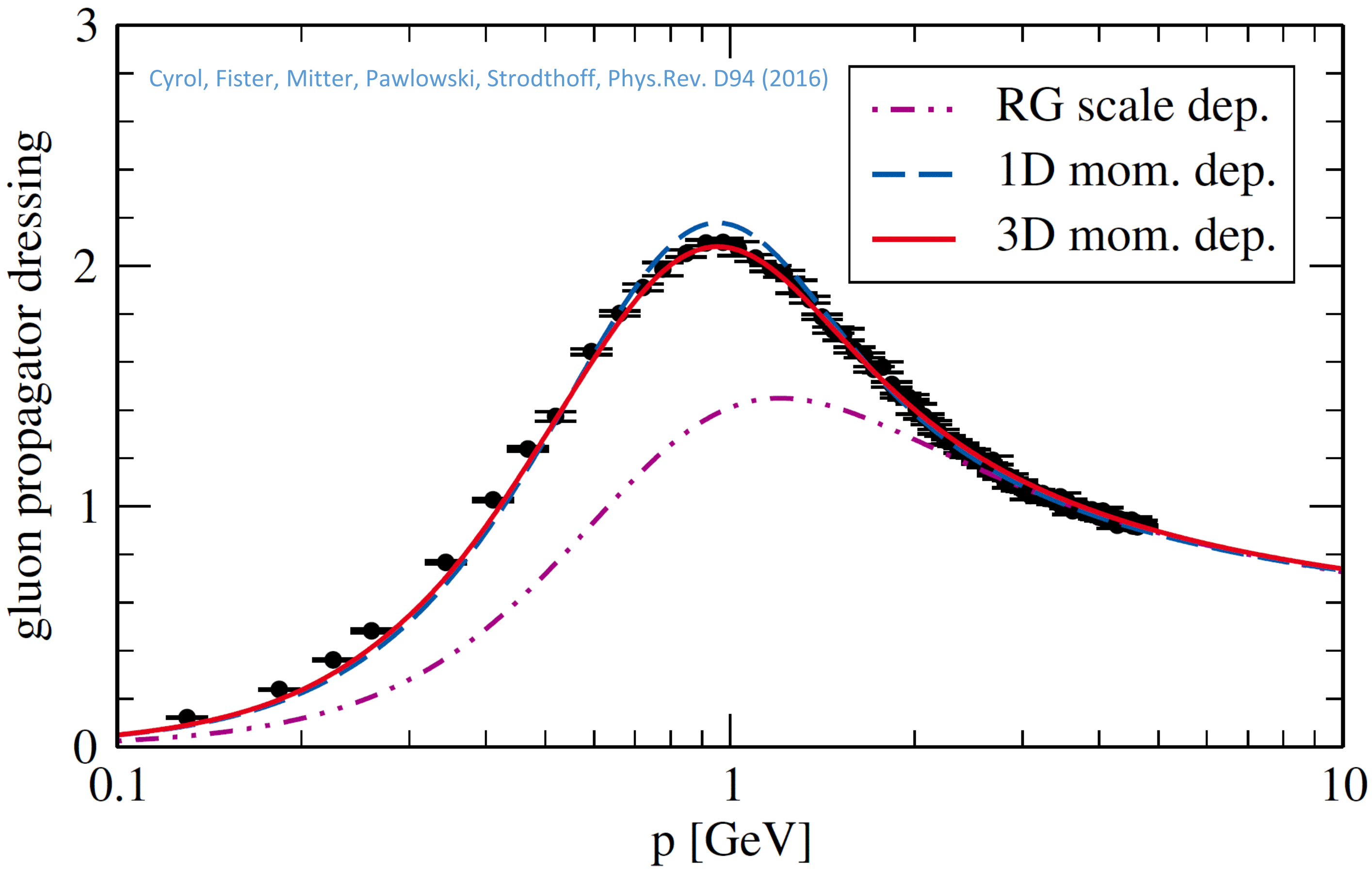

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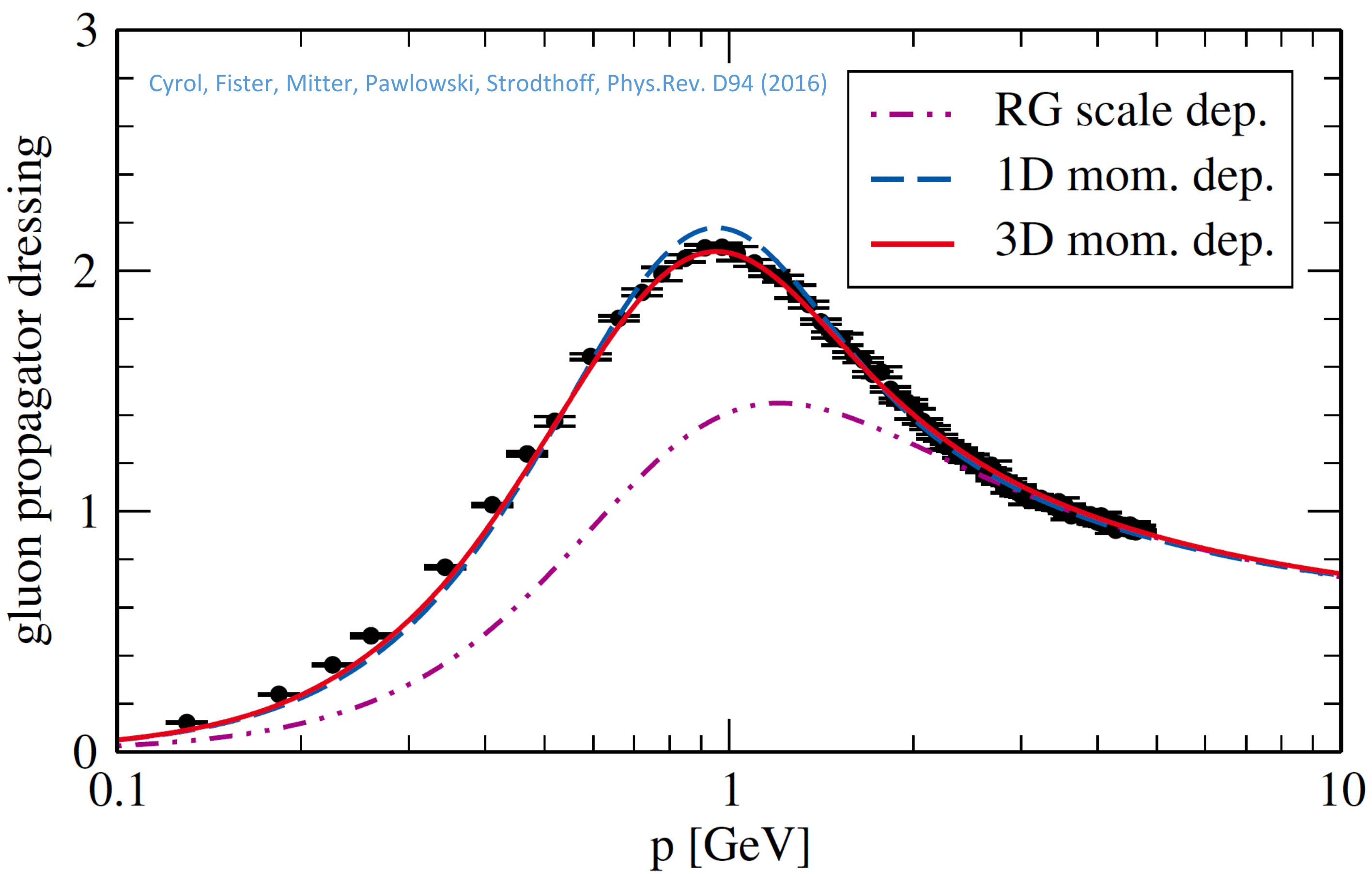
  - First start for improvement, but HMC requires uniqueness of the coefficients



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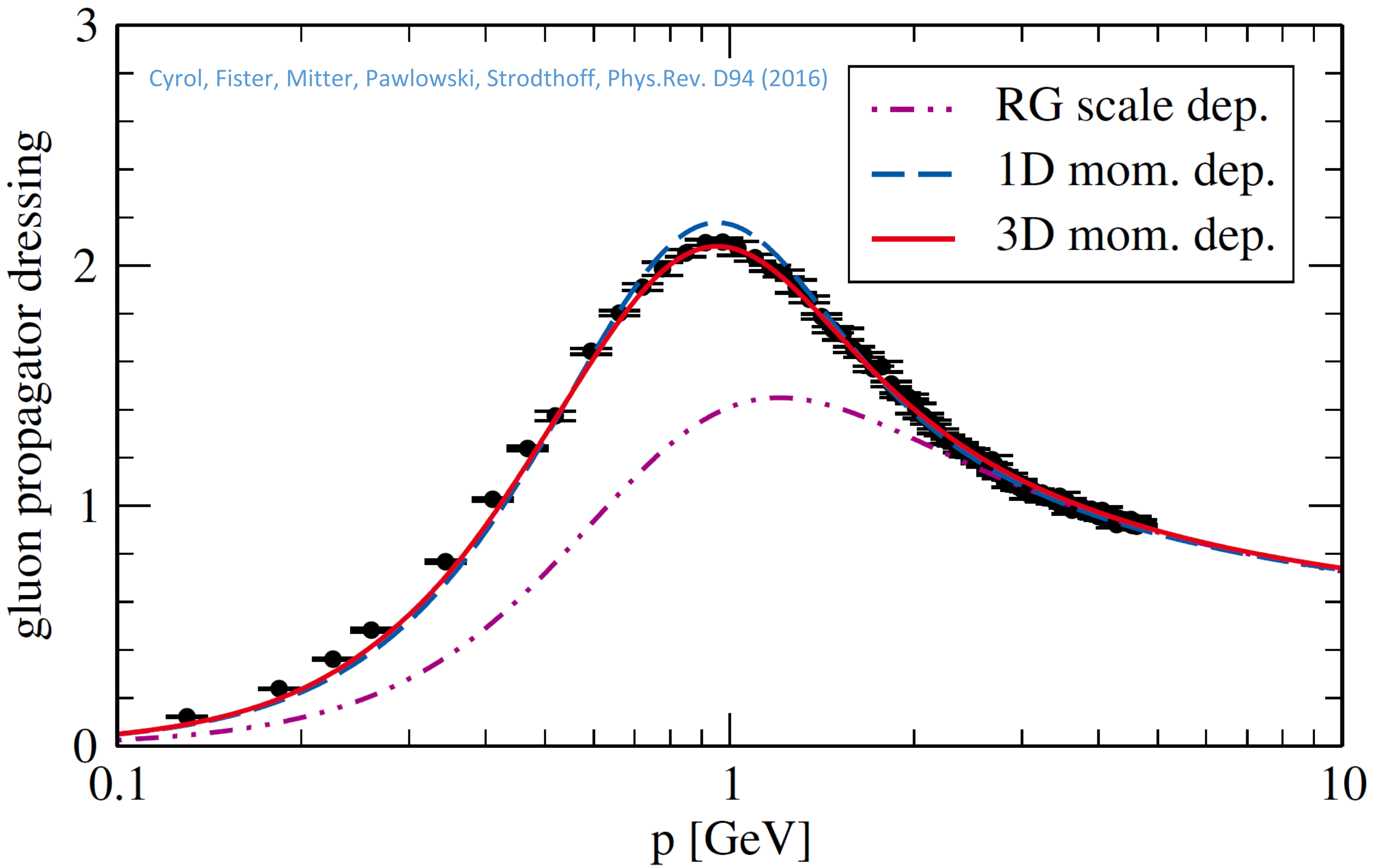
- Shape reliable, quantitative details are not



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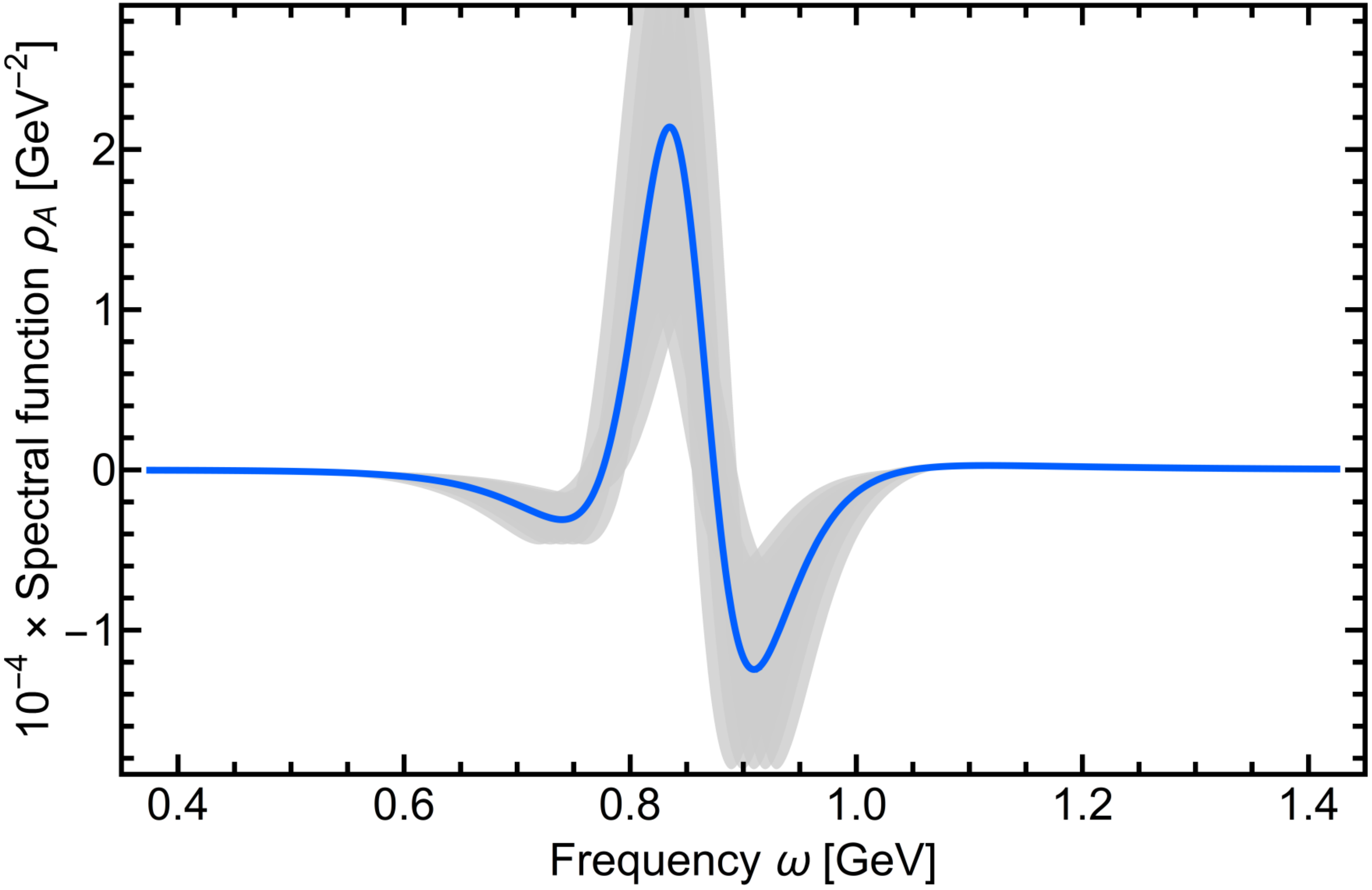
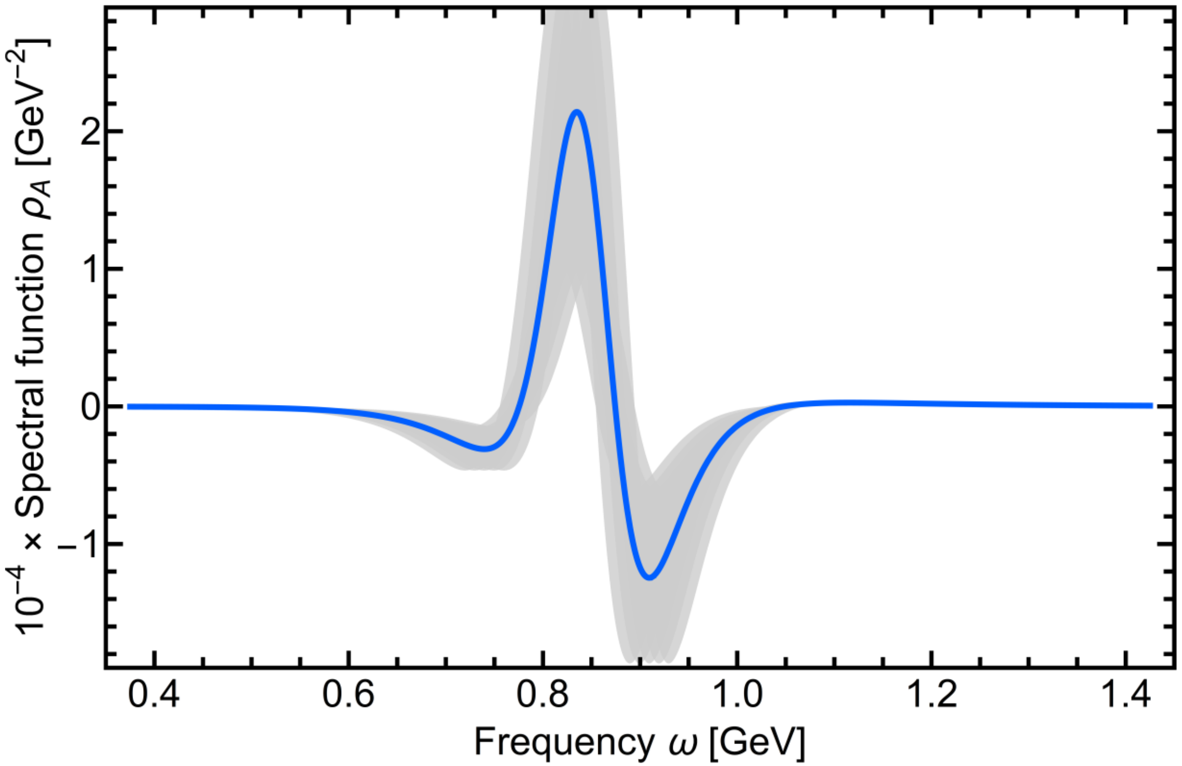
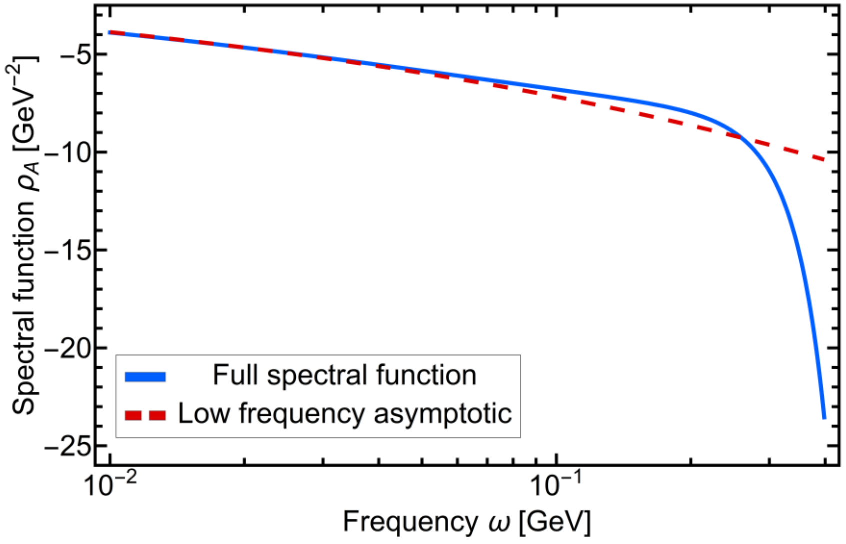
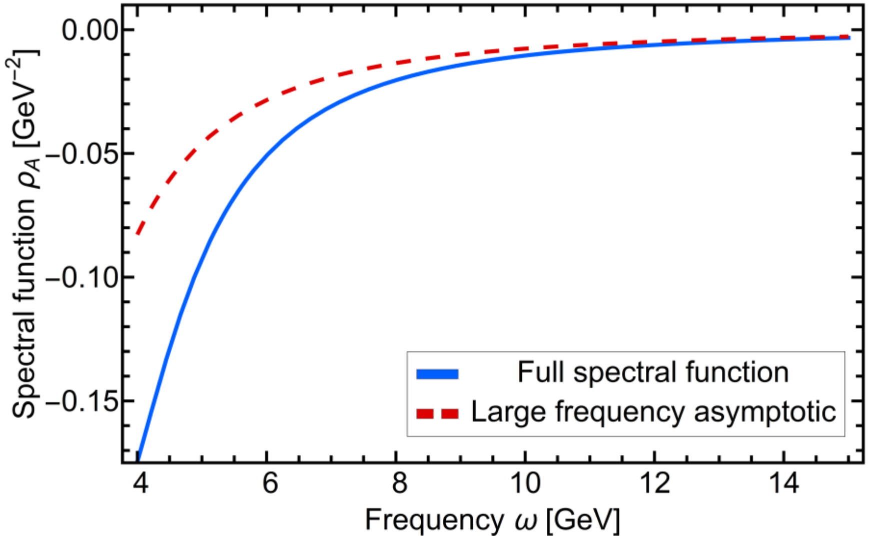


Cyrol, Pawlowski, Rothkopf, NW arxiv:1804.00945

Reconstructing the gluon





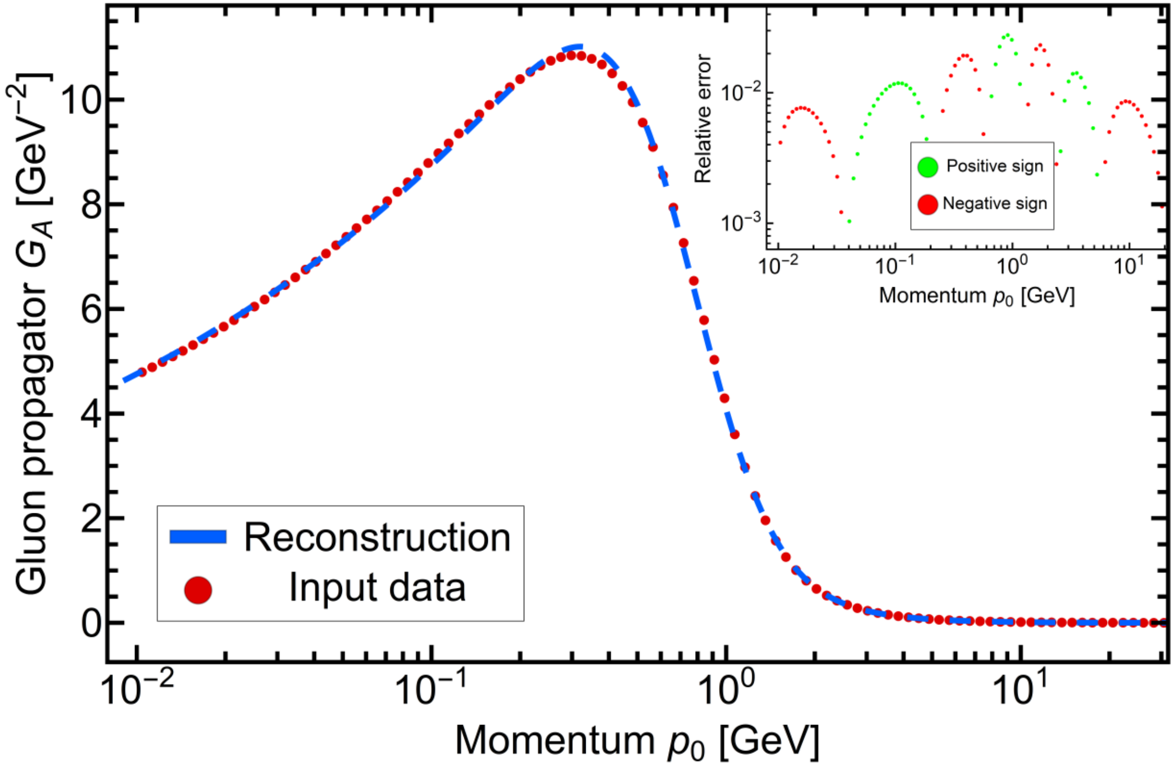


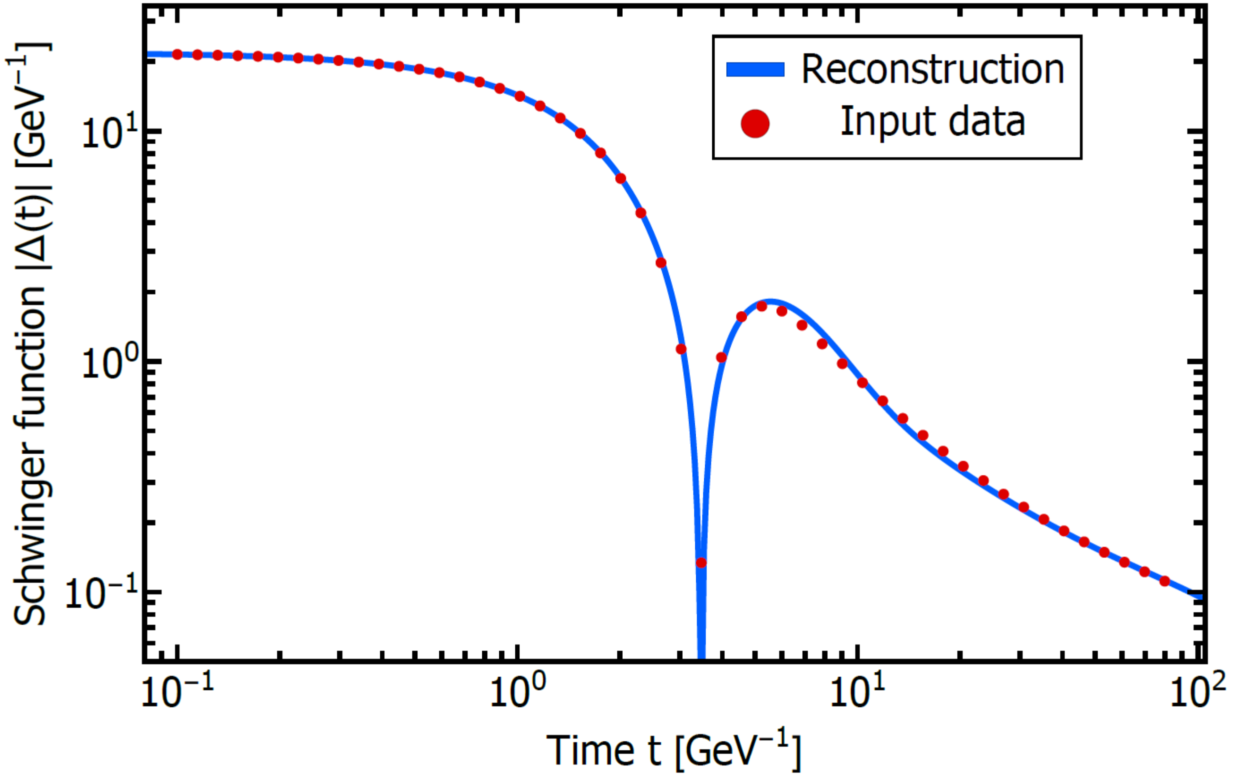
➡ In qualitative agreement with direct DSE calculation and other reconstructions

see e.g. Strauss, Fischer, Kellermann, Phys.Rev.Lett. 109 (2012)
Dudal, Oliveira, Silva, Phys.Rev. D89 (2014)

Cyrol, Pawlowski, Rothkopf, NW arxiv:1804.00945

Reconstructing the gluon





Cyrol, Pawlowski, Rothkopf, NW arxiv:1804.00945

Application

-

Transport coefficients

Transport coefficients

Shear viscosity:

$$\eta = \frac{1}{20} \lim_{\omega \to 0} \frac{\rho_{\pi\pi}(\omega, 0)}{\omega}$$

Bulk viscosity:

$$\zeta = \frac{1}{2} \lim_{\omega \to 0} \frac{\rho_{\mathcal{P}\mathcal{P}}(\omega, 0)}{\omega}$$

Christiansen, Haas, Pawlowski, Strodthoff, PRL (2015)
Pawlowski, NW, work in progress

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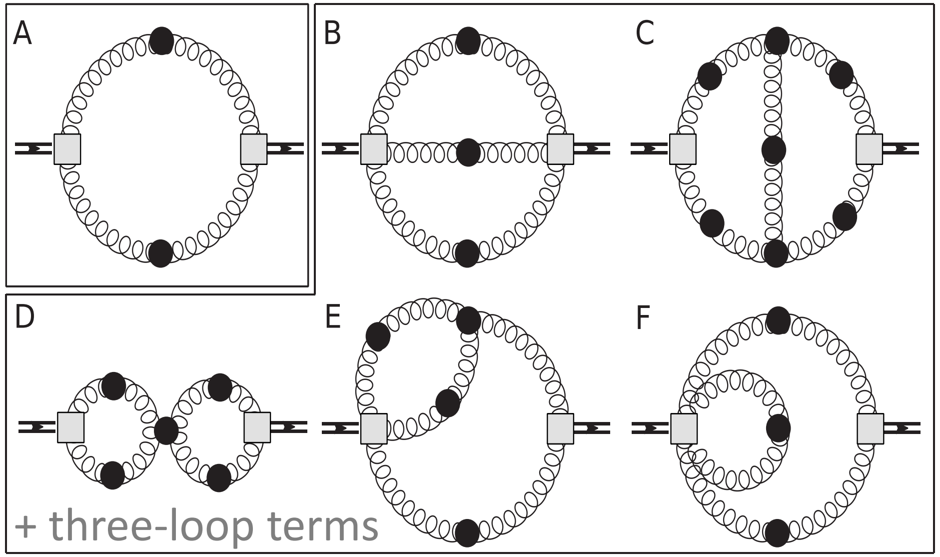
Composite Dyson-Schwinger equation

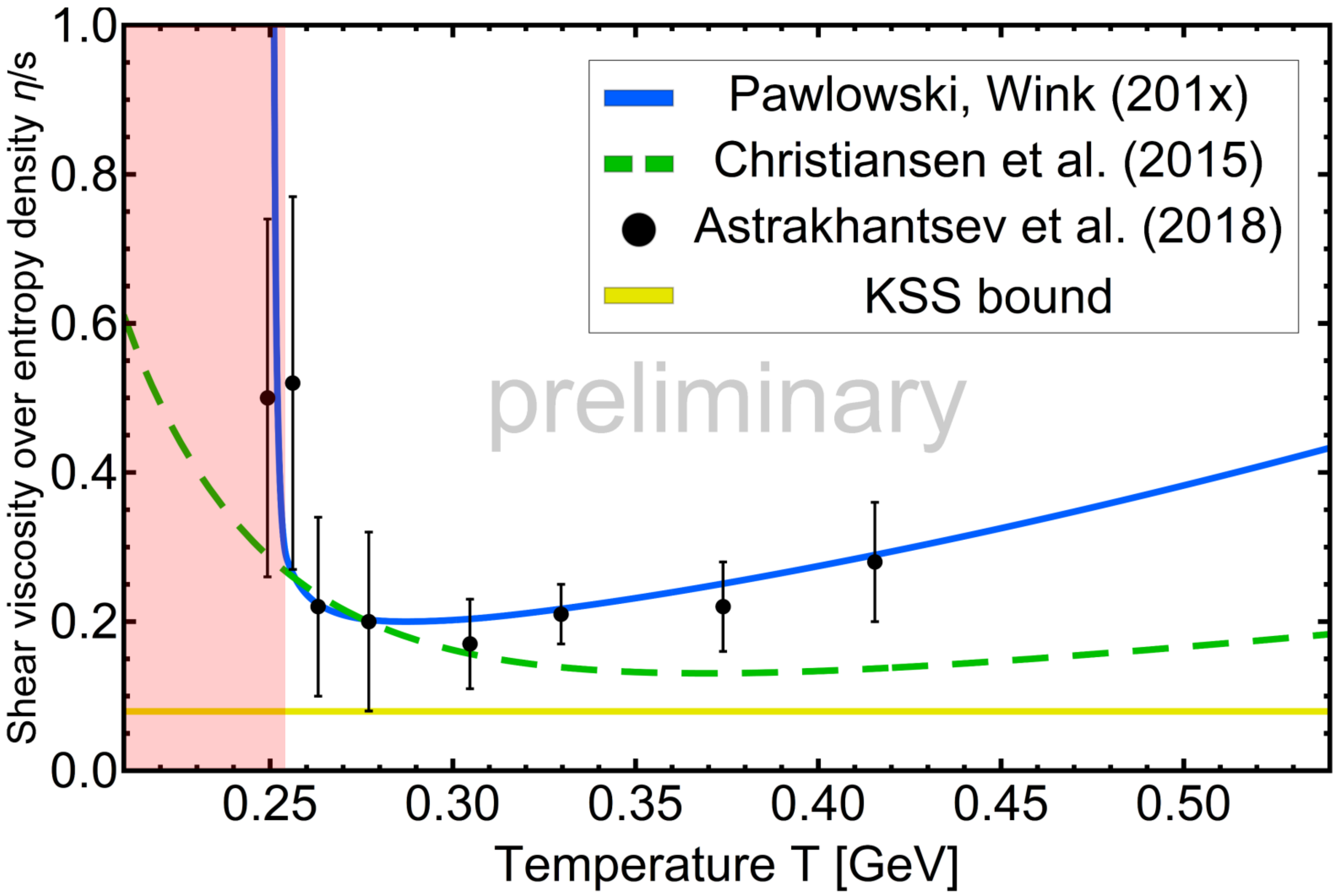
➡ Exact representation with a finite number of loops



Christiansen, Haas, Pawlowski, Strodthoff, PRL (2015)
Pawlowski, NW, work in progress

Transport coefficients

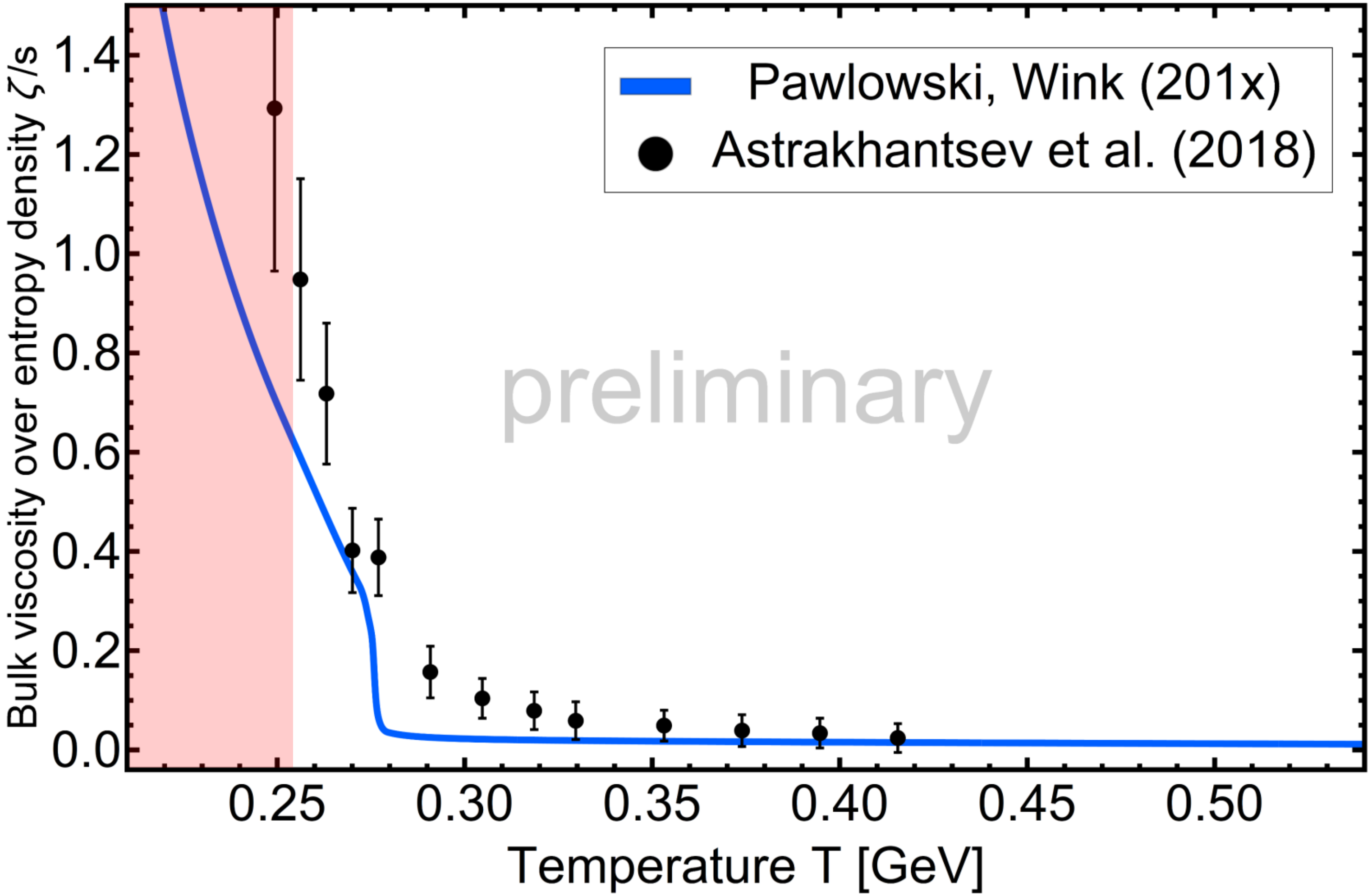
Shear viscosity



Pawlowski, NW , in prep.

Transport coefficients

Bulk viscosity



Pawlowski, NW , in prep.

Summary

- Spectral representations and their implications

- Ways to obtain spectral functions:
  - Spectral functions from direct computation
  - Spectral functions from reconstruction

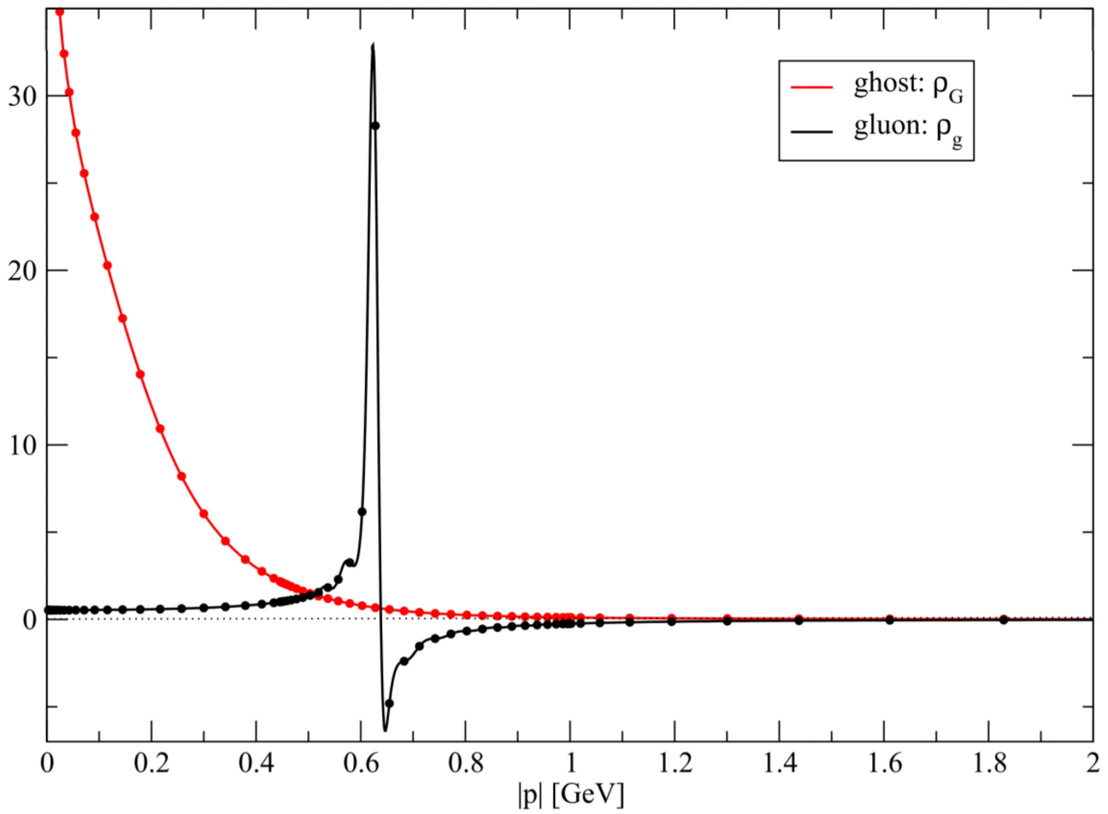
- Applications of spectral functions:
  - Non-equilibrium transport
  - Dimensional regularization + DSE
  - Transport coefficients

Summary

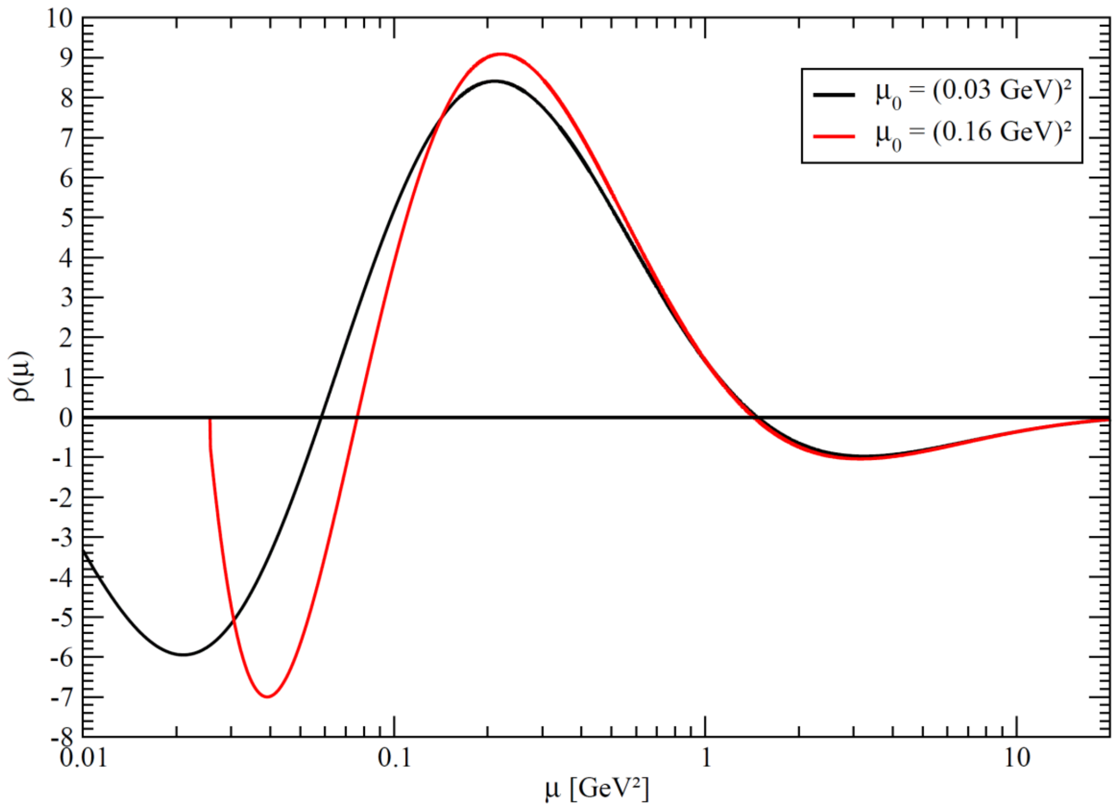
- Spectral representations and their implications
- Ways to obtain spectral functions:
  - Spectral functions from direct computation
  - Spectral functions from reconstruction
- Applications of spectral functions:
  - Non-equilibrium transport
  - Dimensional regularization + DSE
  - Transport coefficients

Thank you for your attention!

Comparison with other works

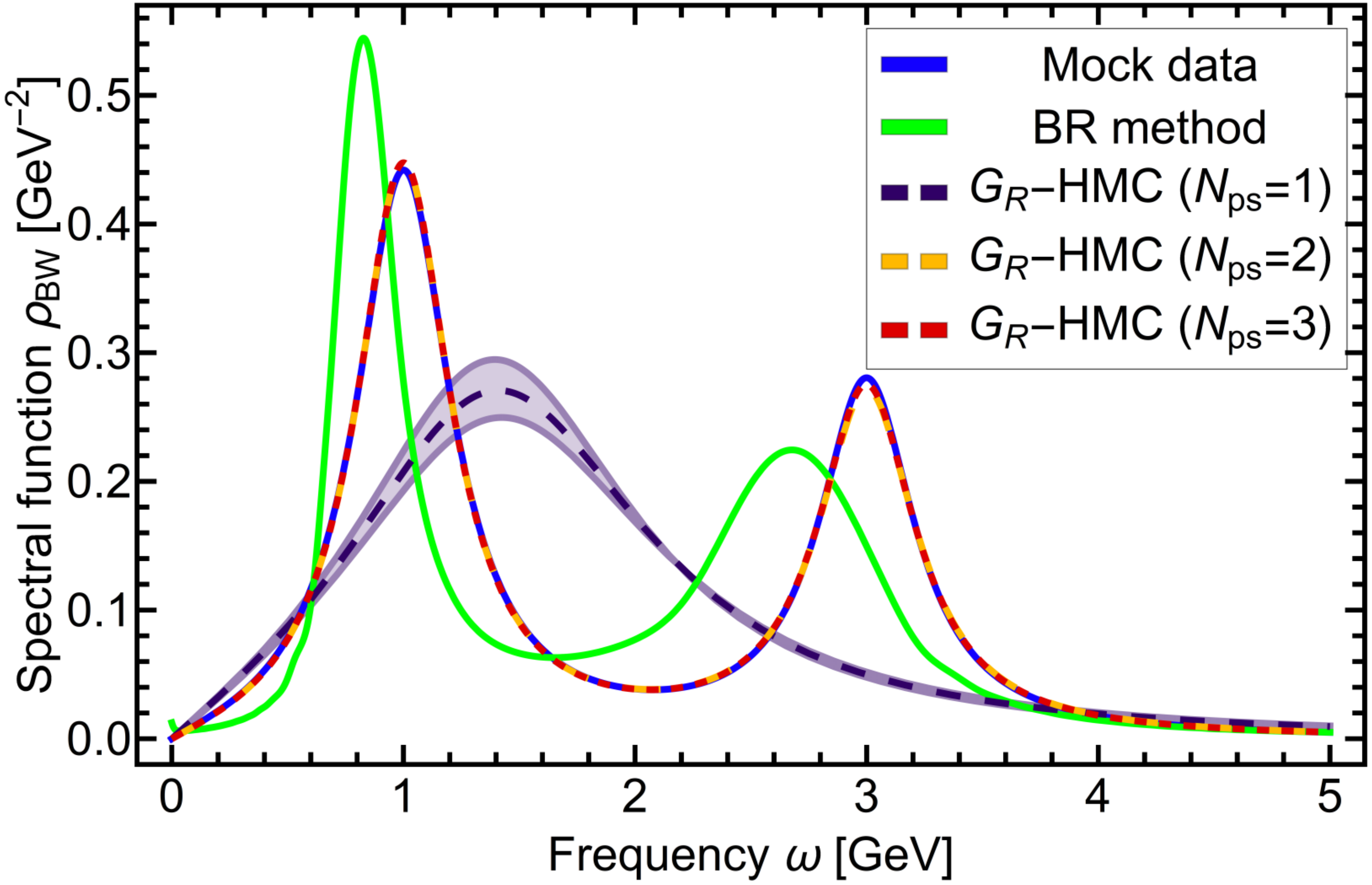


Strauss, Fischer, Kellermann, Phys.Rev.Lett. 109 (2012)



Dudal, Oliveira, Silva, Phys.Rev. D89 (2014)

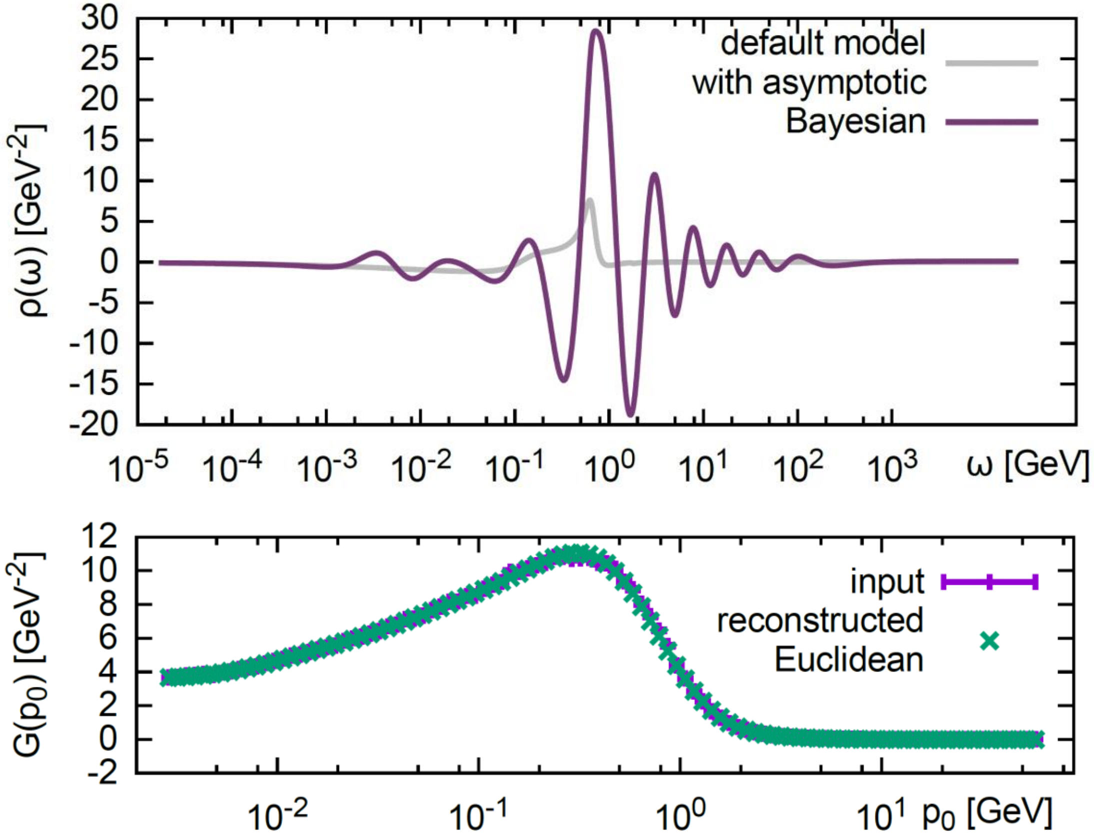
Breit-Wigner benchmark



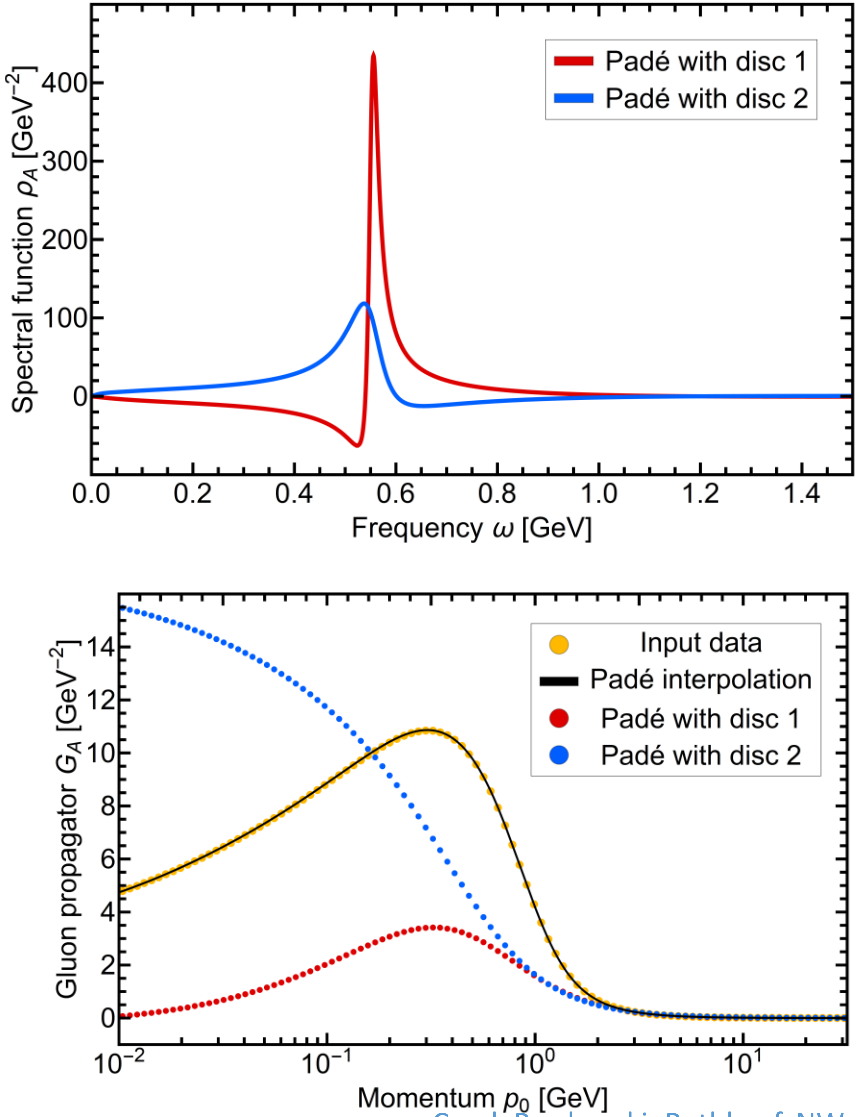
Cyrol, Pawlowski, Rothkopf, NW arxiv:1804.00945

Comparison with other methods

Bayesian reconstruction



Padé reconstruction

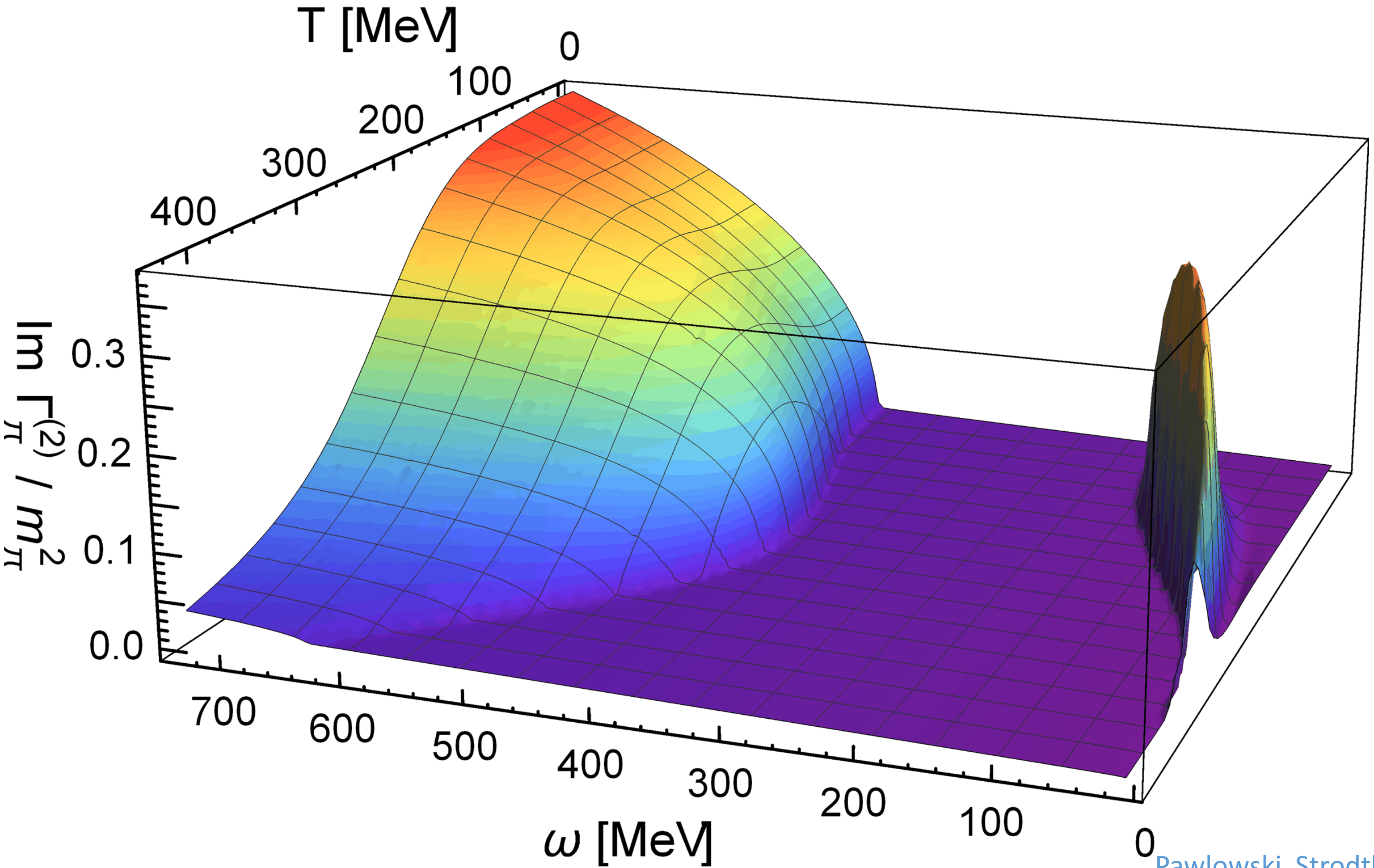


Cyrol, Pawlowski, Rothkopf, NW arxiv:1804.00945

Application to the O(N)-Model

Pion

Imaginary part of the retarded two-point function

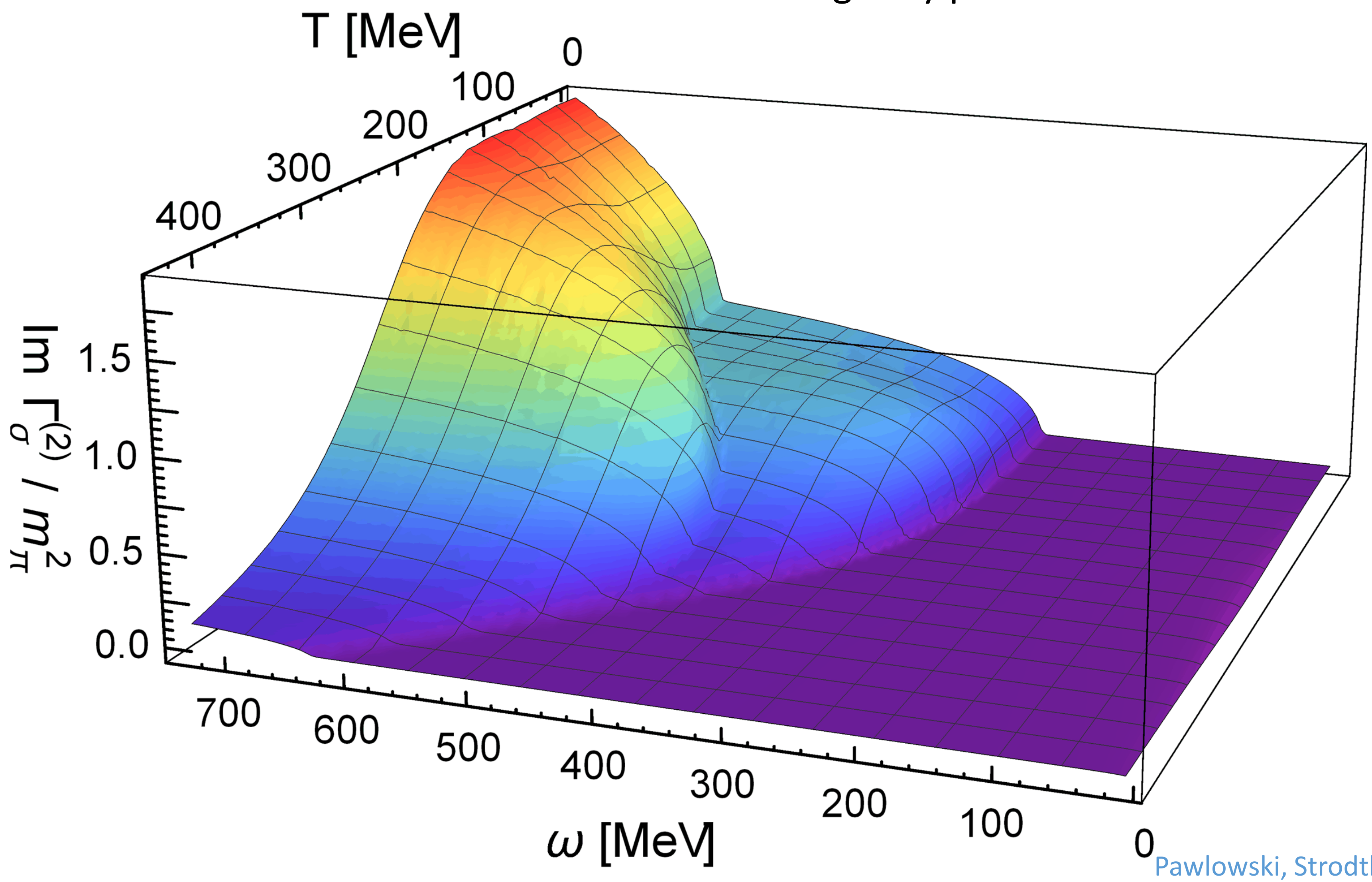


Pawlowski, Strodthoff, NW, arxiv:1711.07444

Application to the O(N)-Model

Sigma meson

Imaginary part of the retarded two-point function

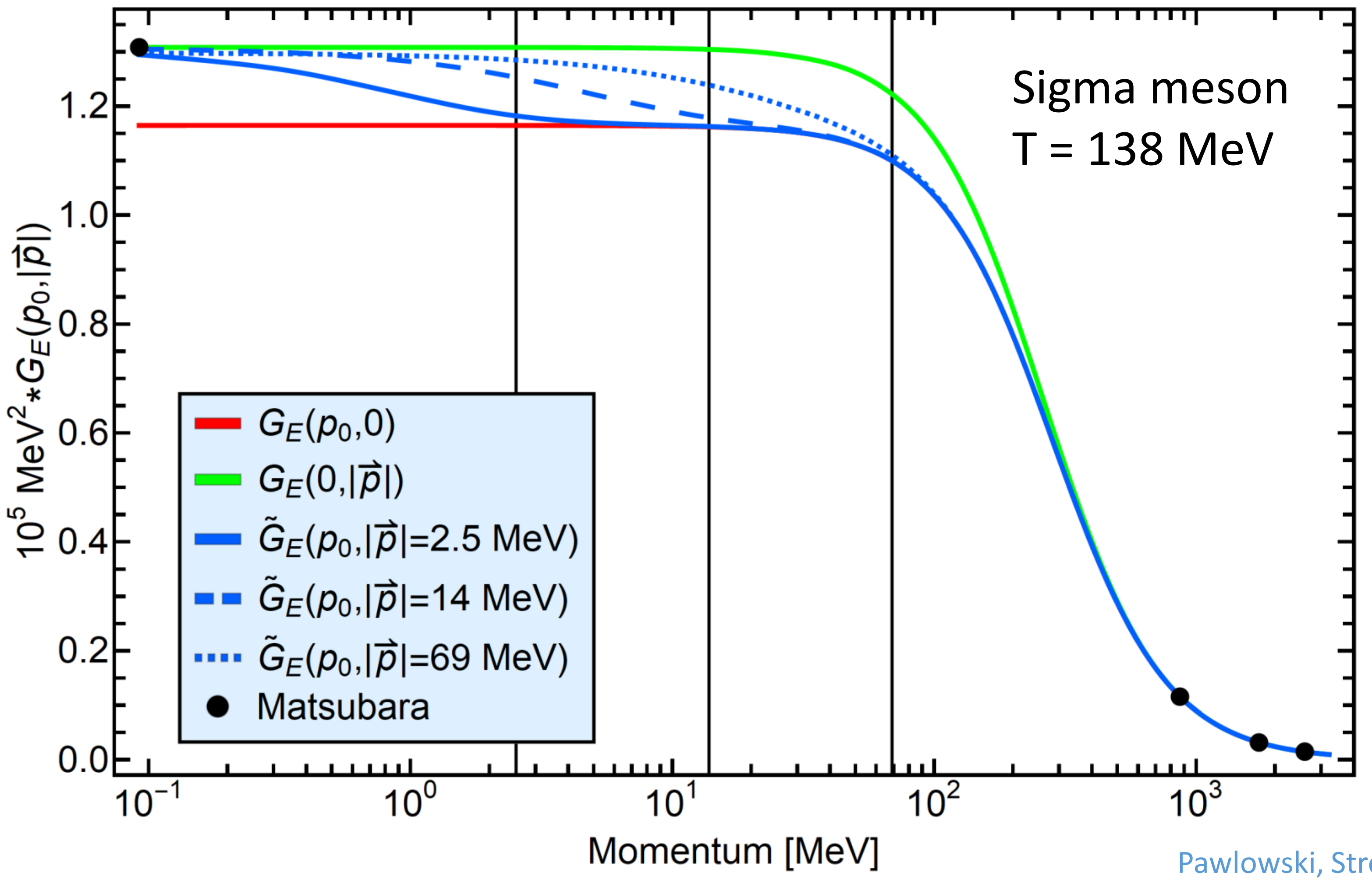


Pawlowski, Strodthoff, NW, arxiv:1711.07444

Application to the O(N)-Model

In medium non-commuting limits

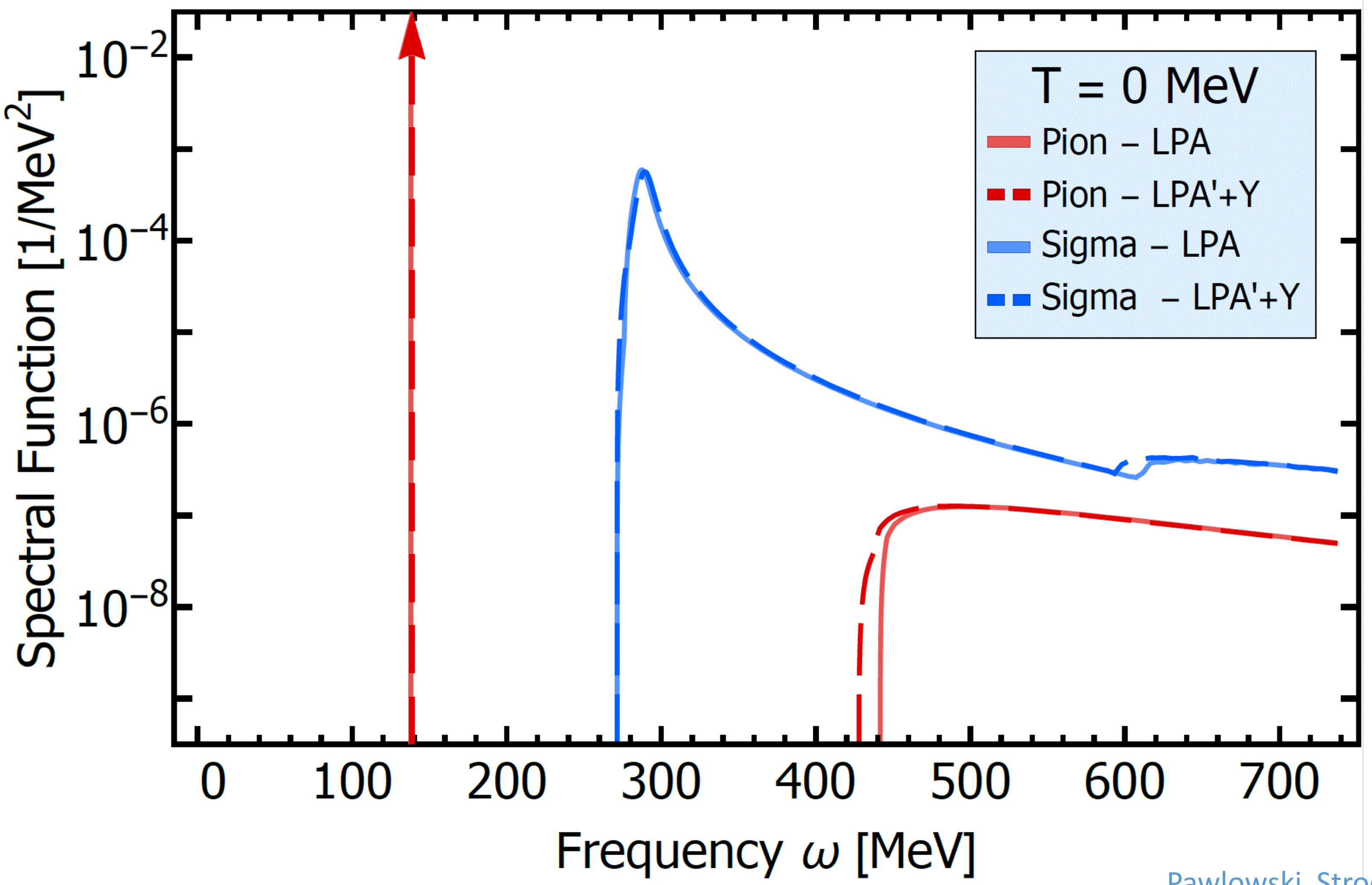
$$\lim_{\vec{p}\to 0}\lim_{p_0\to 0}\Gamma^{(2)}(p_0,\vec{p}) \neq \lim_{p_0\to 0}\lim_{\vec{p}\to 0}\Gamma^{(2)}(p_0,\vec{p})$$



Pawlowski, Strodthoff, NW, arxiv:1711.07444

Application to the O(N)-Model

Finite temperature spectral functions



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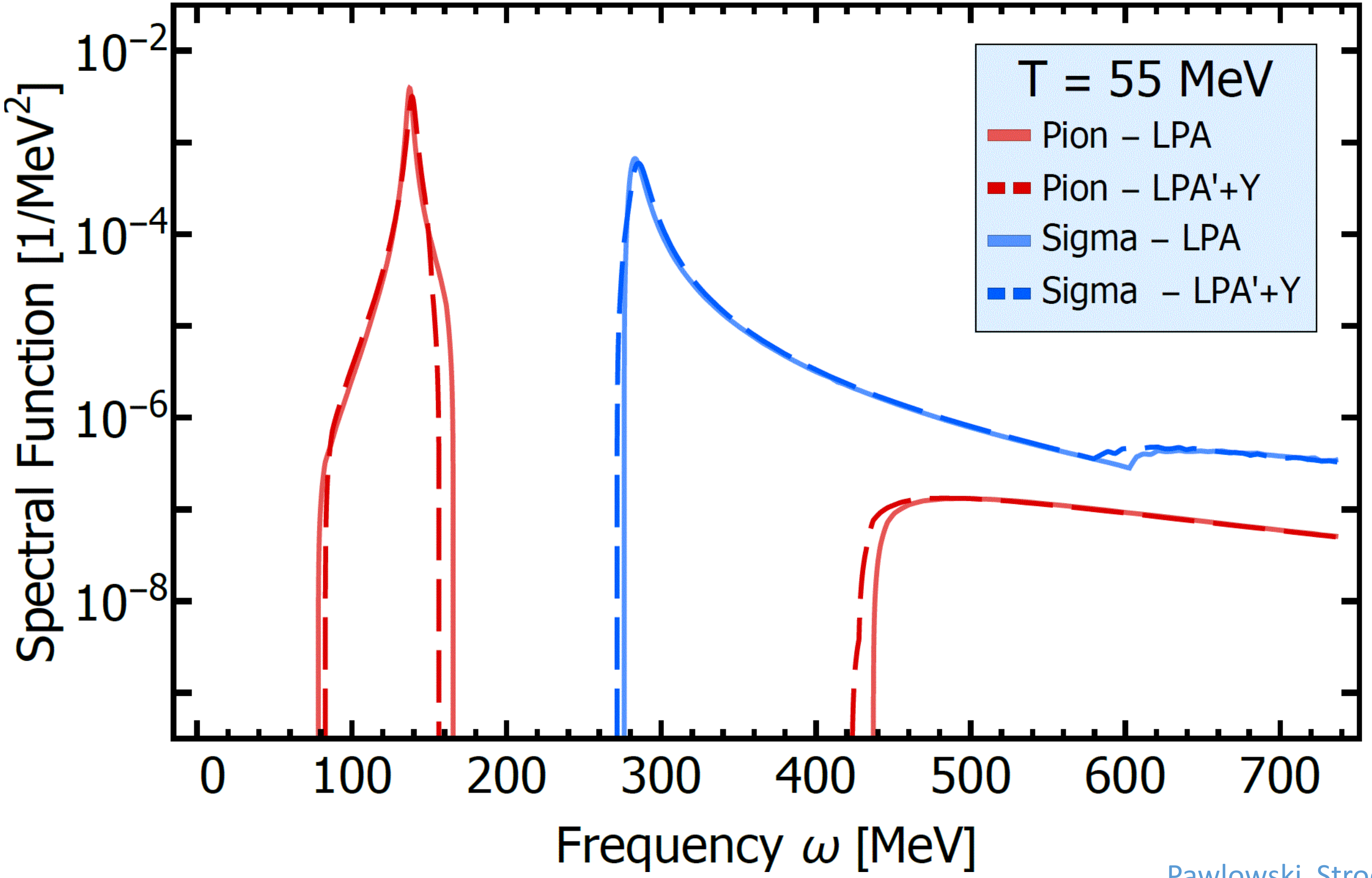

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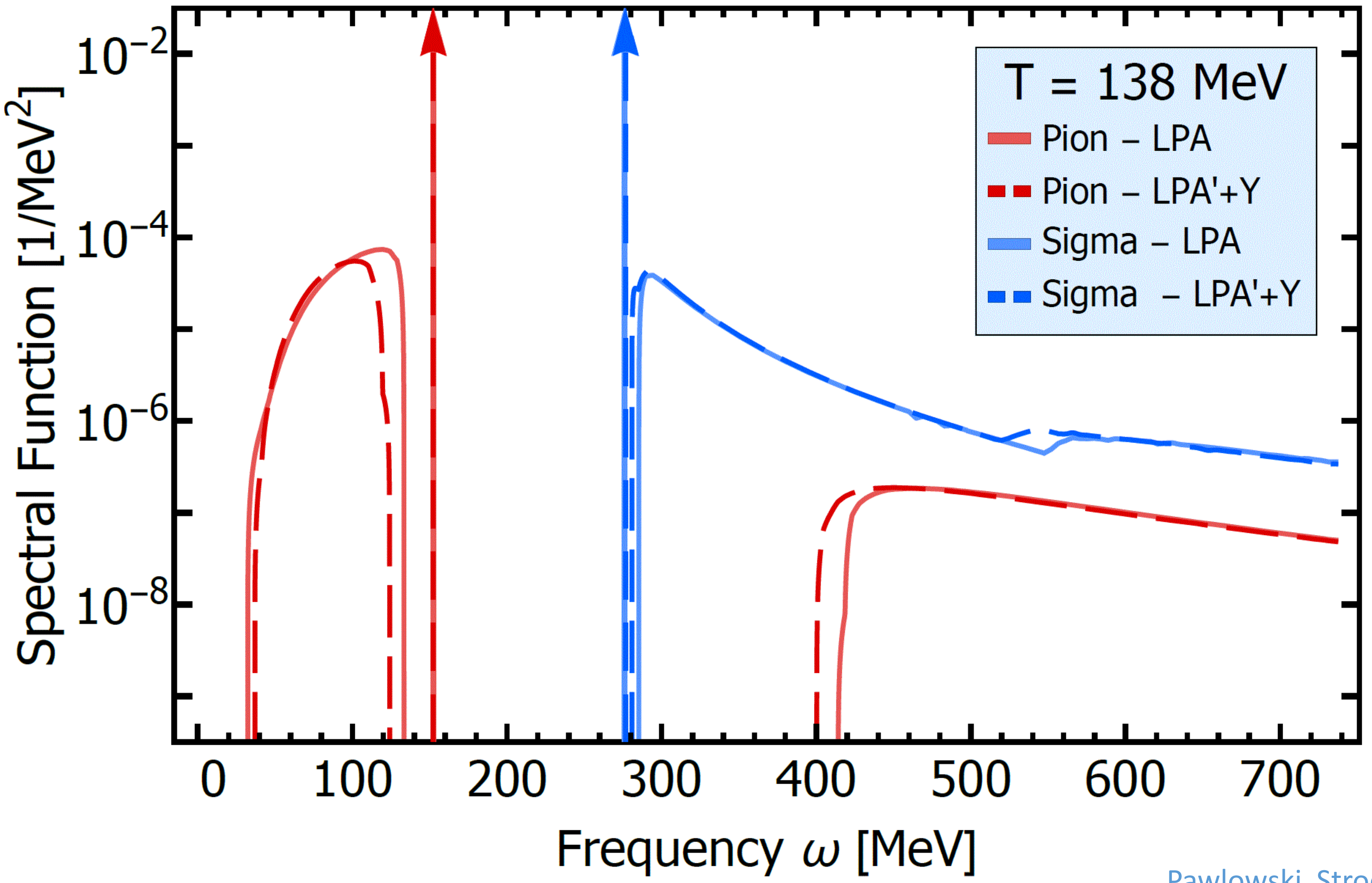

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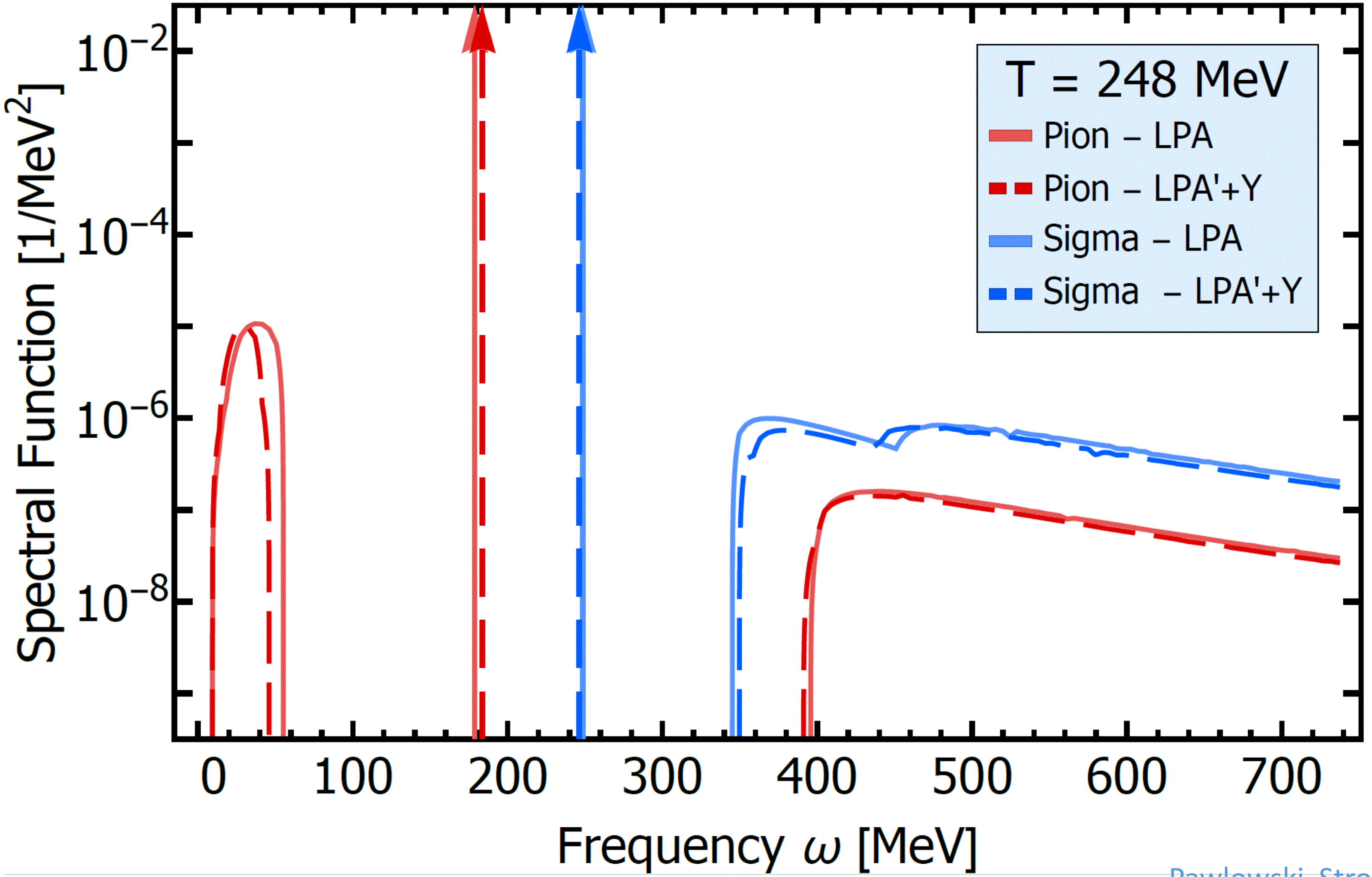

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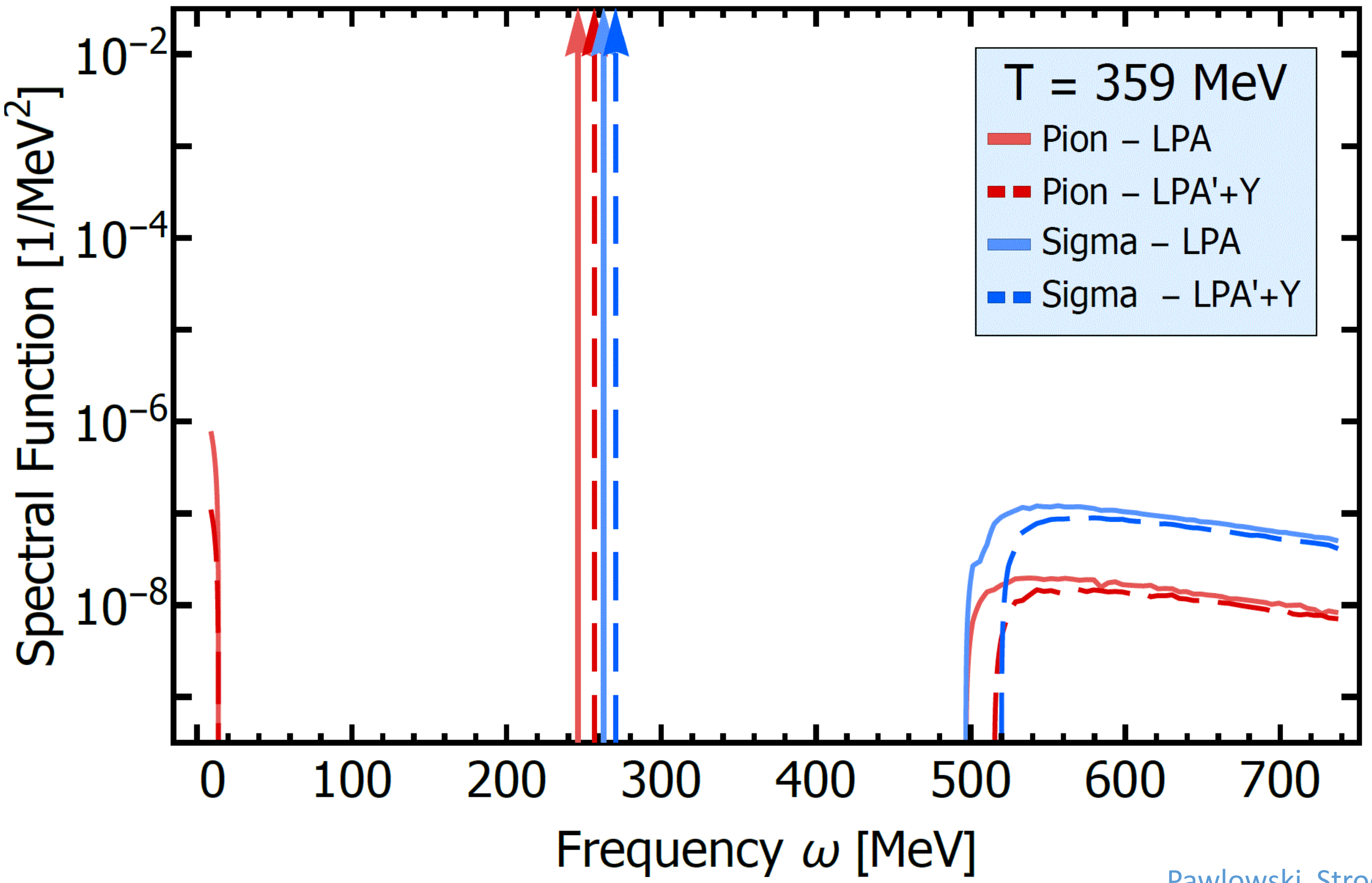
Finite temperature spectral functions

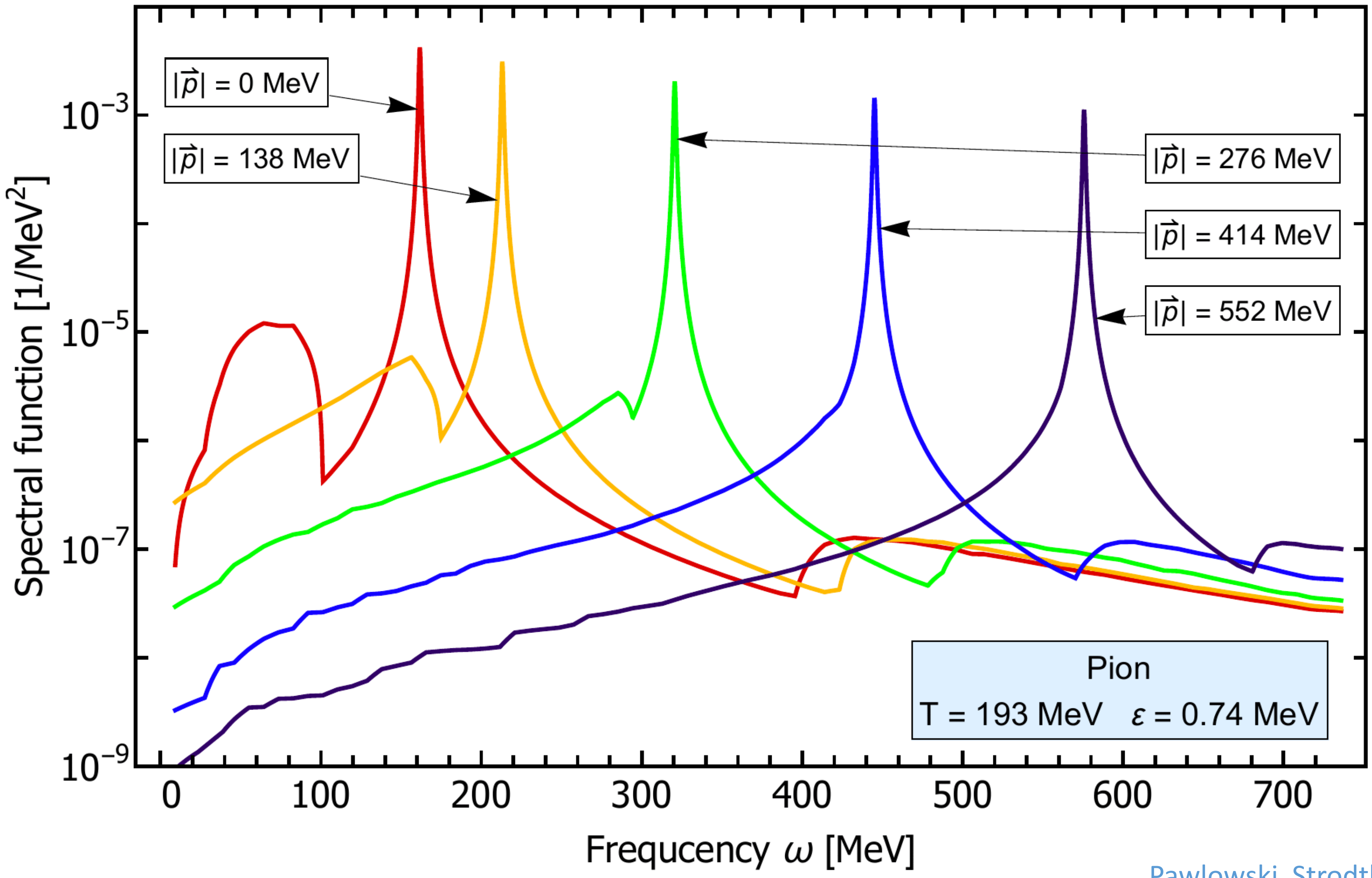


Pawlowski, Strodthoff, NW, arxiv:1711.07444

Application to the O(N)-Model

Pion meson

Finite temperature spectral function for various external momenta



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Application to the O(N)-Model

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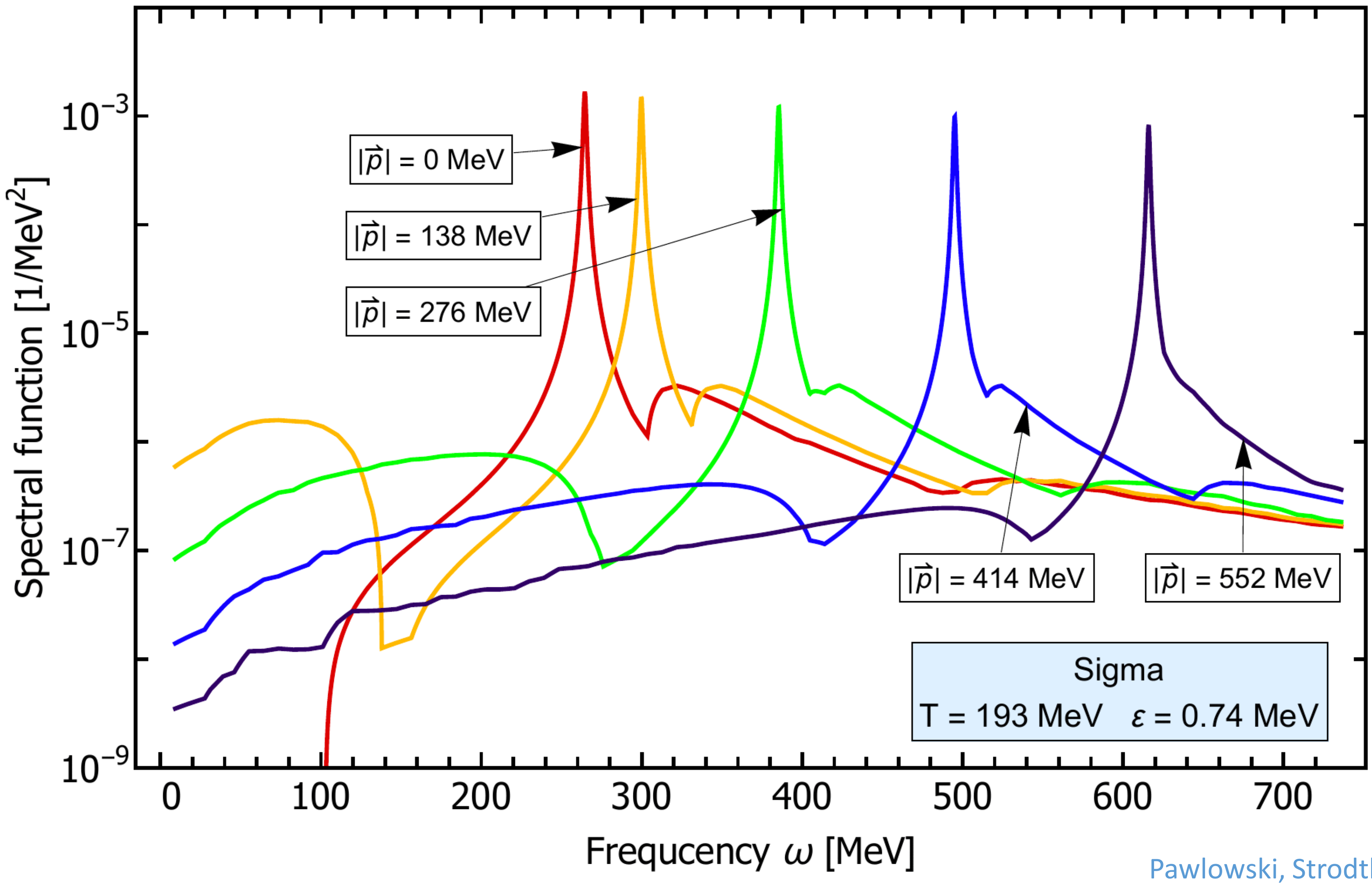

Pawlowski, Strodthoff, NW, arxiv:1711.07444

Spectral representation

Propagator

Spectral representation:

$$G(p_0,\vec{p}) = \int \frac{\mathrm{d}\eta}{2\pi}\,\frac{\rho(\eta,\vec{p})}{\eta - \mathrm{i}p_0} = \int_{\eta>0} \frac{\mathrm{d}\eta}{2\pi}\,2\eta\frac{\rho(\eta,\vec{p})}{\eta^2 + p_0^2}$$

Spectral function:

$$\rho(p_0,\vec{p}) = 2\,\mathrm{Im}\,G_{RA}(p_0,\vec{p})$$

Evans, Phys.Lett. B249 (1990)
Evans, Nucl.Phys. B374 (1992)
Bodeker, Sangel, JCAP 1706 (2017)
Pawlowski, NW, work in progress

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Spectral function:

$$\rho(p_0,\vec{p}) = 2\,\mathrm{Im}\,G_{RA}(p_0,\vec{p})$$

Three-point function

Spectral representation:

$$\Gamma^{(3)}(p_0,r_0) = \int \frac{\mathrm{d}\eta_1}{2\pi}\frac{\mathrm{d}\eta_2}{2\pi}\,\frac{-1}{(\eta_1+\eta_2) - \mathrm{i}(p_0+r_0)}\left[\frac{\rho_1(\eta_1,\eta_2)}{\eta_1 - \mathrm{i}p_0} + \frac{\rho_2(\eta_1,\eta_2)}{\eta_2 - \mathrm{i}r_0}\right]$$

preliminary

Spectral functions:

$$\rho_1 = 2\,\mathrm{Re}\left(\Gamma^{(3)}_{ARA} + \Gamma^{(3)}_{AAR}\right)$$
$$\rho_2 = 2\,\mathrm{Re}\left(\Gamma^{(3)}_{RAA} + \Gamma^{(3)}_{AAR}\right)$$

Degenerate for a identical fields →

$$\rho_1(\eta_1,\eta_2) = \rho_2(\eta_2,\eta_1)$$

Evans, Phys.Lett. B249 (1990)
Evans, Nucl.Phys. B374 (1992)
Bodeker, Sangel, JCAP 1706 (2017)
Pawlowski, NW, work in progress

Analytic continuations

Consider

$$\Gamma^{(n)}(p_1, p_2, \ldots, p_n)$$

Constrained by $\sum \varepsilon_i = 0$

Analytically continue with $p_i \to -\mathrm{i}(\omega_i + \mathrm{i}\varepsilon_i)$

Evans, Nucl.Phys. B374 (1992)
Hou, Wang, Heinz, J.Phys. G24 (1998)
Pawlowski, NW, work in progress

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Two-point function

	$\varepsilon_1/\varepsilon$	$\varepsilon_2/\varepsilon$	
$\Gamma^{(2)}_{\mathrm{RA}}$	$+1$	-1	Retarded
$\Gamma^{(2)}_{\mathrm{AR}}$	-1	$+1$	Advanced

Identities:

$$\Gamma^{(2)}_{\alpha\alpha} = 0 \quad \text{and} \quad \Gamma^{(2)}_{\alpha\beta} = \left(\Gamma^{(2)}_{\bar{\alpha}\bar{\beta}}\right)^*$$

Evans, Nucl.Phys. B374 (1992)
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Pawlowski, NW, work in progress

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Three-point function

	$\varepsilon_1/\varepsilon$	$\varepsilon_2/\varepsilon$	$\varepsilon_3/\varepsilon$
$\Gamma^{(2)}_{\mathrm{RAA}}$	$+2$	-1	-1
$\Gamma^{(2)}_{\mathrm{ARA}}$	-1	$+2$	-1
$\Gamma^{(2)}_{\mathrm{AAR}}$	-1	-1	$+2$
$\Gamma^{(2)}_{\mathrm{ARR}}$	-2	$+1$	$+1$
$\Gamma^{(2)}_{\mathrm{RAR}}$	$+1$	-2	$+1$
$\Gamma^{(2)}_{\mathrm{RRA}}$	$+1$	$+1$	-2

Identities:

$$\Gamma^{(3)}_{\alpha\alpha\alpha} = 0 \quad \text{and} \quad \Gamma^{(3)}_{\alpha\beta\gamma} = \left(\Gamma^{(3)}_{\bar{\alpha}\bar{\beta}\bar{\gamma}}\right)^*$$

Evans, Nucl.Phys. B374 (1992)
Hou, Wang, Heinz, J.Phys. G24 (1998)
Pawlowski, NW, work in progress

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$\Gamma^{(2)}_{\mathrm{ARA}}$	-1	$+2$	-1
$\Gamma^{(2)}_{\mathrm{AAR}}$	-1	-1	$+2$
$\Gamma^{(2)}_{\mathrm{ARR}}$	-2	$+1$	$+1$
$\Gamma^{(2)}_{\mathrm{RAR}}$	$+1$	-2	$+1$
$\Gamma^{(2)}_{\mathrm{RRA}}$	$+1$	$+1$	-2

Identities:

$$\Gamma^{(3)}_{\alpha\alpha\alpha} = 0 \quad \text{and} \quad \Gamma^{(3)}_{\alpha\beta\gamma} = \left(\Gamma^{(3)}_{\bar{\alpha}\bar{\beta}\bar{\gamma}}\right)^*$$

There are $2^n - 2$ n-point functions

of which $2^{n-1} - 1$ are independent

Number of different analytic continuations unknown for general n

Evans, Nucl.Phys. B374 (1992)
Hou, Wang, Heinz, J.Phys. G24 (1998)
Pawlowski, NW, work in progress

Analytic continuations

Consider

$$\Gamma^{(n)}(p_1, p_2, \ldots, p_n)$$

Constrained by $\sum \varepsilon_i = 0$

Analytically continue with $p_i \to -\mathrm{i}(\omega_i + \mathrm{i}\varepsilon_i)$

Four-point function

Signs of individual ε's does not fix signs of all possible sums

More analytic continuations (32) than retarded/advanced basis functions (16)

Evans, Nucl.Phys. B374 (1992)
Aurenche, Becherrawy, Nucl.Phys. B379 (1992)
Hou, Wang, Heinz, J.Phys. G24 (1998)
Pawlowski, NW, work in progress

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More analytic continuations (32) than retarded/advanced basis functions (16)

The 8 simple retarded/advanced functions $\Gamma^{(4)}_{\mathrm{RAAA}}$ ← Obtained from a single analytic continuation

The other 6 retarded/advanced functions $\Gamma^{(4)}_{\mathrm{RRAA}}$ ← Superposition of four analytic continuations

↓

Four possibilities for the signs of

$\varepsilon_1 > 0, \varepsilon_2 > 0$
$\varepsilon_3 < 0, \varepsilon_4 < 0$ → $\varepsilon_1 + \varepsilon_4$, $\varepsilon_2 + \varepsilon_3$ → $\Gamma^{(4)}_{\mathrm{RRAA}}$ is the direct linear superposition

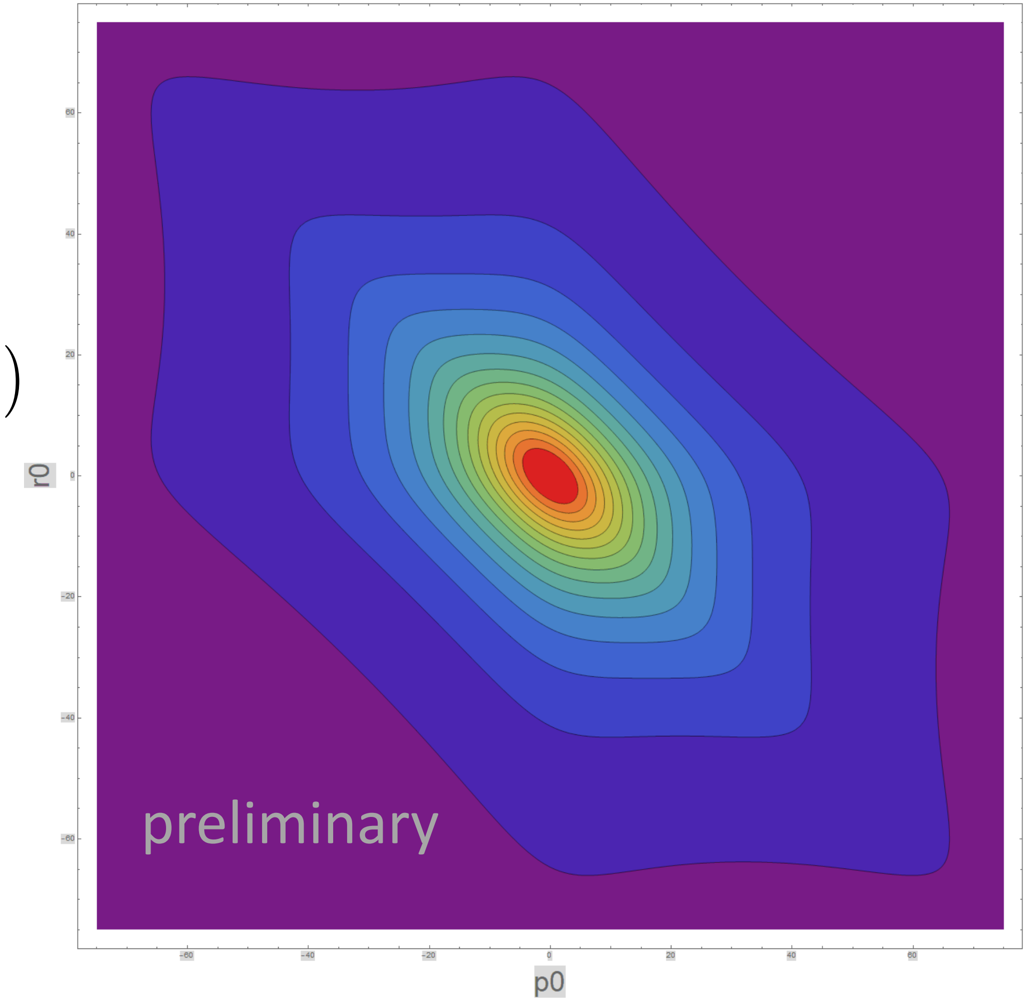
Evans, Nucl.Phys. B374 (1992)
Aurenche, Becherrawy, Nucl.Phys. B379 (1992)
Hou, Wang, Heinz, J.Phys. G24 (1998)
Pawlowski, NW, work in progress

Application to scalar field

1^{st} iteration for a scalar field

Euclidean three point function

$$\Gamma^{(3)}_{\mathrm{Eucl}}(p_0, r_0, \vec{p}=0, \vec{r}=0)$$



Pawlowski, NW, work in progress

Application to scalar field

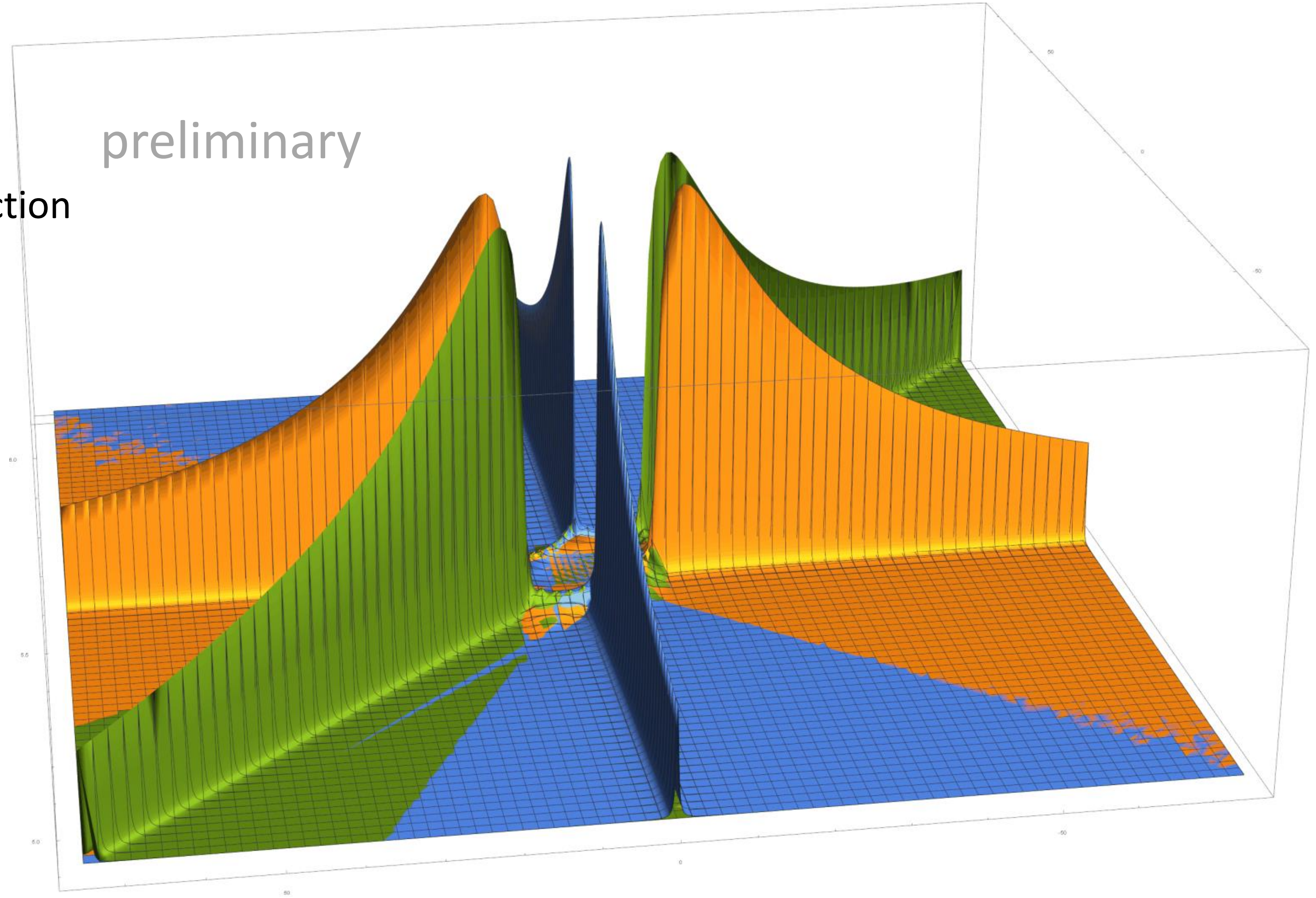
1st iteration for a scalar field

Real part of analytic continued three-point function

$\Gamma^{(3)}_{\mathrm{RAA}}(\omega_1, \omega_2, \vec{p}=0, \vec{r}=0)$ (blue)

$\Gamma^{(3)}_{\mathrm{ARA}}(\omega_1, \omega_2, \vec{p}=0, \vec{r}=0)$ (orange)

$\Gamma^{(3)}_{\mathrm{AAR}}(\omega_1, \omega_2, \vec{p}=0, \vec{r}=0)$ (green)



Pawlowski, NW, work in progress

Application to scalar field

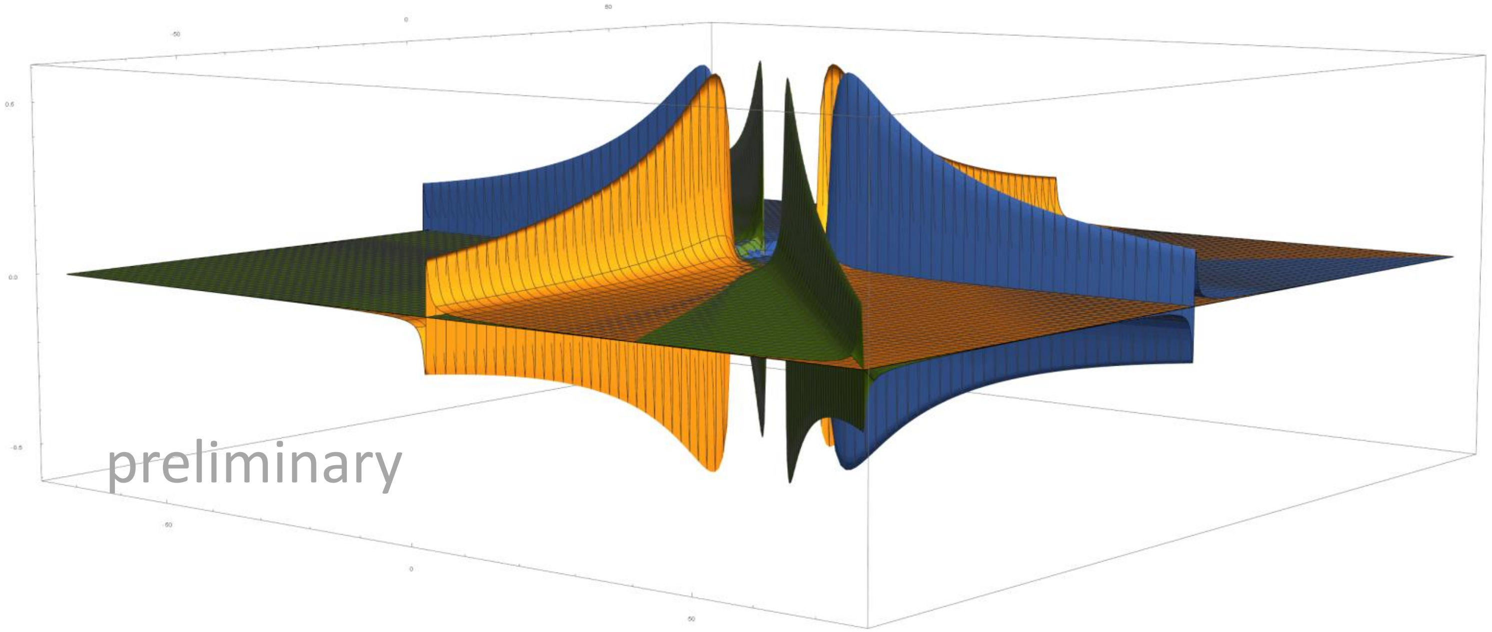
1^{st} iteration for a scalar field

Imaginary part of analytic continued three-point function

$$\Gamma^{(3)}_{\mathrm{RAA}}(\omega_1,\omega_2,\vec{p}=0,\vec{r}=0)$$ (blue)

$$\Gamma^{(3)}_{\mathrm{ARA}}(\omega_1,\omega_2,\vec{p}=0,\vec{r}=0)$$ (orange)

$$\Gamma^{(3)}_{\mathrm{AAR}}(\omega_1,\omega_2,\vec{p}=0,\vec{r}=0)$$ (green)



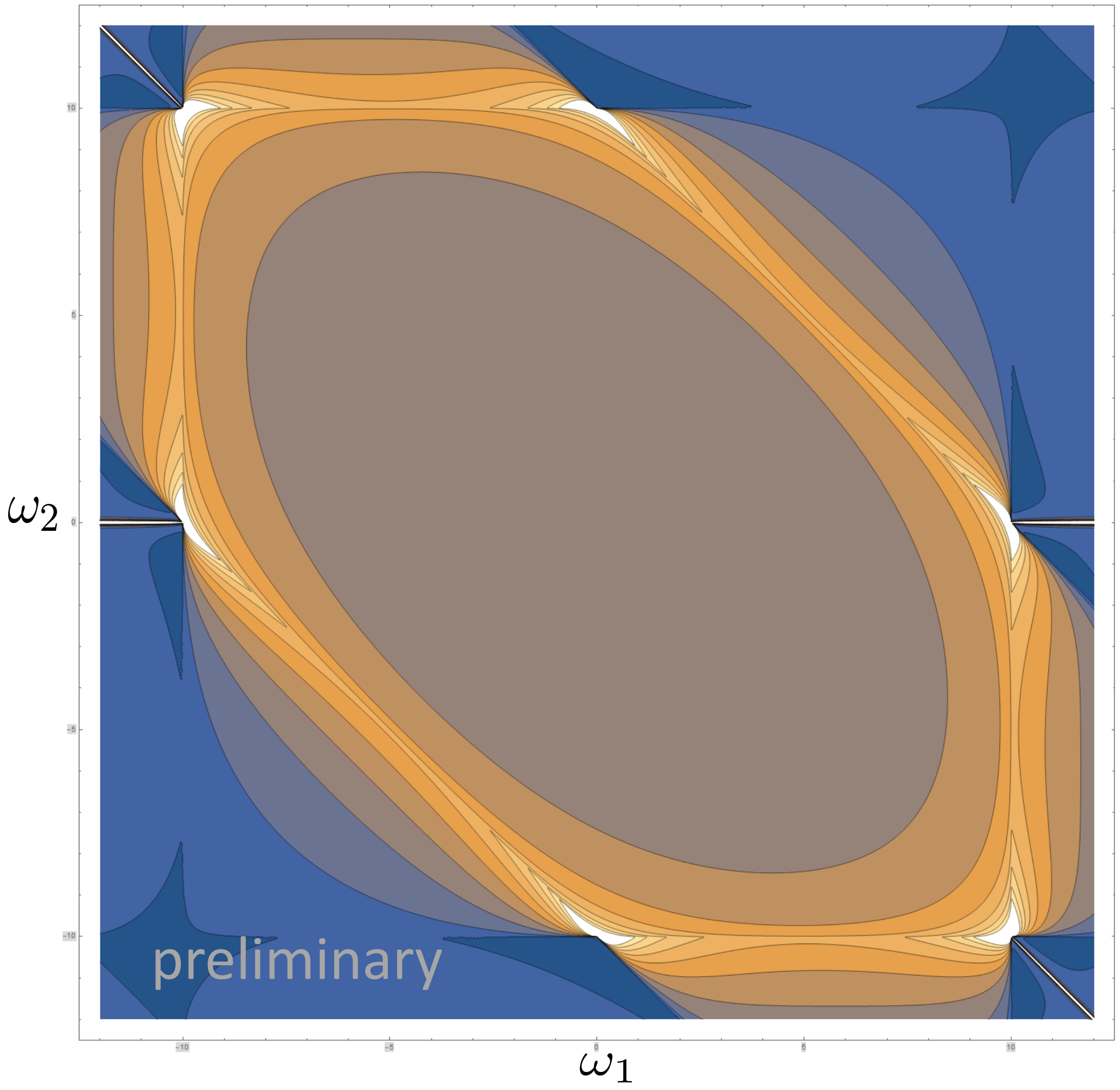
Pawlowski, NW, work in progress

Application to scalar field

1st iteration for a scalar field

Three-point spectral density

$\rho_1(\omega_1, \omega_2)$



Pawlowski, NW, work in progress

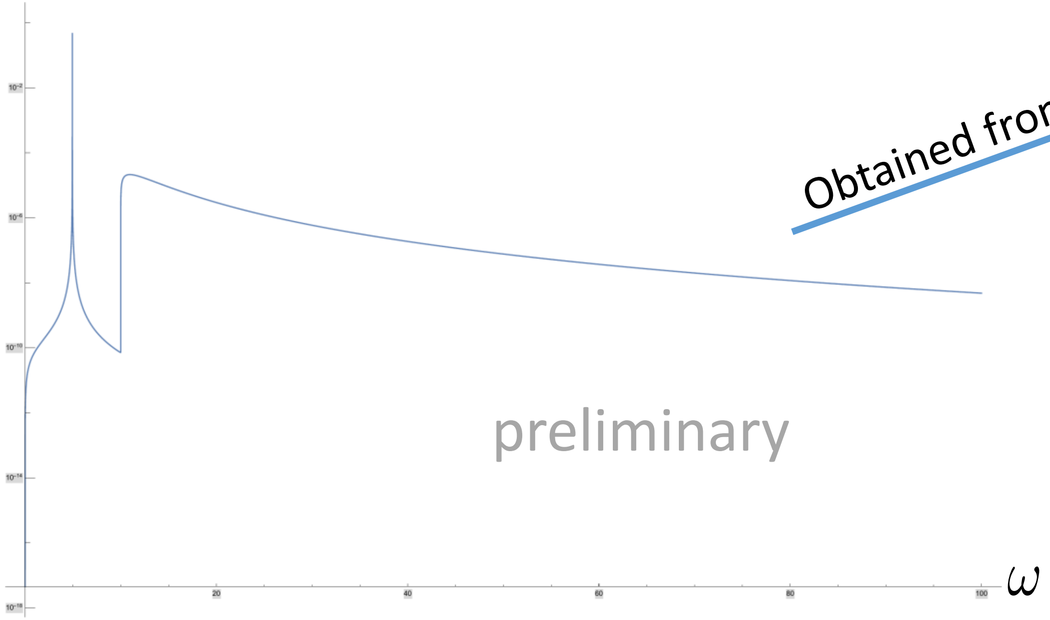
Application to scalar field

1^{st} iteration for a scalar field

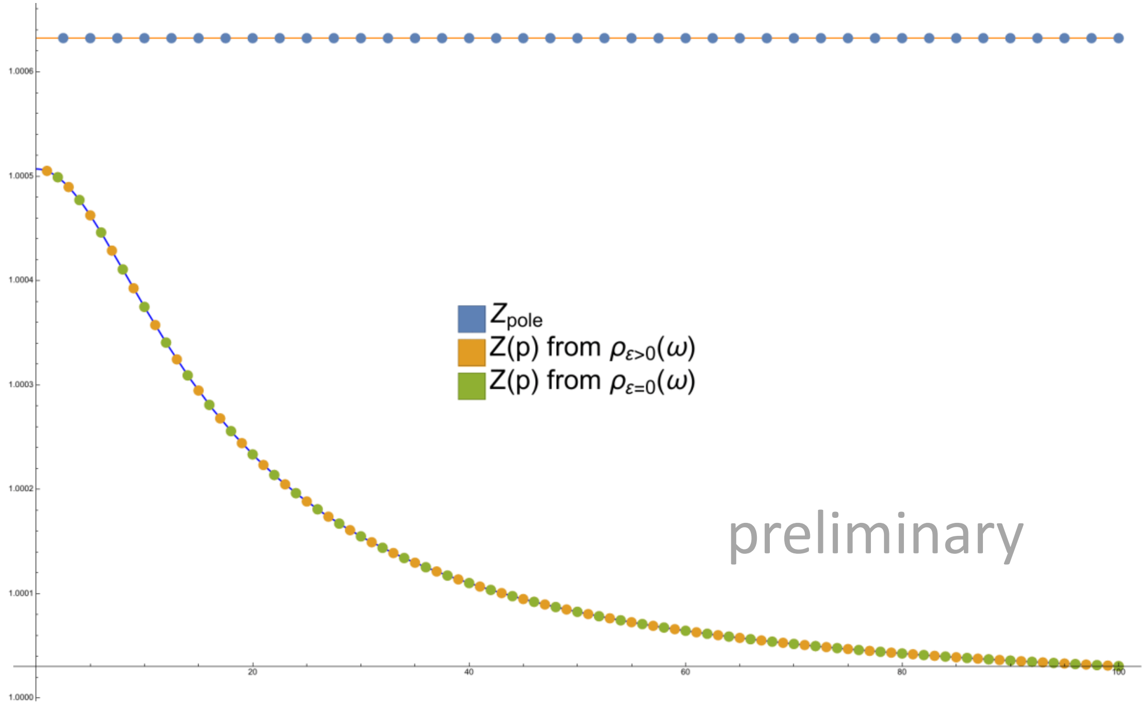
Reconstruction two-point function

Euclidean Dressing

Spectral function



Obtained from



$$G(p_0,\vec{p}) = \sum_{j\in\text{poles}} \frac{R_j(\vec{p})}{p_0 - \mathrm{i}m_j(\vec{p})} + \int_{\text{cut}} \mathrm{d}\eta\, \frac{\rho(\eta,\vec{p})}{\eta - \mathrm{i}p_0}$$

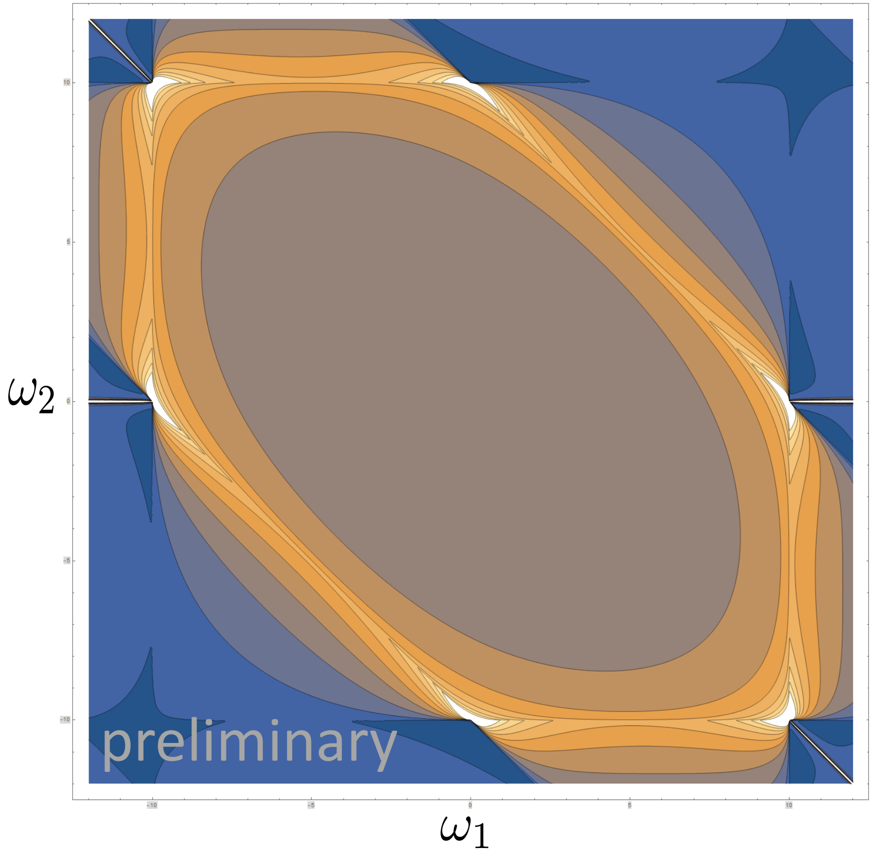
Pawlowski, NW, work in progress

Application to scalar field

1st iteration for a scalar field

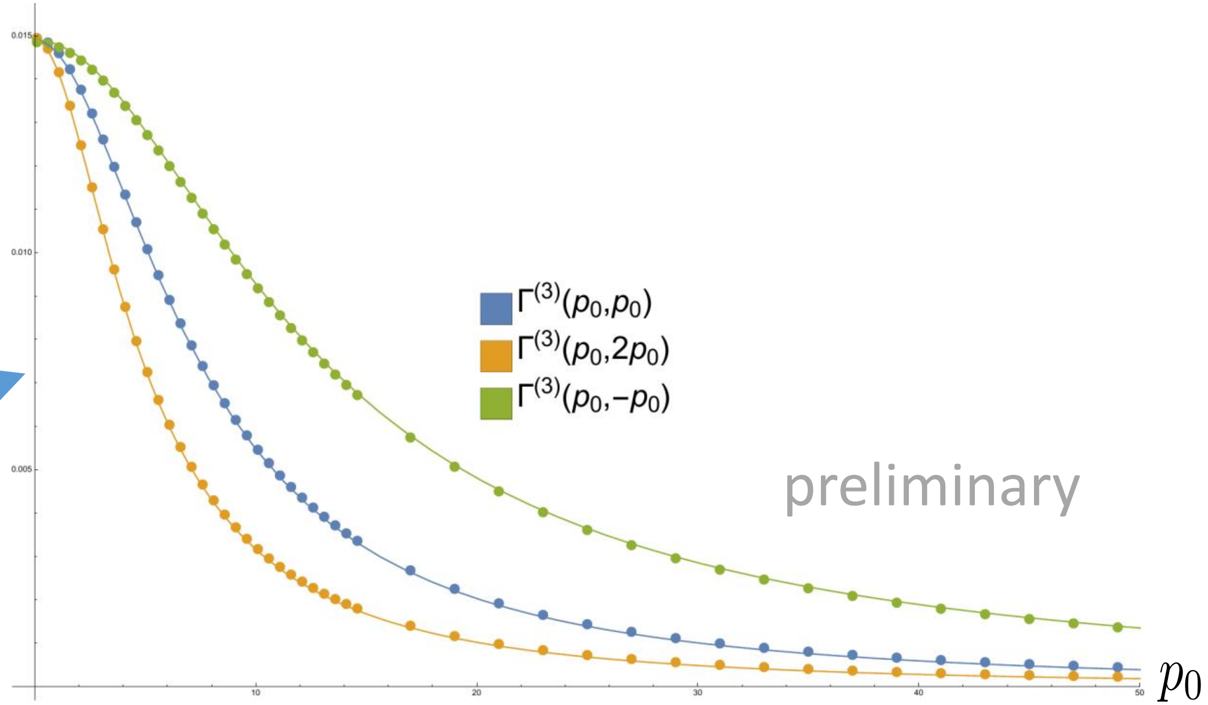
Reconstruction three-point function

Spectral function



Obtained from

Euclidean Dressing



$$\Gamma^{(3)}(p_0, r_0) = \int \frac{d\eta_1}{2\pi} \frac{d\eta_2}{2\pi} \frac{-1}{(\eta_1+\eta_2) - i(p_0+r_0)} \left[\frac{\rho_1(\eta_1,\eta_2)}{\eta_1 - ip_0} + \frac{\rho_2(\eta_1,\eta_2)}{\eta_2 - ir_0} \right]$$

Pawlowski, NW, work in progress