

Status of Standard Model Prediction for Muon $g-2$

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Partially based on
A. Keshavarzi, DN and T. Teubner (**KNT**)
Phys. Rev. D97 (2018) 114025
[arXiv:1802.02995]

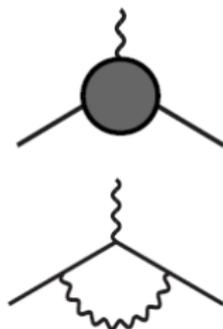
Muon g-2: introduction

Lepton magnetic moment $\vec{\mu}$: $\mathcal{H} = -\vec{\mu} \cdot \vec{B}$

$$\boxed{\vec{\mu} = -g \frac{e}{2m} \vec{s}}, \quad (\vec{s} = \frac{1}{2} \vec{\sigma} \text{ (spin)}, \quad g = 2 + 2F_2(0))$$

where

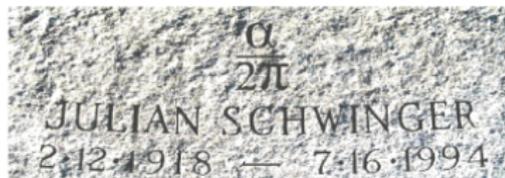
$$\bar{u}(p+q)\Gamma^\mu u(p) = \bar{u}(p+q) \left(\gamma^\mu F_1(q^2) + \frac{i\sigma^{\mu\nu}q_\nu}{2m} F_2(q^2) \right) u(p)$$



Anomalous magnetic moment: $a \equiv (g - 2)/2$ ($= F_2(0)$)

Historically,

- ★ $g = 2$ (tree level, Dirac)
- ★ $a = \alpha/(2\pi)$ (1-loop QED, Schwinger)



Today, still important, since...

- ★ One of the **most precisely measured** quantities:

$$\boxed{a_\mu^{\text{exp}} = 11\,659\,208.9(6.3) \times 10^{-10}} \quad [0.5\text{ppm}] \quad (\text{Bennett et al})$$

- ★ **Extremely useful** in probing/constraining physics beyond the SM

Breakdown of SM prediction for muon g-2

	<u>2011</u>	→	<u>2018</u>	
QED	11658471.81 (0.02)	→	11658471.90 (0.01)	[arXiv:1712.06060]
EW	15.40 (0.20)	→	15.36 (0.10)	[Phys. Rev. D 88 (2013) 053005]
LO HLbL	10.50 (2.60)	→	9.80 (2.60)	[EPJ Web Conf. 118 (2016) 01016]
NLO HLbL			0.30 (0.20)	[Phys. Lett. B 735 (2014) 90]

	<u>HLMNT11</u>	→	<u>KNT18</u>	
LO HVP	694.91 (4.27)	→	693.27 (2.46)	this work
NLO HVP	-9.84 (0.07)	→	-9.82 (0.04)	this work
NNLO HVP			1.24 (0.01)	[Phys. Lett. B 734 (2014) 144]

Theory total	11659182.80 (4.94)	→	11659182.05 (3.56)	this work
Experiment			11659209.10 (6.33)	world avg
Exp - Theory	26.1 (8.0)	→	27.1 (7.3)	this work

Δa_μ 3.3σ → 3.7σ this work

(HVP: Hadronic Vacuum Polarization)
(HLbL: Hadronic Light-by-Light)

(Numbers taken from KNT18,
Phys. Rev. D97 (2018) 114025)

Slide by A. Keshavarzi (Liverpool) at 'Muon g - 2 Workshop' at Mainz, June 18-22, 2018

QED contribution

QED contribution:

$$\begin{aligned} a_\mu(\text{QED}) &= \frac{\alpha}{2\pi} + 0.765857425(17) \left(\frac{\alpha}{\pi}\right)^2 + 24.05050996(32) \left(\frac{\alpha}{\pi}\right)^3 \\ &\quad + 130.8796(63) \left(\frac{\alpha}{\pi}\right)^4 + 753.3(1.0) \left(\frac{\alpha}{\pi}\right)^5 + \dots \\ &= 11658471.895(0.008) \times 10^{-10}, \quad (\text{numbers from PDG 2018}) \end{aligned}$$

where the uncertainty is dominated by that of α .

- 5-loop calculation! (Aoyama, Hayakawa, Kinoshita & Nio)
- The 4-loop corrections $\simeq 38 \times 10^{-10} \simeq \mathcal{O}(a_\mu(\text{exp}) - a_\mu(\text{SM}))$.
- The 4-loop contribution now fully cross-checked by another group. Mass-independent part by S. Laporta (*Phys.Lett.* **B772** (2017) 232), and mass-dependent part by A. Kurz et al (*Nucl. Phys.* **B879** (2014) 1; *Phys. Rev.* **D92** (2015) 073019; *ibid.* **D93** (2016) 053017)
- The 5-loop contribution very small ($\simeq 0.5 \times 10^{-10} \ll a_\mu(\text{exp}) - a_\mu(\text{SM})$)

Electroweak Contribution

Electroweak (EW) contribution:

$$a_{\mu}(\text{EW}) = \underbrace{19.48 \times 10^{-10}}_{\text{1-loop}} + \underbrace{(-4.12(10) \times 10^{-10})}_{\text{2-loop}} + \underbrace{\mathcal{O}(10^{-12})}_{\text{leading log 3-loop}}$$
$$= 15.36(10) \times 10^{-10}, \quad (\text{Number taken from PDG 2018})$$

where the uncertainty mainly comes from quark loops.

- 1-loop result published by many groups (Bardeen-Gastmans-Lautrup, Altarelli-Cabibbo-Maiani, Jackiw-Weinberg, Bars-Yoshimura, Fujikawa-Lee-Sanda) in 1972, and now a textbook exercise (Peskin & Schroeder's textbook, Problems 6.3 (Higgs) and 21.1 (W, Z))
- 2-loop contribution (~ 1700 diagrams in the 't Hooft-Feynman gauge) enhanced by $\ln(m_Z/m_{\mu})$ and also by a factor of $\mathcal{O}(10)$,

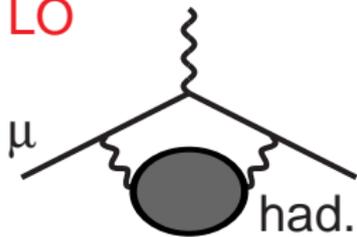
$$a_{\mu}(\text{EW}, \text{2-loop}) \simeq -10 \left(\frac{\alpha}{\pi} \right) a_{\mu}(\text{EW}, \text{1-loop}) \left(\ln \frac{m_Z}{m_{\mu}} + 1 \right),$$

where the factor of 10 appears since many "order one" diagrams accidentally add up coherently.

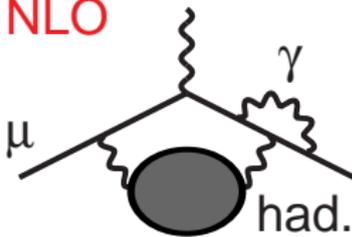
Hadronic Contributions

There are several hadronic contributions:

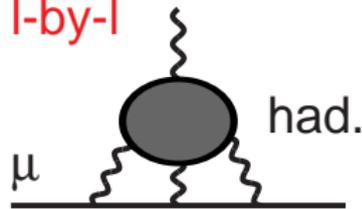
LO



NLO



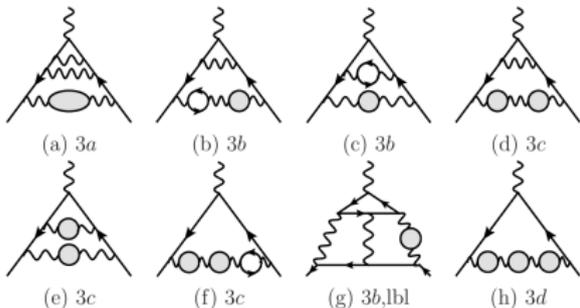
I-by-I



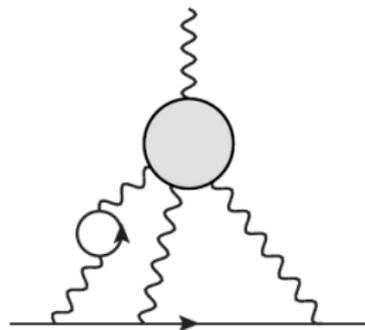
LO: Leading Order (or Vacuum Polarization) Hadronic Contribution

NLO: Next-to-Leading Order Hadronic Contribution

I-by-I: Hadronic light-by-light Contribution



NNLO Hadronic Contributions

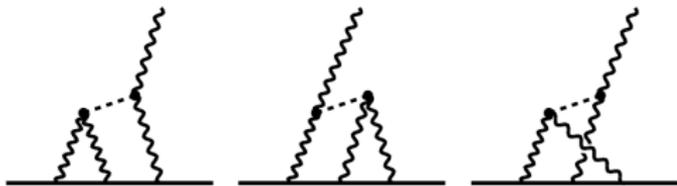


Hadronic I-by-I NLO Contrib.

Modern evaluation of I-by-I contribution

(Melnikov & Vainshtein)

1. First, use the large N_C expansion to find that the leading contribution is the pion pole contribution.

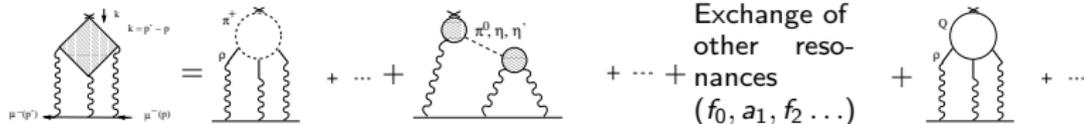


2. Choose the momentum-dependence of the $\pi\gamma\gamma$ coupling (form factor) in such a way that it is consistent with a constraint from QCD (OPE) at the momentum region $q_1^2 \sim q_2^2 \gg q_3^2$. Integrate over the loop momenta.
3. Repeat the above for η, η', a_1, \dots . Basically that's all for the LO in $1/N_C$.
4. As for NLO in $1/N_C$, it depends on authors which diagram is numerically important.

For example,

$$a_{\mu}^{\text{IbyI}} = \begin{cases} (10.5 \pm 2.6) \times 10^{-10} & \text{'Glasgow consensus', arXiv:0901.0306} \\ (9.8 \pm 2.6) \times 10^{-10} & \text{'G.c.' w/ correction by Nyffeler, PRD94(2016)053006} \\ (10.2 \pm 3.9) \times 10^{-10} & \text{Nyffeler, arXiv:1710.09742} \end{cases}$$

HLbL in muon $g - 2$: summary of selected results (model calculations)



de Rafael '94:

Chiral counting: p^4

N_C -counting: 1

Contribution to $a_\mu \times 10^{11}$:

p^6

N_C

p^8

N_C

p^8

N_C

BPP: +83 (32)
 HKS: +90 (15)
 KN: +80 (40)
 MV: +136 (25)
 2007: +110 (40)
 PdRV: +105 (26)
 N,JN: +116 (39)

-19 (13)
 -5 (8)
 0 (10)
 -19 (19)
 -19 (13)

+85 (13)
 +83 (6)
 +83 (12)
 +114 (10)
 +114 (13)
 +99 (16)

-4 (3) [f_0, a_1]
 +1.7 (1.7) [a_1]
 +22 (5) [a_1]
 +8 (12) [f_0, a_1]
 +15 (7) [f_0, a_1]

+21 (3)
 +10 (11)
 0
 +2.3 [c-quark]
 +21 (3)

ud.: -45

ud.: $+\infty$

ud.: +60

ud. = undressed, i.e. point vertices without form factors

Pseudoscalars: numerically dominant contribution (according to most models !).

Recall (in units of 10^{-11}): $\delta a_\mu(\text{HVP}) \approx 40$; $\delta a_\mu(\text{exp [BNL]}) = 63$; $\delta a_\mu(\text{future exp}) = 16$

BPP = Bijmens, Pallante, Prades '96, '02; HKS = Hayakawa, Kinoshita, Sanda '96, '98, '02; KN = Knecht, AN '02; MV = Melnikov, Vainshtein '04; 2007 = Bijmens, Prades; Miller, de Rafael, Roberts; PdRV = Prades, de Rafael, Vainshtein '09 (compilation; "Glasgow consensus"); N,JN = AN '09; Jegerlehner, AN '09 (compilation)

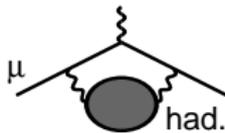
Recent reevaluations of axial vector contribution lead to much smaller estimates than in MV '04:

$a_\mu^{\text{HLbL}; \text{axial}} = (8 \pm 3) \times 10^{-11}$ (Pauk, Vanderhaeghen '14; Jegerlehner '14, '15). Would shift central values of compilations downwards:

$a_\mu^{\text{HLbL}} = (98 \pm 26) \times 10^{-11}$ (PdRV) and $a_\mu^{\text{HLbL}} = (102 \pm 39) \times 10^{-11}$ (N, JN).

LO Hadronic Vacuum Polarization Contribution

The diagram to be evaluated:

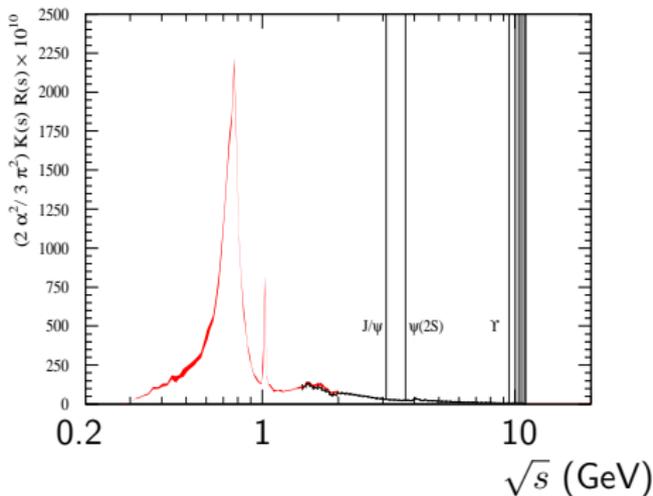


pQCD not useful. Use the **dispersion relation** and the **optical theorem**.

$$\text{had.} = \int \frac{ds}{\pi(s-q^2)} \text{Im had.}$$

$$2 \text{Im had.} = \sum_{\text{had.}} \int d\Phi \left| \text{had.} \right|^2$$

$$a_{\mu}^{\text{had,LO}} = \frac{m_{\mu}^2}{12\pi^3} \int_{s_{\text{th}}}^{\infty} ds \frac{1}{s} \hat{K}(s) \sigma_{\text{had}}(s)$$



- Weight function $\hat{K}(s)/s = \mathcal{O}(1)/s$
 \implies **Lower** energies **more important**
 $\implies \pi^+\pi^-$ channel: 73% of total $a_{\mu}^{\text{had,LO}}$

Channel	Energy range [GeV]	$a_\mu^{\text{had,LOVP}} \times 10^{10}$	$\Delta a_\mu^{(S)}(M_Z^2) \times 10^4$	New data
Chiral perturbation theory (ChPT) threshold contributions				
$\pi^0\gamma$	$m_\pi \leq \sqrt{s} \leq 0.600$	0.12 ± 0.01	0.00 ± 0.00	...
$\pi^+ \pi^-$	$2m_\pi \leq \sqrt{s} \leq 0.305$	0.87 ± 0.02	0.01 ± 0.00	...
$\pi^+ \pi^- \pi^0$	$3m_\pi \leq \sqrt{s} \leq 0.660$	0.01 ± 0.00	0.00 ± 0.00	...
$\eta\gamma$	$m_\eta \leq \sqrt{s} \leq 0.660$	0.00 ± 0.00	0.00 ± 0.00	...
Data based channels ($\sqrt{s} \leq 1.937$ GeV)				
$\pi^0\gamma$	$0.600 \leq \sqrt{s} \leq 1.350$	4.46 ± 0.10	0.36 ± 0.01	[65]
$\pi^+ \pi^-$	$0.305 \leq \sqrt{s} \leq 1.937$	502.97 ± 1.97	34.26 ± 0.12	[34,35]
$\pi^+ \pi^- \pi^0$	$0.660 \leq \sqrt{s} \leq 1.937$	47.79 ± 0.89	4.77 ± 0.08	[36]
$\pi^+ \pi^- \pi^+ \pi^-$	$0.613 \leq \sqrt{s} \leq 1.937$	14.87 ± 0.20	4.02 ± 0.05	[40,42]
$\pi^+ \pi^- \pi^0 \pi^0$	$0.850 \leq \sqrt{s} \leq 1.937$	19.39 ± 0.78	5.00 ± 0.20	[44]
$(2\pi^+ 2\pi^- \pi^0)_{\text{non}}$	$1.013 \leq \sqrt{s} \leq 1.937$	0.99 ± 0.09	0.33 ± 0.03	...
$3\pi^+ 3\pi^-$	$1.313 \leq \sqrt{s} \leq 1.937$	0.23 ± 0.01	0.09 ± 0.01	[66]
$(2\pi^+ 2\pi^- 2\pi^0)_{\text{non}}$	$1.322 \leq \sqrt{s} \leq 1.937$	1.35 ± 0.17	0.51 ± 0.06	...
$K^+ K^-$	$0.988 \leq \sqrt{s} \leq 1.937$	23.03 ± 0.22	3.37 ± 0.03	[45,46,49]
$K_S^0 K_L^0$	$1.004 \leq \sqrt{s} \leq 1.937$	13.04 ± 0.19	1.77 ± 0.03	[50,51]
$KK\pi$	$1.260 \leq \sqrt{s} \leq 1.937$	2.71 ± 0.12	0.89 ± 0.04	[53,54]
$KK2\pi$	$1.350 \leq \sqrt{s} \leq 1.937$	1.93 ± 0.08	0.75 ± 0.03	[50,53,55]
$\eta\gamma$	$0.660 \leq \sqrt{s} \leq 1.760$	0.70 ± 0.02	0.09 ± 0.00	[67]
$\eta\pi^+ \pi^-$	$1.091 \leq \sqrt{s} \leq 1.937$	1.29 ± 0.06	0.39 ± 0.02	[68,69]
$(\eta\pi^+ \pi^- \pi^0)_{\text{non}}$	$1.333 \leq \sqrt{s} \leq 1.937$	0.60 ± 0.15	0.21 ± 0.05	[70]
$\eta 2\pi^+ 2\pi^-$	$1.338 \leq \sqrt{s} \leq 1.937$	0.08 ± 0.01	0.03 ± 0.00	...
$\eta\omega$	$1.333 \leq \sqrt{s} \leq 1.937$	0.31 ± 0.03	0.10 ± 0.01	[70,71]
$\omega(\rightarrow \pi^0\gamma)\pi^0$	$0.920 \leq \sqrt{s} \leq 1.937$	0.88 ± 0.02	0.19 ± 0.00	[72,73]
$\eta\phi$	$1.569 \leq \sqrt{s} \leq 1.937$	0.42 ± 0.03	0.15 ± 0.01	...
$\phi \rightarrow \text{unaccounted}$	$0.988 \leq \sqrt{s} \leq 1.029$	0.04 ± 0.04	0.01 ± 0.01	...
$\eta\omega\pi^0$	$1.550 \leq \sqrt{s} \leq 1.937$	0.35 ± 0.09	0.14 ± 0.04	[74]
$\eta(\rightarrow \text{npp})K\bar{K}_{\text{non}\phi \rightarrow K\bar{K}}$	$1.569 \leq \sqrt{s} \leq 1.937$	0.01 ± 0.02	0.00 ± 0.01	[53,75]
$p\bar{p}$	$1.890 \leq \sqrt{s} \leq 1.937$	0.03 ± 0.00	0.01 ± 0.00	[76]
$n\bar{n}$	$1.912 \leq \sqrt{s} \leq 1.937$	0.03 ± 0.01	0.01 ± 0.00	[77]
Estimated contributions ($\sqrt{s} \leq 1.937$ GeV)				
$(\pi^+ \pi^- 3\pi^0)_{\text{non}}$	$1.013 \leq \sqrt{s} \leq 1.937$	0.50 ± 0.04	0.16 ± 0.01	...
$(\pi^+ \pi^- 4\pi^0)_{\text{non}}$	$1.313 \leq \sqrt{s} \leq 1.937$	0.21 ± 0.21	0.08 ± 0.08	...
$KK3\pi$	$1.569 \leq \sqrt{s} \leq 1.937$	0.03 ± 0.02	0.02 ± 0.01	...
$\omega(\rightarrow \text{npp})2\pi$	$1.285 \leq \sqrt{s} \leq 1.937$	0.10 ± 0.02	0.03 ± 0.01	...
$\omega(\rightarrow \text{npp})3\pi$	$1.322 \leq \sqrt{s} \leq 1.937$	0.17 ± 0.03	0.06 ± 0.01	...
$\omega(\rightarrow \text{npp})KK$	$1.569 \leq \sqrt{s} \leq 1.937$	0.00 ± 0.00	0.00 ± 0.00	...
$\eta\pi^+ \pi^- 2\pi^0$	$1.338 \leq \sqrt{s} \leq 1.937$	0.08 ± 0.04	0.03 ± 0.02	...
Other contributions ($\sqrt{s} > 1.937$ GeV)				
Inclusive channel	$1.937 \leq \sqrt{s} \leq 11.199$	43.67 ± 0.67	82.82 ± 1.05	[56,62,63]
J/ψ	...	6.26 ± 0.19	7.07 ± 0.22	...
ψ'	...	1.58 ± 0.04	2.51 ± 0.06	...
$\Upsilon(1S - 4S)$...	0.09 ± 0.00	1.06 ± 0.02	...
pQCD	$11.199 \leq \sqrt{s} \leq \infty$	2.07 ± 0.00	124.79 ± 0.10	...
Total	$m_\pi \leq \sqrt{s} \leq \infty$	693.26 ± 2.46	276.11 ± 1.11	...

Breakdown of contributions to a_μ (had, LO VP) from various hadronic final states

We have included new data sets from ~ 30 papers, in addition to those included in the HLMNT11 analysis

We have included ~ 30 hadronic final states

At $2 \lesssim \sqrt{s} \lesssim 11$ GeV, we use inclusively measured data

At higher energies $\gtrsim 11$ GeV, we use pQCD

Table from KNT18, Phys. Rev. D97 (2018) 114025

$\sigma(e^+e^- \rightarrow \pi^+\pi^-)$ data

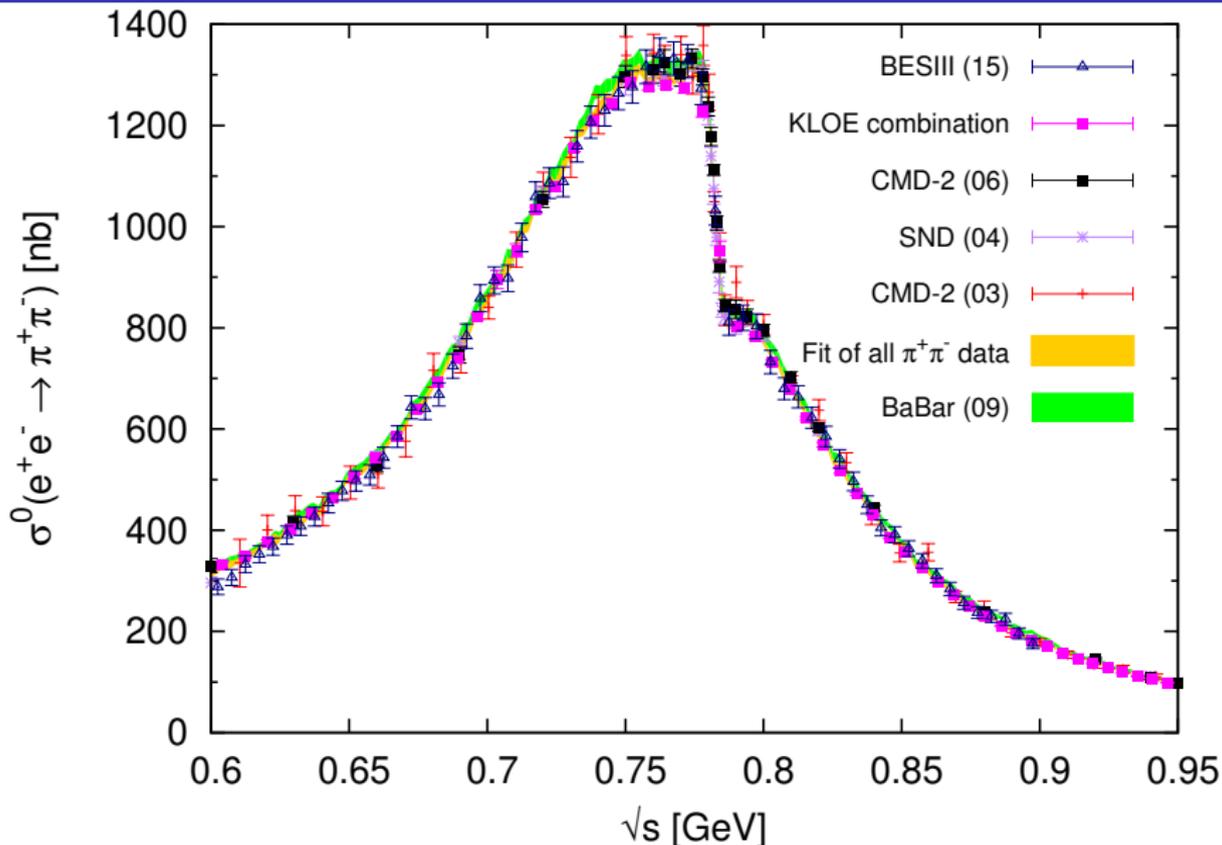


Fig. from KNT18, Phys. Rev. D97 (2018) 114025

$\sigma(e^+e^- \rightarrow \pi^+\pi^-)$: ρ - ω interference region

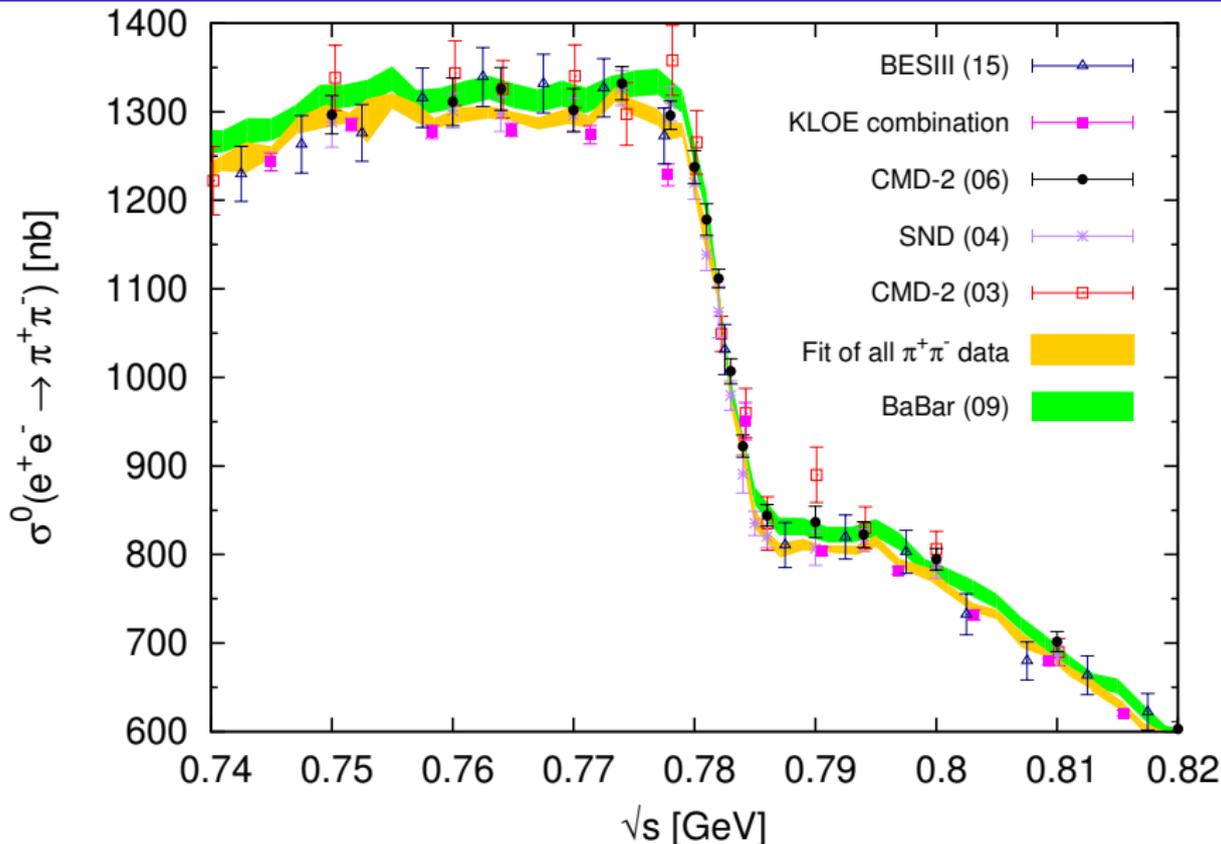


Fig. from KNT18, Phys. Rev. D97 (2018) 114025

$\sigma(e^+e^- \rightarrow \pi^+\pi^-)$: relative differences

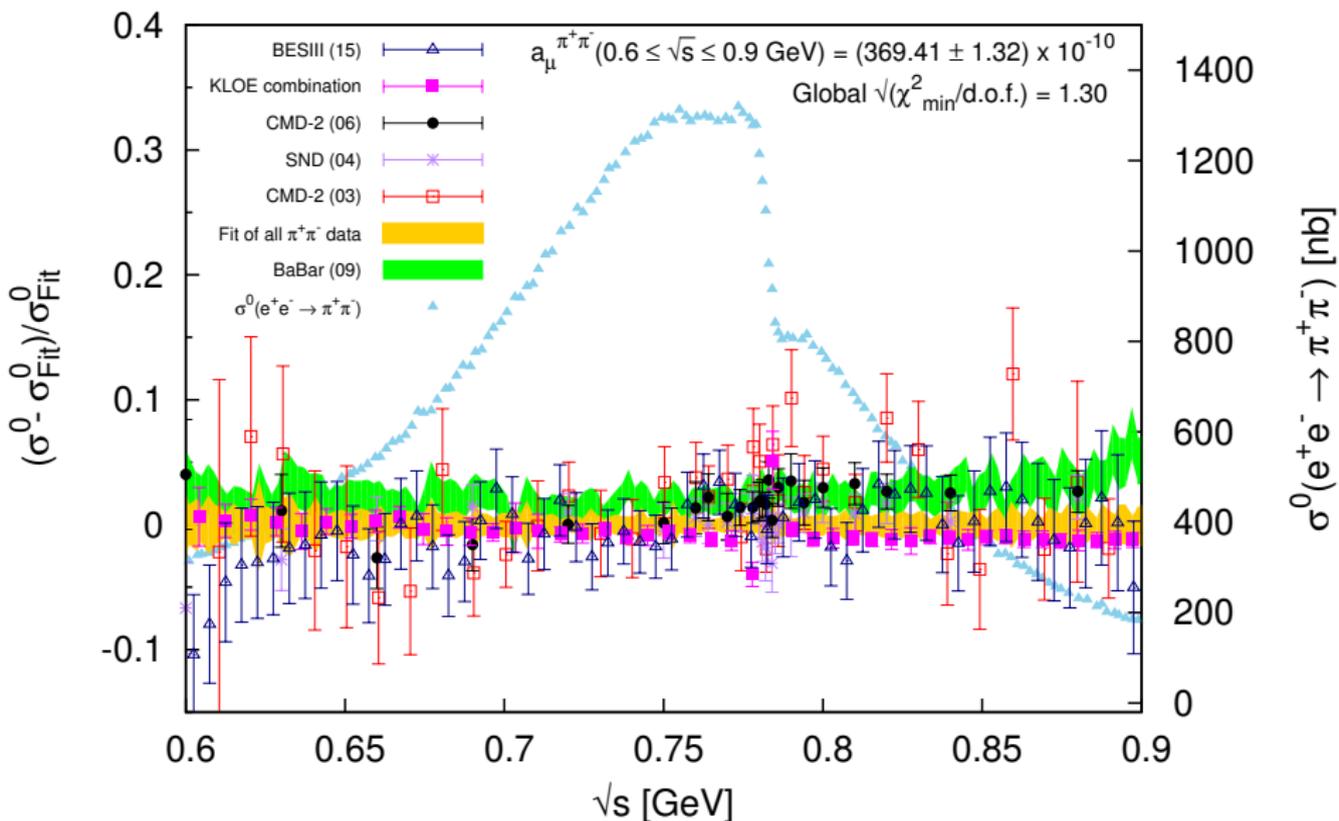


Fig. from KNT18, Phys. Rev. D97 (2018) 114025

Contribution to $(g - 2)_\mu$ from $\pi^+\pi^-$ channel

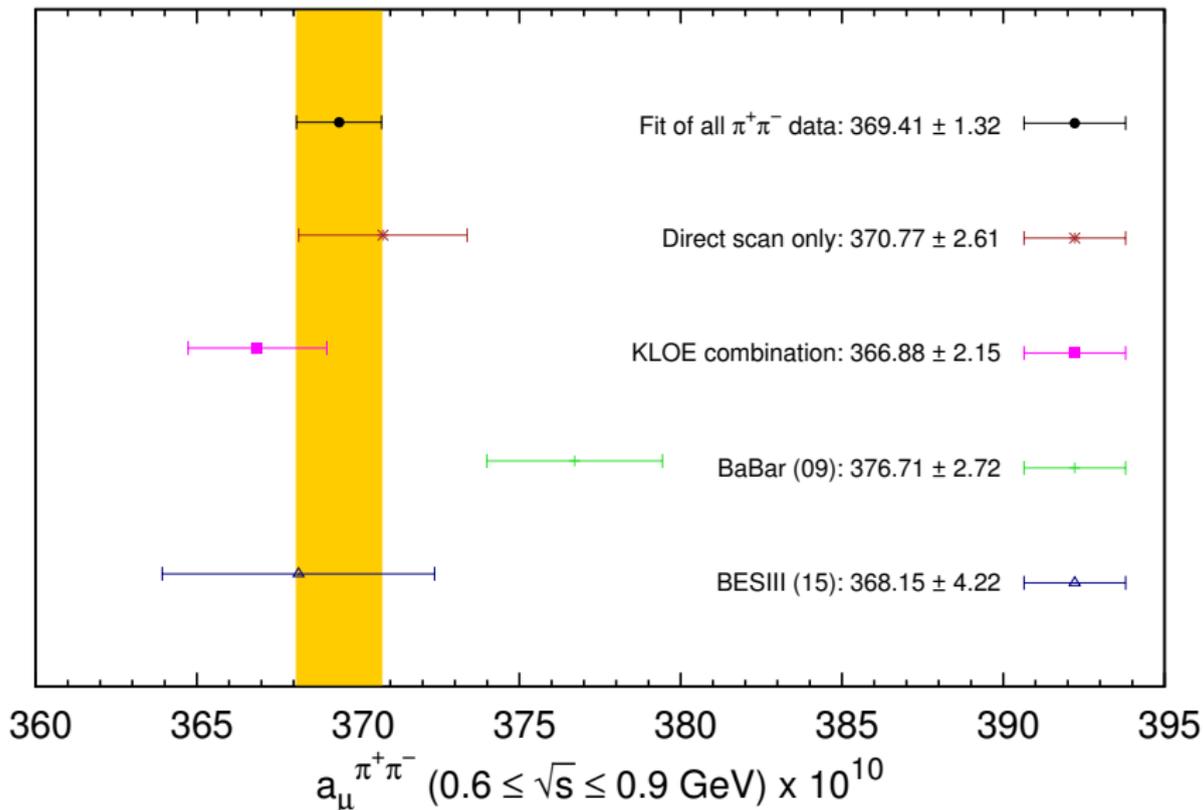
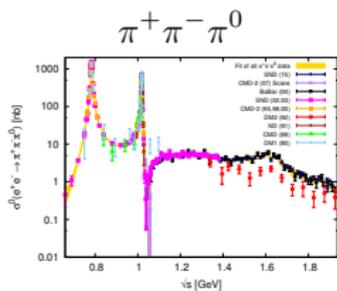


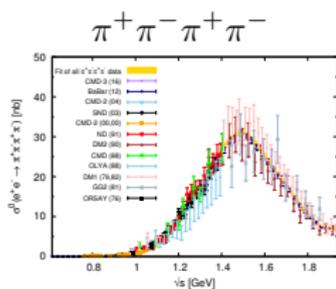
Fig. from KNT18, Phys. Rev. D97 (2018) 114025

Other notable exclusive channels [KNT18: arXiv:1802.02995, PRD (in press)]



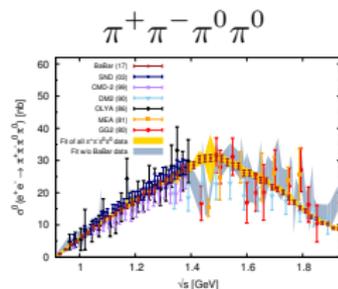
HLMNT11: 47.51 ± 0.99

KNT18: 47.92 ± 0.89



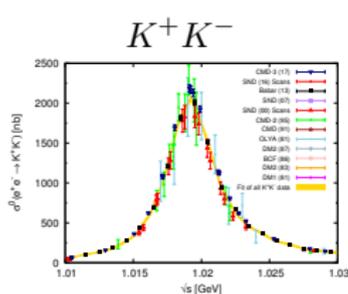
HLMNT11: 14.65 ± 0.47

KNT18: 14.87 ± 0.20



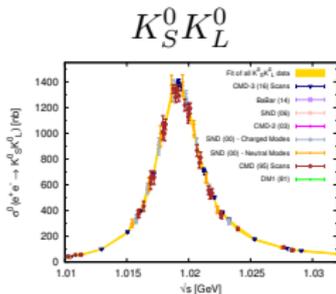
HLMNT11: 20.37 ± 1.26

KNT18: 19.39 ± 0.78



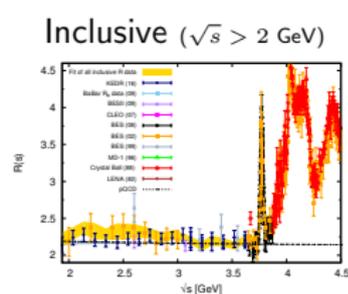
HLMNT11: 22.15 ± 0.46

KNT18: 23.03 ± 0.22



HLMNT11: 13.33 ± 0.16

KNT18: 13.04 ± 0.19



HLMNT11: 41.40 ± 0.87

KNT18: 41.27 ± 0.62

Slide by A. Keshavarzi (Liverpool) at 'Muon $g - 2$ Workshop' at Mainz, June 18-22, 2018

KNT18 $a_\mu^{\text{had, VP}}$ update

$$\text{HLMNT(11): } 694.91 \pm 4.27$$



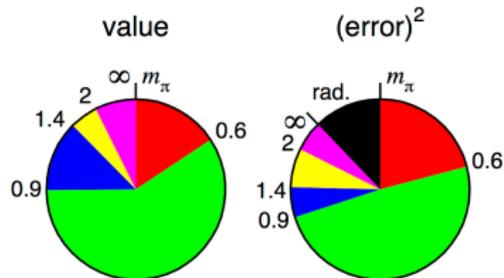
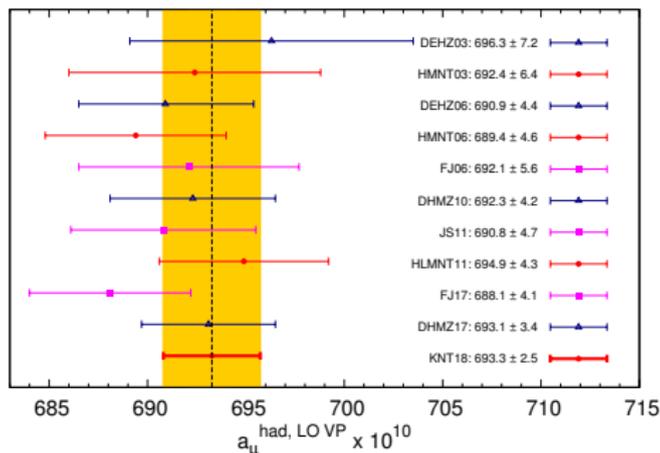
$$\text{This work: } a_\mu^{\text{had, LO VP}} = 693.27 \pm 1.19_{\text{stat}} \pm 2.01_{\text{sys}} \pm 0.22_{\text{vp}} \pm 0.71_{\text{fsr}}$$

$$= 693.27 \pm 2.34_{\text{exp}} \pm 0.74_{\text{rad}}$$

$$= 693.27 \pm 2.46_{\text{tot}}$$

$$a_\mu^{\text{had, NLO VP}} = -9.82 \pm 0.04_{\text{tot}}$$

⇒ Accuracy better than 0.4%
(uncertainties include all available correlations)



⇒ 2π dominance

Slide by A. Keshavarzi (Liverpool) at 'Muon $g-2$ HVP Workshop' at KEK, Feb. 12-14, 2018

Breakdown of SM prediction for muon g-2

	<u>2011</u>	→	<u>2018</u>	
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NLO HLbL			0.30 (0.20)	[Phys. Lett. B 735 (2014) 90]

	<u>HLMNT11</u>	→	<u>KNT18</u>	
LO HVP	694.91 (4.27)	→	693.27 (2.46)	this work
NLO HVP	-9.84 (0.07)	→	-9.82 (0.04)	this work
NNLO HVP			1.24 (0.01)	[Phys. Lett. B 734 (2014) 144]

Theory total	11659182.80 (4.94)	→	11659182.05 (3.56)	this work
Experiment			11659209.10 (6.33)	world avg
Exp - Theory	26.1 (8.0)	→	27.1 (7.3)	this work

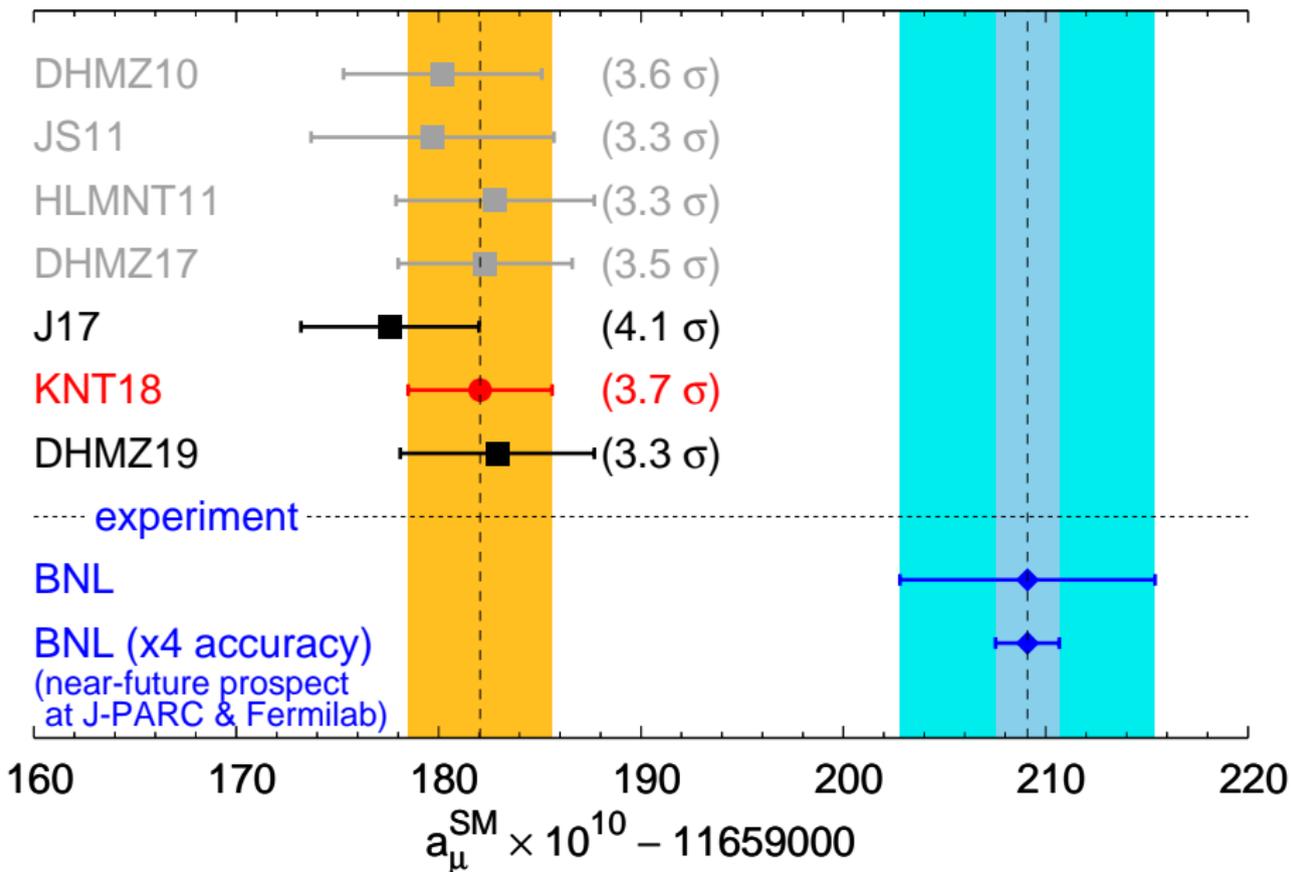
Δa_μ 3.3σ → 3.7σ this work

(HVP: Hadronic Vacuum Polarization)
(HLbL: Hadronic Light-by-Light)

(Numbers taken from KNT18,
Phys. Rev. D97 (2018) 114025)

Slide by A. Keshavarzi (Liverpool) at 'Muon g - 2 Workshop' at Mainz, June 18-22, 2018

Exp. value of muon g-2 vs SM prediction



Comparison with Other Work

Contributions from major channels to $a_\mu(\text{LO,had})$ for $\sqrt{s} < 1.8\text{GeV}$:

channel	KNT18	DHMZ19	diff
$\pi^+\pi^-$	503.74 ± 1.96	507.80 ± 3.35	-4.06
$\pi^+\pi^-\pi^0$	47.70 ± 0.89	46.20 ± 1.45	1.50
K^+K^-	23.00 ± 0.22	23.08 ± 0.44	-0.08
$\pi^+\pi^-2\pi^0$	18.15 ± 0.74	18.01 ± 0.55	0.14
$2\pi^+2\pi^-$	13.99 ± 0.19	13.68 ± 0.31	0.31
$K_S^0K_L^0$	13.04 ± 0.19	12.82 ± 0.24	0.22
$\pi^0\gamma$	4.58 ± 0.10	4.29 ± 0.10	0.29
\vdots	\vdots	\vdots	\vdots

“DHMZ19” = M. Davier et al, arXiv:1908.00921

Difference in the $\pi^+\pi^-$ channel is mainly from the way to combine the data sets.

KNT18: Global χ^2 minimization

DHMZ19: Takes the average of “all but KLOE” and “all but BaBar” as the mean value, and counts the half of the diff of the two as an additional systematic uncertainty.

Comparison with Lattice Results

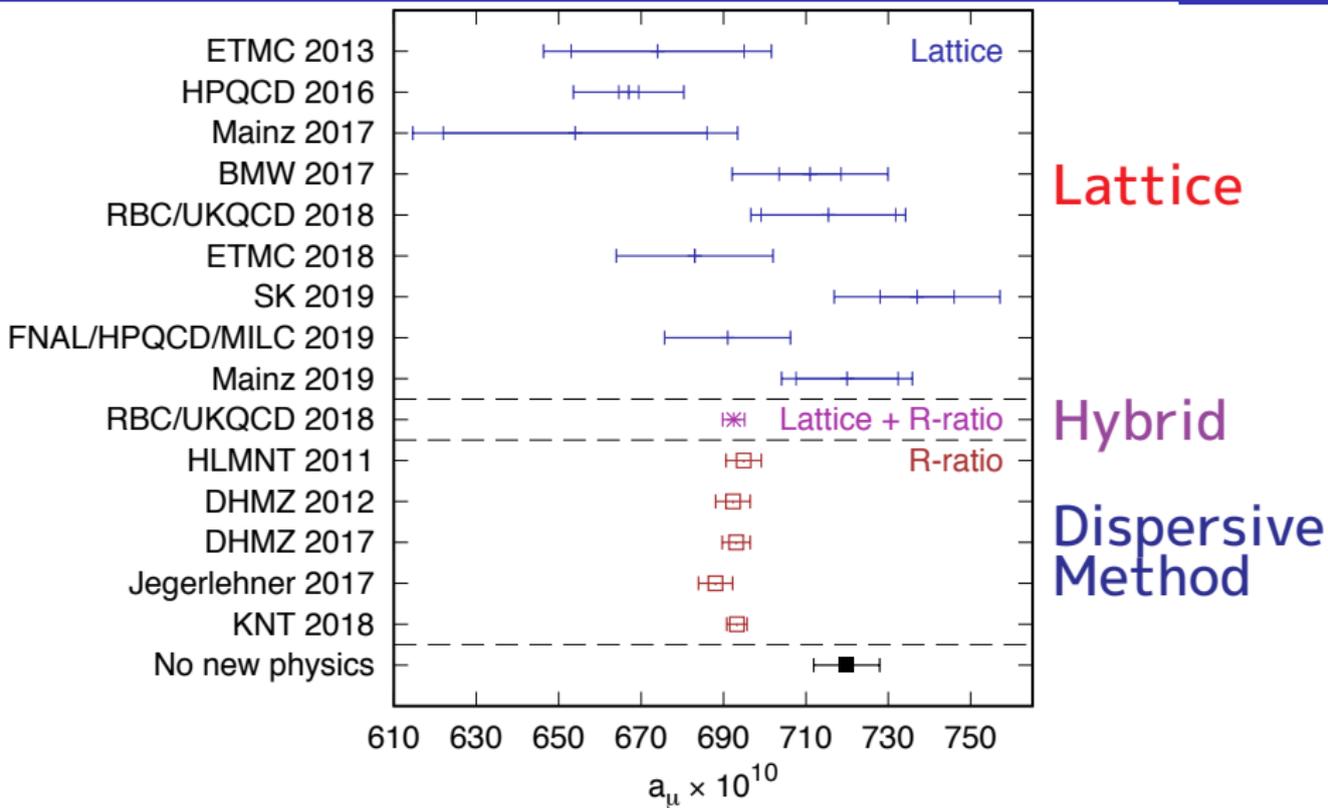


Fig. by C. Lehner (BNL), talk at Lattice 2019

Some Preliminary Results (NOT included in KNT18)

New data updates [preliminary]



There have been some notable data updates from SND and BaBar:

SND (arXiv:1809.07631)

$$e^+e^- \rightarrow \pi^+\gamma, 1.075 \leq \sqrt{s} < 2 \text{ GeV}$$

It extends the upper border of the π^0 gamma data from 1.35 GeV to 1.935 GeV.

$$\text{KNT18: } a_\mu^{\pi^+\gamma} = 4.46 \pm 0.08, \chi^2_{\text{min}}/d.o.f. = 1.44$$

$$\text{Now [preliminary]: } a_\mu^{\pi^+\gamma} = 4.46 \pm 0.08, \chi^2_{\text{min}}/d.o.f. = 1.41$$

→ Negligible changes, consolidation of previous estimate.

BaBar (arXiv:1810.11962)

- $e^+e^- \rightarrow \pi^+\pi^-\pi^0\pi^0\pi^0, 1.125 \leq \sqrt{s} \leq 4.325 \text{ GeV}$
- $e^+e^- \rightarrow \pi^+\pi^-\eta, 1.075 \leq \sqrt{s} \leq 3.025 \text{ GeV}$
- $e^+e^- \rightarrow \omega\pi^+\pi^0, 1.125 \leq \sqrt{s} \leq 4.325 \text{ GeV}$
- $e^+e^- \rightarrow \pi^+\pi^-\pi^0\pi^0\eta, 1.625 \leq \sqrt{s} \leq 4.325 \text{ GeV}$
- $e^+e^- \rightarrow \omega\eta\pi^0, 1.525 \leq \sqrt{s} \leq 4.325 \text{ GeV}$

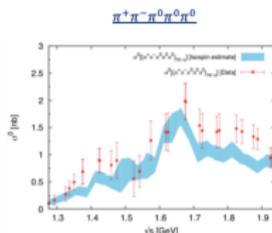
→ The BaBar updates in particular as they update modes/final states that were previously estimated via isospin relations.



New data updates [preliminary]



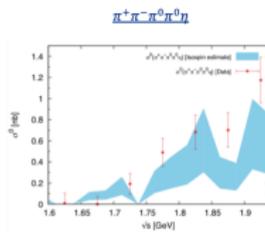
BaBar (arXiv:1810.11962)



$$\text{KNT18: } a_\mu^{\pi^+\pi^-\pi^0\pi^0} = 0.50 \pm 0.04$$

$$\text{Now: } a_\mu^{\pi^+\pi^-\pi^0\pi^0} = 0.64 \pm 0.11$$

[preliminary]



$$\text{KNT18: } a_\mu^{\pi^+\pi^-\pi^0\eta} = 0.08 \pm 0.04$$

$$\text{Now: } a_\mu^{\pi^+\pi^-\pi^0\eta} = 0.12 \pm 0.02$$

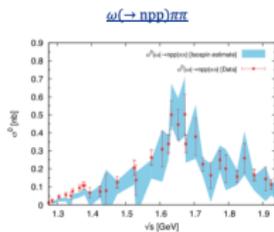
[preliminary]



New data updates [preliminary]



BaBar (arXiv:1810.11962)



$$\text{KNT18: } a_\mu^{\omega(-npp)\pi\pi} = 0.10 \pm 0.02$$

$$\text{Now: } a_\mu^{\omega(-npp)\pi\pi} = 0.11 \pm 0.01$$

[preliminary]

$$\text{KNT18: } a_\mu^{\pi^+\pi^-\eta} = 1.29 \pm 0.06$$

$$\text{Now: } a_\mu^{\pi^+\pi^-\eta} = 1.30 \pm 0.06$$

[preliminary]

$$\text{KNT18: } a_\mu^{\omega\eta\pi^0} = 0.35 \pm 0.09$$

$$\text{Now: } a_\mu^{\omega\eta\pi^0} = 0.24 \pm 0.05$$

[preliminary]

→ These changes have a minor effect overall:

$$\text{KNT18: } a_\mu^{\text{had, LOVP}} = 693.26 \pm 2.46$$

$$\text{Now: } a_\mu^{\text{had, LOVP}} = 693.23 \pm 2.46$$

[preliminary]

But, good that isospin estimates are further consolidated...



In addition, from very new data of $e^+e^- \rightarrow 3\pi^+3\pi^-\pi^0$ from CMD-3 (arXiv:1902.06449),

$$a_\mu(3\pi^+3\pi^-\pi^0) = (0.020 \pm 0.004) \times 10^{-10}$$

(preliminary)

Slides by A. Keshavarzi at "SchwingerFest2018"
December 3-5, 2018

Summary

- Standard Model prediction for $(g - 2)_\mu$: $\gtrsim 3.5\sigma$ deviation from measured value \implies New Physics?
- Recent data-driven evaluations of hadronic vacuum polarization contributions seem convergent
- To better establish the $g - 2$ anomaly, better data for $e^+e^- \rightarrow \pi^+\pi^-$ welcome (from CMD-3, SND, Belle II, ...)
- Lattice calculations still suffer from large uncertainties (but a hybrid approach is useful)
- Input from ChPT-assisted dispersive method, lattice & exp. at space-like q^2 also very welcome!
- New exp. at Fermilab and J-PARC expected to reduce the uncertainty of $(g - 2)_\mu$ by a factor of 4