The computational challenge of lattice chiral symmetry - Is it worth the expense?

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This is a reversal of ordering predicted by simple quark models.

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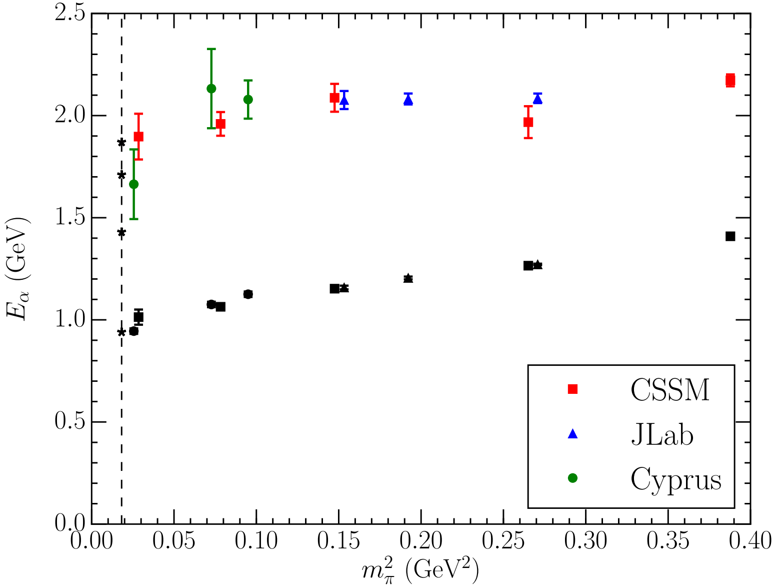
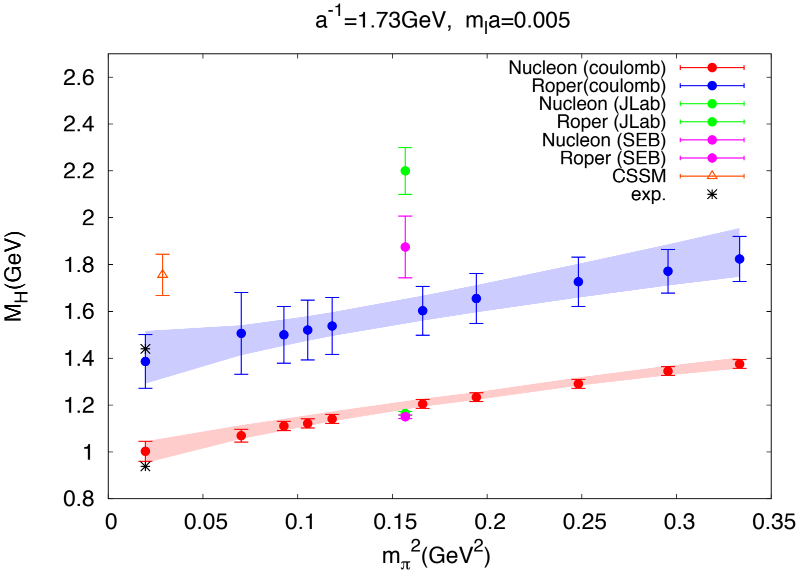
We aim to carefully asses the role chiral symmetry plays in understanding the Roper in lattice QCD.

Positive parity nucleon spectrum



χQCD results



Summary of fermion actions

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Naive finite difference

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Naive finite difference $\implies$ fermion doubling

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Not straightforward...

No-Go theorem

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It is not possible to find a lattice Dirac operator D that simultaneously satisfies the following conditions:

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It is not possible to find a lattice Dirac operator D that simultaneously satisfies the following conditions:

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4. Chiral symmetry.

Ginsparg-Wilson Relation

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Solution found in 90's - the overlap fermion action.

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- The matrix sign function is expensive to evaluate.
- $D_o \sim \mathcal{O}(100)$ times more expensive than Wilson-type fermions.

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Chiral fermions:

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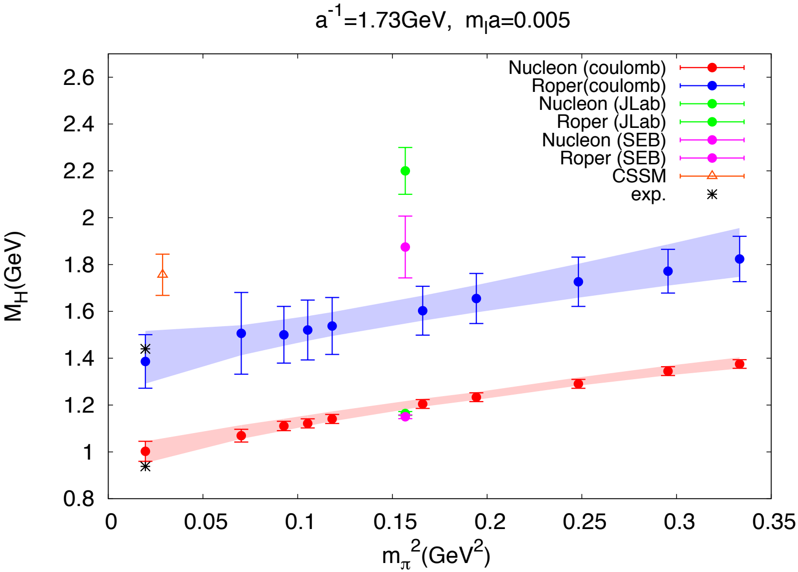

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Does the Overlap action deliver a spectrum 300 MeV lower than the NP-improved Clover?

Waiting for the dust to settle...

J. Segovia and C. D. Roberts, "Dissecting nucleon transition electromagnetic form factors," Phys. Rev. C **94** (2016) no.4, 042201 [arXiv:1607.04405 [nucl-th]].

G. Eichmann, H. Sanchis-Alepuz, R. Williams, R. Alkofer and C. S. Fischer, "Baryons as relativistic three-quark bound states," Prog. Part. Nucl. Phys. **91** (2016) 1 [arXiv:1606.09602 [hep-ph]].

G. Eichmann, C. S. Fischer and H. Sanchis-Alepuz, "Light baryons and their excitations," Phys. Rev. D **94** (2016) no.9, 094033 [arXiv:1607.05748 [hep-ph]].

G. Yang, J. Ping and J. Segovia, "The S- and P-Wave Low-Lying Baryons in the Chiral Quark Model," Few Body Syst. **59** (2018) no.6, 113 [arXiv:1709.09315 [hep-ph]].

V. D. Burkert and C. D. Roberts, "Colloquium : Roper resonance: Toward a solution to the fifty year puzzle," Rev. Mod. Phys. **91** (2019) no.1, 011003 [arXiv:1710.02549 [nucl-ex]].

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Perform simulations at three valence quark masses:

$$m_\pi = 0.435(4),\ 0.577(4),\ 0.698(4)\ \text{GeV}.$$

Variational correlation matrix analysis

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Construct the Dirac-traced correlation function at $\vec{p} = 0$

$$G_{ij}(t) = \sum_{\alpha} \lambda_i^{\alpha} \, \bar{\lambda}_j^{\alpha} \, \mathrm{e}^{-m_\alpha t} \, ,$$

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Find a linear combination of creation/annihilation operators of interpolators

$$\bar{\phi}^{\alpha} = \bar{\chi}_j \, u_j^{\alpha} \qquad \text{and} \qquad \phi^{\alpha} = \chi_i \, v_i^{\alpha} \, ,$$

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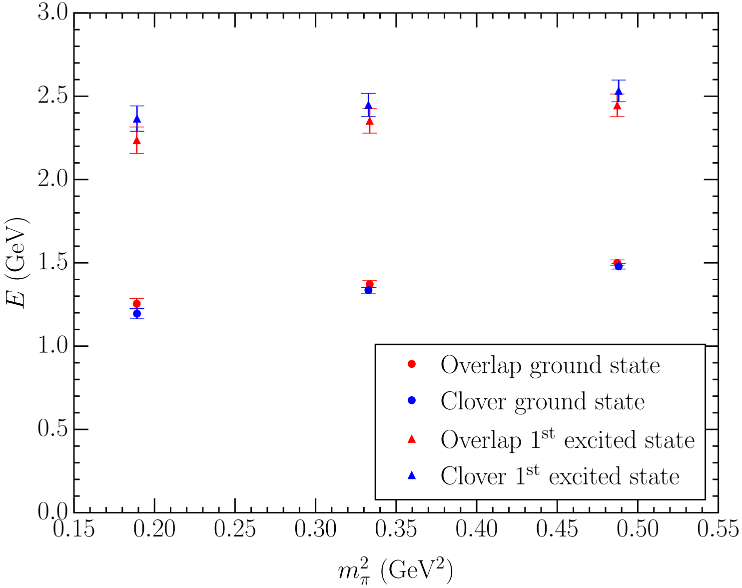
$$G_{ij}(t_0 + dt)\, u_j^{\alpha} = \mathrm{e}^{-m_{\alpha} dt}\, G_{ij}(t_0)\, u_j^{\alpha} \,,$$

and solve the GEVP to obtain the eigenstate projected correlator

$$G^{\alpha}(t) = v_i^{\alpha}\, G_{ij}(t)\, u_j^{\alpha} \,.$$

Positive parity nucleon spectrum for $t_0 = 1$, $t = t_0 + dt = 4$



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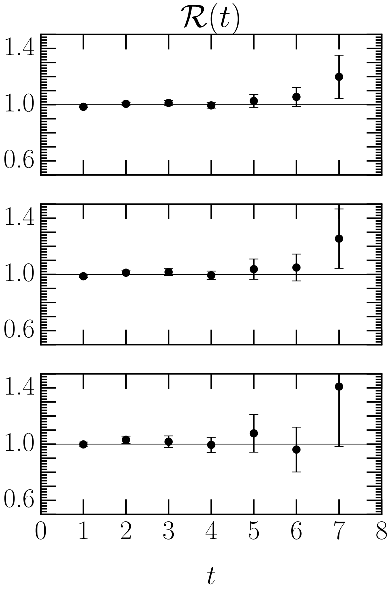
$$\mathcal{R}(t) = \frac{R^{\text{clover}}_{1/0}(t)}{R^{\text{overlap}}_{1/0}(t)}$$

and compare with 1.

$\mathcal{R}(t)$ for $t_0 = 1$, $t = t_0 + dt = 4$



Heaviest

Middle

Lightest

χ^2/d.o.f. of $\mathcal{R}(t) = 1$ for $2 \leqslant t \leqslant 6$.

m_π/GeV	χ^2/d.o.f.
0.698(4)	0.757
0.577(4)	0.850
0.435(4)	1.002

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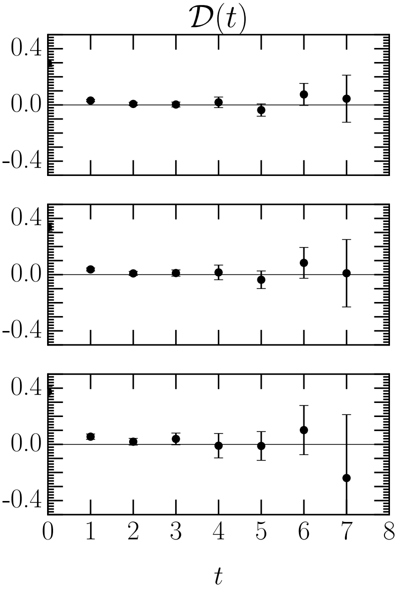
$$\Delta M_{\mathrm{eff}}(t) = \ln\left(\frac{G_{1/0}(t)}{G_{1/0}(t+1)}\right)\,.$$

Calculate the difference

$$\mathcal{D}(t) = \Delta M_{\mathrm{eff}}^{\mathrm{clover}}(t) - \Delta M_{\mathrm{eff}}^{\mathrm{overlap}}(t)$$

and compare with 0 GeV.

$\mathcal{D}(t)$ for $t_0 = 1$, $t = t_0 + dt = 4$



Heaviest

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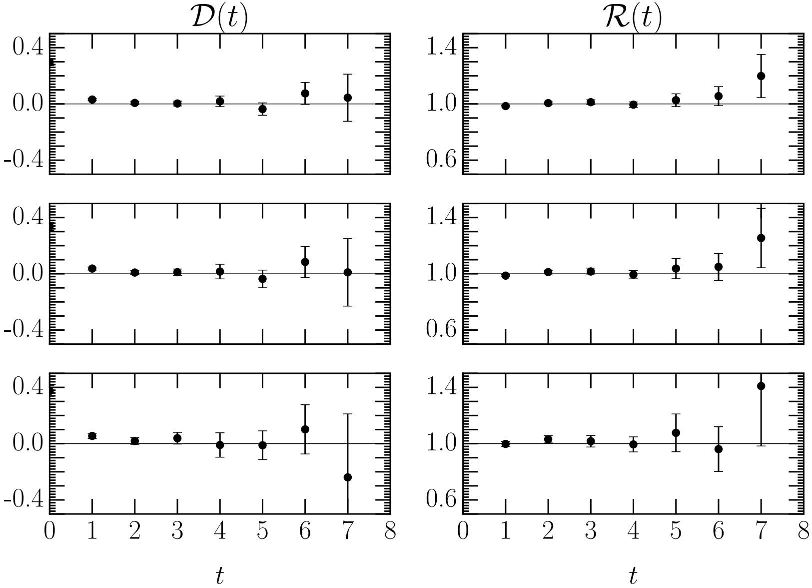
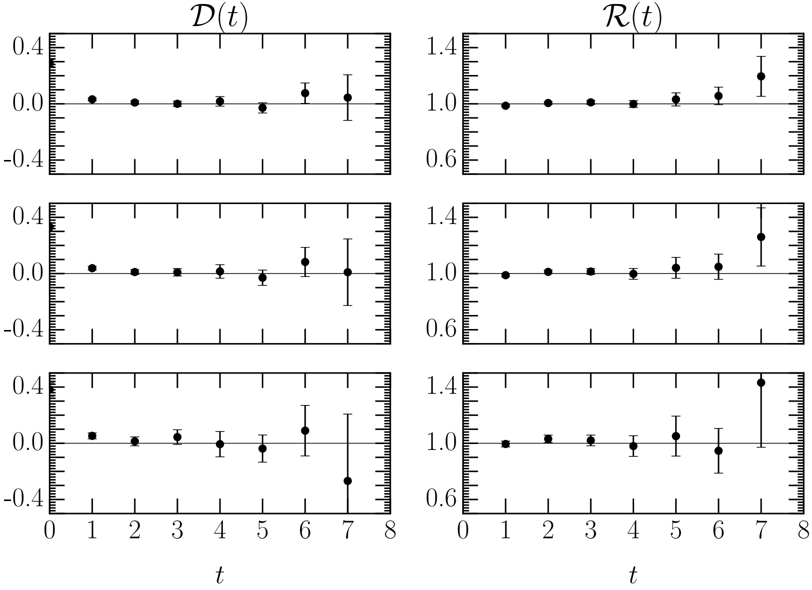
χ^2/d.o.f. of $\mathcal{D}(t) = 0$ for $2 \leqslant t \leqslant 6$.

m_π/GeV	χ^2/d.o.f.
0.698(4)	0.842
0.577(4)	0.595
0.435(4)	0.619

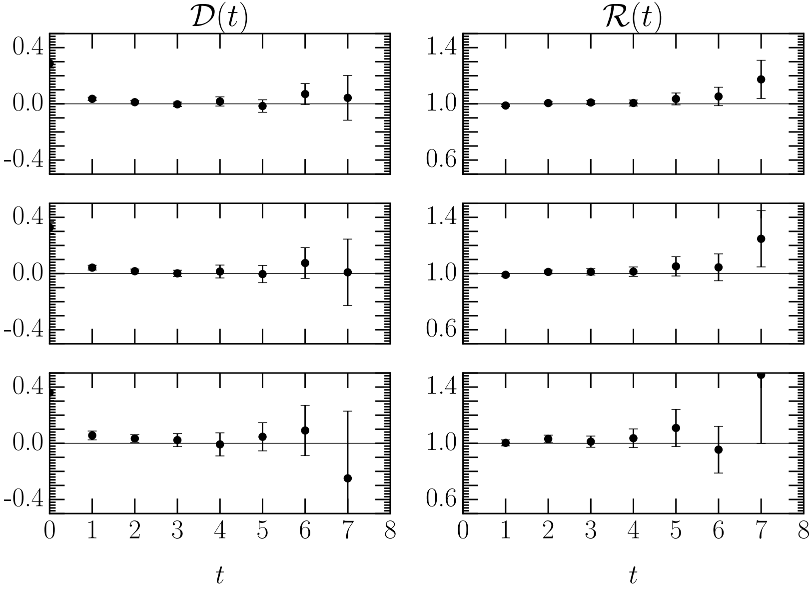
$\mathcal{D}(t)$ & $\mathcal{R}(t)$ for $t_0 = 1$, $t = t_0 + dt = 4$



$\mathcal{D}(t)$ & $\mathcal{R}(t)$ for $t_0 = 1$, $t = t_0 + dt = 5$



$\mathcal{D}(t)$ & $\mathcal{R}(t)$ for $t_0 = 2$, $t = t_0 + dt = 4$



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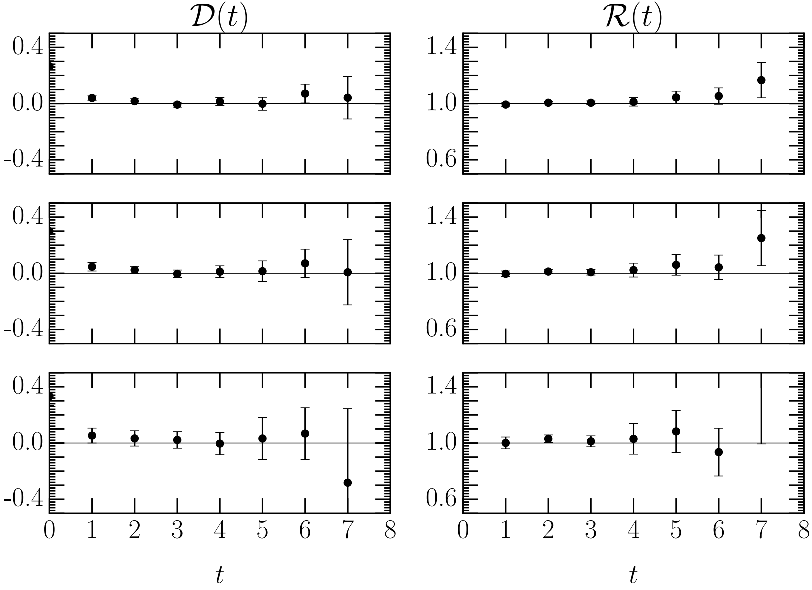

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Find no evidence that chiral symmetry plays a significant role in understanfding the Roper on the lattice.

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