PhaseTracer
-- calculating cosmological phase transition properties

Yang Zhang
Monash University

Outline

- **Introduction**
  - Electroweak phase transition

- **Methods to calculate transition strength**

- **Examples in PhaseTracer**
  - 1-D test model
  - 2-D test model
  - Z2 Scalar Singlet Model
  - Next-to-Minimal Supersymmetric Standard Model

- **Summary**
To explain the baryon asymmetry of the Universe (BAU)

\[ \eta \equiv \frac{n_B - n\bar{B}}{n_\gamma} \approx 6 \times 10^{-10}, \]

Sakharov Criteria for baryogenesis must be satisfied:

- Baryon number violation,
- C-symmetry and CP-symmetry violation,
- Interactions out of thermal equilibrium.

- Realized by Electroweak phase transition
Introduction

In the broken phase, the rate of sphaleron transitions must be strongly suppressed. The bound can be translated into bound on the strength of transition:

$$\frac{E_{\text{sph}}(T_c)}{T_c} > 37 \rightarrow \gamma \equiv \frac{\Delta \phi(T_c)}{T_c} \approx \frac{1}{36} \frac{E_{\text{sph}}(T_c)}{T_c} > 1.0$$

arXiv:1206.2942
## Calculation of transition strength

- **SM + a Z2 symmetric real scalar singlet** (arXiv:1611.02073)

\[
V(H, s, T) = (\mu_h^2 + c_h T^2)H^\dagger H + \lambda_h (H^\dagger H)^2 + \frac{\lambda_{hs}}{2}(H^\dagger H)s^2 + \frac{(\mu_s^2 + c_s T^2)}{2}s^2 + \frac{\lambda_s}{4}s^4
\]

- **Method 1**
  - Analytic calculation

  \[
  \frac{\partial V}{\partial v} \bigg|_{v_h, v_s=0} = \frac{\partial V}{\partial v} \bigg|_{v_s, v_h=0} = 0
  \]

  \[
  v_h = -\frac{\mu_h^2 + c_h T^2}{\lambda_h}, \quad v_s = -\frac{\mu_s^2 + c_s T^2}{\lambda_s}
  \]

  \[
  V(v_h, 0, T_c) = V(0, v_s, T_c)
  \]

- **Method 2**

- **Method 3**

\[
T_c^2 = \frac{\lambda_s c_h \mu_h^2 - \lambda_h c_s \mu_s^2 - \sqrt{\lambda_h \lambda_s} \left| c_s \mu_h^2 - c_h \mu_s^2 \right|}{\lambda_s c_h^2 - \lambda_h c_s^2}
\]
## Calculation of transition strength

<table>
<thead>
<tr>
<th>Method 1</th>
<th>Method 2</th>
<th>Method 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bisection</td>
<td>Starting with an un-broken phase at high temperature and a broken phase at low temperature, find a temperature that these two phase degenerate using bisection method.</td>
<td></td>
</tr>
</tbody>
</table>

> Used in BSMPT (arXiv: 1803.02846)
Calculation of transition strength

Method 1
Method 2
Method 3

Trace minimums

Starting with a minimum at low/high temperature, trace the location of minimum at different temperatures.

➢ Used in CosmoTransitions (arXiv: 1109.4189)
Calculation of transition strength

- Find more than one transitions.
- The phase structure can be used to calculate nucleation temperature, the time scale of phase transition ...

Method 1
Analytic calculation
- Fast, Stable, Many assumptions

Method 2
Bisection
- Slow, Unstable, No assumption

Method 3
Trace minimums
Calculation of transition strength

Method 1
- Fast, Stable, Many assumptions
- Analytic calculation

Method 2
- Slow, Unstable, No assumption
- Bisection

Method 3
- Trace minimums

<table>
<thead>
<tr>
<th>Method</th>
<th>2D simplified potential</th>
<th>NMSSM</th>
</tr>
</thead>
<tbody>
<tr>
<td>CosmoTransitions</td>
<td>4.97 seconds</td>
<td>4 minutes</td>
</tr>
</tbody>
</table>
Based on method 3, we develop a C++ program, *PhaseTracer*, to map phase structure and calculate transition strength. Comparing with CosmoTransition (the transition strength calculation part), Phasetracer is optimized for scan:

- **Faster**

We performed a comprehensive scan for NMSSM with PhaseTracer, and collected more than 3 million valid points.
Example: 1-D test model

\[ V(\phi, T) = -100\phi^2 - 10\phi^3 + 0.1\phi^4 + 0.1\phi^2T^2 , \]

\$ ./bin/run_1D_test_model

=== phase key = 0 ===
Maximum temperature = 61.7344
Minimum temperature = 0
Field at tmax = [38.3267]
Field at tmin = [81.1602]
Potential at tmax = 65720
Potential at tmin = -1.66587e+06
Ended at tmax = Jump in fields indicated end of phase
Ended at tmin = Reached tstop

=== phase key = 1 ===
Maximum temperature = 1000
Minimum temperature = 0
Field at tmax = [2.0146e-06]
Field at tmin = [-6.16063]
Potential at tmax = 4.05456e-07
Potential at tmin = -1313.12
Ended at tmax = Reached tstop
Ended at tmin = Reached tstop

found 1 transition

=== transition from 2 to 1 ===
true vacuum = [49.8362]
false vacuum = [2.0146e-06]
changed = [true]
TC = 59.2295
gamma = 0.841408

Automatically generated by PhaseTracer
Example: 2-D test model

The examples implemented in CosmoTransitions:

\[ V(\phi_1, \phi_2, T) = V_0(\phi_1, \phi_2) + V_1(\phi_1, \phi_2) + V_{1T}(\phi_1, \phi_2, T). \]

\[ V_0(\phi_1, \phi_2) = \frac{1}{8} \frac{m_1^2}{v^2} (\phi_1^2 - v^2) + \frac{1}{8} \frac{m_2^2}{v^2} (\phi_2^2 - v^2) - \mu^2 \phi_1 \phi_2 \]

\[ V_1(\phi) = \pm \frac{1}{64\pi^2} \sum_i n_i \, m_i^4(\phi) \left[ \log \frac{m_i^2(\phi)}{Q^2} - c_i \right], \]

\[ V_{1T}(\phi, T) = \frac{T^4}{2\pi^2} \sum_i n_i J^+_i \left( \frac{m_i(\phi)}{T} \right) \]

<table>
<thead>
<tr>
<th></th>
<th>Tc(GeV)</th>
<th>False VEV</th>
<th>True VEV</th>
<th>Elapsed time</th>
</tr>
</thead>
<tbody>
<tr>
<td>CosmoTransitions</td>
<td>109.408</td>
<td>(220.0, -150.0)</td>
<td>(263.5, 314.7)</td>
<td>4.97 seconds</td>
</tr>
<tr>
<td>PhaseTracer</td>
<td>109.408</td>
<td>(220.0, -150.0)</td>
<td>(263.5, 314.7)</td>
<td>0.07 second</td>
</tr>
</tbody>
</table>
Example: Z2 Scalar Singlet Model

\[ V(H, s) = (\mu_h^2 + c_h T^2) H^\dagger H + \lambda_h (H^\dagger H)^2 + \frac{\lambda_h s}{2} (H^\dagger H)s^2 + \frac{(\mu_s^2 + c_s T^2)}{2} s^2 + \frac{\lambda_s}{4} s^4 \]

\[ T_c^2 = \frac{\lambda_s c_h \mu_h^2 - \lambda_h c_s \mu_s^2 - \sqrt{\lambda_h \lambda_s} |c_s \mu_h^2 - c_h \mu_s^2|}{\lambda_s c_h^2 - \lambda_h c_s^2} \]

\[ \lambda_s = 0.1 \]

Obtained from PhaseTracer

*Dependent on setting*
Example: NMSSM

Strong first-order phase transitions in the NMSSM - a comprehensive survey (arXiv:1908.11847)
Peter Athron, Csaba Balazs, Andrew Fowlie, Giancarlo Pozzo, Graham White, Yang Zhang

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Range</th>
<th>Metric</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda$</td>
<td>$0, \pi/2$</td>
<td>flat</td>
</tr>
<tr>
<td>$</td>
<td>\kappa</td>
<td>$</td>
</tr>
<tr>
<td>$</td>
<td>A_\lambda</td>
<td>$</td>
</tr>
<tr>
<td>$</td>
<td>A_\kappa</td>
<td>$</td>
</tr>
<tr>
<td>$</td>
<td>A_t</td>
<td>$</td>
</tr>
<tr>
<td>$m_{\text{SUSY}}$</td>
<td>$1, 10,\text{TeV}$</td>
<td>log</td>
</tr>
<tr>
<td>$</td>
<td>v_S</td>
<td>$</td>
</tr>
<tr>
<td>$\tan\beta$</td>
<td>$1, 60$</td>
<td>log</td>
</tr>
</tbody>
</table>

$$\chi^2 = \chi^2_{\text{Higgs}} + \chi^2_{\text{SFOPT}} + \chi^2_{\text{LEP}}$$
Example: NMSSM

From arXiv:1908.11847
Summary

PhaseTracer

• Find cosmological phases and calculate transition properties.
• Relatively fast and stable, suitable for scan.
• Easily linked to FlexibleSUSY to obtain field-dependent masses.

Upcoming

• PhaseTracer will link to BubbleProfiler to get actions, and then calculate nucleation temperature and other transition properties.
• The codes will be public soon.