Presentation of STXS Results for Reinterpretation

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Introduction

Higgs couplings measurements can be reinterpreted in specific theory models
⇒ Increasingly relevant as measurements become finer-grained and more precise, e.g. EFT reinterpretation of STXS.

Within ATLAS, can be done at the workspace level. For external use, provide

- **Central values + uncertainties of POIs**
- **POI correlation matrix**

→ Full description of the measurement in the Gaussian approximation.

However:

→ Doesn't account for non-Gaussian behavior ⇒ See previous presentation

→ Effect of NPs already absorbed through profiling, cannot be separated out

⇒ If reinterpretation involves systematics that are correlated with experimental ones, the correlation cannot be included.

→ e.g. theory uncertainties

Toy Model

Check the effect of these limitations on a toy model, loosely modeling the couplings $H\to\gamma\gamma$ measurement.

$\to$ **2-bin counting measurement** (0-jet/2-jet)

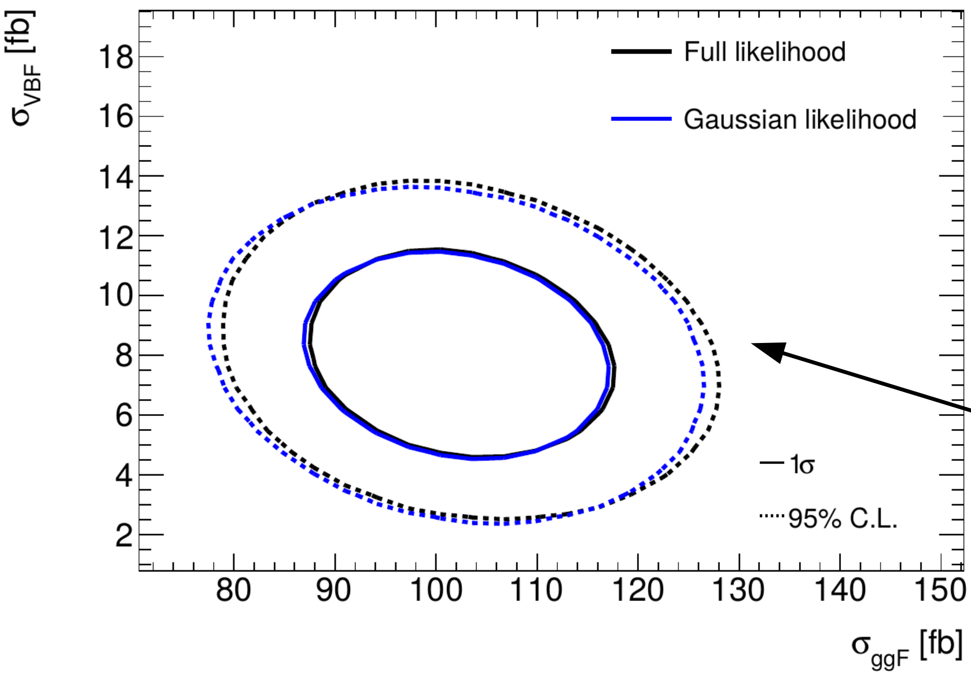
$\to$ measure $\boldsymbol{\sigma_{ggF}}$, $\boldsymbol{\sigma_{VBF}}$ (actually $\sigma \times B\gamma\gamma$)

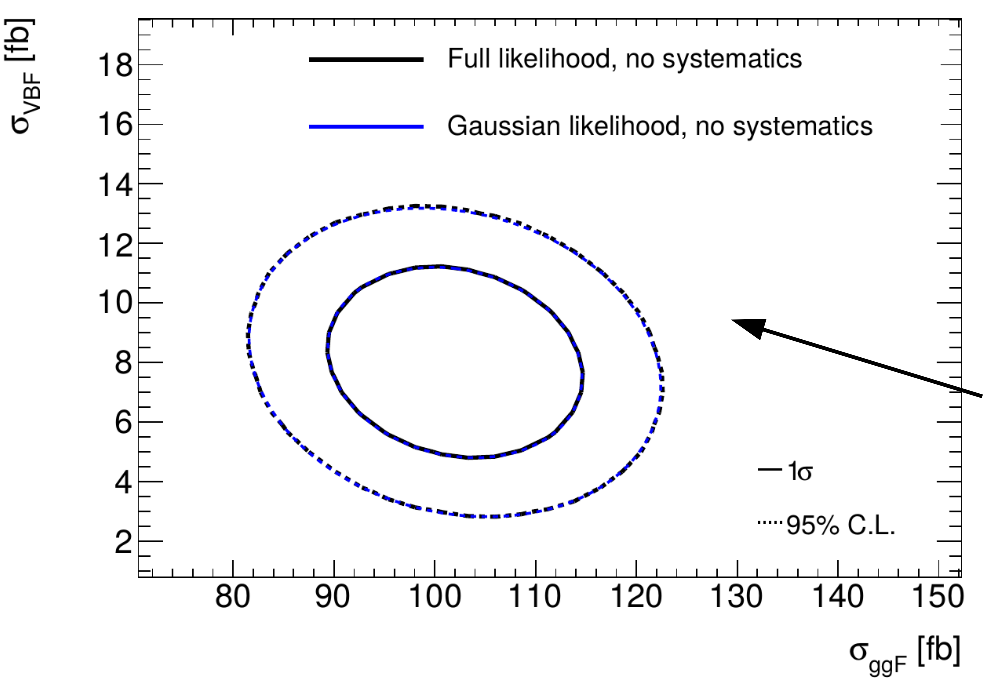
Parameter values:

- σ_{ggF} = 102 fb with 96% / 4% split between 0-jet and 2-jet bins
- σ_{VBF} = 8 fb, all in the 2-jet bin
- Acceptance x efficiency: 40% in both bins
- Adjust background levels to match stat uncertainties from $H\to\gamma\gamma$.
- $L = 150\ \mathrm{fb}^{-1}$
- **Systematics**: Log-normal impact, values:

NP	ggF/0-jet Acceptance	ggF/2-jet Acceptance	VBF Acceptance	Experimental syst., 0-jet	Experimental syst., 2-jet
0-jet	2%	-	2%	5%	-
2-jet	-	15%	-	-	5%

(σ_{ggF}, σ_{VBF}) Measurement





SM expected results:

$$\sigma_{\mathrm{ggF}} = 102^{+8.2}_{-8.5}(\mathrm{stat})^{5.7}_{-4.1}(\mathrm{exp})^{2.3}_{-1.7}(\mathrm{theo})\ \mathrm{fb}$$

$$\sigma_{\mathrm{VBF}} = 8.0 \pm 2.1(\mathrm{stat})^{+0.7}_{-0.5}(\mathrm{exp})^{+0.7}_{-0.4}(\mathrm{theo})\ \mathrm{fb}$$

→ Good agreement with H→γγ results

Compare:

- **Profile likelihood scan using the full likelihood**
- **PLR scan using a Gaussian likelihood** based on HESSE covariance matrix at best-fit point.

→ Measurement is not quite Gaussian, but not catastrophically so.

Same exercise without systematics:

⇒ Excellent agreement

→ Most non-Gaussianity comes from the log-normal uncertainties

Reinterpretation

$$\sigma_{ggF} = \mu_{ggF}\ \sigma_{ggF,SM}$$
$$\sigma_{VBF} = \mu_{VBF}\ \sigma_{VBF,SM}$$

Reinterpretation in simple (μ_{ggF}, μ_{VBF}) framework.

→ Include 5% uncertainties on σ_{SM} values

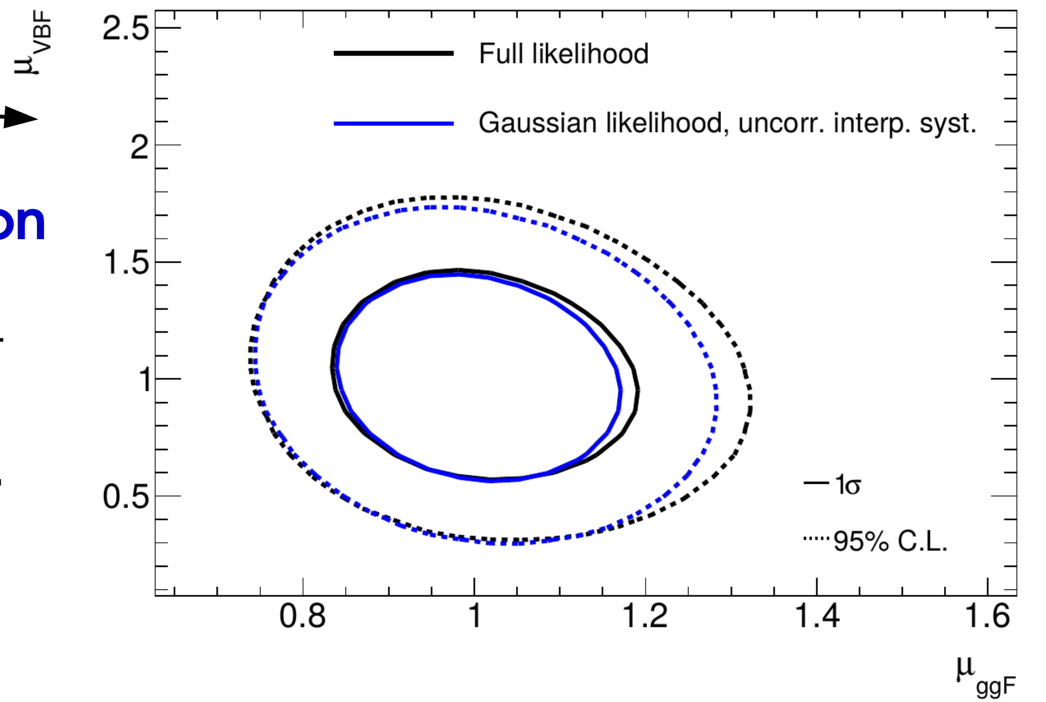
→ Correlate $\sigma_{ggF,SM}$ uncertainty with ggF/0jet acceptance uncertainty

→ Correlate $\sigma_{VBF,SM}$ uncertainty with VBF acceptance uncertainty

Compare

- **Full likelihood**
- **Gaussian L + "external" interpretation systematics (*)**
  → No correlation with measurement

→ Larger difference observed than for Gaussian approx. in σ measurement



Expected due to

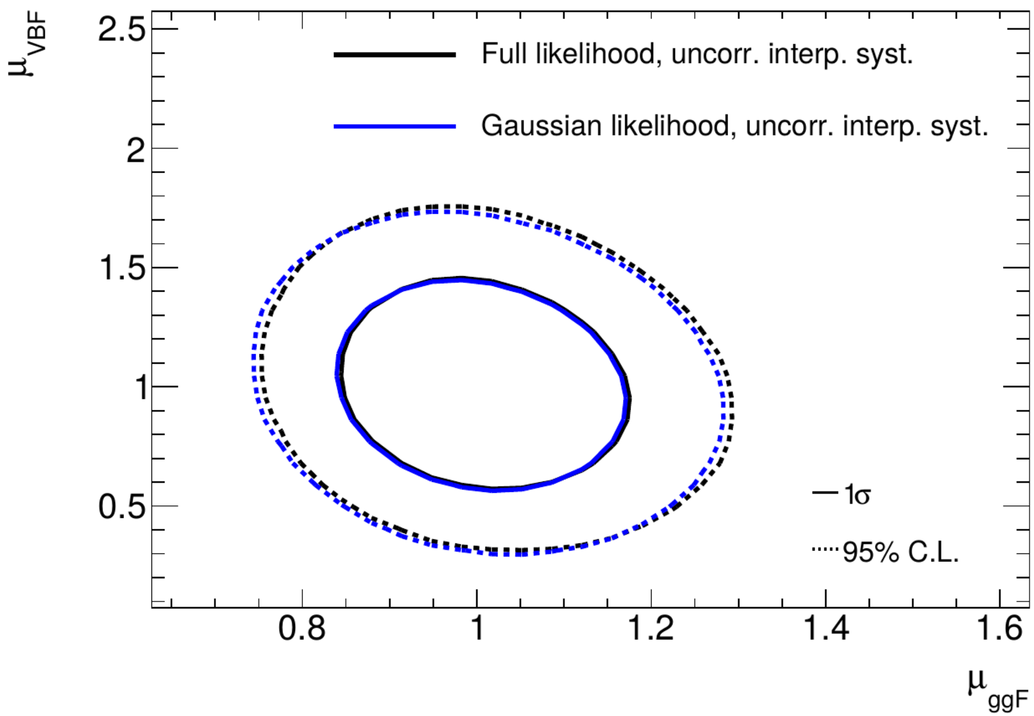
1. Non-Gaussian effects

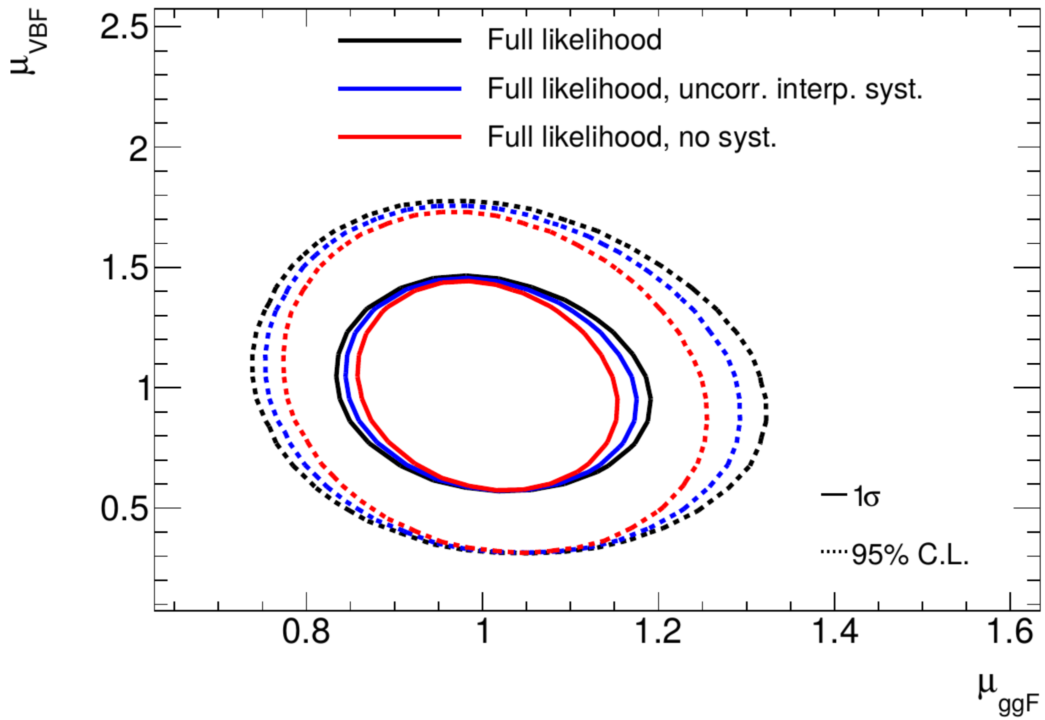
2. Uncorrelated syst. add in quadrature, vs. correlated syst add linearly

(*) $L_{\mathrm{Gaus}}(\sigma_{\mathrm{ggF}}, \sigma_{\mathrm{VBF}}) \rightarrow L_{\mathrm{Gauss}}(\mu_{\mathrm{ggF}}\, \sigma_{\mathrm{ggF,SM}}(1+\delta_{\mathrm{ggF}})^{\theta_{\mathrm{ggF}}}, \mu_{\mathrm{VBF}}\, \sigma_{\mathrm{VBF,SM}} \log(1+\delta_{\mathrm{VBF}})^{\theta_{\mathrm{VBF}}}) G(0;\theta_{\mathrm{ggF}},1) G(0;\theta_{\mathrm{VBF}},1)$

Full vs. Gaussian comparisons

Implement uncorrelated interpretation systematics **in full likelihood** to compare with Gaussian case:





- Difference partially due to non-Gaussianity but larger effect from lack of correlation of systematics
- How to properly correlate interpretation syst. in the Gaussian case as well ?

Correlating NPs: Full Covariance

Option 1: provide the full covariance matrix

$$\begin{array}{c c} & \begin{array}{cc} \mu & \theta \end{array} \\ C = & \left[\begin{array}{cc} C_\mu & C_{\mu\theta} \\ C_{\mu\theta}^T & C_\theta \end{array}\right] \end{array} \begin{array}{c} \\ \mu \\ \theta \end{array}$$

Can use it to build a Gaussian version of the likelihood:

$$L_{\text{Gauss}}(\mu,\theta) = \exp\left(-\frac{1}{2}\begin{bmatrix}\mu-\hat{\mu}\\ \theta-\hat{\theta}\end{bmatrix}^T H \begin{bmatrix}\mu-\hat{\mu}\\ \theta-\hat{\theta}\end{bmatrix}\right)$$

Stat-only covariance matrix

Impact of NPs on POIs

$$\begin{array}{c c} & \begin{array}{cc} \mu & \theta \end{array} \\ H = C^{-1} = & \left[\begin{array}{c|c} C_{\text{stat}}^{-1} & \Delta \\ \hline \Delta^T & H_\theta \end{array}\right] \end{array} \begin{array}{c} \\ \mu \\ \theta \end{array}$$

Post-fit NP uncertainties (should be ~Identity)

Profile θ in the Gaussian likelihood:

Then POIs have Hessian matrix

$$H_\mu' = C_{\text{stat}}^{-1} - \Delta\, H_\theta^{-1}\, \Delta^T = \boldsymbol{C_\mu^{-1}}$$

So profiling $L_{\text{Gauss}}(\mu, \theta)$ gives back *the same covariance matrix as profiling the full likelihood*

For reinterpretations, can reparameterize, extend, ... :

$$L(\mu,\theta) \rightarrow L_{\text{Gauss}}(\mu(\kappa_i,\theta),\theta)\, L_{\text{ext}}(\kappa_i,\theta)$$

Correlating NPs: Decomposed Covariance

Option 2 : Provide covariance matrices for stat-only case, and after freeing each relevant NP one by one:

$$C = C_{\text{stat}} + C_{\theta_1} + C_{\theta_2} + \dots$$

$$C_{\theta_1} = C_{\text{stat}+\theta_1} - C_{\text{stat}}$$

→ **Generalizes uncertainty decomposition**

→ **Provides impact of each NP**

Can reconstruct an approximate form of the full Hessian:

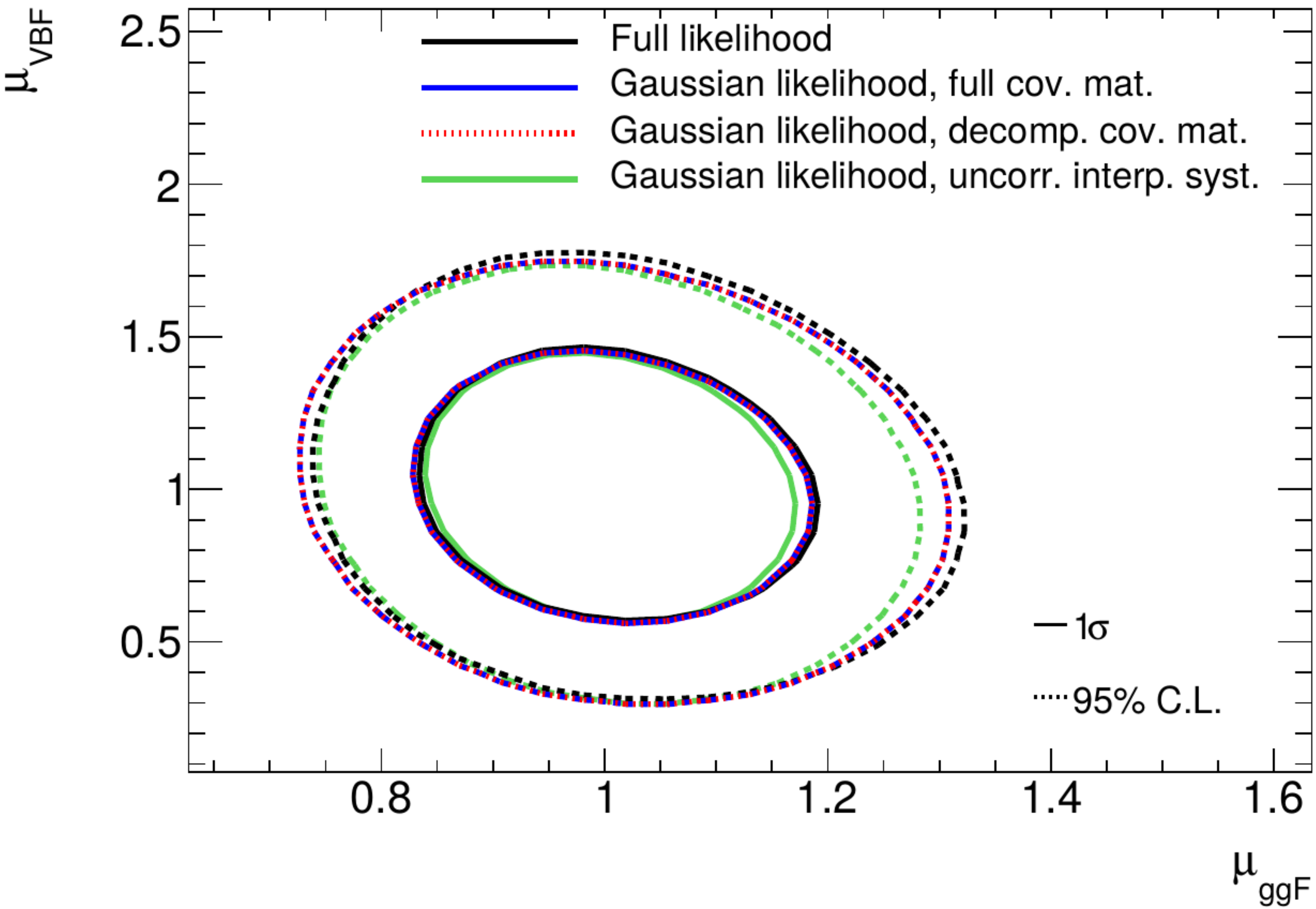
$$H = \begin{bmatrix} C_{\text{stat}}^{-1} & \pm C_{\text{stat}}^{-1}\sqrt{C_{\theta_1}} & \pm C_{\text{stat}}^{-1}\sqrt{C_{\theta_2}} \\ \pm C_{\text{stat}}^{-1}\sqrt{C_{\theta_1}} & 1+\sqrt{C_{\theta_1}}^{T} C_{\text{stat}}^{-1}\sqrt{C_{\theta_1}} & 0 \\ \pm C_{\text{stat}}^{-1}\sqrt{C_{\theta_2}} & 0 & 1+\sqrt{C_{\theta_1}}^{T} C_{\text{stat}}^{-1}\sqrt{C_{\theta_1}} \end{bmatrix}$$

→ **Requires to provide separately the signs of the impacts of NPs on POIs**

→ **Doesn't account for correlations between the POIs**

→ Formulas above valid at leading order in syst/stat, but can also be computed exactly

Application to the Toy Example



- Very similar results from both options
- Match the full likelihood results, up to non-Gaussian effects

Summary

In this example, both options do what was expected:

→ **Describe well the Gaussian approximation to the likelihood**

→ **Allow proper correlations between measurement and interpretation**

→ However non-Gaussian behavior is not quite negligible, mainly due to log-normal implementation of systematics.

⇒ **Need to check other cases**

Is it something to pursue for couplings results ? Ideas on a preferred option ?

Full covariance matrix

⊕ Provides all the information in one go

⊖ Large matrix : $(n_{\text{POIs}} + n_{\text{theory NPs}})^2$

Covariance matrix decomposition

⊕ Generalizes uncertainty decomposition

→ integrates better in existing results ?

⊕ Smaller matrices $n_{\text{POIs}}^2 \times n_{\text{theory NPs}}$.

⊖ Not quite the full information : missing NP ↔ NP correlations

⊖ Needs separate signs matrix : sign of impact for each (POI, NP) pair