Quantum annealing algorithms for track pattern recognition

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High Luminosity LHC

- HL-LHC coming soon! (2026~)
  - High luminosity \( L = 5 \times 10^{34} \text{ cm}^{-2}\text{s}^{-2} \)
  - High pileup \( \langle \mu \rangle = 200 \)
  - High readout rate
- CPU consumption will dramatically increase
  - especially track reconstruction due to the high pileup

New ideas required
Track reconstruction

- Find the correct set of hits belonging to the same particle
  - Hits are distributed according to known physics rule: helix curve, scattering with material…
- Combinatorial optimization problem
  - HL-LHC environment (μ=200)
    - $10^4$ particles, ~10 detector layers. i.e. $10^5$ hits
  - Does quantum computing favor such a problem?

![Hit and Track candidate diagram](image-url)
Quantum Computer

- Arrange gates for each problem
- General-purpose computer

Universal quantum circuit

Quantum annealer

- Find the minimum energy state of a given Hamiltonian
- Suitable for an optimization problem

Focus on quantum annealer
Quantum annealing

- Construct Hamiltonian taking minimum energy at a target state
- Start with the superposition of all possible states under an external transverse B-field, then move to the optimal state.

What is the best assignment for qubit?
- How is Hamiltonian designed?
Qubit assignment

- **Doublets**
  - Original paper (Stimpfl-Abele, et. al., 1991)
  - Zigzag pattern, many qubits required due to many possible fake candidates

- **Triplets**
  - This work (arxiv: 1902.08324)
  - Strong fake reduction, resulting in a reasonable number of qubits
Hamiltonian

\[ E = \alpha \left( \sum_{i}^{N} T_i \right) - \left( \sum_{i,j} S_{i,j} T_i T_j \right) + \zeta \left( \sum_{i,j} T_i T_j \right), \quad T \in \{0, 1\} \]

- bias weight
- Connection strength
- Avoid conflicts, zigzag pattern, holes

Quadratic Unconstrained Binary Optimization (QUBO)

\[ O(a, b, T) = \sum_{i}^{N} a_i T_i + \sum_{i}^{N} \sum_{j < i}^{N} b_{ij} T_i T_j \]

\[ b_{ij} = \begin{cases} -S(T_i, T_j) & T \in \{0, 1\} \\ \zeta & \end{cases} \]

qubit
Overview

Quantum annealing

HEPQPR.Qallse: https://github.com/derlin/hepqpr-qallse
D-Wave Quantum Annealing Machine

- Superconducting qubits (cool down to 15 mK)
- 2048 qubits (D-wave 2000Q)
- Chimera graph
  - 16x16 units, 8 qubits / unit
  - 6016 couplers
  - ~64-bit full connection
- Annealing time 1-2000 μs

Next-generation machine:
- Pegasus processor
- 5000 qubits
- 2020 mid

\[ O(a, b, T) = \sum_{i}^{N} a_i T_i + \sum_{i}^{N} \sum_{j<i}^{N} b_{ij} T_i T_j \]

qubit: \( T_i \)  
bias weight: \( a_i \)  
coupling strength: \( b_{ij} \)

Chimera graph

Minor embedding
Solving

- Large QUBOs split into sub-QUBOs due to limited number of qubits
  - Solve each small QUBOs using D-wave hardware
- Repeat annealing to guarantee an optimal solution

4900 particles (60% of HL-LHC)

Energy of solution vs total time

- New minimum energy solution
- New solution but not minimum

Construct track candidates using doublets in final triplets
Results

1600 particles (20% of HL-LHC) - 11000 hits

<table>
<thead>
<tr>
<th>Input</th>
<th>Doublet selection</th>
<th>Annealing</th>
</tr>
</thead>
<tbody>
<tr>
<td>390000 Doublets</td>
<td>2445 Doublets</td>
<td>1424 Doublets</td>
</tr>
<tr>
<td>Purity</td>
<td>0.22 %</td>
<td>98.5 %</td>
</tr>
<tr>
<td>Efficiency</td>
<td>99.5 %</td>
<td>96.4 %</td>
</tr>
</tbody>
</table>
Results

- Reference solver: neal = simulated annealing using CPU
- >90% efficiency / purity below 6000 particles environment
- Equivalent performance with the classical annealing (neal)
Digital annealing

- **Aiming to larger & easier-to-use annealing device**
  - Limit from QUBO size at tracking

- **Is a digital annealing a candidate?**
  - Simulated annealing using logic circuits on a chip

- **Fujitsu Digital annealer**
  - Anneal by artificial fluctuation
  - Work at normal temperature, no quantum noise
  - 8192-bit full connection (2nd generation chips)
  - Precision of weight in QUBO: 64 bit

- **We’ve demonstrated our track finding algorithm with the 1st generation chips (1024 qubits)**
  - Equivalent performance with the classical annealing algorithm (neal)
  - Results are stable
  - Annealing time: 0.5 sec (depends on problem sizes)
    - ~4sec for CPU pre-processing
Summary

- HL-LHC poses a **significant challenge to computing**
  - especially track reconstruction
- One of the new ideas is **quantum annealing**
- Hamiltonian and QUBO are constructed based on triplet as qubits.
- Successful results for quantum annealing using **D-wave machine and Fujitsu digital annealer**

**Future work**
- QUBO Optimization
  - Optimize the coupling strength, bias term, conflict term,...
  - Other qubits: quadraplet, doublet, hit,...
- Comparison between different devices
  - D-wave
  - Digital annealer (Fujitsu, Hitachi, …)