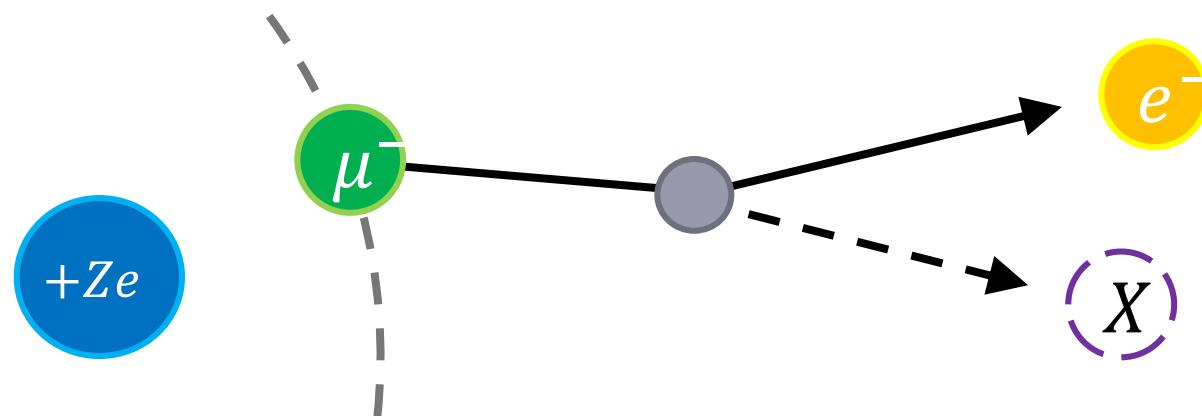


Muon decay with light boson emission in muon atoms

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Contents

- ◆ charged lepton flavor violation
 & light new particles
- ◆ searches for $\mu \rightarrow e + X$ (*invisible*)
- ◆ $\mu^- \rightarrow e^- X$ in muonic atom
 - Advantages of muonic atom
 - Formulation for e^- spectrum
 - Numerical results

CLFV modes in muonic atom

(Charged Lepton Flavor Violation)

◆ $\mu^- \rightarrow e^-$ conversion ($\mu^-(Z, A) \rightarrow e^-(Z, A)$)

- the nucleus can absorb the momentum without taking much energy
- clear signal : e^- with $E_e \simeq m_\mu - B_\mu$
- current constraints $\text{Br}(\mu^- \text{Ti} \rightarrow e^- \text{Ti}) < 6.1 \times 10^{-13}$ (SINDRUM II, 1998)
 $\text{Br}(\mu^- \text{Au} \rightarrow e^- \text{Au}) < 7 \times 10^{-13}$ (SINDRUM II, 2006)
- future experiments

$$\text{COMET, Mu2e} \Rightarrow \text{Br}(\mu^- \text{Al} \rightarrow e^- \text{Al}) \sim 10^{-17}$$

$$\text{DeeMe} \Rightarrow \text{Br}(\mu^- \text{C(SiC)} \rightarrow e^- \text{C(SiC)}) \sim 10^{-14}$$

◆ $\mu^- \rightarrow e^+$ conversion ($\mu^-(Z, A) \rightarrow e^+(Z - 2, A)^*$)

$$\text{Br}(\mu^- \text{Ti} \rightarrow e^+ \text{Ca}^*) < 3.6 \times 10^{-11} \quad (\text{SINDRUM II, 1998})$$

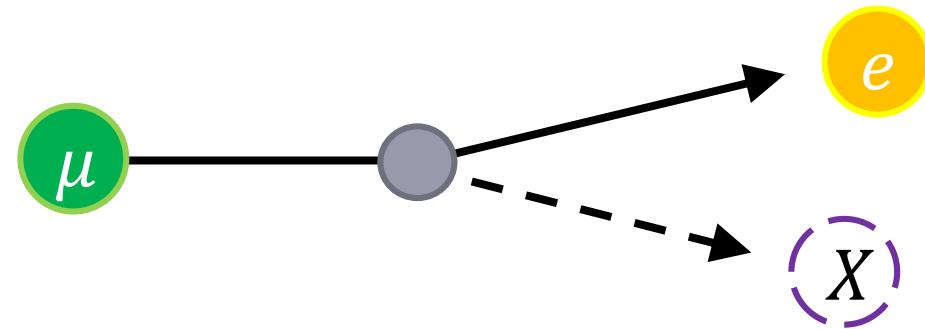
◆ $\mu^- e^- \rightarrow e^- e^-$ proposed by M. Koike *et al.*, PRL 105, 121601 (2010).

◆ $\mu^- \rightarrow e^- X$ Topic of this talk

Light invisible X with CLFV

Properties of an unknown boson X

- ✓ light ($m_X < m_\mu$)
- ✓ neutral
- ✓ with μ - e - X coupling



➤ Theoretical examples of X

- light (pseudo-)scalar : **majoron, familon, axion(-like) particle, ...**
- **light gauge boson**

Example : Majoron

- Singlet Majoron model

Y. Chikashige, R.N. Mohapatra, & R.D. Peccei, PLB**98** (1981) 265.

$$\mathcal{L} = \mathcal{L}_{\text{SM}} + i\bar{N}_R \gamma^\mu \partial_\mu N_R + (\partial_\mu \sigma)^\dagger (\partial^\mu \sigma) - V(\sigma)$$

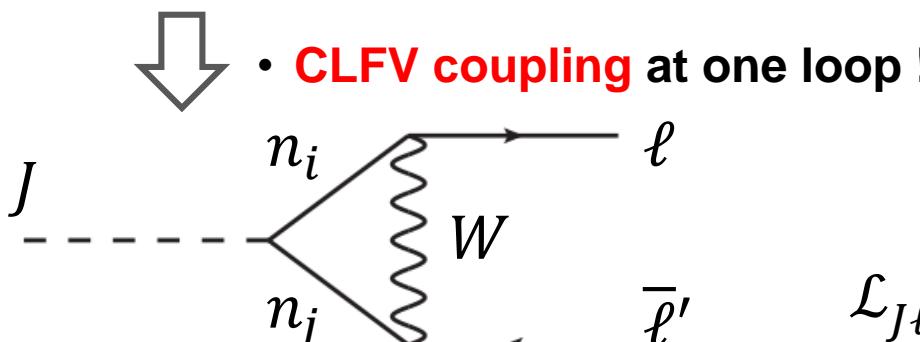
$$- \left(\bar{L} y N_R H + \frac{1}{2} \bar{N}_R^c \lambda N_R \sigma + \text{h. c.} \right)$$

N_R : right-handed neutrino
 σ : scalar with $L = -2$

SSB of lepton #
 $\sigma(x) = f + \rho(x) + iJ(x)$

- majorana mass of neutrino
- interaction of majoron with neutrino

J : majoron
(NG boson of lepton #)



n_i : Majorana neutrino mass eigenstate

$$K \equiv \frac{m_D m_D^\dagger}{v f}$$

$$\mathcal{L}_{J\ell'\ell} \simeq \frac{im_\ell}{8\pi\nu} K_{\ell'\ell} J \bar{\ell}' P_R \ell$$

for $m_\ell \gg m_{\ell'} \quad (\ell \neq \ell')$

$\mu^+ \rightarrow e^+ X$ searches

$$m_X < m_\mu$$

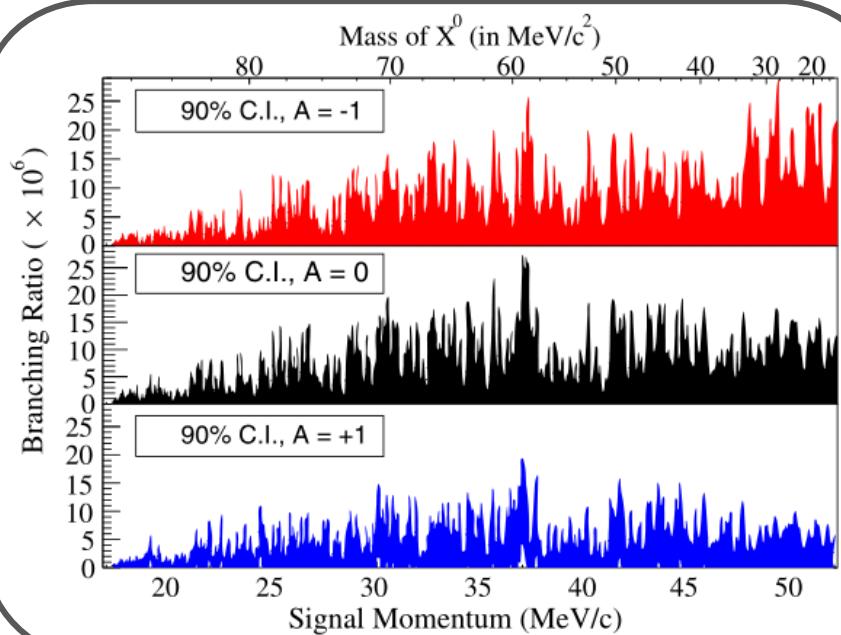
➤ A. Jodidio et al. PRD **34**, 1967 (1986).

- $1.8 \times 10^7 \mu^+$ that was highly polarized
- search for e^+ emitted in opposite direction for μ^+ polarization
- $\text{Br}(\mu^+ \rightarrow e^+ X) < 2.6 \times 10^{-6}$ for $m_X = 0$

➤ TWIST Collab.

PRD **91**, 052020 (2015).

- $5.8 \times 10^8 \mu^+$
- for various m_X
& various angular property
($d\Gamma/d\cos\theta \propto 1 - AP_\mu \cos\theta$)
- $\text{Br} < 2.1 \times 10^{-5}$ ($m_X = 0, A = 0$)



➤ Mu3e Collab.

A. Schöning, Talk at Flavour and Dark Matter Workshop, Heidelberg, September 28 (2017).

- $\text{Br} < 10^{-8}$ (for $25\text{MeV} < m_X < 95\text{MeV}$)

$\mu^- \rightarrow e^- X$ in a muonic atom

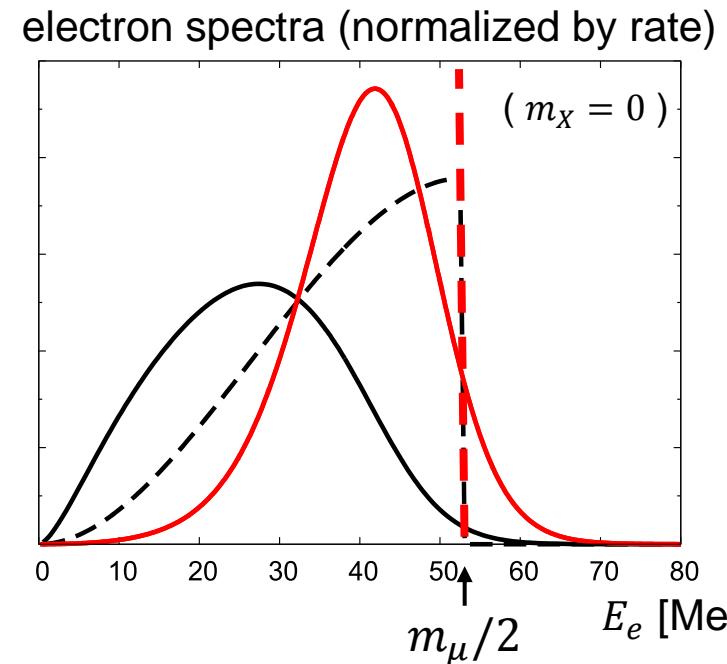
cf. X. G. i Tormo *et al.*, PRD **84**, 113010 (2011).
 & H. Natori, Talk at 73th JPS meeting (2018).

Advantages over free muon decay

1. less background

- : $\mu^+ \rightarrow e^+ X$ (free)
- - - : $\mu^+ \rightarrow e^+ \nu_e \bar{\nu}_\mu$ (free)
- : $\mu^- \rightarrow e^- X$ (μ -gold)
- : $\mu^- \rightarrow e^- \bar{\nu}_e \nu_\mu$ (μ -gold)

- different peak positions of signal & BG



2. more information : “spectrum”, “dependence on nucleus”, ...

3. huge # of muonic atoms in coming experiments (COMET, Mu2e, DeeMe)

Disadvantages

- ✓ non-monochromatic signal
- ✓ shorter life time of muonic atom

Spectrum near end-point (rough estimation)

DIO with **two neutrino emission**
 (Decay In Orbit)

$$\mathcal{L} = G(\bar{\mu}\gamma_\alpha\nu_\mu)(\bar{\nu}_e\gamma^\alpha e)$$

$$\frac{d\Gamma}{dE_e} \propto \int \frac{d^3 p_\nu}{E_\nu} \frac{d^3 \bar{p}_\nu}{\bar{E}_\nu} \delta(E_\nu + \bar{E}_\nu + E_e - E_{tot}) \cdots p_\nu \cdots \bar{p}_\nu$$

$$\sim \Delta^5 \quad \text{near end-point}$$

$$\Delta \equiv \frac{E_{endpoint} - E_e}{m_\mu}$$

DIO with **one boson emission**

$$\mathcal{L} = g(\bar{\mu}\gamma_\alpha e)\partial^\alpha X$$

$$\frac{d\Gamma}{dE_e} \propto \int \frac{d^3 p_X}{E_X} \delta(E_X + E_e - E_{tot}) |\cdots p_X|^2$$

$$\sim \Delta^3 \quad \text{near end-point}$$

Investigation of e^- spectrum near **endpoint** could be a good probe for $\mu^- \rightarrow e^- X$.

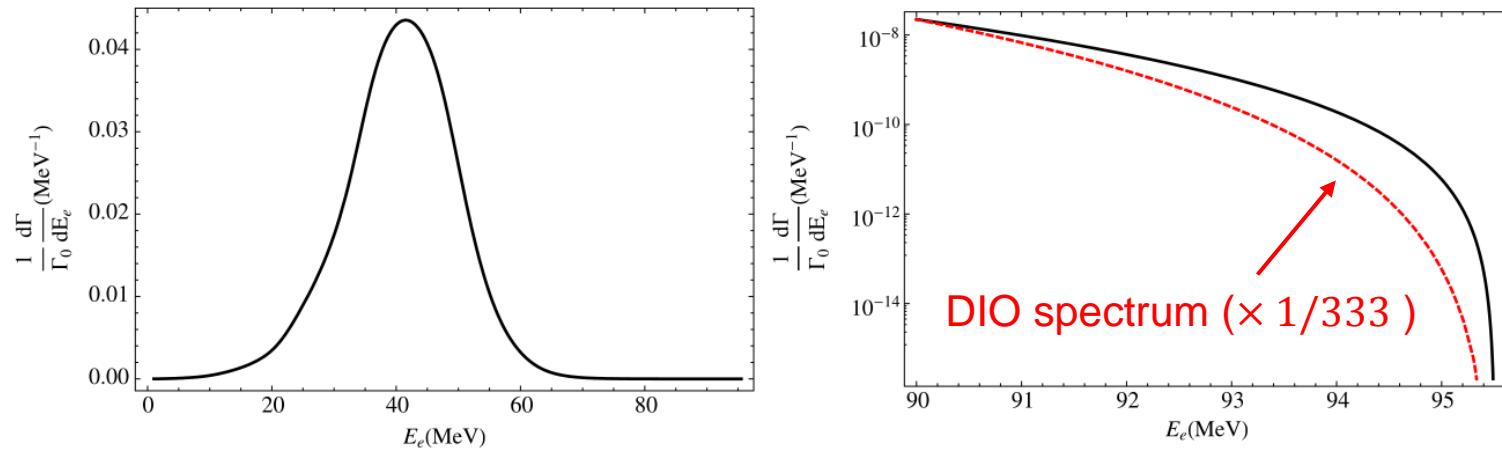
→ Need to take into account relativistic effect for bound μ^- ,
 distortion of emitted e^- in Coulomb potential,
 finite size of nucleus, nuclear recoil, ...

Previous work X. G. i Tormo *et al.*, PRD **84**, 113010 (2011).

- assuming that massless X has yukawa-type CLFV interaction

$$\mathcal{L}_I = g(\bar{\mu}e)X \quad (g : \text{coupling})$$

electron spectrum of $\mu^- \rightarrow e^- X$ (gold)



- ✓ The result of the past μ -e conv. corresponds to $\text{Br}(\mu^+ \rightarrow e^+ X) < 3 \times 10^{-3}$.
- ✓ Sensitivity of COMET & Mu2e : $\text{Br}(\mu^+ \rightarrow e^+ X) \sim 2 \times 10^{-5}$



same level as the current limit of free μ^+ search ($\sim 10^{-5}$)

Aim of this work

$\mu^- \rightarrow e^- X$ in a muonic atom

X. G. i Tormo *et al.*

CLFV interaction : $\mathcal{L}_I = g(\bar{\mu}e)X$



This work

- ✓ consider various possibilities of CLFV interactions

If e^- spectrum depends on CLFV interaction,

we could determine the new physics model by observation.

Effective models

A. Scalar X

- ◆ yukawa coupling (e.g. majoron induced by R-parity violation, ...)
already analyzed by X. G. i Tormo *et al.*, PRD **84**, 113010 (2011).

$$\mathcal{L}_{S0} = g_{S0} (\bar{e}\mu) X + [H.c.]$$

- ◆ derivative coupling (e.g. majoron, familon, axion, ...)

$$\mathcal{L}_{S1} = \frac{g_{S1}}{\Lambda_{S1}} (\bar{e}\gamma_\alpha\mu) \partial^\alpha X + [H.c.]$$

B. Vector X

- ◆ dipole coupling

$$\mathcal{L}_{V1} = \frac{g_{V1}}{\Lambda_{V1}} (\bar{e}\sigma_{\alpha\beta}\mu) X^{\alpha\beta} + [H.c.]$$

$$X^{\alpha\beta} = \partial^\alpha X^\beta - \partial^\beta X^\alpha$$

Formulation for decay rate

$$\Gamma = \int \frac{d^3 p_e}{(2\pi)^3 2E_e} \frac{d^3 p_X}{(2\pi)^3 2E_X} (2\pi) \delta(m_\mu - E_e - E_X) \times \sum_{spins} |\langle \psi_e^{s_e}(\mathbf{p}_e) \phi_X^{s_X}(\mathbf{p}_X) | \mathcal{L} | \psi_\mu^{s_\mu}(1S) \rangle|^2$$

partial wave expansion for the electron in the final state

$$\psi_e^{p,s} = \sum_{\kappa,\mu,m} 4\pi i^{l_\kappa} (l_\kappa, m, 1/2, s | j_\kappa, \mu) Y_{l_\kappa, m}^*(\hat{p}) e^{-i\delta_\kappa} \psi_p^{\kappa, \mu}$$

Dirac eq. for radial wave functions

$$\frac{dg_\kappa(r)}{dr} + \frac{1+\kappa}{r} g_\kappa(r) - (E + m + e\phi(r)) f_\kappa(r) = 0$$

$$\frac{df_\kappa(r)}{dr} + \frac{1-\kappa}{r} f_\kappa(r) + (E - m + e\phi(r)) g_\kappa(r) = 0$$

$$\psi_p^{\kappa, \mu}(\mathbf{r}) = \begin{pmatrix} g_\kappa(r) \chi_\kappa^\mu(\hat{r}) \\ i f_\kappa(r) \chi_{-\kappa}^\mu(\hat{r}) \end{pmatrix}$$

ϕ : nuclear Coulomb potential

Electron spectrum

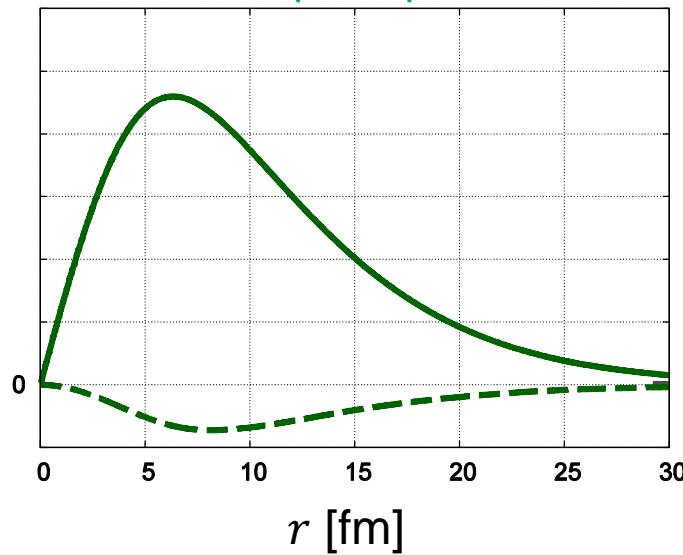
◆ yukawa coupling

$$\frac{d\Gamma}{dE_e} = \frac{g_{S0}^2}{4\pi^2} \frac{p_e p_X}{m_\mu^2} \sum_\kappa (2j_\kappa + 1) |I_\kappa|^2$$

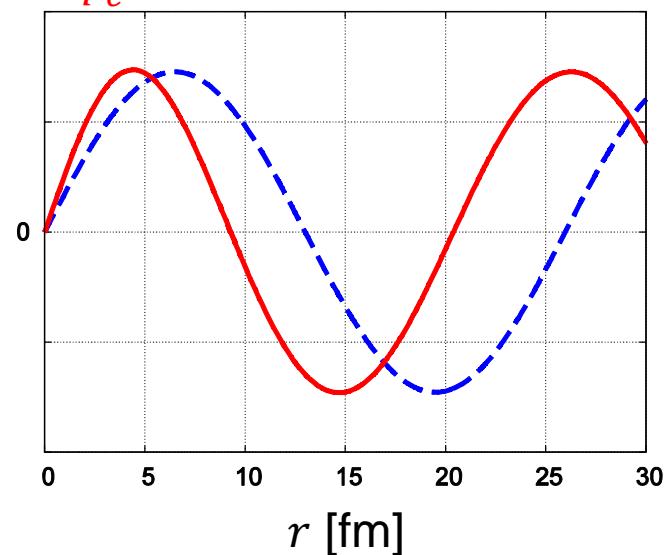
κ : angular momentum of e^-

$$I_\kappa = m_\mu \int_0^\infty dr r^2 j_{l_\kappa}(p_X r) \{ g_{p_e}^\kappa(r) g_\mu^{1s}(r) - f_{p_e}^\kappa(r) f_\mu^{1s}(r) \}$$

rg_μ^{1s}, rf_μ^{1s}

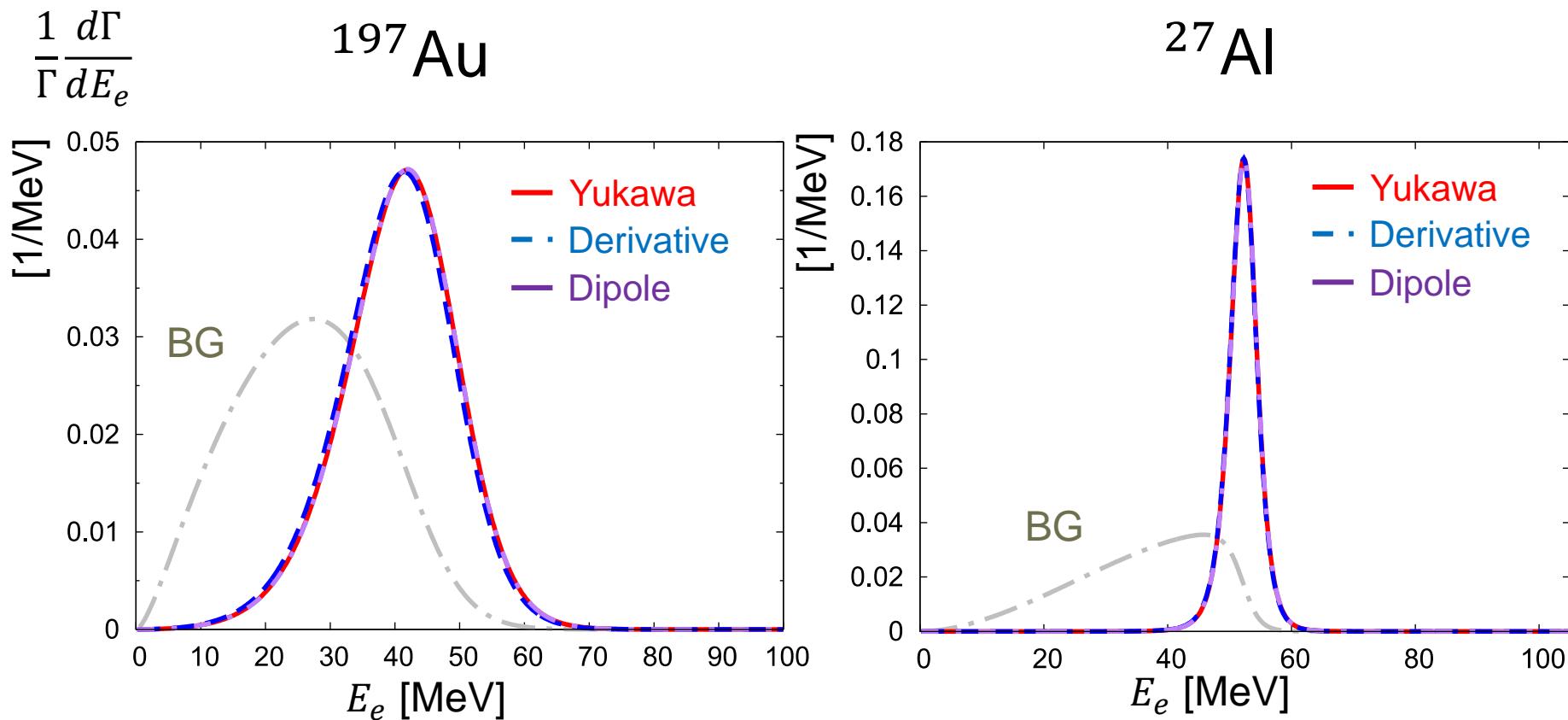


$rg_{p_e=48\text{MeV}}^{\kappa=-1}, rj_0(48\text{MeV} \times r)$



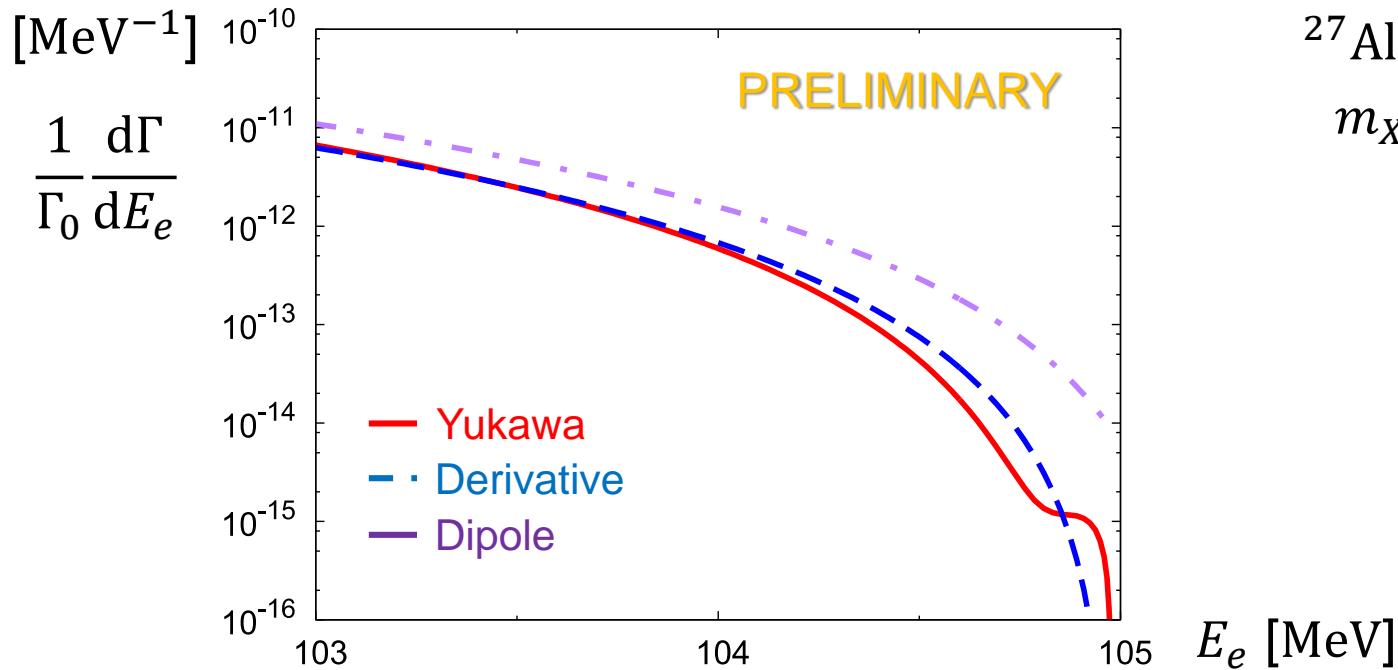
^{208}Pb

e^- spectrum ($m_X = 0$)



- Spectrum does not strongly depend on properties of X .
- The sharper peak is obtained for the lighter nucleus.

Spectrum near end-point



$$\frac{m_\mu}{\Gamma_0} \frac{d\Gamma}{dE_e} = \sum_i a_i \Delta^i$$

$$\Delta = \frac{E_{\text{endpoint}} - E_e}{m_\mu}$$

	Yukawa	Derivative	Dipole
a_1	3.4×10^{-10}	8.4×10^{-16}	3.0×10^{-9}
a_2	-3.1×10^{-7}	-2.3×10^{-12}	-2.6×10^{-7}
a_3	9.7×10^{-5}	8.1×10^{-5}	3.3×10^{-4}
a_4	1.2×10^{-3}	8.7×10^{-4}	-2.8×10^{-2}

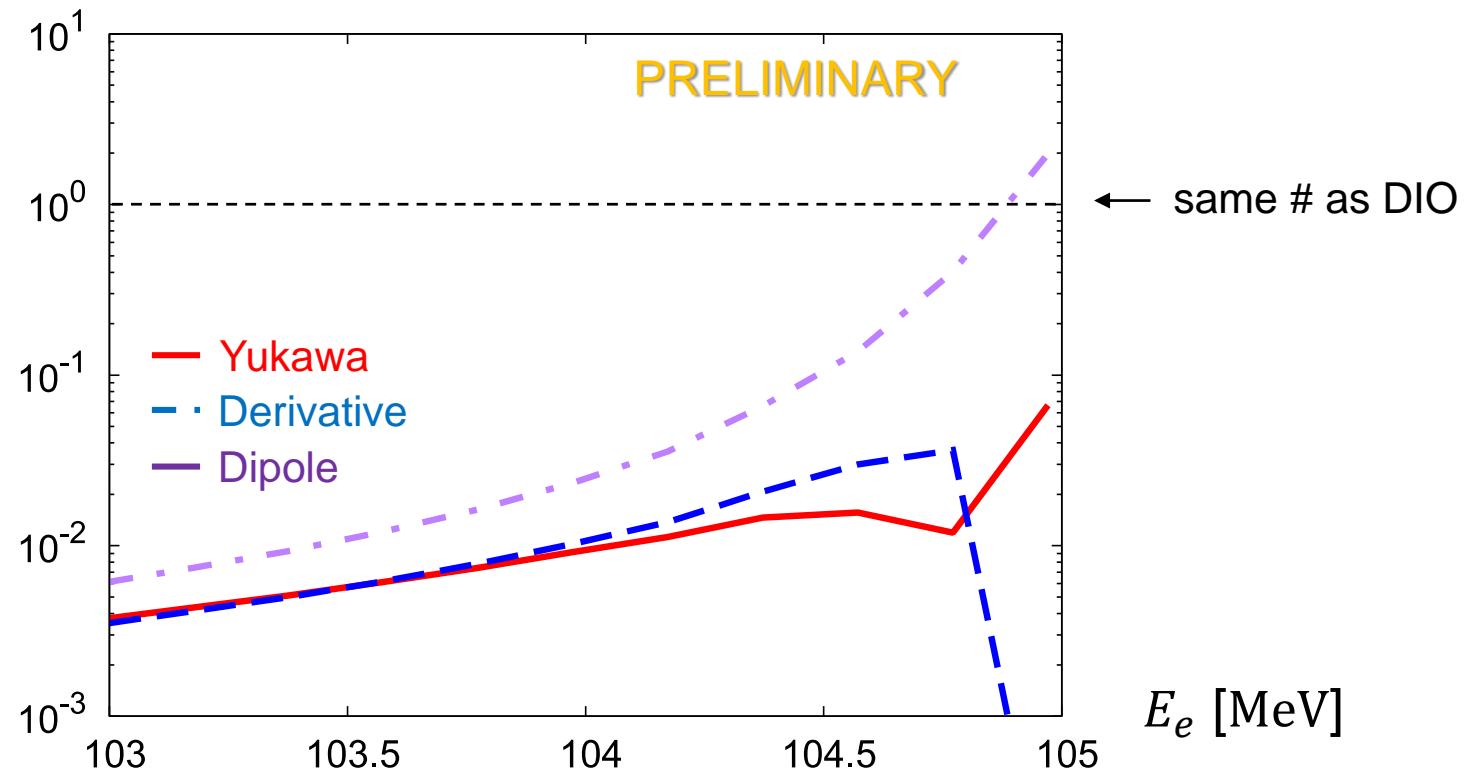
Ratio to background

$$\text{Br}(\mu^+ \rightarrow e^+ X) < 2.6 \times 10^{-6}$$



$$\frac{d\Gamma_{\mu \rightarrow eX}}{dE_e} / \frac{d\Gamma_{\mu \rightarrow e\nu\nu}}{dE_e} \text{ (allowed maximum)}$$

^{27}Al target
 $m_X = 0$

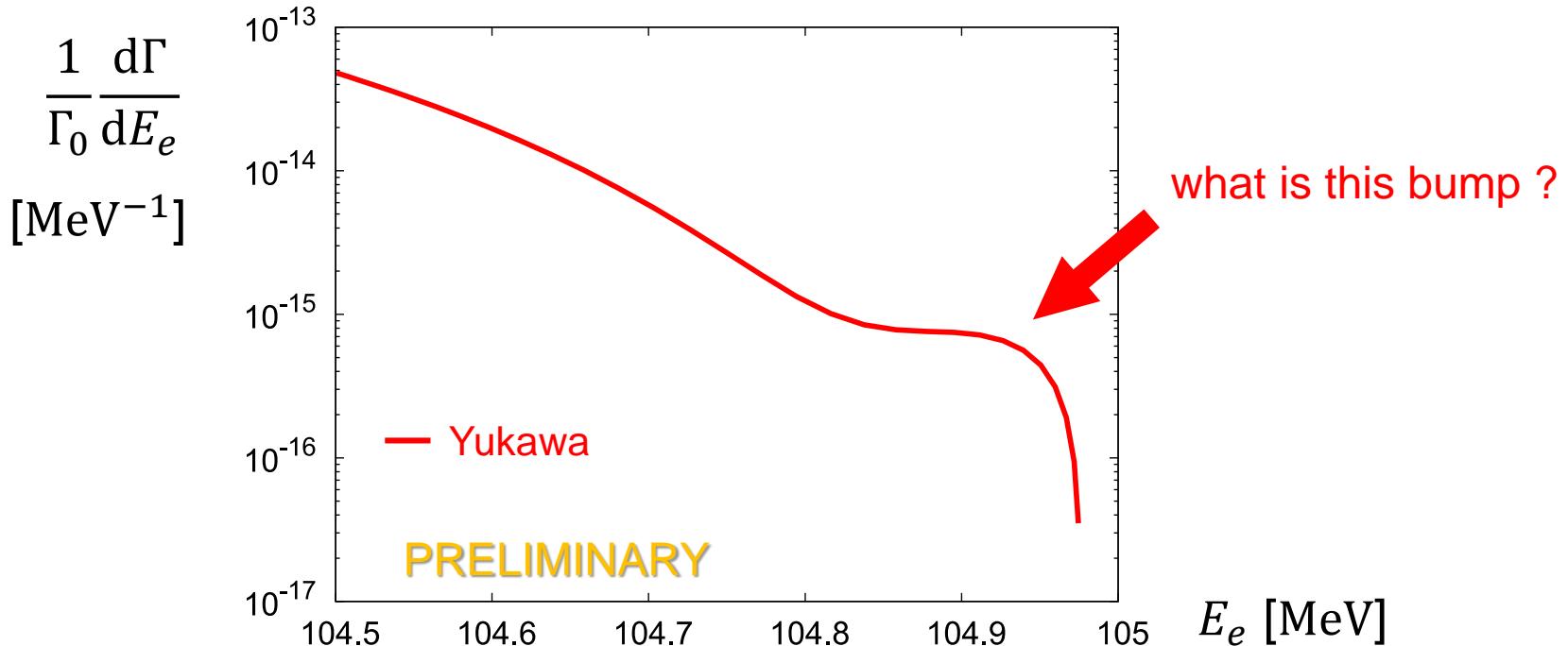


Summary

- $\mu^- \rightarrow e^- X$ in a muonic atom
 - ✓ Promising probe of a light neutral boson with CLFV
(e.g. majoron)
 - ✓ Advantages over free muon
 - less background
 - more information (spectrum, Z-dependence, ...)
 - many muonic atoms in coming experiments (COMET, Mu2e, DeeMe)
 - ✓ Findings in e^- spectrum
 - the rough shape does not depend on the type of CLFV interaction
 - dipole-case : large tail
 - yukawa-case : characteristic behavior near endpoint
 - ✓ Detailed simulation is in progress with members of COMET

Backups

Characteristic behavior of spectrum



$$\frac{d\Gamma}{dE_e} = \frac{g_Y^2}{4\pi^2} p_e p_X \sum_{\kappa} (2j_{\kappa} + 1) |I_{\kappa}|^2$$

$$I_{\kappa} = \int_0^{\infty} dr r^2 j_{l_{\kappa}}(p_X r) \{ g_{p_e}^{\kappa}(r) g_{\mu}^{1s}(r) - f_{p_e}^{\kappa}(r) f_{\mu}^{1s}(r) \}$$

Characteristic behavior of spectrum

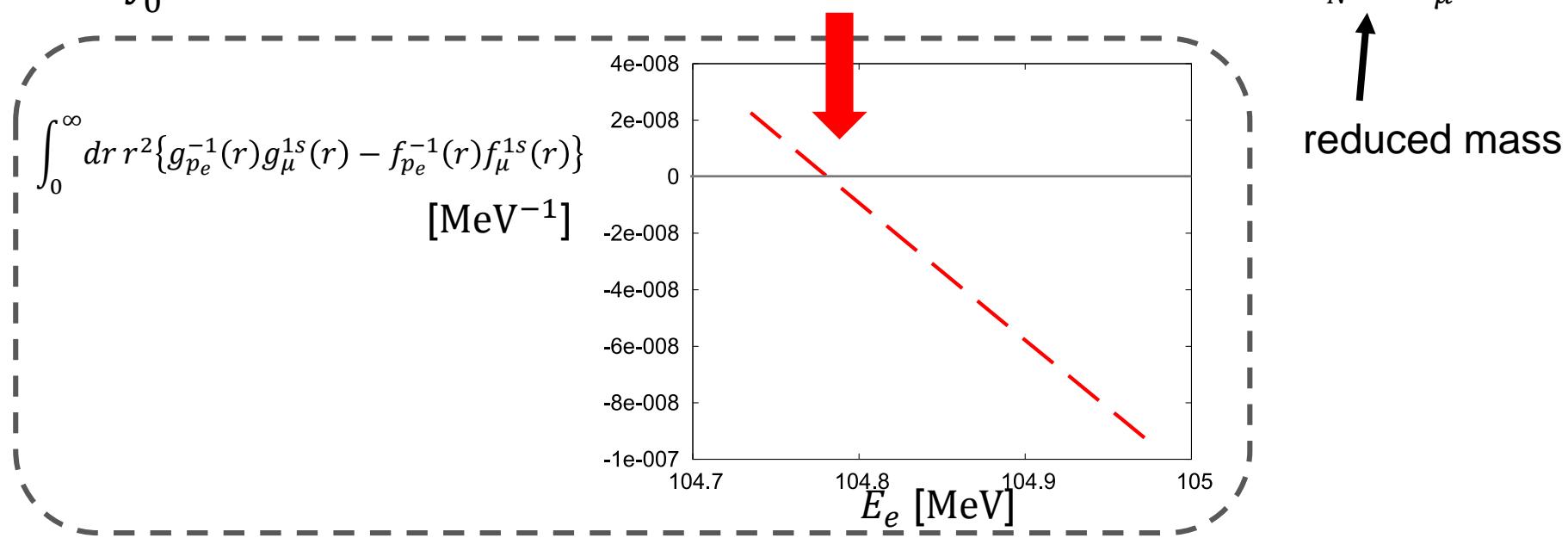
- ✓ Main contribution comes from s-wave of emitted electron.

→ s-wave ($\kappa = -1$) amplitude

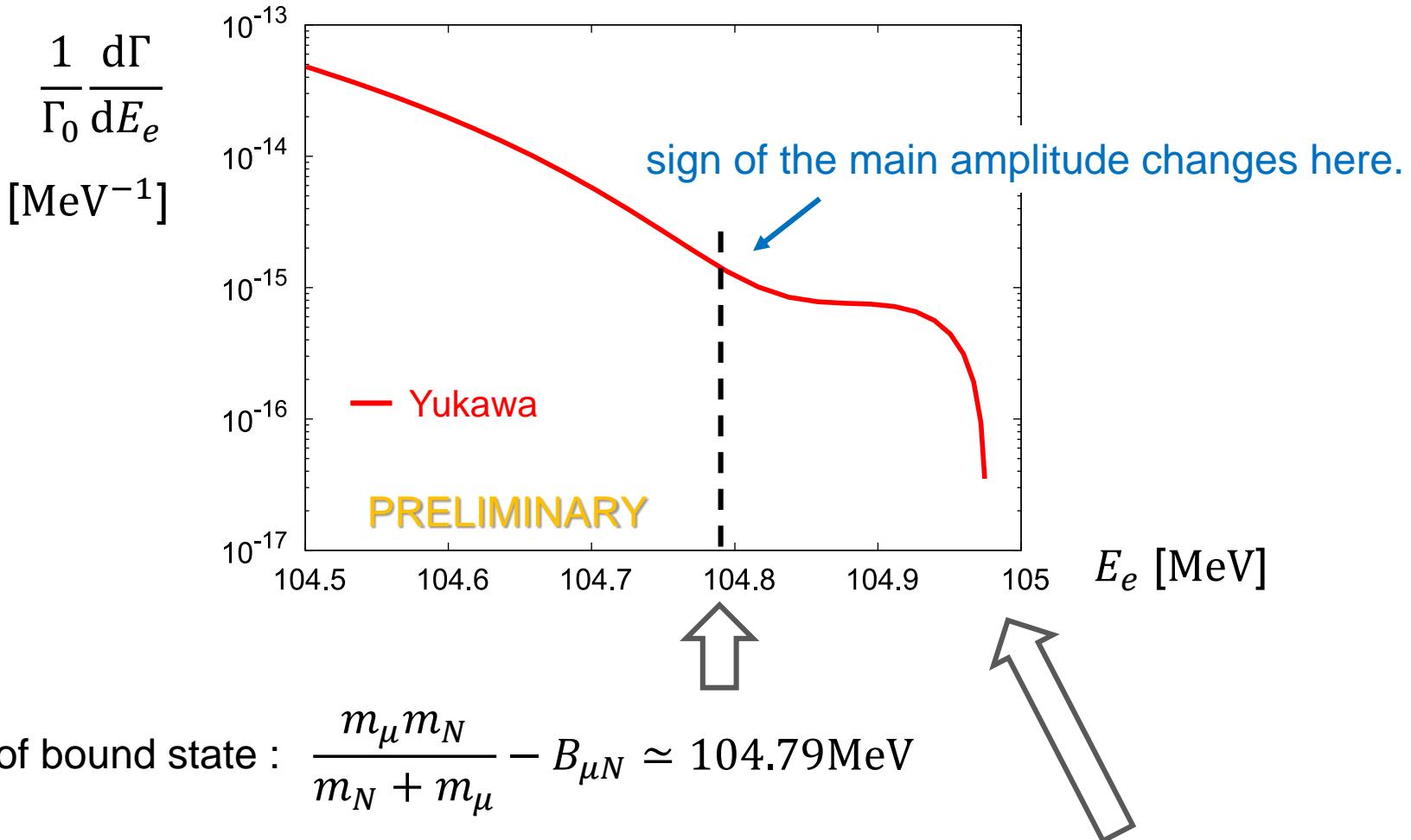
$$I_{-1} = m_\mu \int_0^\infty dr r^2 j_0(p_X r) \{g_{p_e}^{-1}(r) g_\mu^{1s}(r) - f_{p_e}^{-1}(r) f_\mu^{1s}(r)\}$$

↓ $j_0(p_X r) \simeq \text{const. near the endpoint}$

✓ $\int_0^\infty dr r^2 \{g_{p_e}^{-1}(r) g_\mu^{1s}(r) - f_{p_e}^{-1}(r) f_\mu^{1s}(r)\} = 0 \quad \text{if} \quad E_e = E_\mu = \frac{m_\mu m_N}{m_N + m_\mu} - B_{\mu N}$

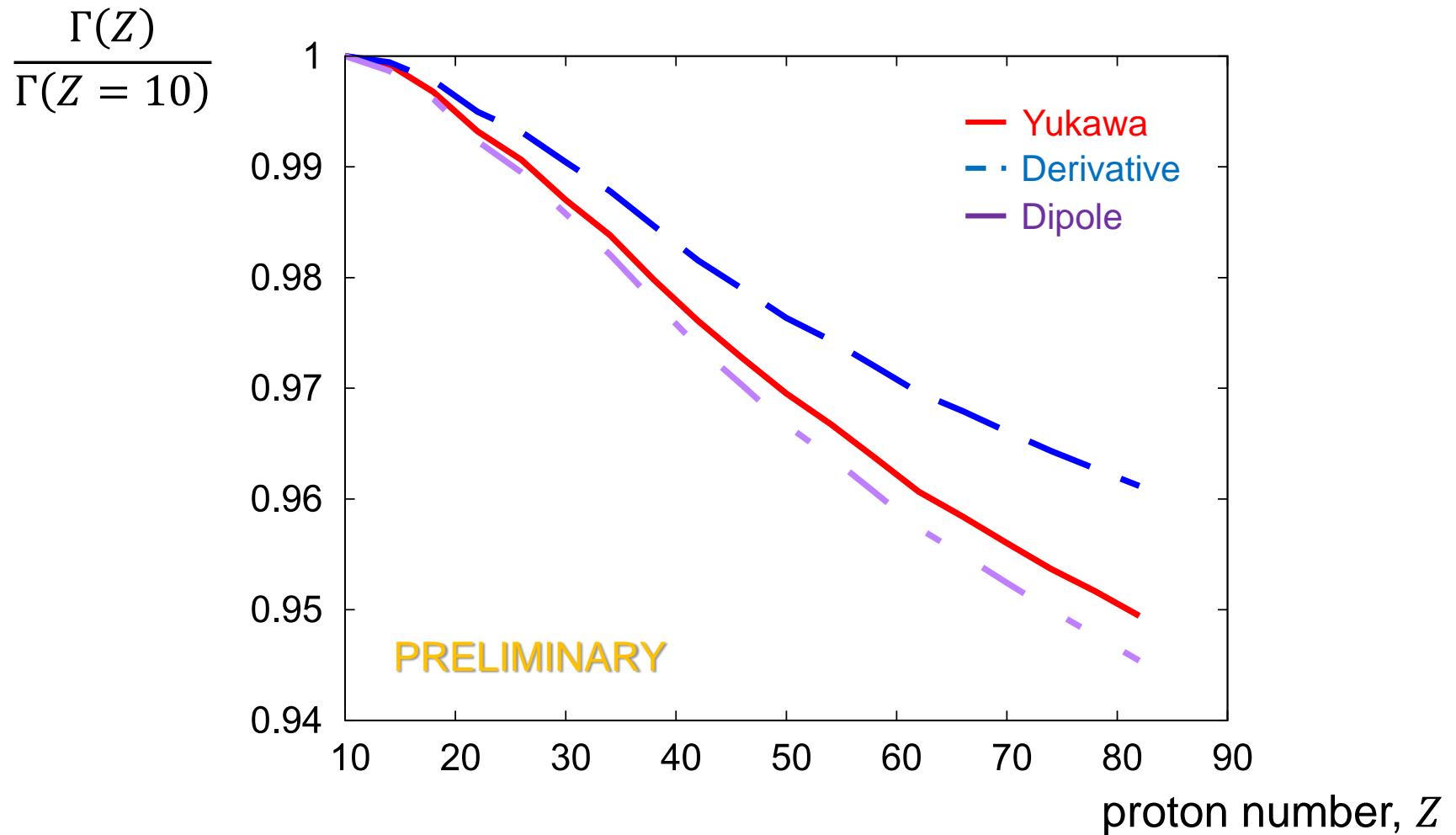


Characteristic behavior of spectrum



Maximum energy of electron : $\frac{2m_N(m_\mu - B_{\mu N}) + (m_\mu - B_{\mu N})^2}{2(m_N + m_\mu - B_{\mu N})} \simeq 104.97 \text{ MeV}$

Nuclear dependence ($m_x = 0$)



Charged Lepton Flavor Violation (CLFV)

Mode	Upper bound	Experiment (Year)
$\mu^+ \rightarrow e^+ \gamma$	4.2×10^{-13}	MEG (2016)
$\mu^+ \rightarrow e^+ e^+ e^-$	1.0×10^{-12}	SINDRUM (1988)
$\mu^- \text{Au} \rightarrow e^- \text{Au}$	7×10^{-13}	SINDRUM II (2006)
$\mu^+ \rightarrow e^+ X, X \rightarrow \text{inv.}$	$O(10^{-5})$	TWIST (2015)
$\mu^+ \rightarrow e^+ \gamma X, X \rightarrow \text{inv.}$	$O(10^{-9})$	Crystal Box (1988)
$\mu^+ \rightarrow e^+ X, X \rightarrow e^+ e^-$	$O(10^{-12})$	SINDRUM (1986)
$\mu^+ \rightarrow e^+ X, X \rightarrow \gamma\gamma$	$O(10^{-10})$	MEG (2012)
$\tau \rightarrow eX(\mu X), X \rightarrow \text{inv.}$	$O(10^{-2})$	ARGUS (1995)

Effective models

A. Scalar X

- ◆ yukawa coupling (e.g. majoron induced by R-parity violation, ...)
already analyzed by X. G. i Tormo *et al.*, PRD **84**, 113010 (2011).

$$\mathcal{L}_{S0} = g_{S0}(\bar{e}\mu)X + [H.c.]$$

- ◆ derivative coupling (e.g. majoron, familon, axion, ...)

$$\mathcal{L}_{S1} = \frac{g_{S1}}{\Lambda_{S1}}(\bar{e}\gamma_\alpha\mu)\partial^\alpha X + [H.c.]$$

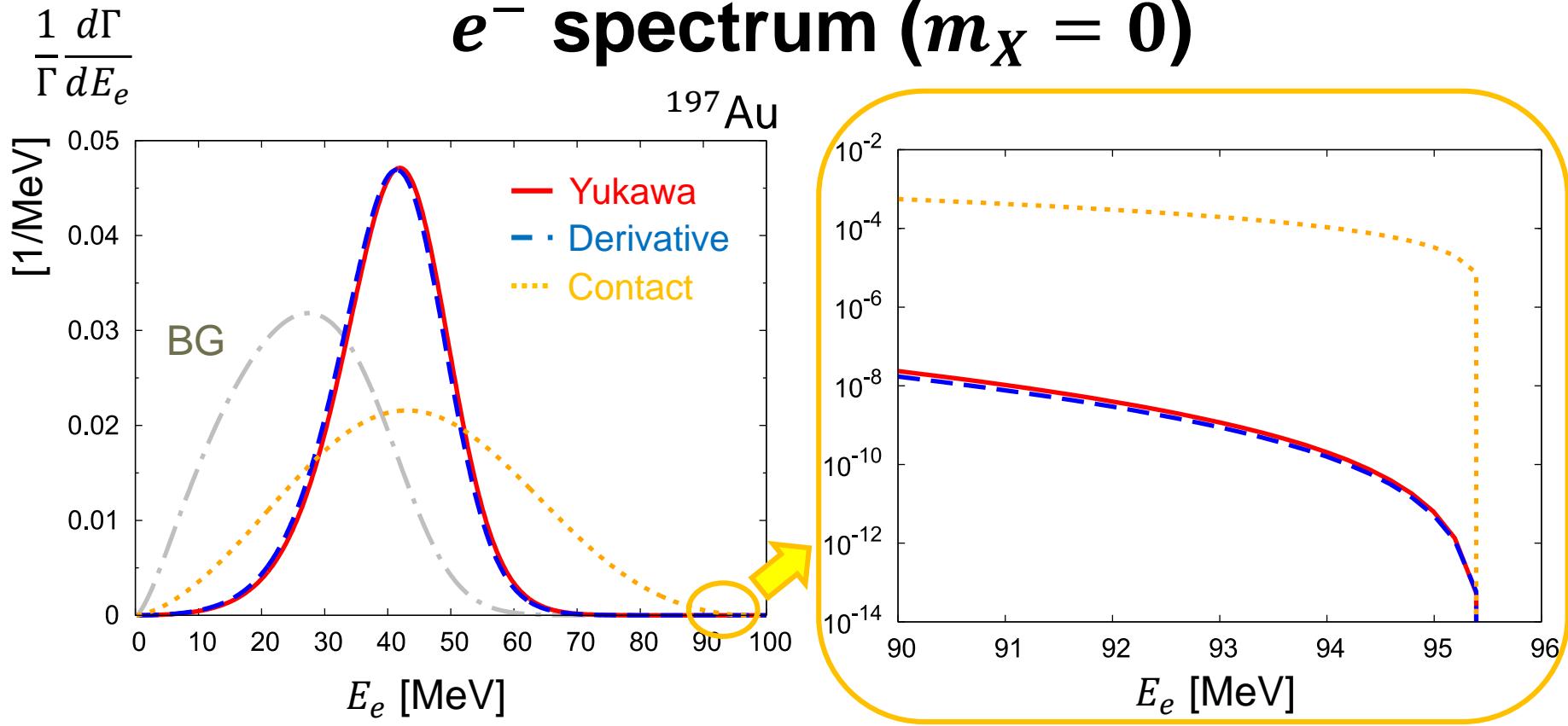
B. Vector X

$$\mathcal{L}_{V0} = g_{V0}(\bar{e}\gamma_\alpha\mu)X^\alpha + [H.c.] \quad (\text{for only massive } X)$$

- ◆ dipole coupling

$$\mathcal{L}_{V1} = \frac{g_{V1}}{\Lambda_{V1}}(\bar{e}\sigma_{\alpha\beta}\mu)X^{\alpha\beta} + [H.c.]$$
$$X^{\alpha\beta} = \partial^\alpha X^\beta - \partial^\beta X^\alpha$$

e^- spectrum ($m_X = 0$)

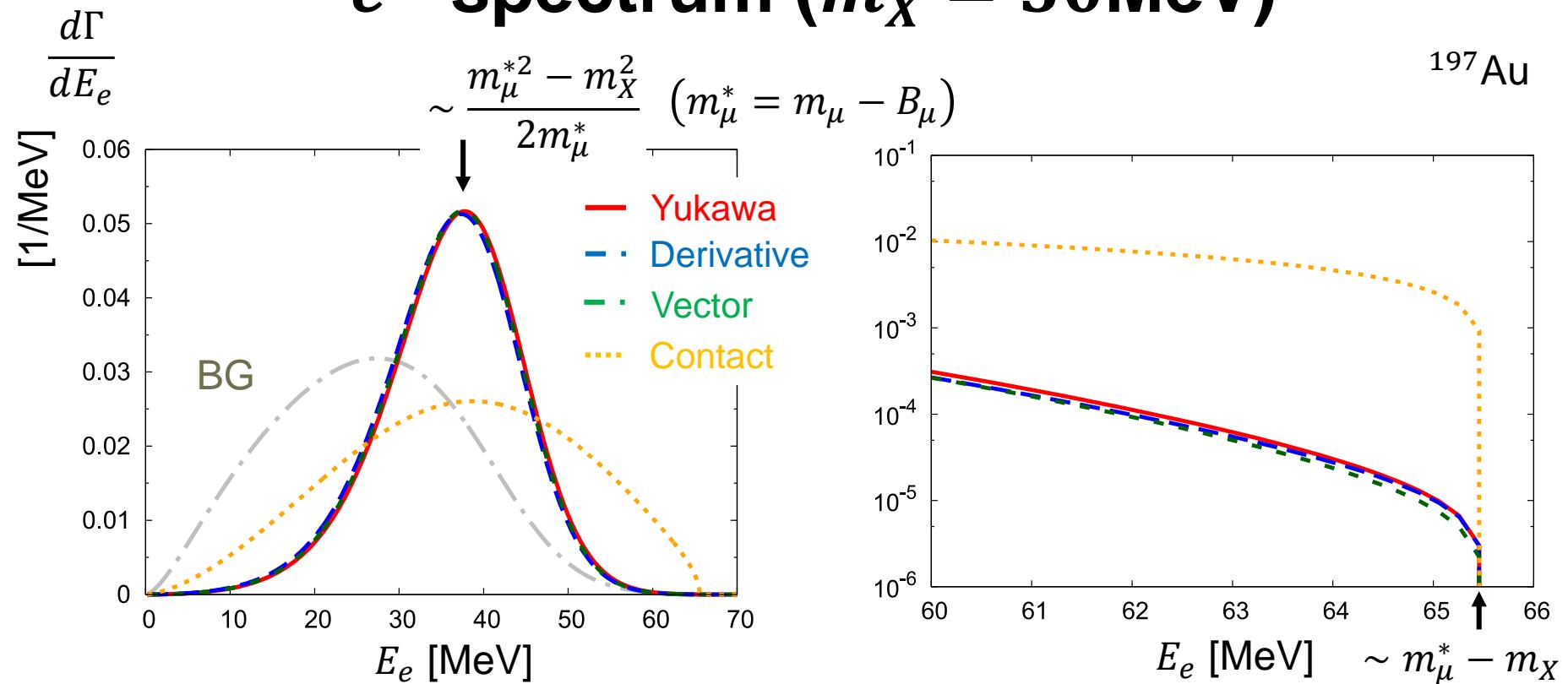


rate of high energy electron

$$f(E_{Low}) = \frac{1}{\Gamma} \int_{E_{Low}}^{E_{EndPoint}} dE_e \frac{d\Gamma}{dE_e}$$

Model	$f(70\text{MeV})$	$f(80\text{MeV})$	$f(90\text{MeV})$
Yukawa	4.8×10^{-4}	1.0×10^{-5}	2.6×10^{-8}
Derivative	3.7×10^{-4}	7.2×10^{-6}	1.9×10^{-8}
Contact	7.4×10^{-2}	1.8×10^{-2}	1.3×10^{-3}

e^- spectrum ($m_X = 30\text{MeV}$)



rate of high energy electron

$$f(E_{Low}) = \frac{1}{\Gamma} \int_{E_{Low}}^{E_{EndPoint}} dE_e \frac{d\Gamma}{dE_e}$$

Model	$f(50\text{MeV})$	$f(60\text{MeV})$
Yukawa	3.4×10^{-2}	5.5×10^{-4}
Derivative	3.0×10^{-2}	4.8×10^{-4}
Vector	3.1×10^{-2}	4.6×10^{-4}
Contact	2.0×10^{-1}	3.5×10^{-2}

Electron spectrum

◆ derivative coupling

κ : angular momentum of e^-

$$\frac{d\Gamma}{dE_e} = \frac{g_{S1}^2}{4\pi^2} \frac{p_e p_X}{\Lambda_{S1}^2} \sum_{\kappa} (2j_{\kappa} + 1) |I_{\kappa}^0 + I_{\kappa}^{1;+} + I_{\kappa}^{1;-}|^2$$

$$I_{\kappa}^0 = E_X \int_0^{\infty} dr r^2 j_{l_{\kappa}}(p_X r) \{ g_{p_e}^{\kappa}(r) g_{\mu}^{1s}(r) + f_{p_e}^{\kappa}(r) f_{\mu}^{1s}(r) \}$$

$$I_{\kappa}^{1;+} = p_X \int_0^{\infty} dr r^2 j_{l_{\kappa}+1}(p_X r) \left\{ \frac{\kappa - l_{\kappa}}{2l_{\kappa} + 1} f_{p_e}^{\kappa}(r) g_{\mu}^{1s}(r) + \frac{2 + \kappa + l_{\kappa}}{2l_{\kappa} + 1} g_{p_e}^{\kappa}(r) f_{\mu}^{1s}(r) \right\}$$

$$I_{\kappa}^{1;-} = p_X \int_0^{\infty} dr r^2 j_{l_{\kappa}-1}(p_X r) \left\{ \frac{1 + \kappa + l_{\kappa}}{2l_{\kappa} + 1} f_{p_e}^{\kappa}(r) g_{\mu}^{1s}(r) + \frac{1 + \kappa - l_{\kappa}}{2l_{\kappa} + 1} g_{p_e}^{\kappa}(r) f_{\mu}^{1s}(r) \right\}$$

Electron spectrum

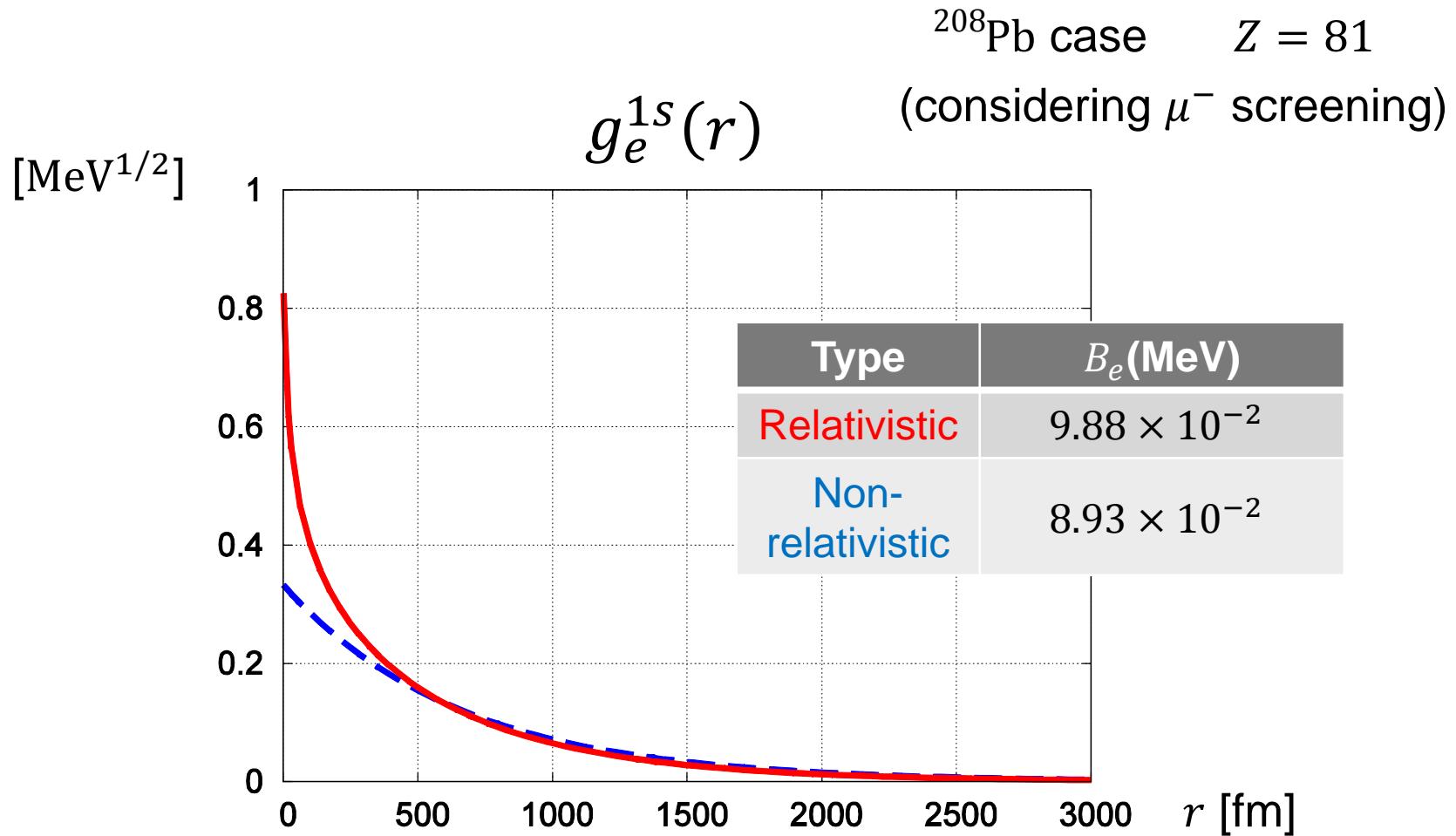
◆ dipole

$$\begin{aligned}
 \frac{d\Gamma}{dE_e} = & \frac{g_{DP}^2}{\pi^2 \Lambda_{DP}^2} p_e p_X \sum_{\kappa} (2j_{\kappa} + 1) \sum_{l_X=|j_{\kappa}-1/2|}^{j_{\kappa}+1/2} \\
 & \times \left[\left| E_X A_{\kappa, l_X-1, l_X} - \sqrt{\frac{l_X + 1}{2l_X + 1}} p_X B_{\kappa, l_X, l_X} \right|^2 + \left| E_X A_{\kappa, l_X+1, l_X} - \sqrt{\frac{l_X}{2l_X + 1}} p_X B_{\kappa, l_X, l_X} \right|^2 \right. \\
 & + \left| \sqrt{\frac{l_X + 1}{2l_X + 1}} E_X A_{\kappa, l_X, l_X} - p_X B_{\kappa, l_X-1, l_X} \right|^2 + \left| \sqrt{\frac{l_X}{2l_X + 1}} E_X A_{\kappa, l_X, l_X} - p_X B_{\kappa, l_X+1, l_X} \right|^2 \\
 & \left. - \frac{p_X^2}{\sqrt{2l_X + 1}} \left(\left| \sqrt{l_X} A_{\kappa, l_X-1, l_X} + \sqrt{l_X + 1} A_{\kappa, l_X+1, l_X} \right|^2 + \left| \sqrt{l_X} B_{\kappa, l_X-1, l_X} + \sqrt{l_X + 1} B_{\kappa, l_X+1, l_X} \right|^2 \right) \right]
 \end{aligned}$$

$$A_{\kappa, l_X, J} = \int_0^\infty dr r^2 j_{l_X}(p_X r) \{ g_{p_e}^{\kappa}(r) f_{\mu}^{1s}(r) V_{l_X, J}^{\kappa, +1} + f_{p_e}^{\kappa}(r) g_{\mu}^{1s}(r) V_{l_X, J}^{\kappa, -1} \} \frac{1 - (-1)^{l_{\kappa} + l_X}}{2}$$

$$B_{\kappa, l_X, J} = \int_0^\infty dr r^2 j_{l_X}(p_X r) \{ g_{p_e}^{\kappa}(r) g_{\mu}^{1s}(r) V_{l_X, J}^{\kappa, -1} + f_{p_e}^{\kappa}(r) f_{\mu}^{1s}(r) V_{l_X, J}^{\kappa, +1} \} \frac{1 - (-1)^{l_{\kappa} + l_X}}{2}$$

Radial wave function (bound e^-)



Relativity enhances the value near the origin.

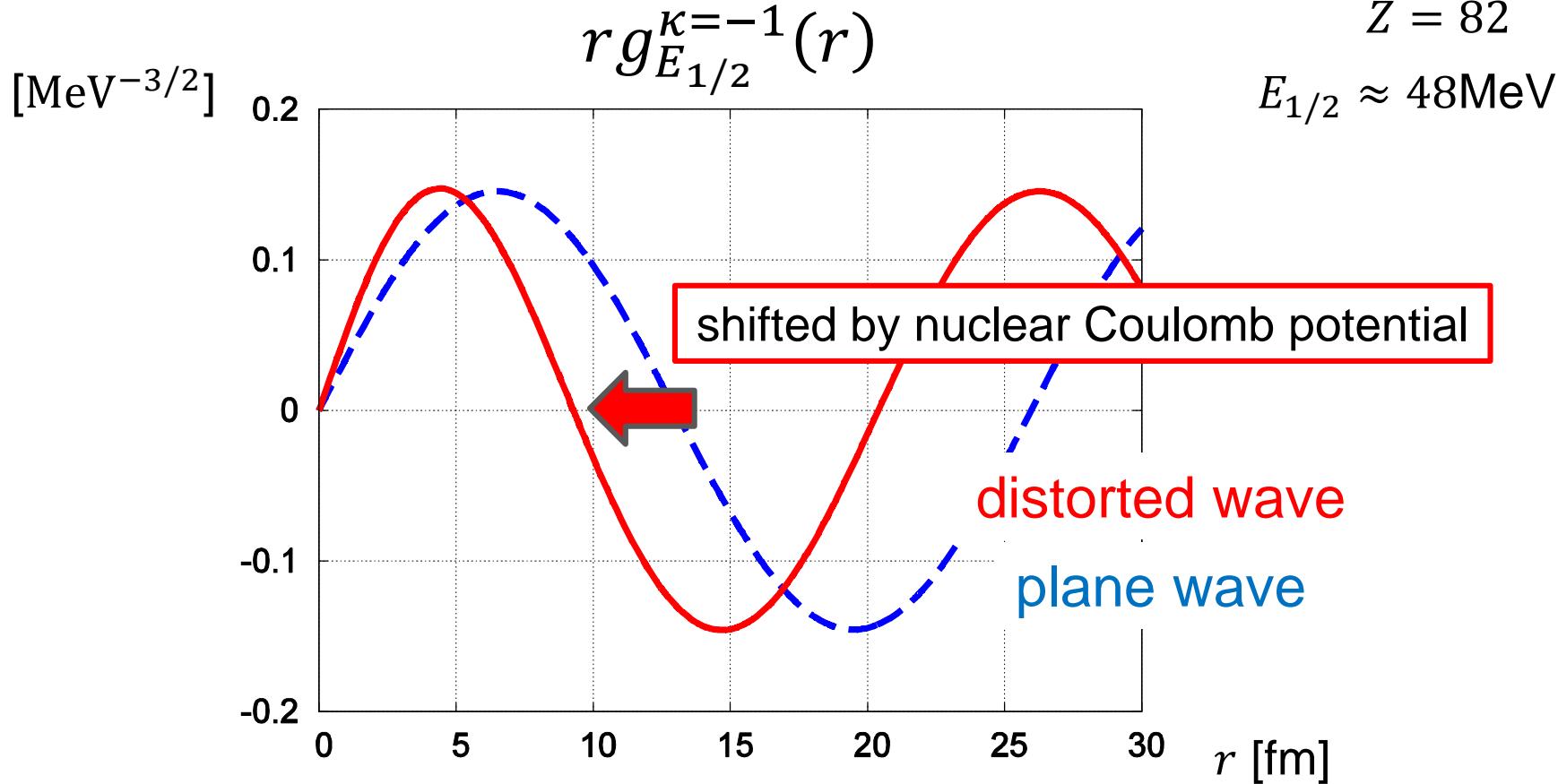
Radial wave function (scattering e^-)

e.g. $\kappa = -1$ partial wave

^{208}Pb case

$Z = 82$

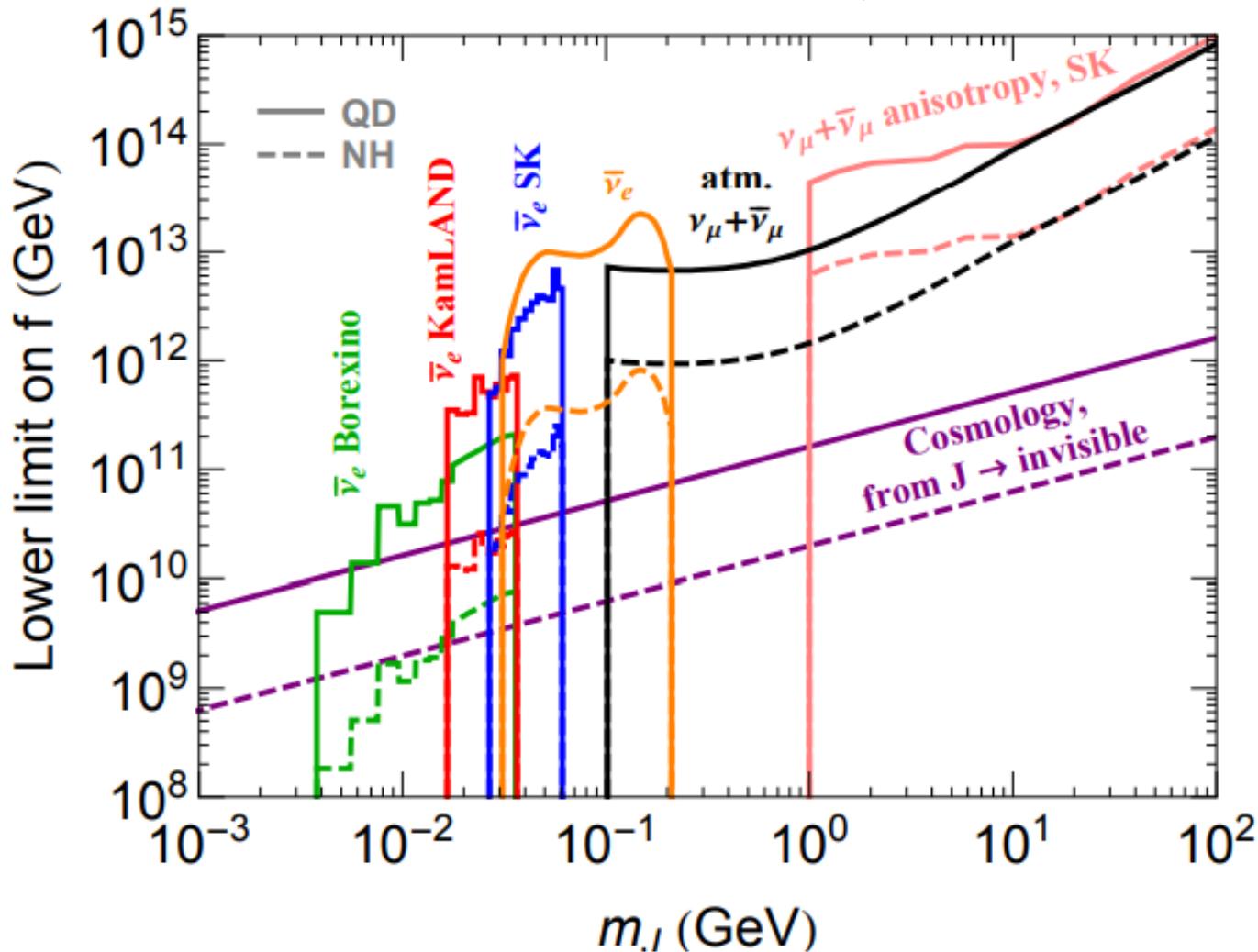
$E_{1/2} \approx 48\text{MeV}$



- ① enhanced value near the origin
- ② local momentum increased effectively

Constraint for Majoron parameter

C. Garcia-Cely & J. Heeck, JHEP 05 (2017) 102.



Constraint for Majoron parameter

C. Garcia-Cely & J. Heeck, JHEP 05 (2017) 102.

