Investigating the anomalous magnetic moment on the lattice

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Background

Muon anomalous magnetic moment $(g-2)$ is an extremely sensitive test of the SM

Experimentally measured to high accuracy 0.54 ppm

$$11659209.1(5.4)(3.3) \times 10^{-10}$$

Calculated with SM to high accuracy 0.51 ppm

$$11659182.04(3.56) \times 10^{-10}$$

Note the $3-4\sigma$ discrepancy

Background

Two new experiments aim to improve experimental result

- Fermilab E989 [Started March 2018, run 1 results early 2020]
- J-PARC E34 [Expected start early 2020]

Aim to improve experimental results fourfold

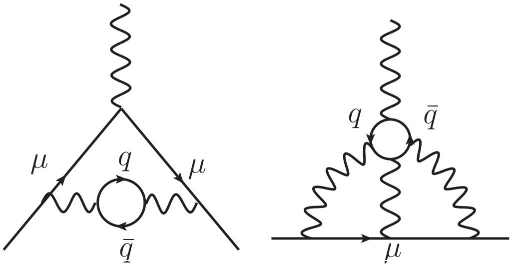
Standard Model calculations needs similar improvements

SM Calculation

Current best estimate

$$a_\mu^{SM} = (\text{QED} \pm 0.0007 + \text{EW} \pm 0.10 + \text{Hadronic} \pm 5.02) \times 10^{-10}$$

- QED and EW terms are well constrained
- Largest uncertainty come from Hadronic term
- We are interested in the leading order HVP term



Lattice

Current best values for HVP come from e^+e^- scattering cross sections.

$$2.5 \times 10^{-10} \text{ uncertainty}$$

However, there is some tension in scattering results.

Lattice provides an independent method from first principles

Current best lattice results $\sim 18 \times 10^{-10}$ uncertainty. $\sim 2\%$ of a_μ^{HVP}

As lattice results approach new levels of precision, we need to investigate QED effects

In this study we use fully dynamical QCD+QED

Lattice Setup

Simulate with 6 ensembles

- $32^3 \times 64$
- $48^3 \times 96$

Lattice spacing $a = 0.068 fm$

Partially-quenched quark masses

$$260 \leq m_\pi^{q\bar{q}} \leq 770 MeV$$

Non-compact QED with exaggerated QED coupling

$$\alpha_{QED} = \frac{e^2}{4\pi} \sim 0.1$$

Lattice Setup

Use the novel QCDSF mass tuning

Keep average quark mass fixed at physical value

$$\bar{m}^{phys} = \frac{1}{3}\left(m_{up} + m_{down} + m_{strange}\right)$$

QED inclusion effects quark mass

Use 'Dashen Scheme', define $SU(3)_{sym}$ via $m_\pi^{u\bar{u}} = m_\pi^{d\bar{d}} = m_\pi^{n\bar{n}}$

- Neutral, $q = 0$ $m_\pi^{n\bar{n}} = 408(3) MeV$
- Down, $q = -\frac{1}{3}$ $m_\pi^{d\bar{d}} = 409(1) MeV$
- Up, $q = \frac{2}{3}$ $m_\pi^{u\bar{u}} = 407(3) MeV$

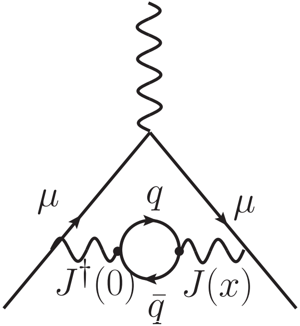
Accessing a_μ^{HVP}

Time-momentum representation

$$a_\mu^{HVP} = 4\alpha^2 \int_0^\infty dt G(t) \tilde{K}\left(t; m_\mu^2\right)$$

$G(t)$ is the vector-vector 2-pt function

$$G(t) = \frac{1}{3} \sum \int d^3x \left\langle J(x) J^\dagger(0) \right\rangle$$



$\tilde{K}$ is known kernel
[Bernecker-Meyer arXiv:1107.4388]

Note: requires long-time integral.

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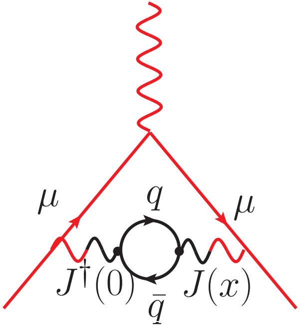

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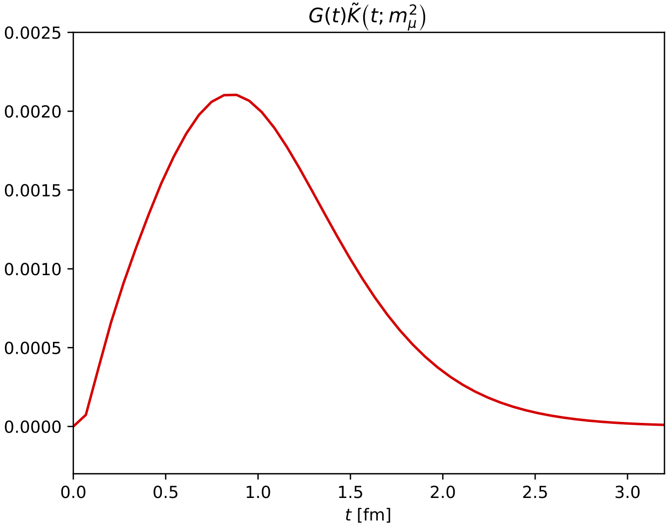

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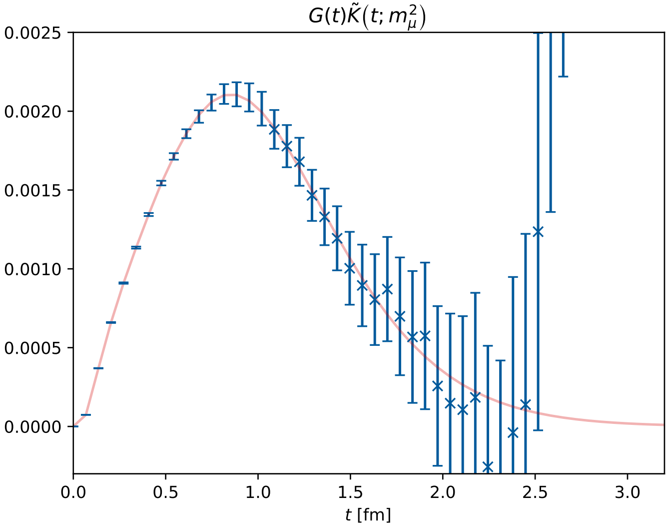

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Lattice data very noisy at large t

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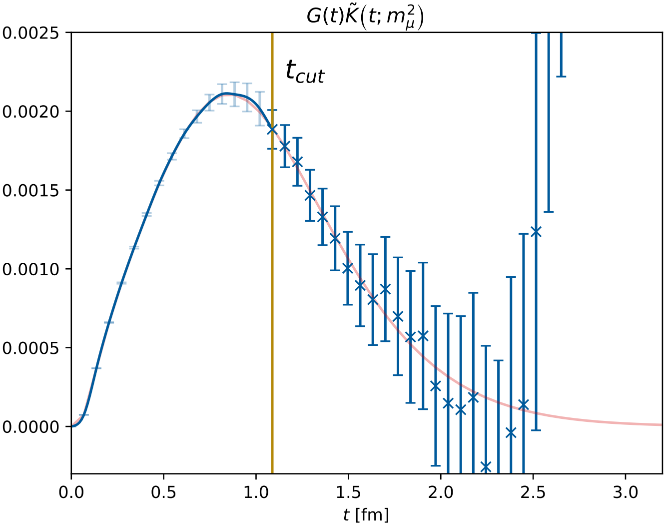

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$$G(t) = \begin{cases} G(t) & t \leq t_{cut} \\ & \end{cases}$$

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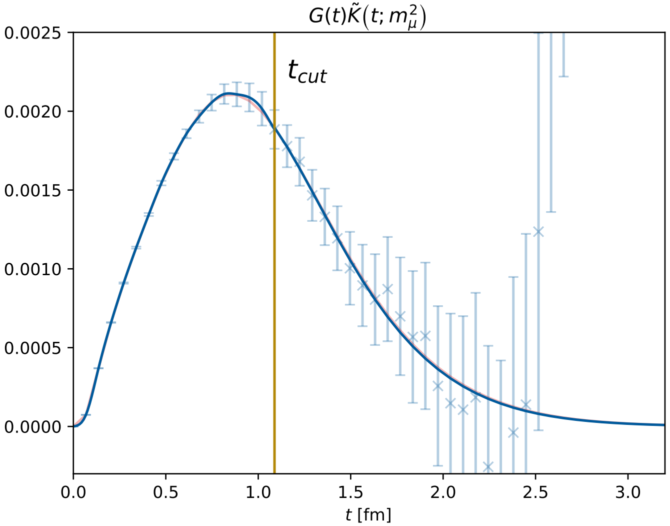

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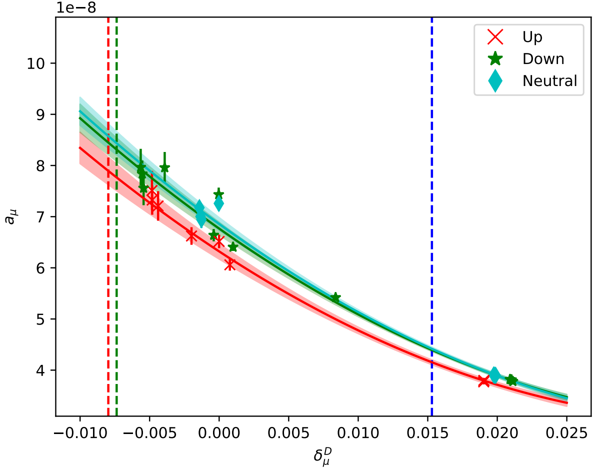
Lattice data very noisy at large t

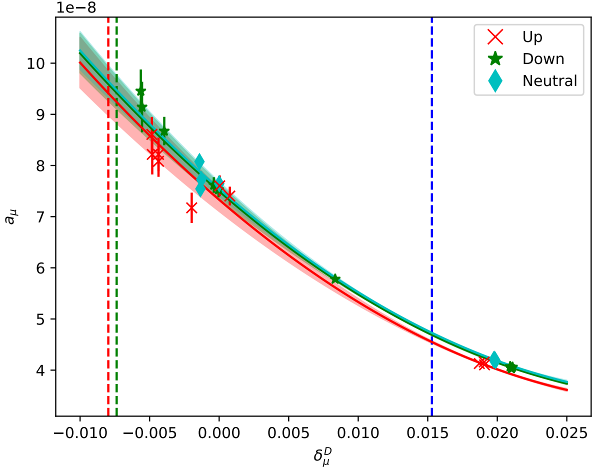
$$G(t) = \begin{cases} G(t) & t \leq t_{cut} \\ Ae^{-mt} & t > t_{cut} \end{cases}$$

Results - a_μ^{HVP}



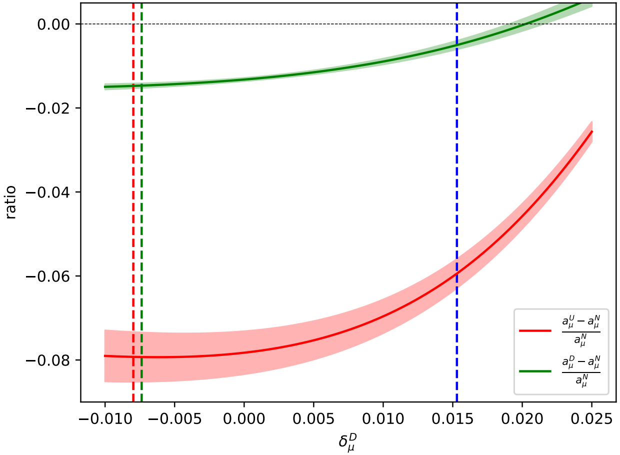
$32^3 \times 64$ Lattice results

Results - a_μ^{HVP}



$48^3 \times 96$ Lattice results

Results - Charge Effect



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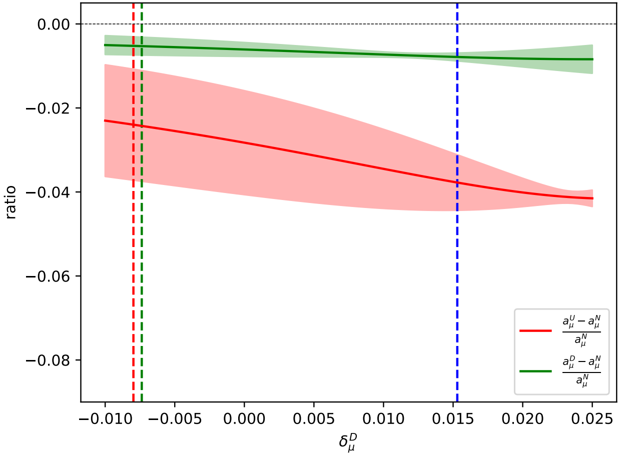
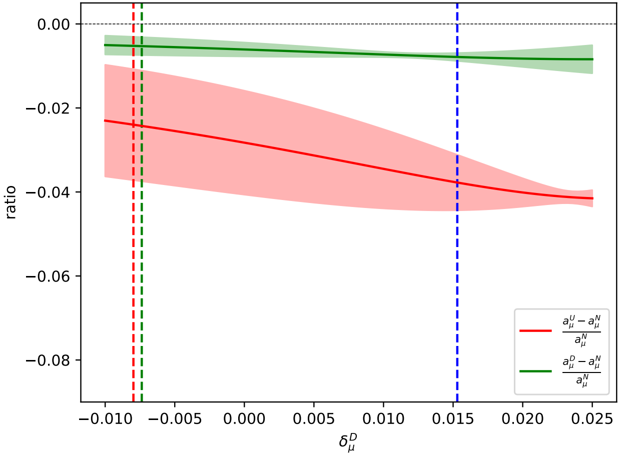


$48^3 \times 96$ Lattice results

QED contributions $\sim$ 2% effect

Results - Charge Effect

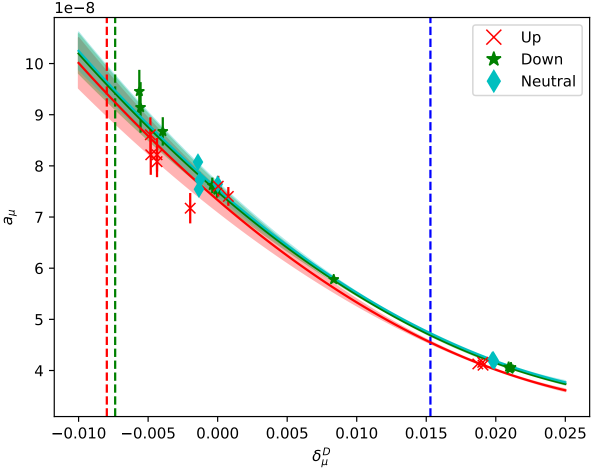


$48^3 \times 96$ Lattice results

QED contributions $\sim$ 0.2% effect

Remember $\alpha_{QED} \sim 10$ times physical.

Results - Charge Effects



Combine quark contributions

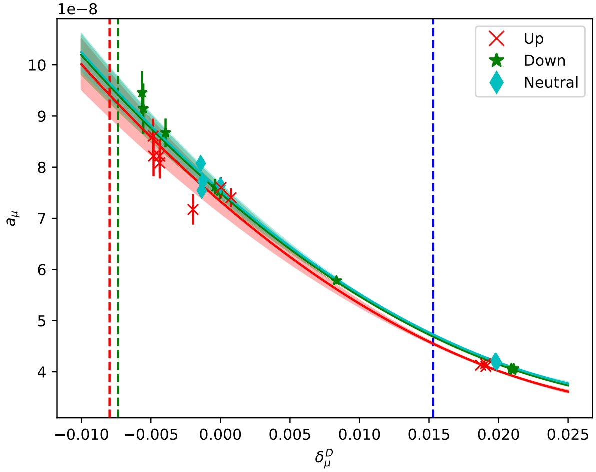
$$a^{HVP}_{\mu,\text{charged}} = \frac{4}{9}(u_c) + \frac{1}{9}(d_c) + \frac{1}{9}(s_c)$$

$$a^{HVP}_{\mu,\text{neutral}} = \frac{4}{9}(u_0) + \frac{1}{9}(d_0) + \frac{1}{9}(s_0)$$

Take the ratio → charge effects

$2\% \pm \ 1\%$

Results - Charge Effects



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Take the ratio $\rightarrow$ charge effects $0.2\% \pm 0.1\%$

Remember $\alpha_{QED} \sim 10$ times physical.

Summary

Remember

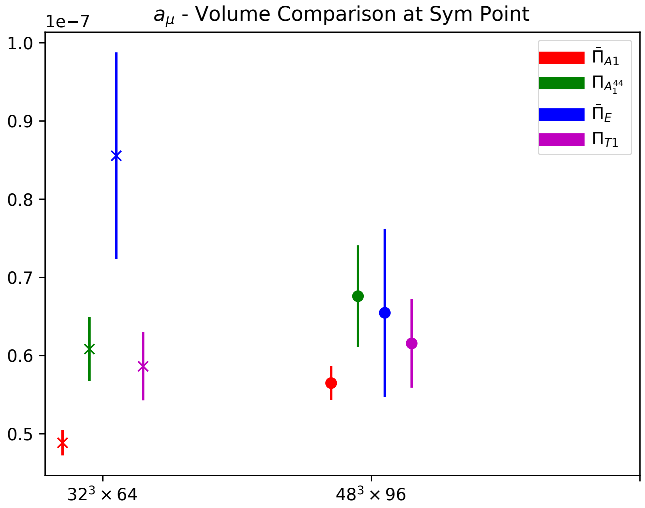
- current lattice results sits at around 2%
- current e^+e^- scattering results at around 0.5%
- QED effects around 0.2%

If lattice results are to reach desired precision, need to consider QED corrections.

Summary

Thank you for listening

BONUS: Finite volume effects



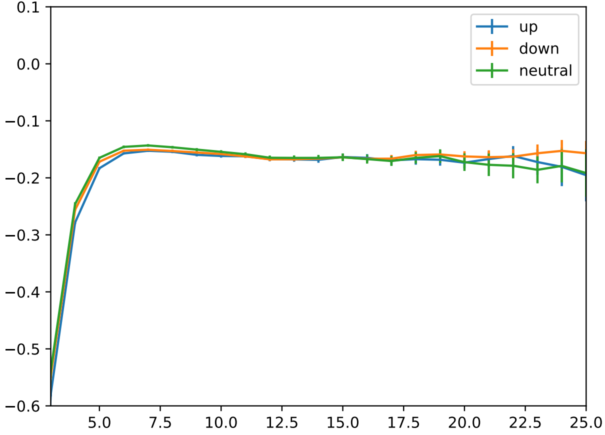
$$A_1 : \sum_i \bar{\Pi}_{ii} = \left(3q^2 - \vec{q}^{\,2}\right) \bar{\Pi}_{A_1},$$

$$T_1 : \bar{\Pi}_{4i} = -\left(q_4 q_i\right) \bar{\Pi}_{T_1}$$

$$A_1^{44} : \bar{\Pi}_{44} = \left(\vec{q}^{\,2}\right) \bar{\Pi}_{A_1^{44}},$$

$$E : \bar{\Pi}_{ii} - \sum_i \bar{\Pi}_{ii}/3 = \left(-q_i^2 + \vec{q}^{\,2}/3\right) \bar{\Pi}_E .$$

BONUS: Z_V calculation



$$J^R_{\mu,f} = Z_V^{m_f} J_{\mu,f} \left(1 + c_V \frac{\partial_\nu T_{\mu\nu,f}}{J_{\mu,f}} \right) .$$

$\mathcal{O}(a)$ improvements not fully included, but appear charge independent.