



Emittance Exchange in MICE

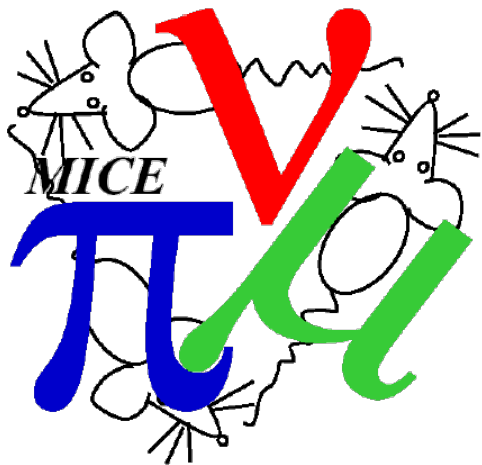
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on behalf of the
MICE Collaboration

NuFact 2019

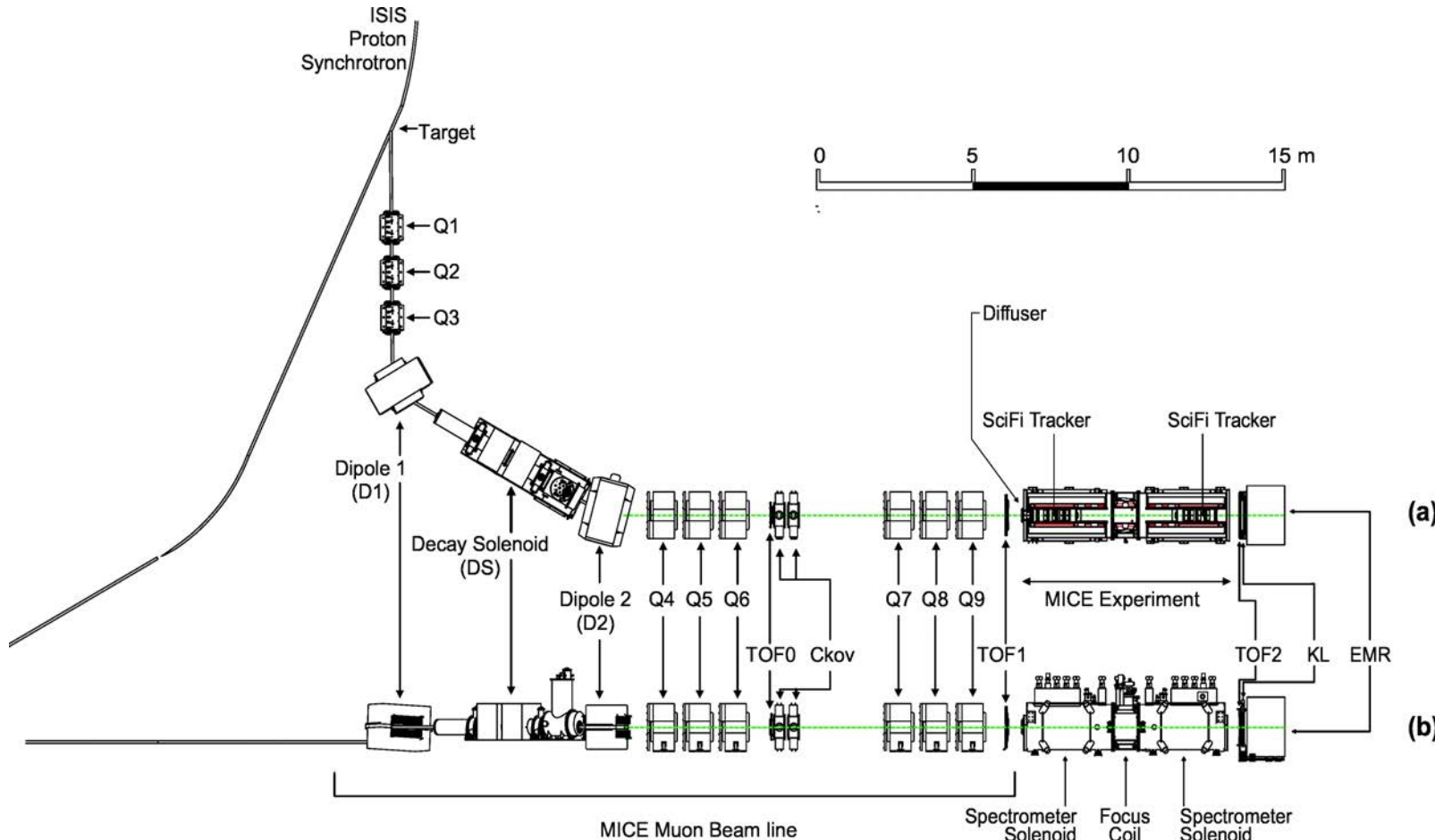
Daegu, Korea



Overview

- Muon Ionization Cooling Experiment (MICE)
- Neutrino Factory/Muon Collider – The need for cooling
- Emittance Exchange
- Kernel Density Estimation (KDE)
- Change in Phase-Space density

Muon Ionization Cooling Experiment



Proton beam impacts Target

Pions produced captured in quadrupole triplet

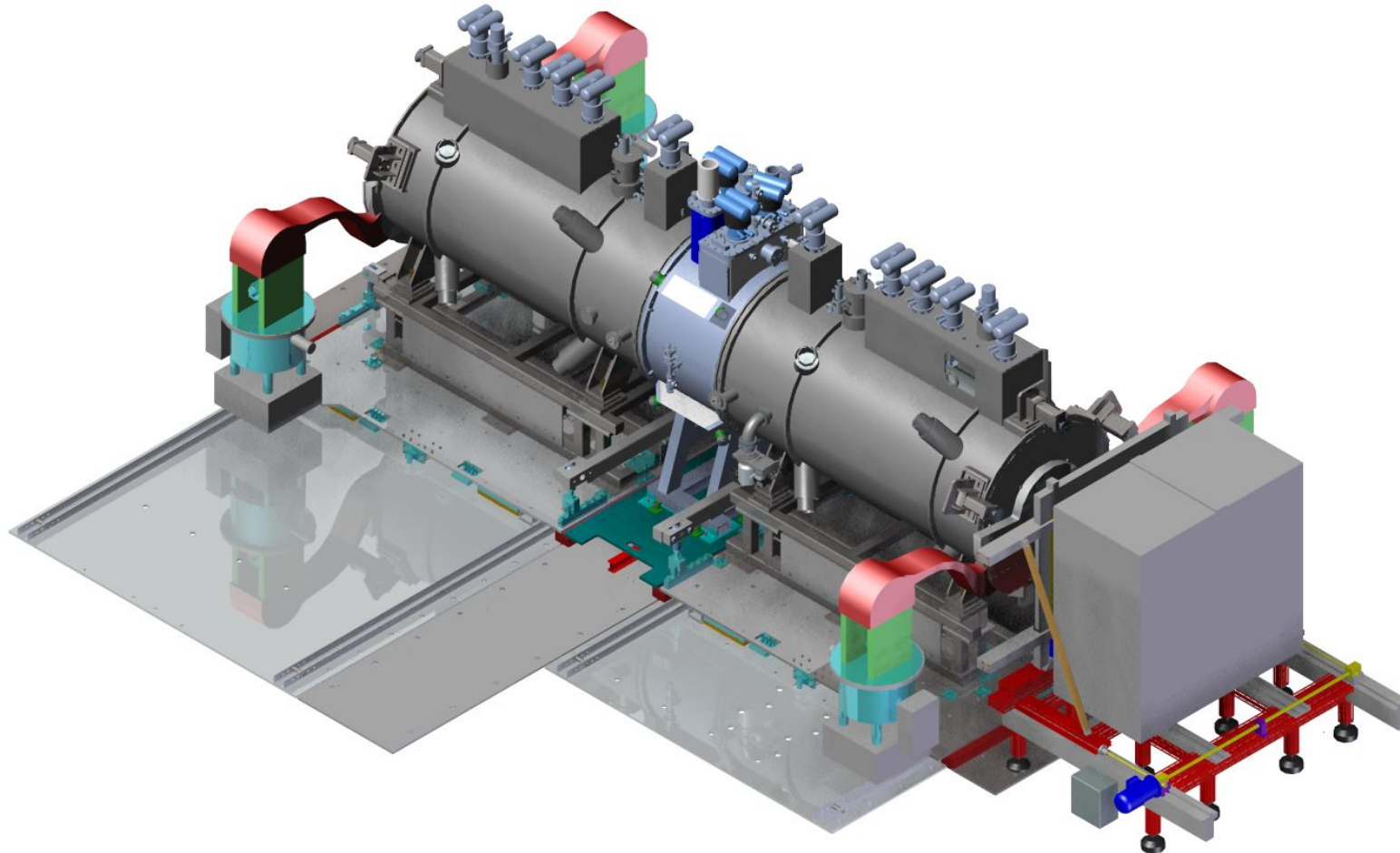
Dipole selects Pions of the desired momentum

Pions decay to muons in decay solenoid

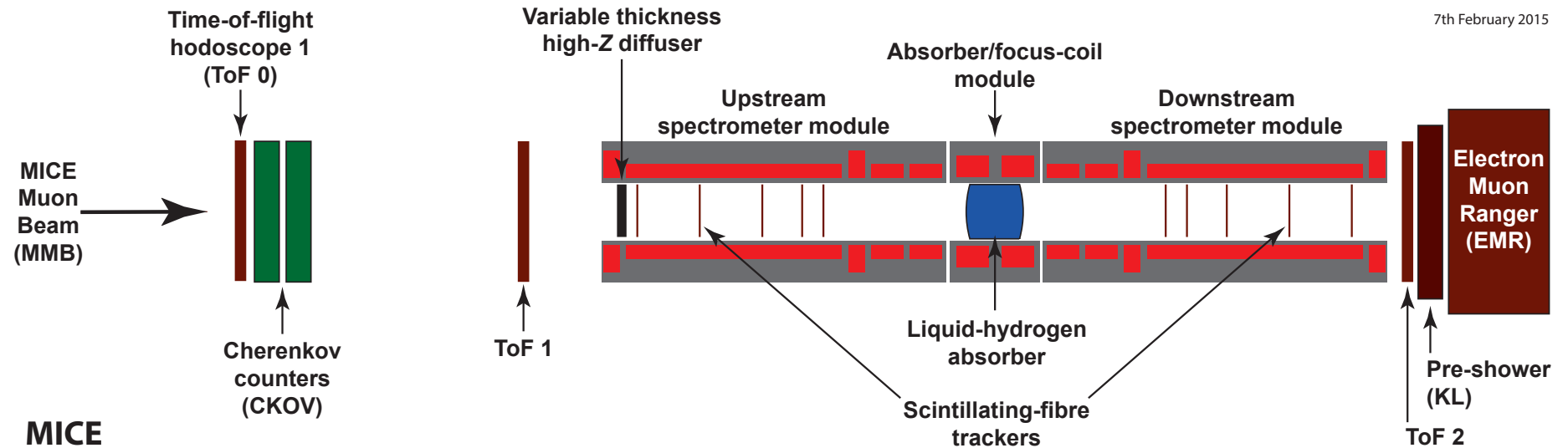
Second dipole selects muons of desired momentum

Particles delivered to cooling channel

MICE Cooling Channel



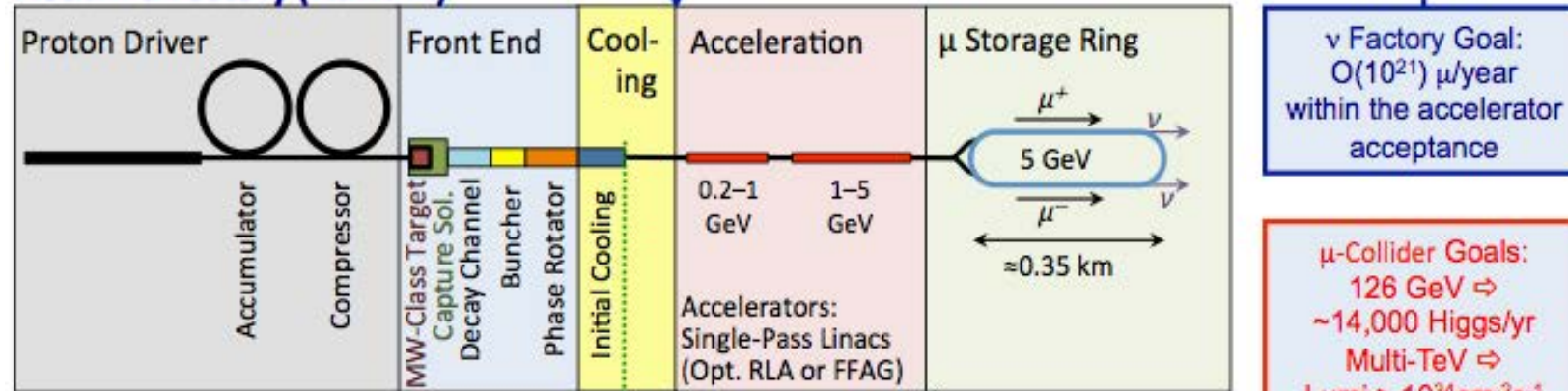
MICE Cooling Channel



- Time-of-flight Detectors used to separate muons from pions and electrons
- Diffuser used to create beam of desired emittance
- Two trackers of 5 stations of three planes of scintillating fibres either side of absorber
- Particle-by-particle measurement of position and momentum
- Beam assembled from particle-by-particle measurements
- LiH and LH2 absorbers used to demonstrate ionization cooling
- Polyethylene wedge used to investigate Emittance Exchange

Neutrino Factory/Muon Collider

Neutrino Factory (NuMAX)



ν Factory Goal:
 $O(10^{21})$ μ /year
within the accelerator
acceptance

Initial Cooling:

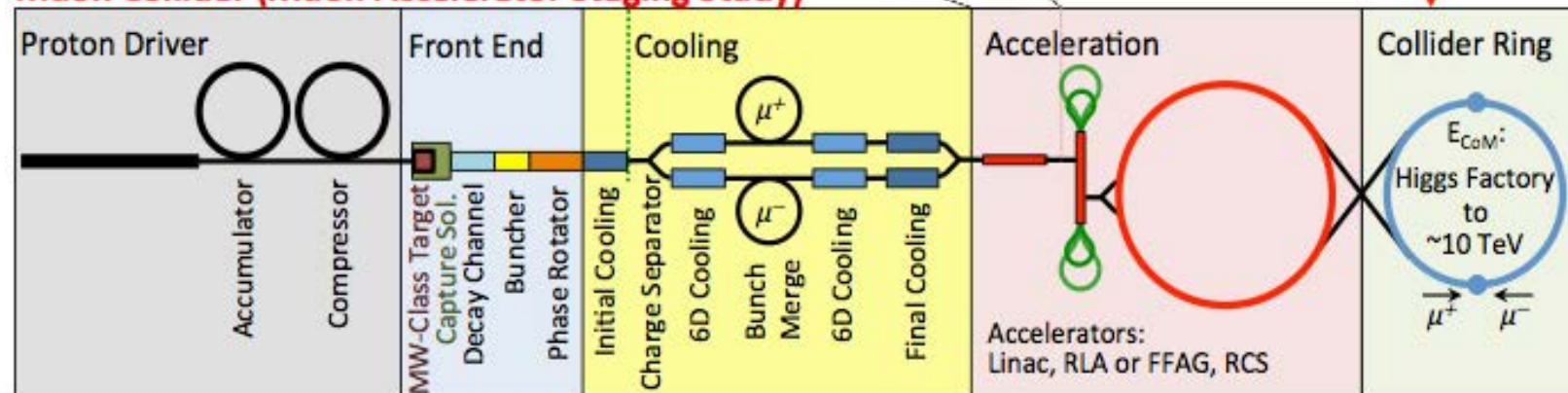
Transverse Ionization cooling

μ -Collider Goals:
126 GeV \Rightarrow
 $\sim 14,000$ Higgs/yr
Multi-TeV \Rightarrow
Lumi $> 10^{34} \text{ cm}^{-2} \text{ s}^{-1}$



Share same complex

Muon Collider (Muon Accelerator Staging Study)

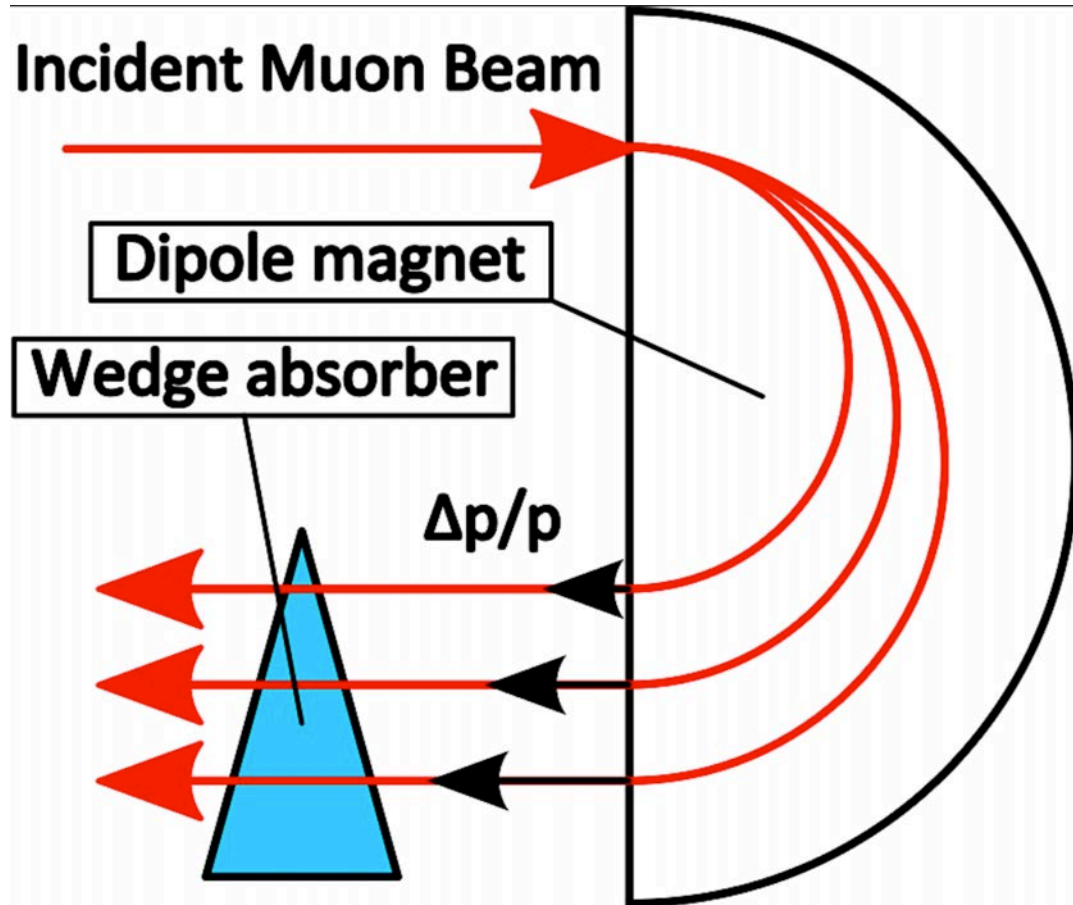


6D Cooling:

Emittance Exchange

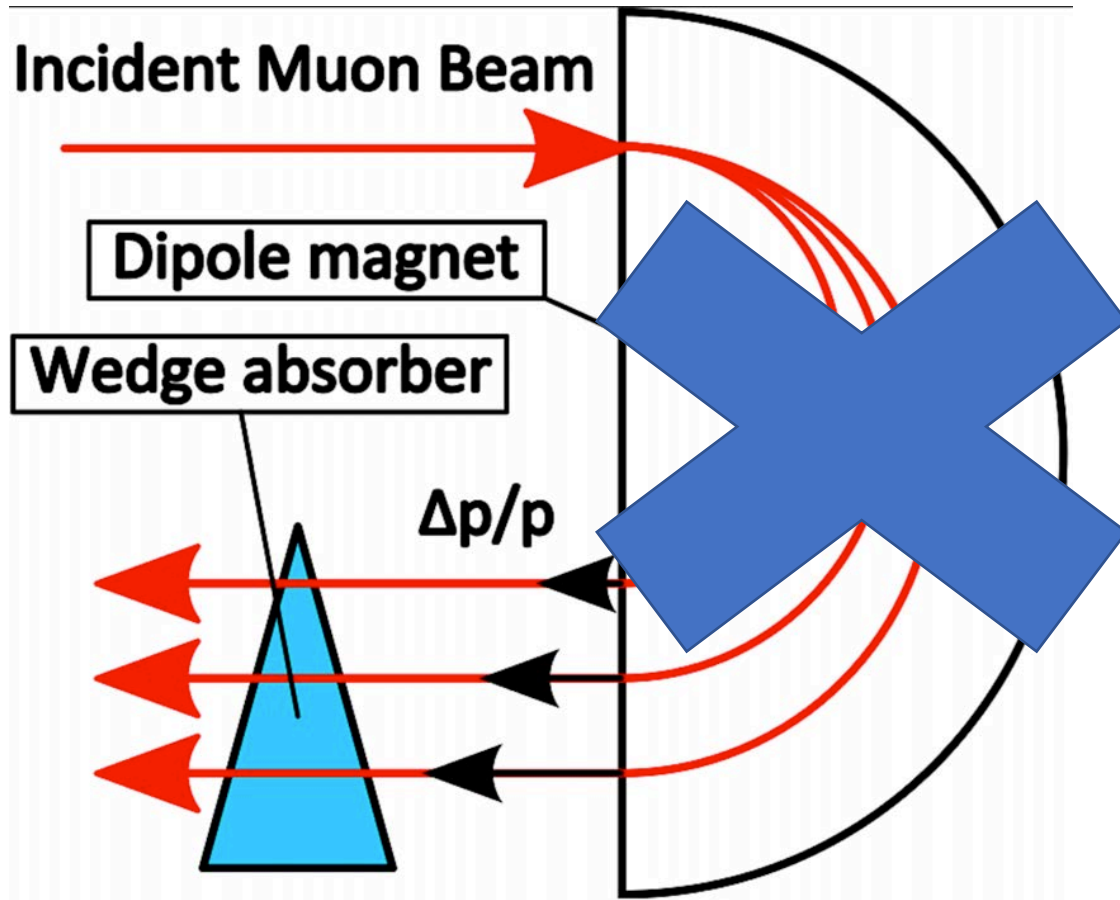


Emittance Exchange



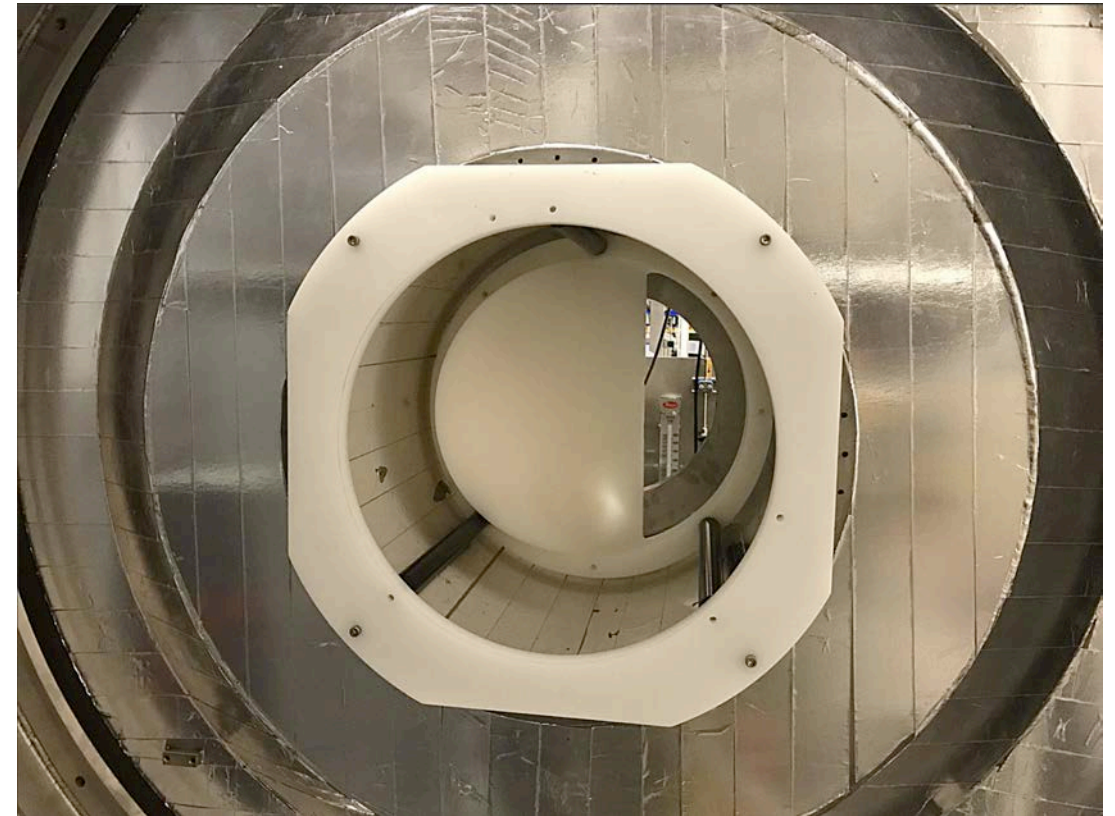
- Dipole Magnet creates position/momentum correlation
- Wedge reduces momentum spread
- Transverse beam emittance is increased in exchange for a longitudinal emittance reduction
- The transverse emittance can then be reduced via passing the muon beam through a flat absorber, causing an overall 6D cooling effect

Emittance Exchange in MICE



- MICE only has the wedge
- Data has been collected with the wedge in the cooling channel
- Change in phase-space density measured by comparing upstream and downstream tracker measurements
- MICE beam is assembled from particle-by-particle measurements
- Beam with dispersion allows us to show Emittance Exchange (increase in longitudinal phase-space density)
- Beam with no dispersion allows us to show Reverse Emittance Exchange (decrease in longitudinal phase-space density through the wedge)

Wedge in MICE

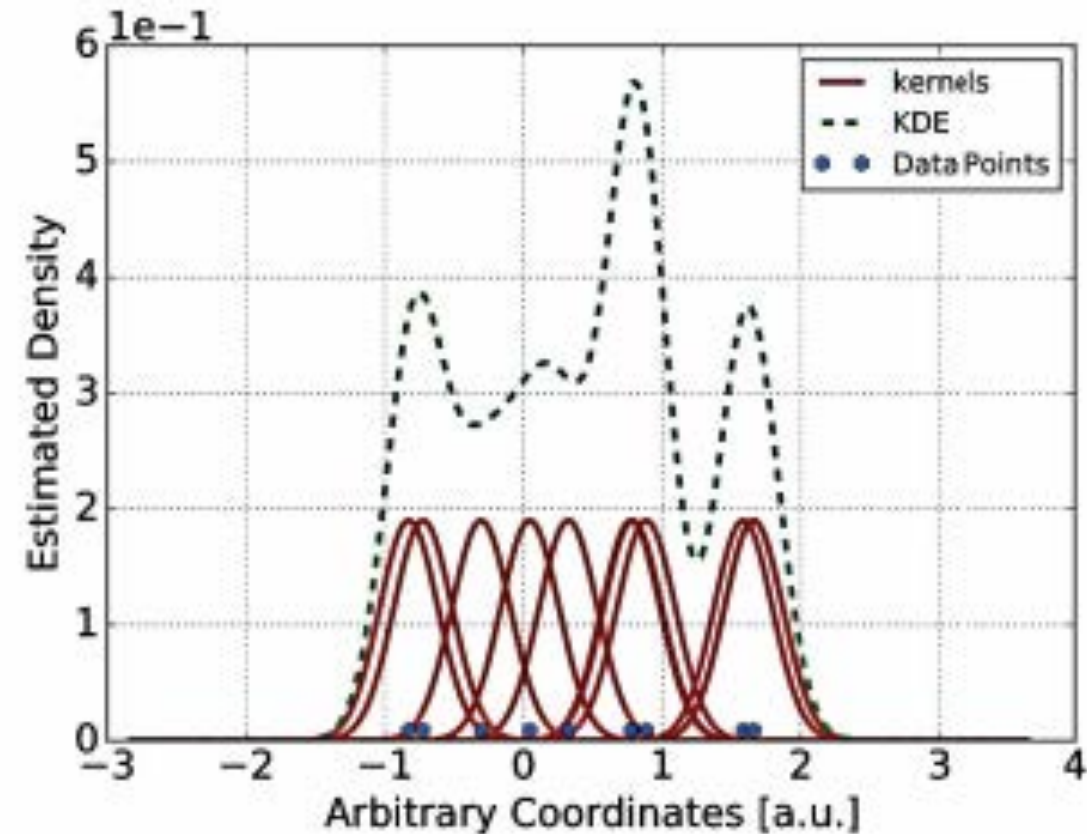


Kernel Density Estimation

- Let each data point be represented by a kernel (e.g gaussian, uniform, epanechnikov)
- Sum of over all kernels to obtain the particle distribution
- The density estimate is obtained by applying:

$$\hat{\rho}(\vec{x}) = \frac{1}{nh^d} \sum_{i=1}^n K\left(\frac{\vec{x} - \vec{X}_i}{h}\right)$$

where K is the kernel choice, h is the bandwidth of the kernel, d is the dimension and n is the number of points



Phase Space Density

- RMS emittance measurements can be affected by the scraping of the beam and beam filamentation
- Beam scraping gives a false “cooling effect”, both when the absorber is present and absent
- Beam Filamentation gives an apparent “heating effect”
- Phase Space Density Estimation is a non-parametric technique to estimate the underlying probability density, the probability that a particle will be realized at a particular phase space density.
- Scraping and Filamentation take place in lower density regions. Solution is to pick the core of the beam where the beam density is highest, and analyse the change in the core of the beam before and after the absorber.

Phase Space Density – Core of Beam

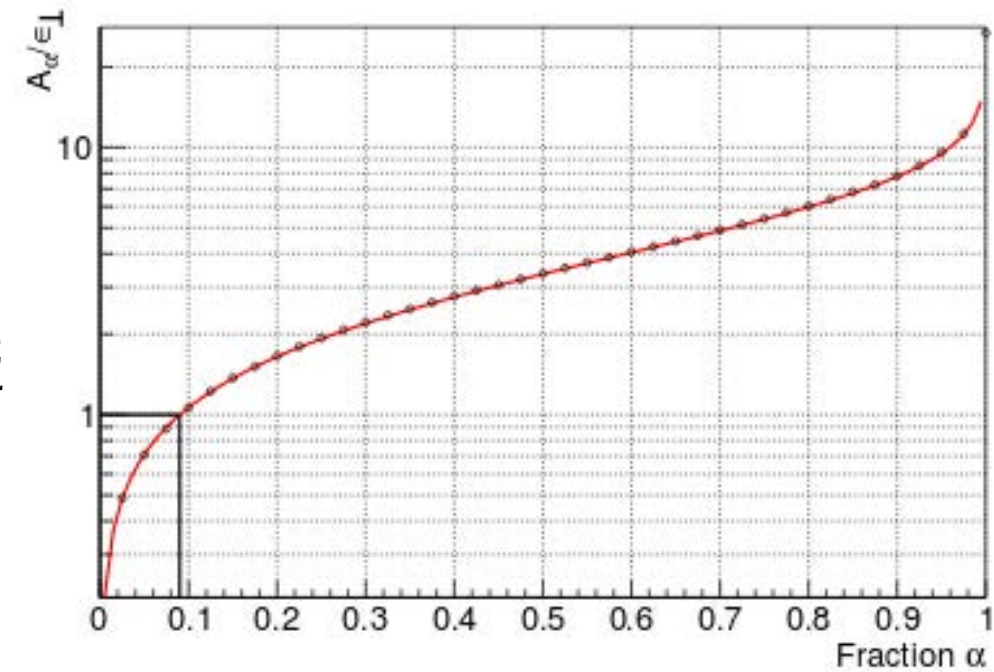
The α -amplitude, A_α , the largest amplitude of the core fraction α of the sample, is related to the volume of the smallest ellipsoid that encompasses all the subsample points by

$$A_\alpha = \epsilon_\perp \chi_4^2(\alpha)$$

where ϵ_\perp is the transverse normalised emittance and χ_4^2 the α -quantile of a 4 degree of freedom χ^2 distribution.

In four dimensions and for $\alpha = 9\%$, the α -amplitude corresponds exactly to the normalized RMS emittance.

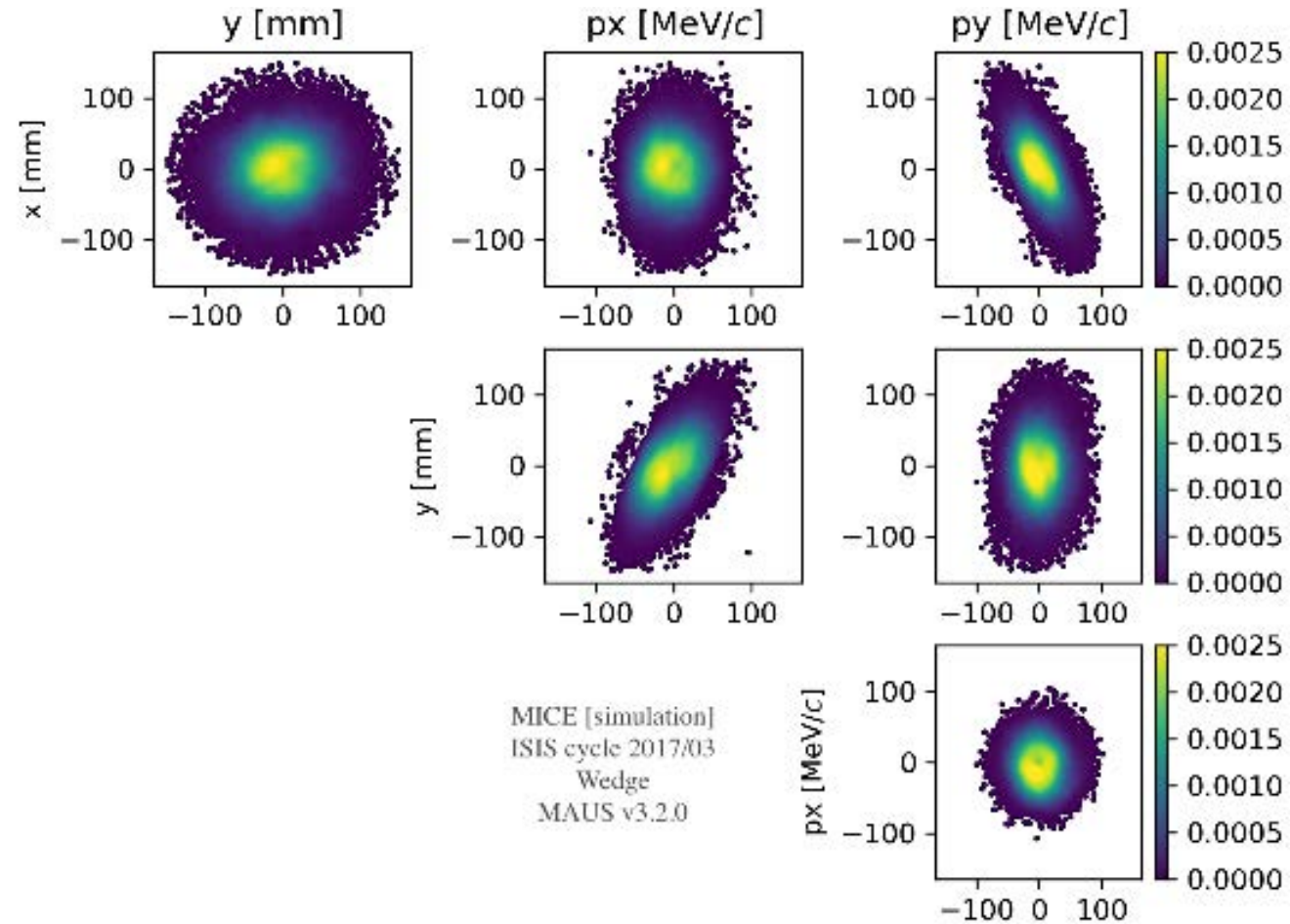
In two and six dimensions the α -amplitude corresponds to the normalized RMS emittance at 24% and 2% respectively.



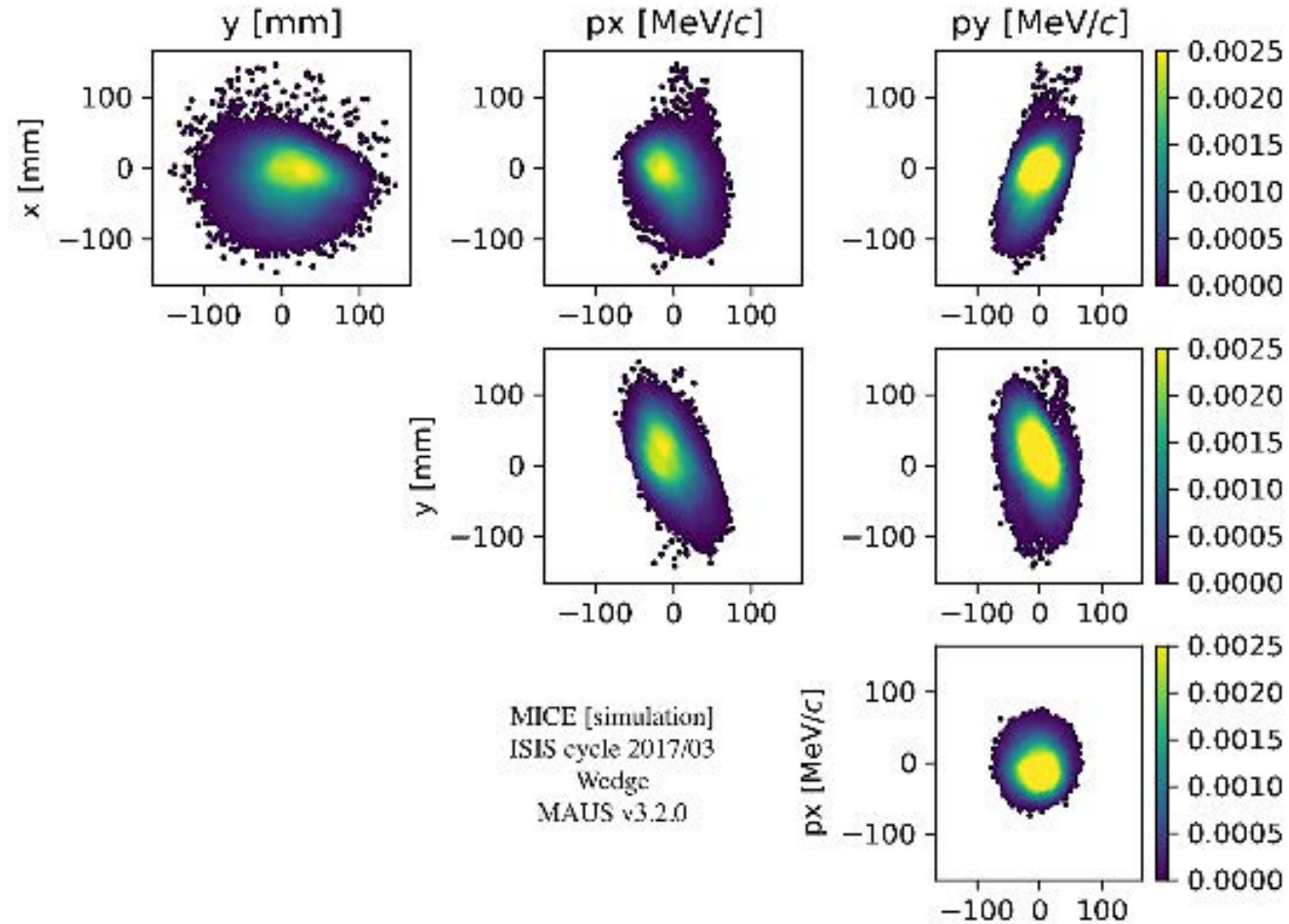
Simulation of Phase Space Density change

- Simulation in G4 Beamline with 20,000 muons.
- Tracking particles in the MICE cooling channel from the upstream reference plane to the downstream reference plane through the wedge using the same magnet configurations as were used during data taking.
- Input distribution made by taking the desired dispersion at the wedge and tracking those particles backwards to the upstream reference plane without the wedge. These particles are now used as the input distribution with the signs reversed of p_x , p_y and t .
- Simulation will track x , y , p_x , p_y , Δt and ΔE .
- The density is then calculated with the desired α -quantile of particles selected. The density of the remaining particles is then recalculated. The following plots show the change in density through the wedge.

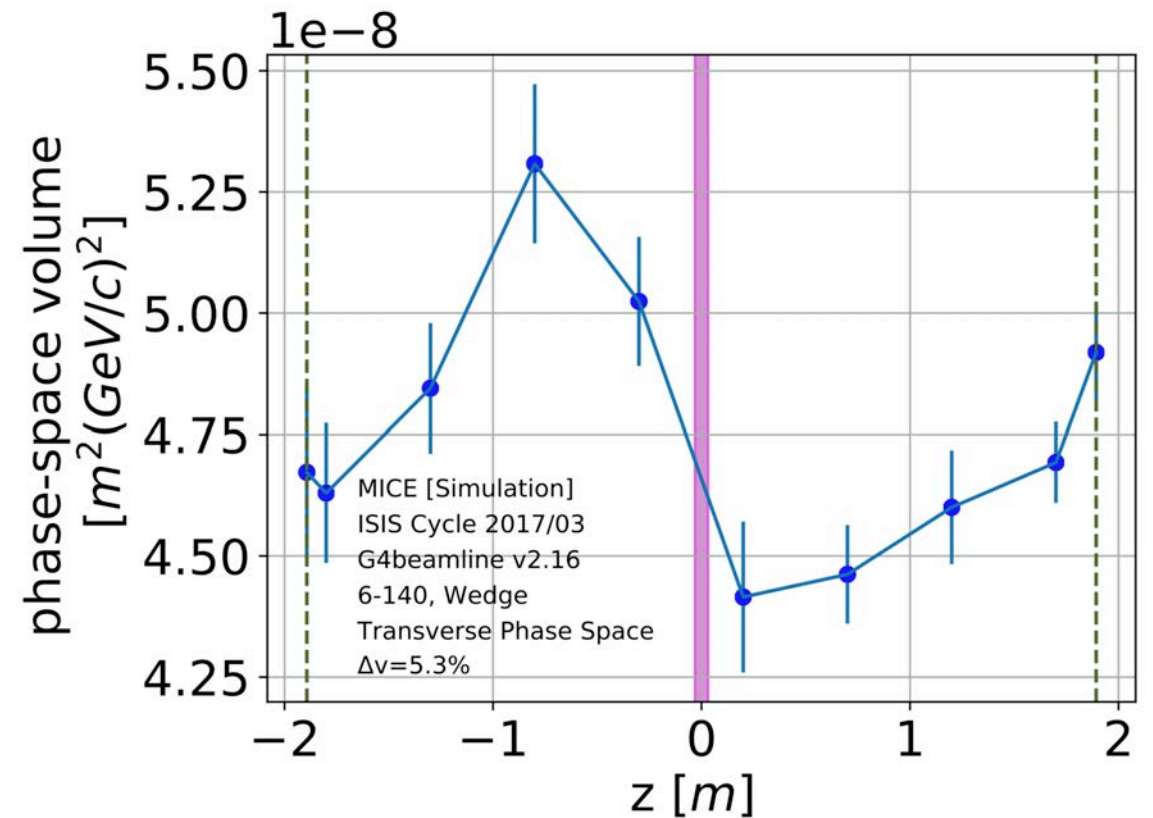
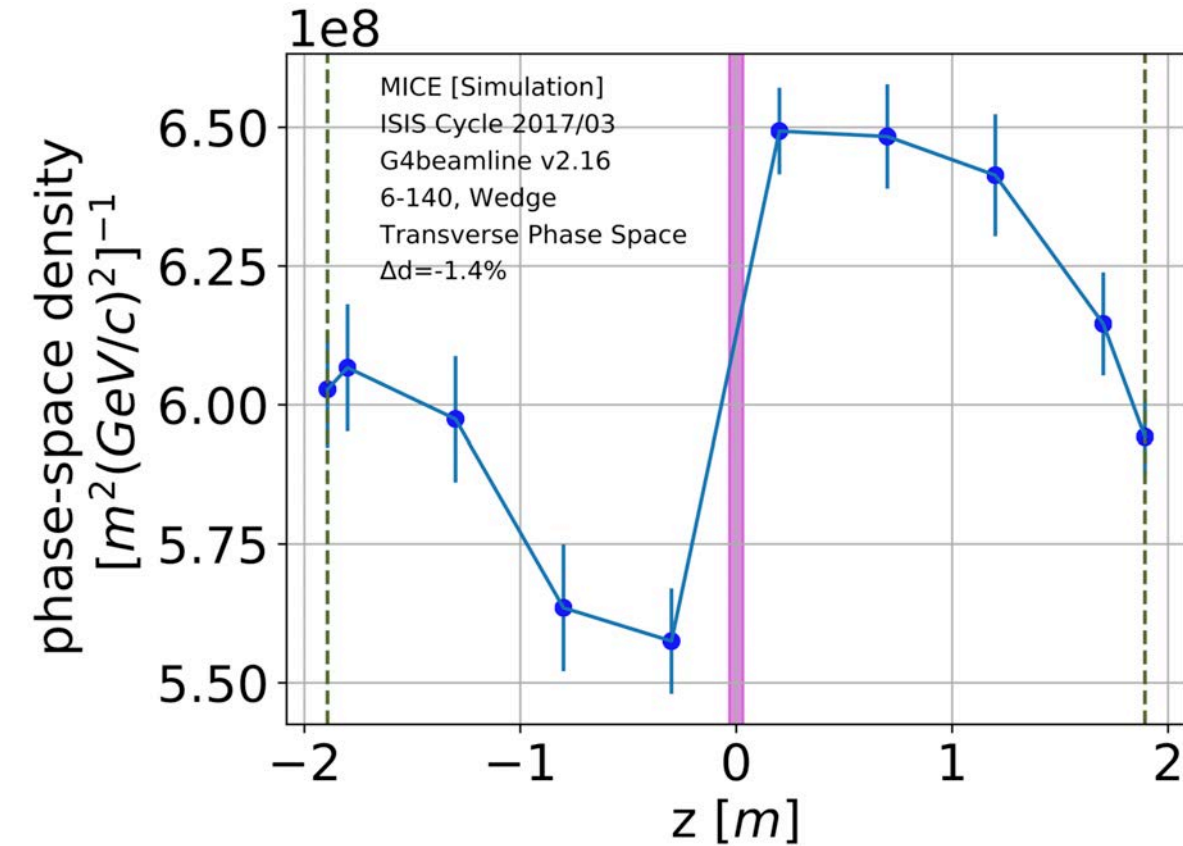
Phase Space Plots of an Upstream Distribution



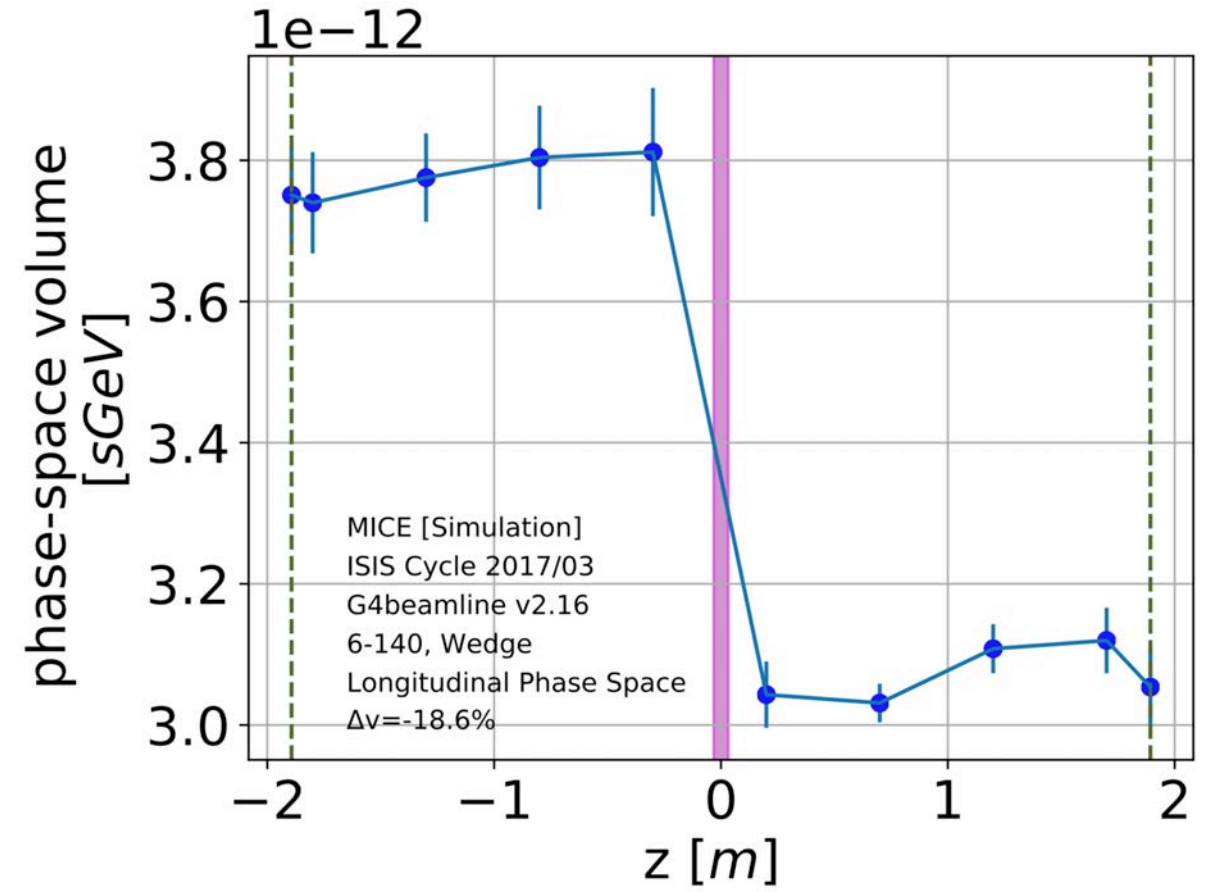
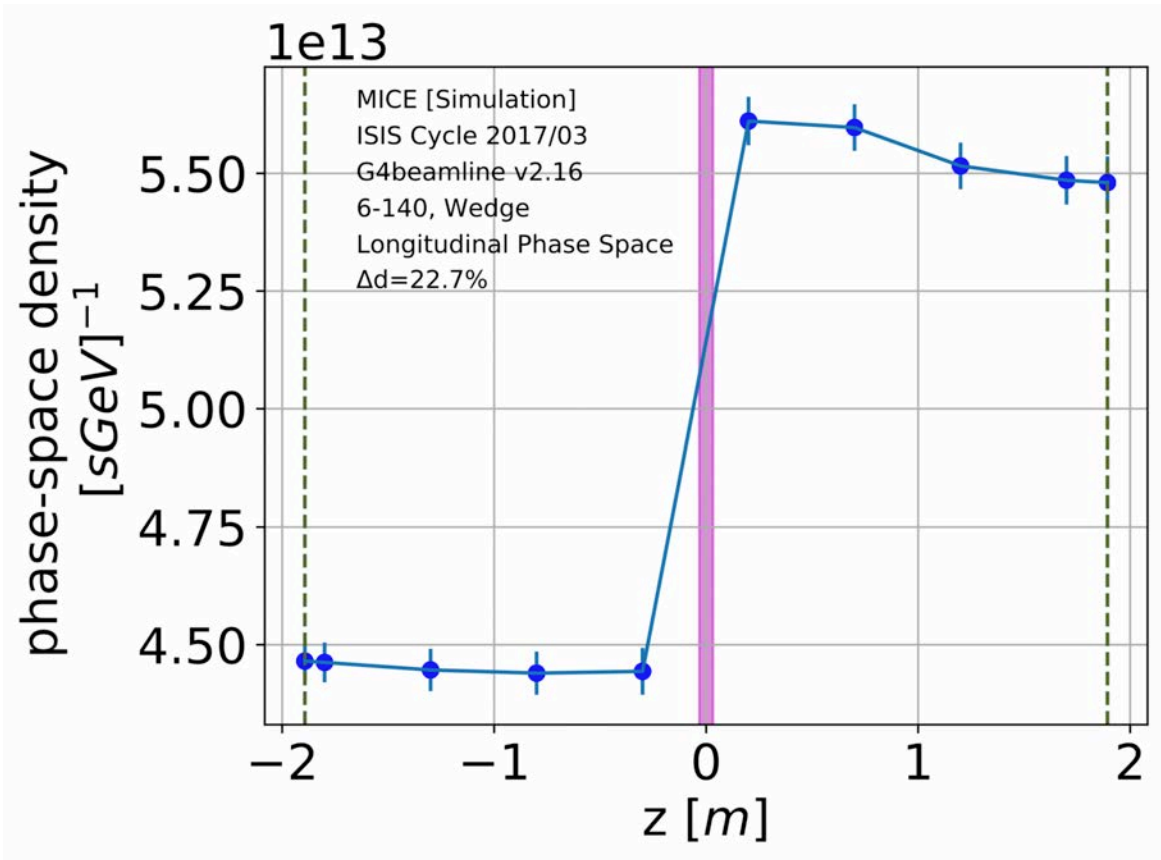
Phase Space Plots of a Downstream Distribution



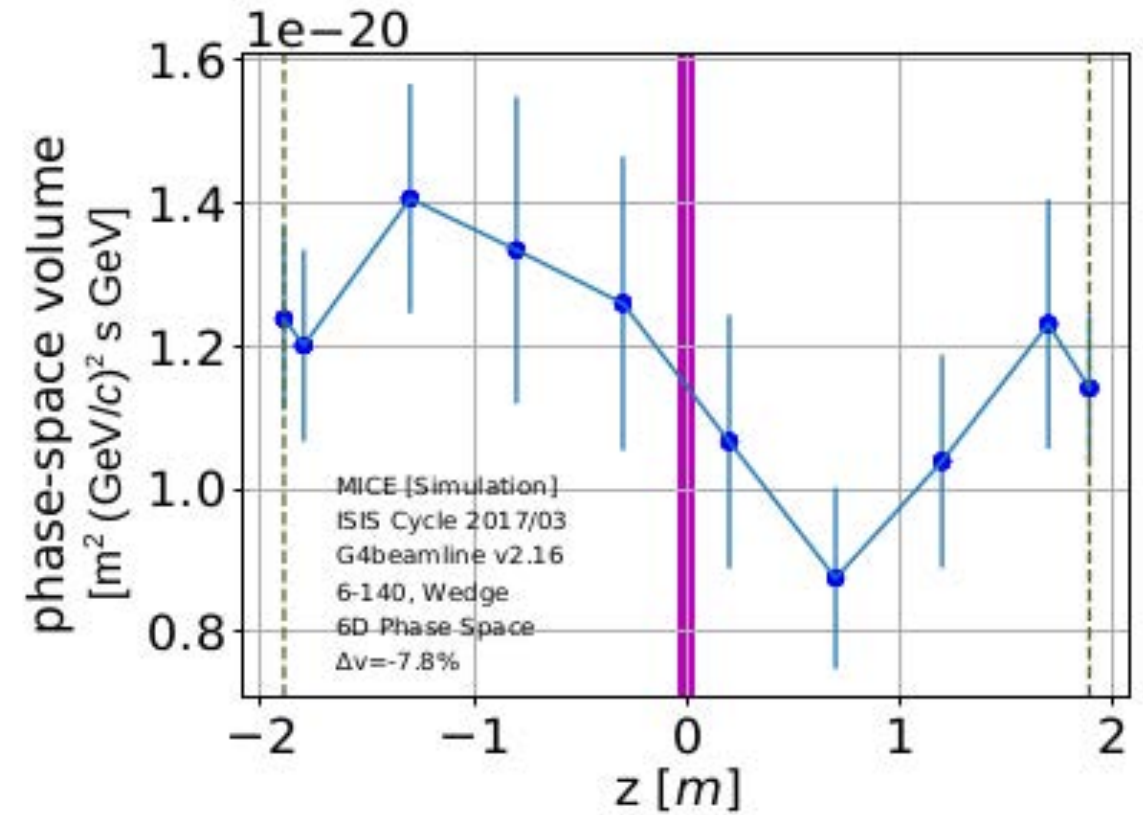
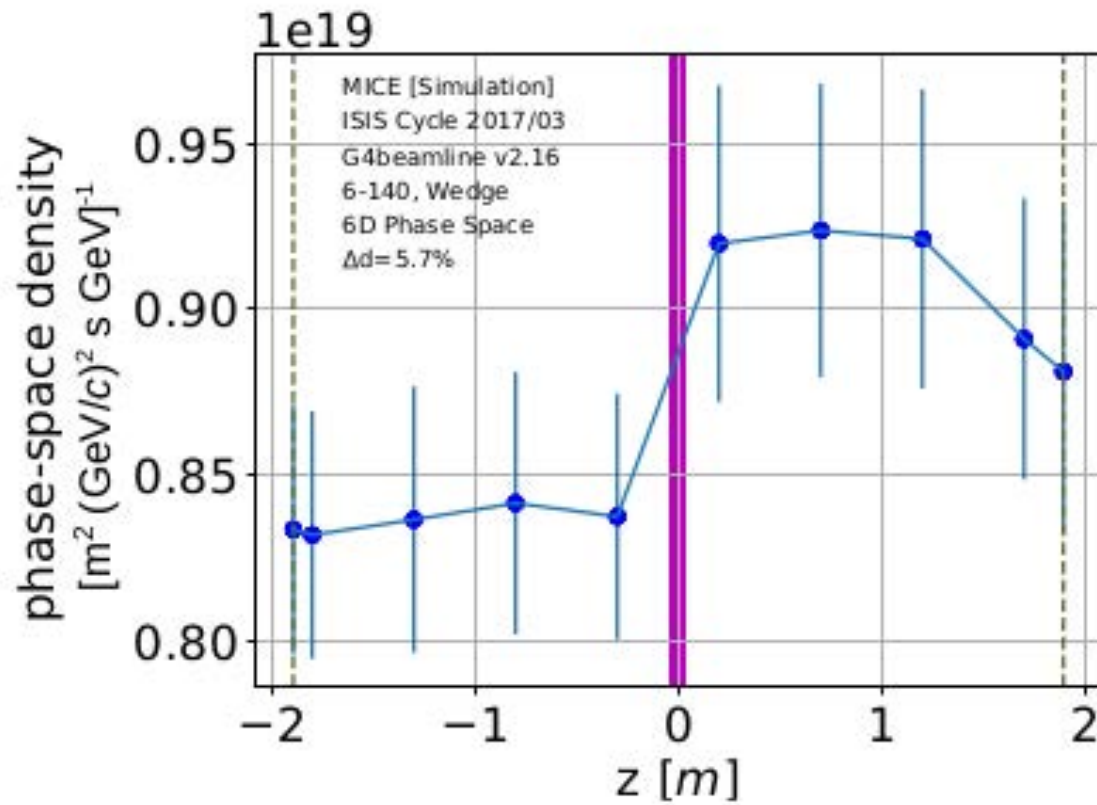
Transverse Phase Space Density



Longitudinal Phase Space Density



6D Phase Space Density



Conclusion

- MICE has collected data of a muon beam passing through a wedge.
- Simulations have shown a decrease in transverse and an increase in overall phase-space density.
- This results in overall 6D cooling, thereby showing Emittance Exchange.
- Reverse Emittance Exchange has also been demonstrated.
- The next task will be to apply the phase space density analysis to MICE data.

THE END